HADRONIC g-2 OF THE MUON: A THEORETICAL QCD DETERMINATION

C.A. Dominguez

Centre for Theoretical & Mathematical Physics
Department of Physics
University of Cape Town

6th Internationa Workshop on HEP in the LHC Era, UTFSM, Valparaiso
January 2016

S. Bodenstein et al. PRD **85**,014029 (2012); **88**, 014005 (2013)

g-2

$\mu_B = e\,\hbar/2\,m\,c = \{e\,\hbar/2\,m\,c\}\;g\;s$

$s = ½\,;\;g = 2$

$a = (g-2)/2$

$$a = (g-2)/2 \neq 0$$

$$a_e|_{EXP} = 1\,159\,652\,180.73\ (0.28) \times 10^{-12}$$

$$a_e = 0.001\ldots\ !!!$$

$$a_e|_{THY} = 1\,159\,652\,181.13\ (0.86) \times 10^{-12}$$

$$a_e|_{EXP} - a_e|_{THY} = -0.40\ (0.88) \times 10^{-12}$$

MUON

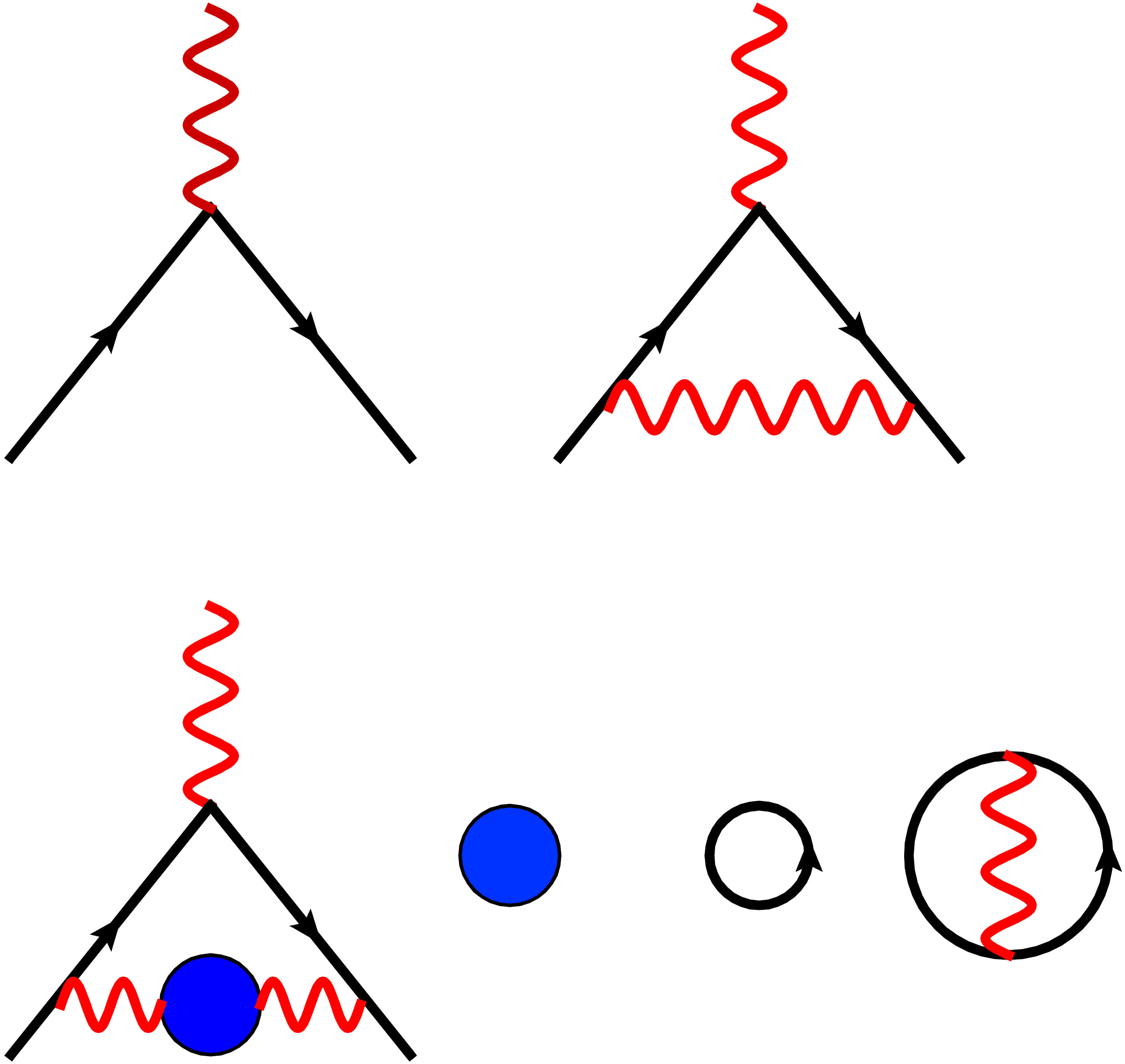
$$a_\mu|_{EXP} = 1\ 165\ 920\ 8.9\ (6.3) \times 10^{-10}$$

$$a_\mu = 0.001\ldots\ !!!$$

$$a_\mu|_{THY} = 1\ 165\ 918\ 1.8\ (7.6) \times 10^{-10}$$

$$a_\mu|_{EXP} - a_\mu|_{THY} = 27.1\ (9.9) \times 10^{-10}$$

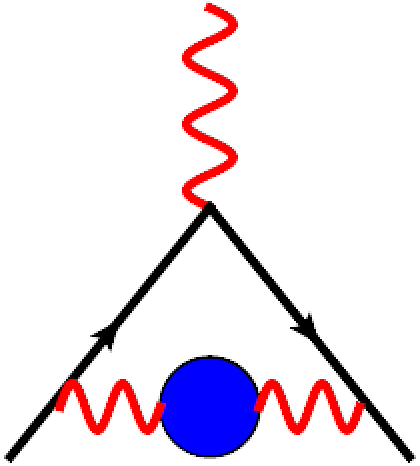
$$\Delta a_\mu : 2.7\ \sigma$$



CONTRIBUTIONS TO a = (g-2)/2

$a_{QED} \quad a_{HAD} \quad a_{EW}$

$$a_{HAD} = a|_{HAD}(LO) + a|_{HAD}(HO) + a|_{HAD}(LBL)$$



$$a_\mu|_{QED} = 11\ 658\ 471.8853 \pm\ 0.3650$$

$$a_\mu|_{EW} = 15.4\ (2)$$

$$a_\mu|_{HAD}\ (HO) = -\ 9.84\ (7)$$

$$a_\mu|_{HAD}\ (LBL) = 11.6\ (4.0)$$

$$a_\mu|_{HAD}\ (LO) = 692.7\ (6.5)$$

$$a_\mu|_{THY} = 11\ 659\ 181.8 \pm 7.6$$

$$a_\mu|_{EXP} - a_\mu|_{THY} = 27.1\ (9.9) \times 10^{-10} \quad [\Delta a_\mu : 2.7\ \sigma]$$

$a_\mu|_{HAD}$ (LO) = 692.7 (6.5)

RELIES ENTIRELY ON

(unreliable) DATA

$e^+ e^- \rightarrow$ hadrons

(uds) + (charm) + (bottom)

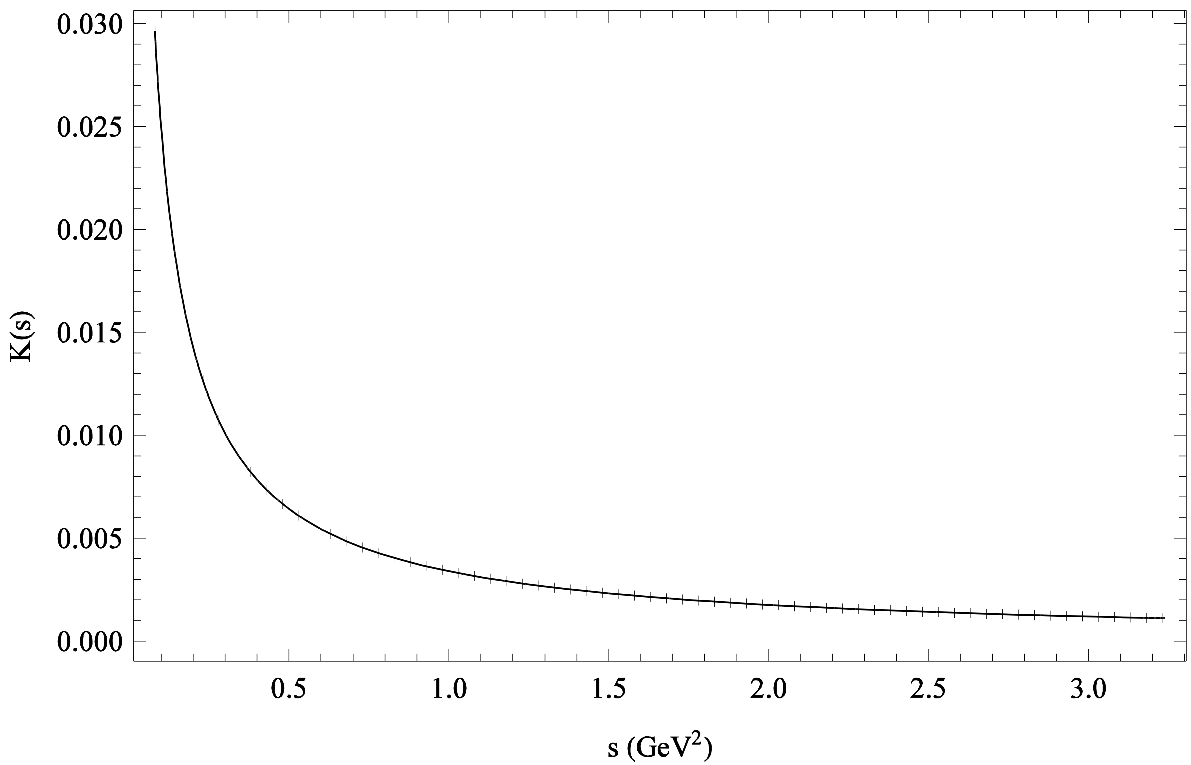
$$a^{\text{HAD}} = \frac{\alpha^2}{3\pi^2} \int_{s_{th}}^{\infty} \frac{ds}{s}\, K(s)\, R(s) \quad (s=E^2)$$

$$R(s) \equiv \frac{\sigma(e+\,e- \ \to hadrons}{\sigma(e+\,e- \ \to leptons} \propto \text{Im}\,\Pi(s)$$

$$\Pi_{\mu\nu}(q^2) = i\int d^4x\, e^{iqx} < 0|T(J_\mu(x)\, J_\nu{}^+(0))|0 >$$

$$= (-g_{\mu\nu} q^2 + q_\mu q_\nu)\, \Pi_{EM}(q^2)$$

$$\text{Im}\,\Pi_{EM}(q^2) = \frac{1}{8\pi}\left[1 + \frac{\alpha_s(s)}{\pi} + \ldots\right]$$

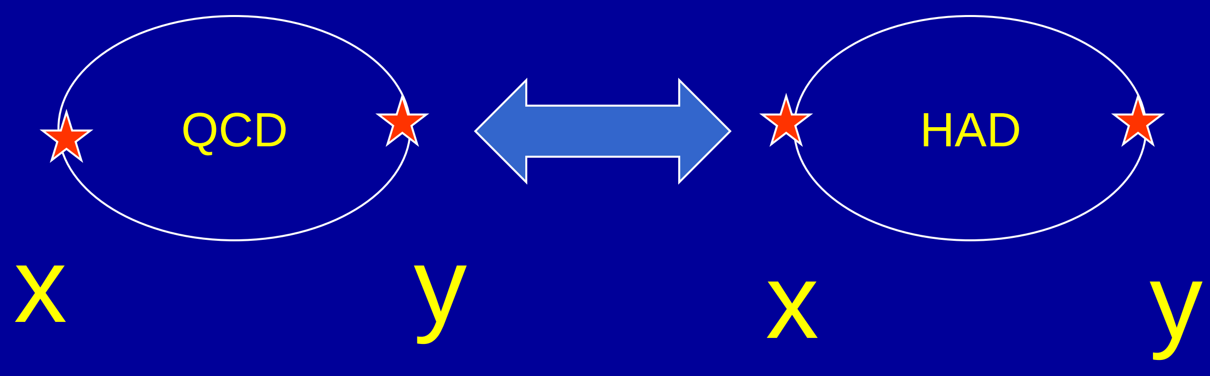


K(s)
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0.5
1.0
1.5
2.0
2.5
3.0
s (GeV^2)

A THEORETICAL CALCULATION OF

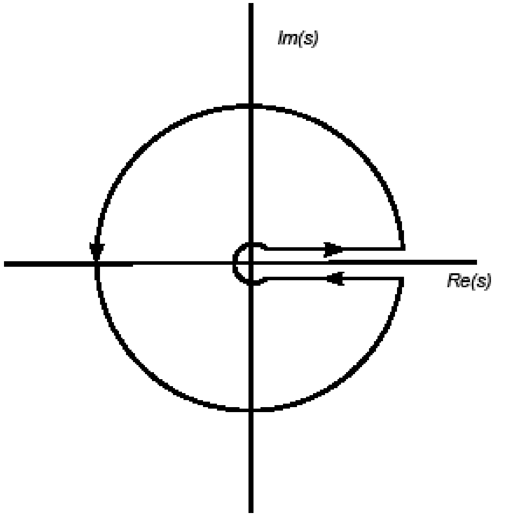
$$a_\mu|_{HAD}\,(LO) = a_\mu|_{uds} + a_\mu|_c + a_\mu|_b$$

$$a_\mu|_c\ (LQCD)$$

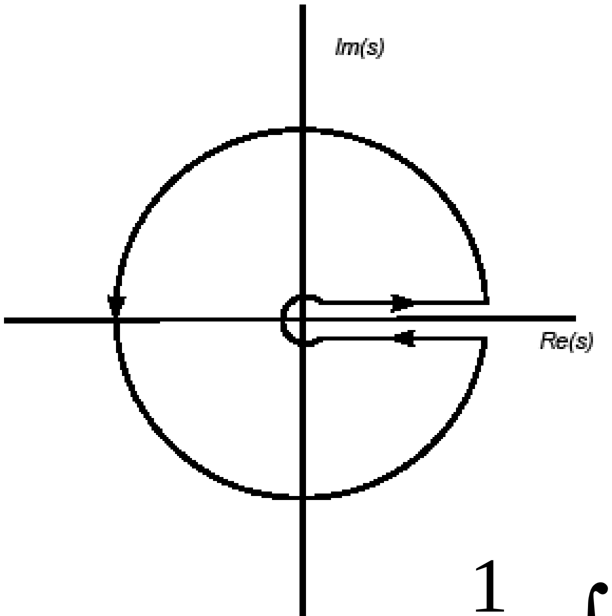


CAUCHY'S THEOREM IN THE COMPLEX ENERGY^2 PLANE

COMPLEX SQUARED-ENERGY S-PLANE



QUARK-HADRON DUALITY

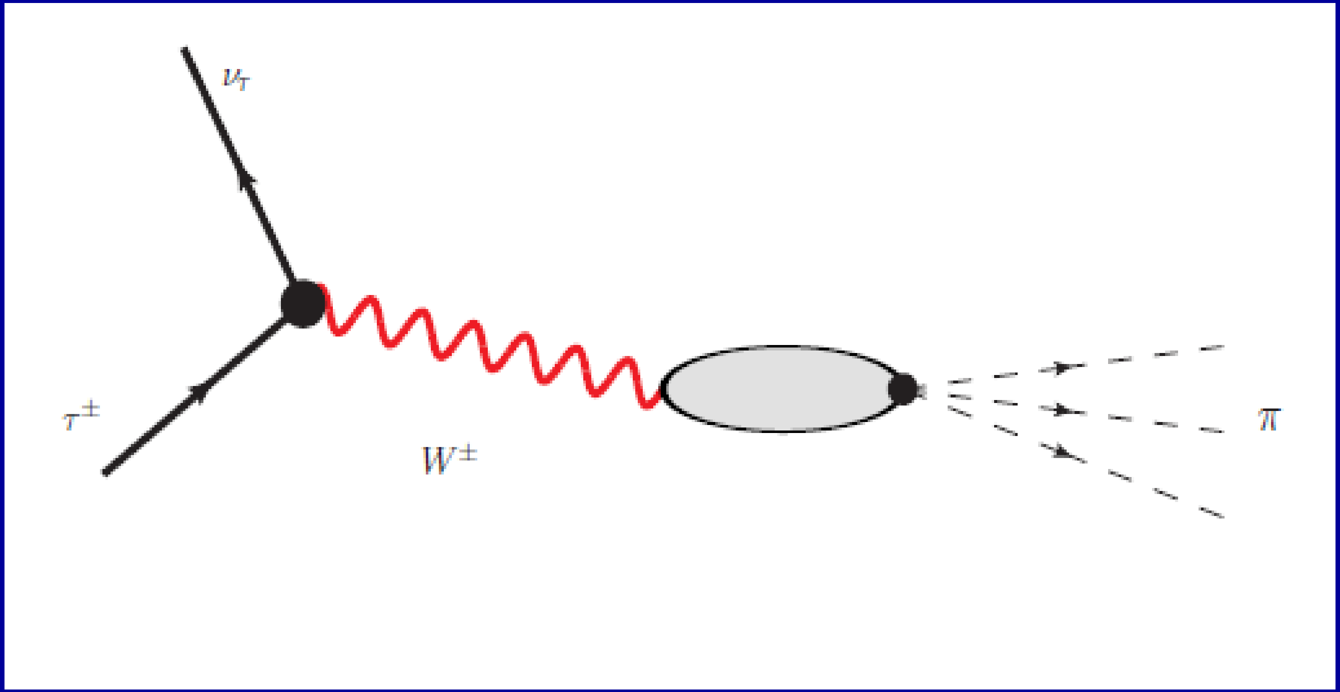


$$\oint_C \Pi(s)\, ds = 0$$

$$-\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\, \Pi(s) = \int_{s_{th}}^{s_0} ds\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(s)$$

$$-\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\,\Pi_{QCD}(s) = \int_{s_{th}}^{s_0} ds\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)\,|_{HAD}$$

τ→ hadrons (π's)



$\tau \rightarrow$ hadrons

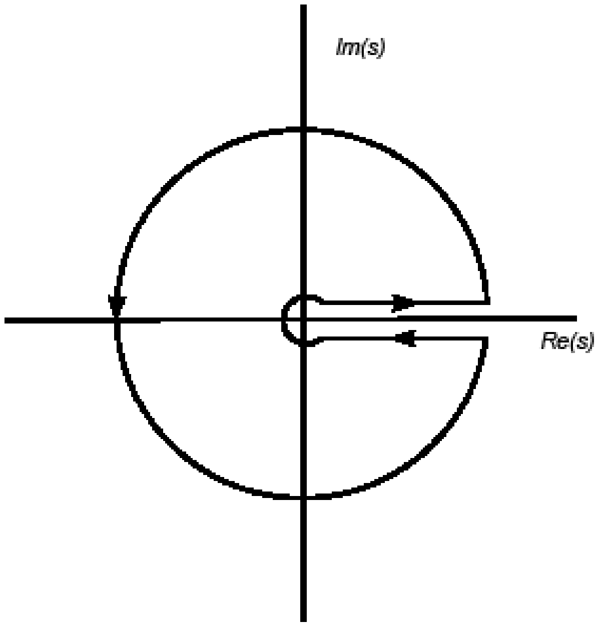
$R_\tau = \sigma(\tau \rightarrow \text{hadrons}) / \sigma(\tau \rightarrow \text{leptons})$

$$R_\tau \propto \int_0^{M_\tau^2} ds \ \mathrm{K}_\tau(s)\, \mathrm{Im}\,\Pi(s)\,|_{QCD}$$

$$\mathrm{Im}\,\Pi(s)\,|_{QCD} = \sum_n c_n\, \alpha_s^n \qquad (n = 0,1,\ldots 4)$$

CAUCHY'S THEOREM

$$\oint_C \Pi(s)\, ds = 0$$



$$\oint_C \Pi(s)\, ds = \sum_i (\text{Residue Pole})_i$$

$$\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\, \Pi(s) + \int_{s_{th}}^{s_0} ds\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(s) = \sum_i \mathrm{Res}_\mathrm{i}$$

$$\mathrm{K(s)} \rightarrow K_1(s) = \frac{a_1}{s} + \frac{a_2}{s^2}$$

$$\int_{s_{th}}^{s_0} \frac{ds}{s}\, K_1(\mathrm{s})\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(\mathrm{s}) = \mathrm{Res}\, [\Pi(\mathrm{s})\, \frac{K_1(s)}{s}\,]_{\mathrm{s}=0}$$

$$-\frac{1}{2\,\pi\, i} \oint_{|s|=s_0} \frac{ds}{s}\, K_1(\mathrm{s})\, \Pi(\mathrm{s})$$

PERTURBATIVE QCD EXPANSION (HEAVY QUARKS)

$$z = \frac{s}{4{m_Q}^2}$$

$$\Pi(s)_{PQCD} \propto \sum_{n\geq 0} C_n\, z^n$$

PQCD up to 4-loop level

$a^{HAD}|_c = 14.4 \pm 0.1 \times 10^{-10}$ $a^{HAD}|_b = 0.29 \pm 0.01 \times 10^{-10}$
(S. Bodenstein et al. 2012)

- **LQCD**
- **(2014)**

- $a^{HAD}|_c = 14.1 \pm 0.1 \times 10^{-10}$ **(ETM Coll.)**

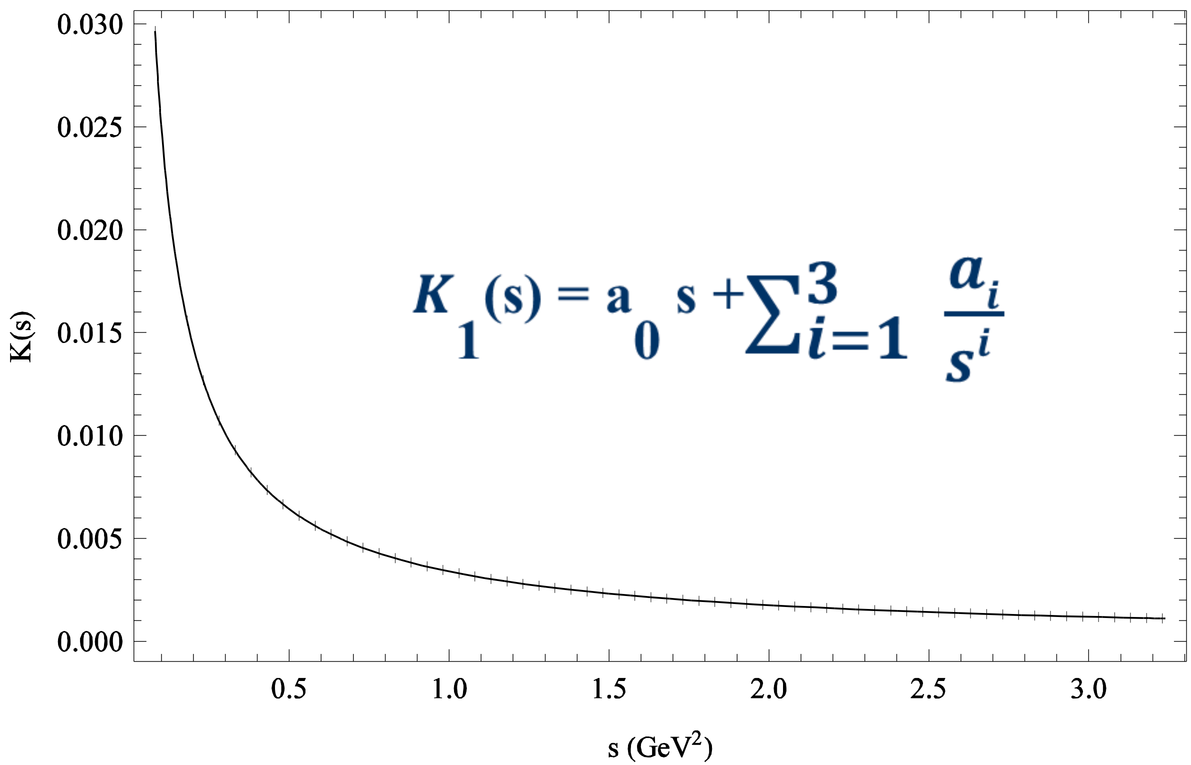
- $a^{HAD}|_c = 14.42 \pm 0.39 \times 10^{-10}$ **(HPQCDColl.)**

uds

$$\mathbf{K(s)} \rightarrow \boldsymbol{K_1}\mathbf{(s)} = \mathbf{a_0\, s} + \sum_{n=1}^{3} \frac{a_n}{s^n}$$

$$\int_{s_{th}}^{s_0} \frac{ds}{s}\; K_1(\mathrm{s})\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(\mathrm{s}) = \mathrm{Res}\, \left[\Pi(\mathrm{s})\, \frac{K_1(s)}{s}\right]_{\mathrm{s}=0}$$

$$-\frac{1}{2\,\pi\, i} \oint_{|s|=s_0} \frac{ds}{s}\, K_1(\mathrm{s})\, \Pi(\mathrm{s})$$



$K_1(s) = a_0 s + \sum_{i=1}^{3} \frac{a_i}{s^i}$
K(s)
s (GeV^2)
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0.5
1.0
1.5
2.0
2.5
3.0

$$\text{Res}\,[\Pi(\text{s})\,\frac{K_1((s)}{s}\,]_{\text{s=0}} = \lim_{s\to 0} \Sigma\, \frac{a_n}{n!}\,(\frac{d}{ds})^{\text{n}}\,\Pi(\text{s})$$

NO LOW ENERGY THEOREM FOR

$$\Pi(s)|_{uds}$$

$$\mathrm{R(s)} \equiv \frac{\sigma(e+e- \rightarrow hadrons}{\sigma(e+e- \rightarrow leptons} \propto \mathrm{Im}\,\Pi(s)$$

$$\mathrm{Im}\,\Pi(s) \propto |F_\pi(s)|^2$$

NEED A MODEL OF $F_\pi(s)$ ($s \leq 0$)

MUST REPRODUCE $< r^2_\pi >$

QCD ($N_c \to \infty$) **(Phys. Lett. B 512, 331 (2001)**

$$F_\pi(s) \to < r^2_\pi > = 0.436 \pm 0.004 \text{ fm}^2$$

$$< r^2_\pi >_{|EXP} = 0.439 \pm 0.008 \text{ fm}^2$$

FITS DATA (0/ -10 GeV^2): $\chi^2|_{DOF} = 1.5$

$\Pi(s)|_{uds}$: MODEL: $N_c = \infty$

Phys. Lett. **B** 512, 331 (2001) CAD

$$a_{QCD\infty} = 116\ 592\ 10.6 \pm 9.8\ \times 10^{-10}$$

$$a_{EXP} = 116\ 592\ 08.9 \pm 6.3 \times 10^{-10}$$

Phys. Rev. D **85,** 014029 (2012) [S.Bodenstein, CAD, K. Schilcher]