Light-cone QCD sum rules for soft contribution to exclusive Drell-Yan process

$$\pi^- p \to \ell^+ \ell^- n$$

Kazuhiro Tanaka (Juntendo U/KEK)

High momentum beam line at J-PARC

- Primary beam (proton)

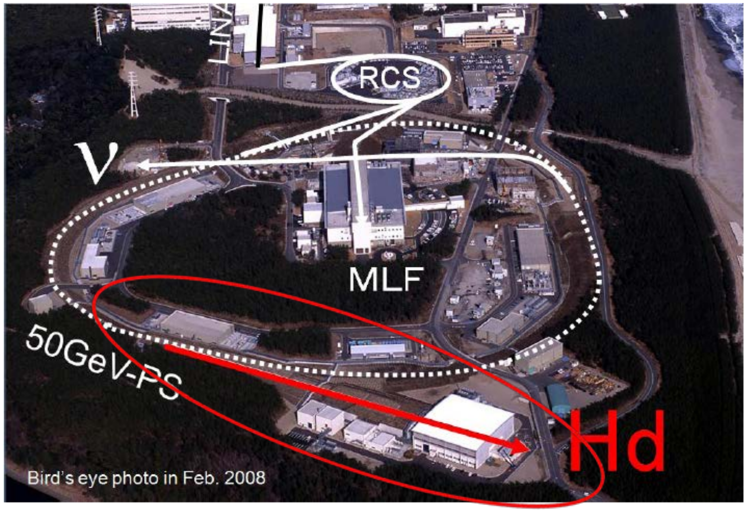
$$E = 30\text{GeV} \ (\to 50\text{GeV?})$$

$$L = 10^{35}\,\text{cm}^{-2}\text{s}^{-1}$$

$\leftrightarrow$ PANDA (anti-proton)

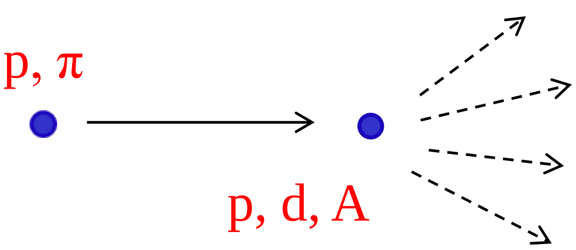
$$E \le 15\text{GeV}\ ,\ \ L = 10^{32}\,\text{cm}^{-2}\text{s}^{-1}$$

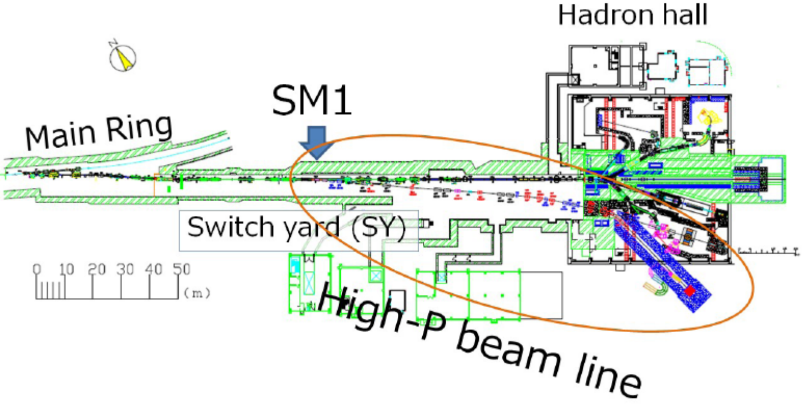
Hadron Facility at J-PARC



- Secondary beam (pion)

E = 15-20GeV





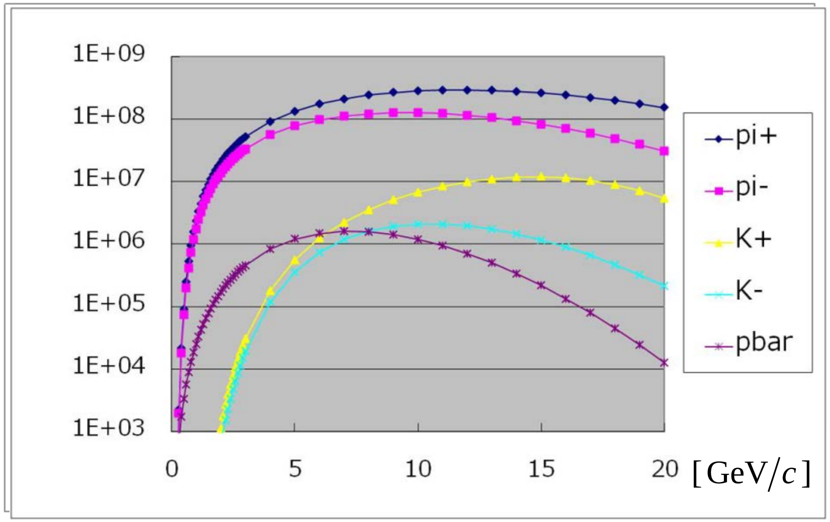
Unseparated Secondary Beam Intensity

High-momentum beamline

- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

beam loss limit @ SM1:15kW

(limited by the thickness of the tunnel wall)



0° extraction angle

high intensity

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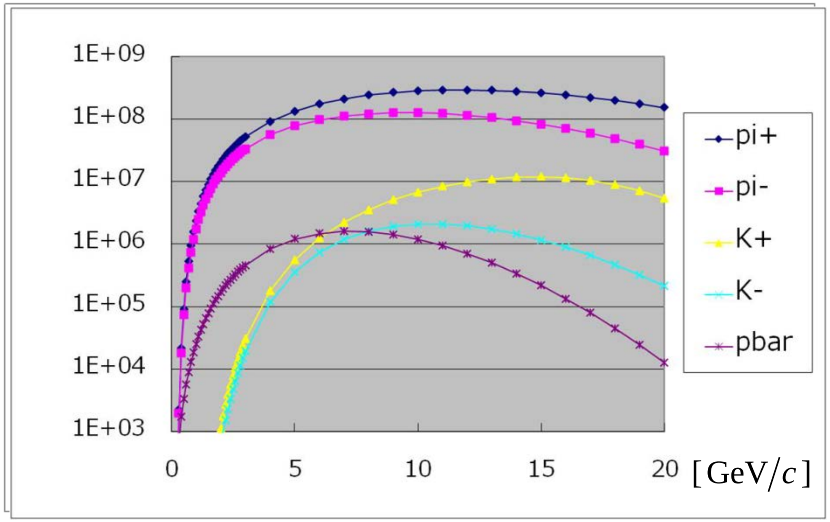


0° extraction angle

high intensity

not too high energy $d\sigma \sim 1/s^a$

best suited to study meson-induced hard exclusive processes

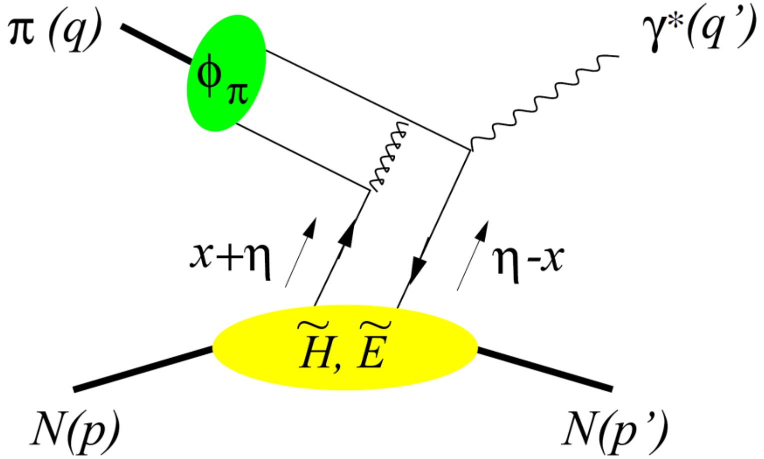
Exclusive lepton pair production in πN scattering

$$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$$

Berger, Diehl, Pire, PLB523(2001)265

"exclusive limit of DY"

small $t = (q - q')^2$



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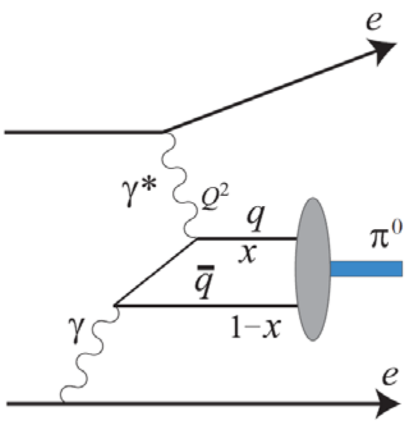

@Belle, Babar

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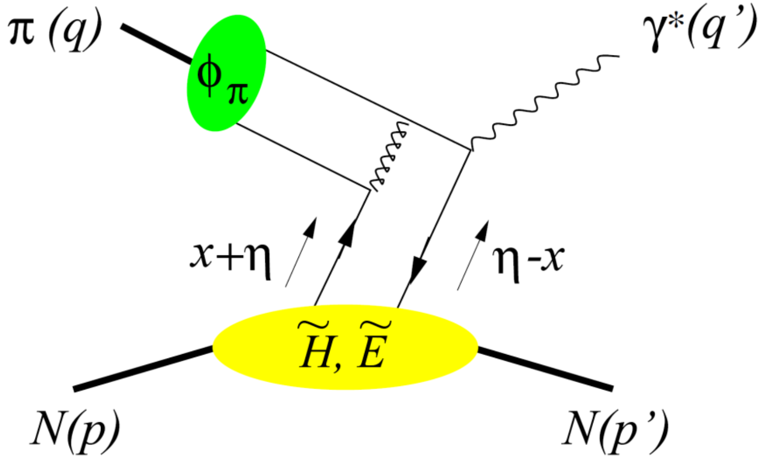

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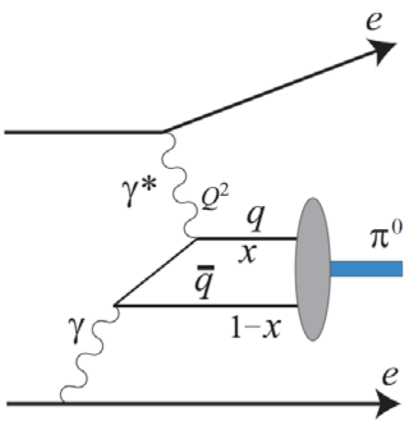

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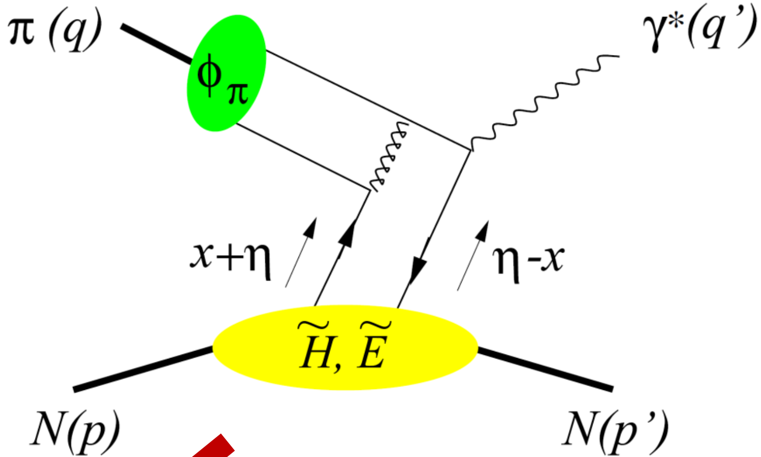

small $t = (q - q')^2$

$t \to 0$

$\Delta q(x)$

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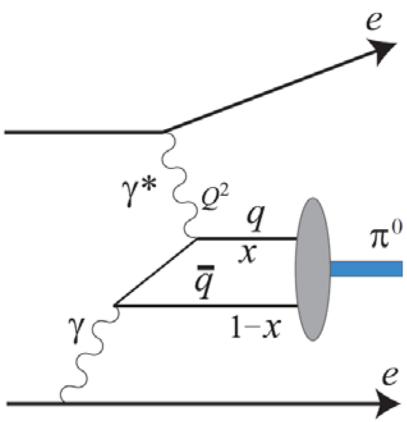

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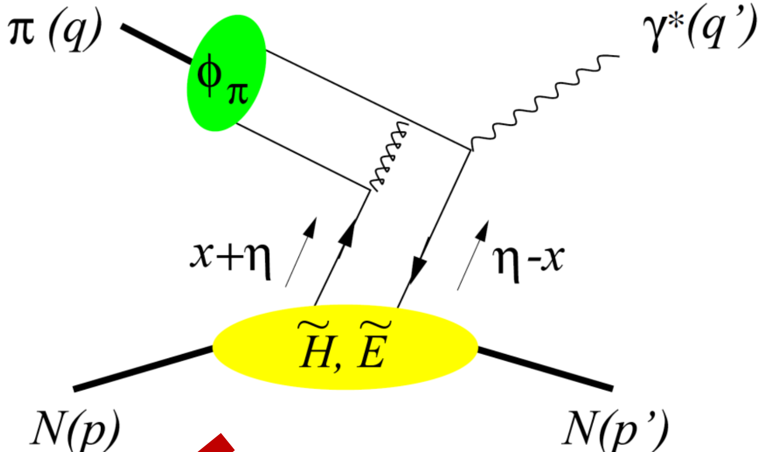

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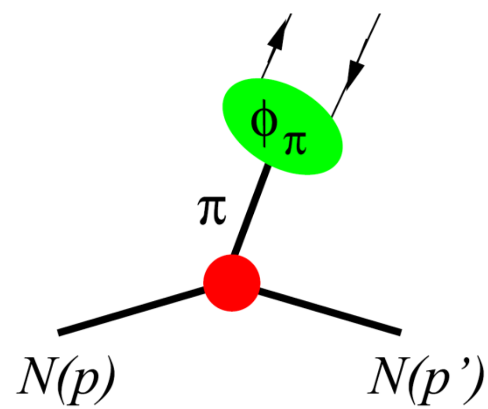
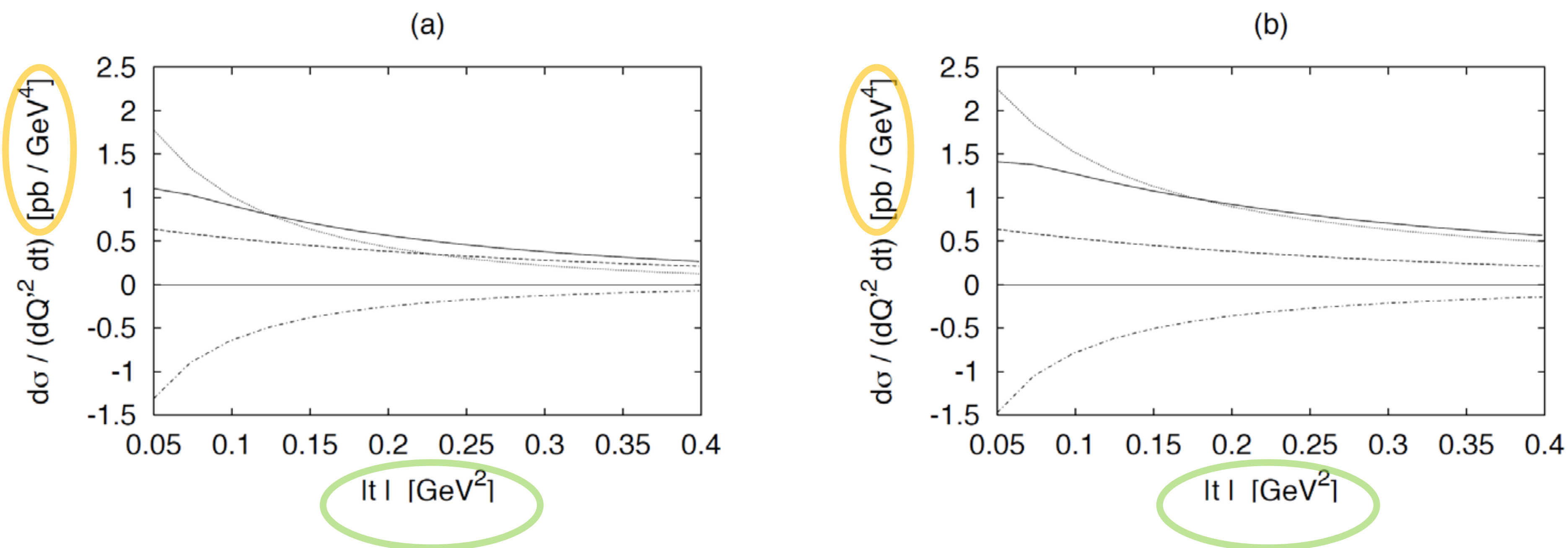


LO Estimates

Bjorken variable $\tau = \frac{Q'^2}{s - M^2}$

Berger, Diehl, Pire, PLB523(2001)265

$Q'^2 = 5GeV^2 \qquad \tau = 0.2$



(dashed) $= |\tilde{\mathcal{H}}|^2$; **(dash-dotted)** $=$ **Re**$(\tilde{\mathcal{H}}^* \tilde{\mathcal{E}})$; **(dotted)** $= |\tilde{\mathcal{E}}|^2$

$$\frac{d\sigma}{dQ'^2\,dt}(\pi^- p \to \gamma^* n) = \frac{4\pi\alpha_{em}^2}{27}\frac{\tau^2}{Q'^8} f_\pi^2 \left[(1-\eta^2)\,|\tilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \,\mathrm{Re}\left(\tilde{\mathcal{H}}^{du*}\tilde{\mathcal{E}}^{du}\right) - \eta^2 \frac{t}{4M^2} |\tilde{\mathcal{E}}^{du}|^2 \right]$$

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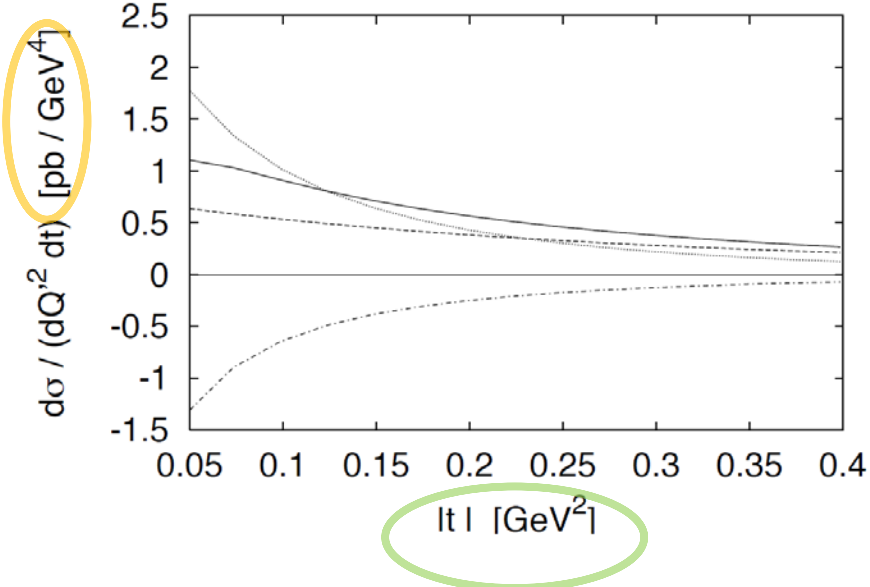
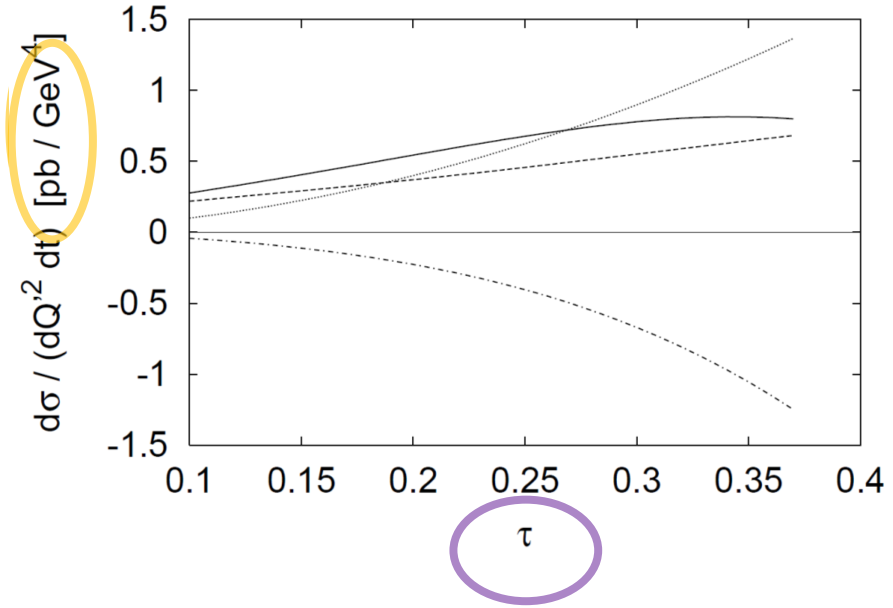


$|t| = 0.2\ \mathrm{GeV}^2$



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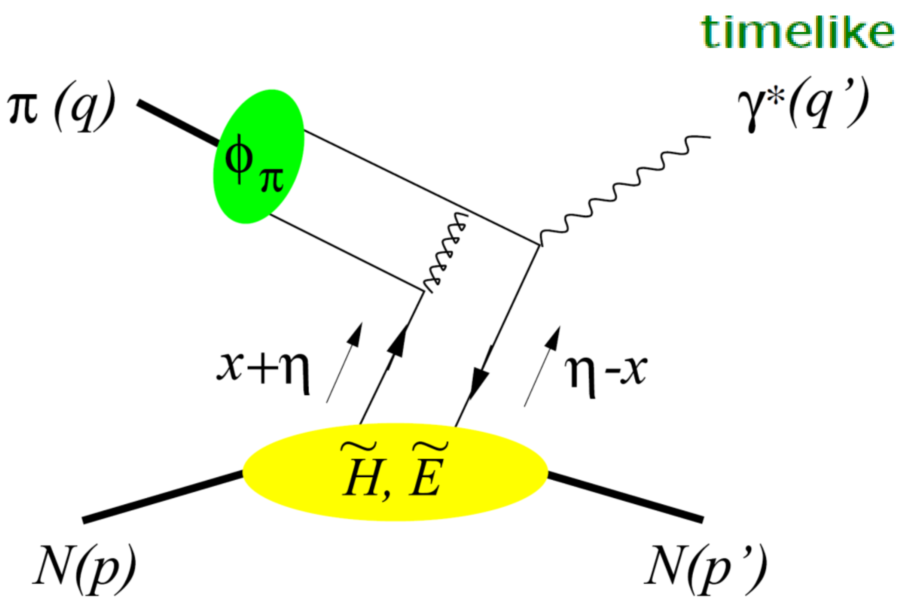
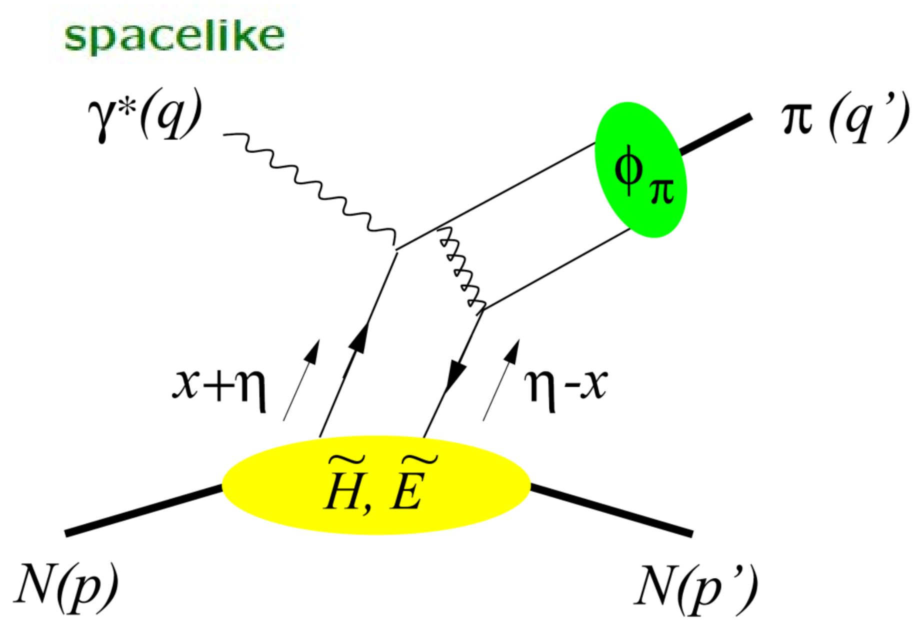
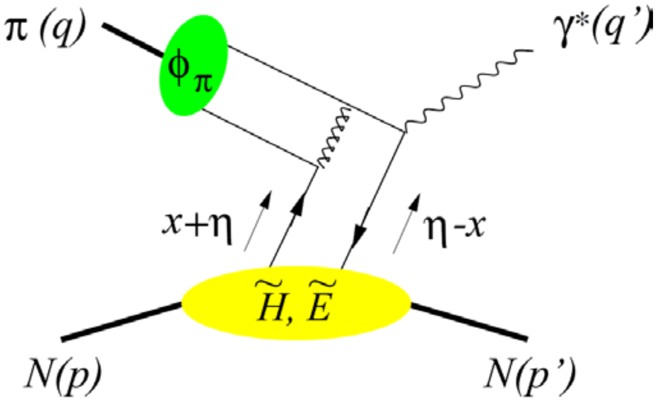
Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions



exDY@J-PARC



DVMP@JLab



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Berger, Diehl, Pire, PLB523(2001)

$$\frac{d\sigma}{dQ'^2\, dt\, d(\cos\theta)\, d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2 \sin^2\theta$$

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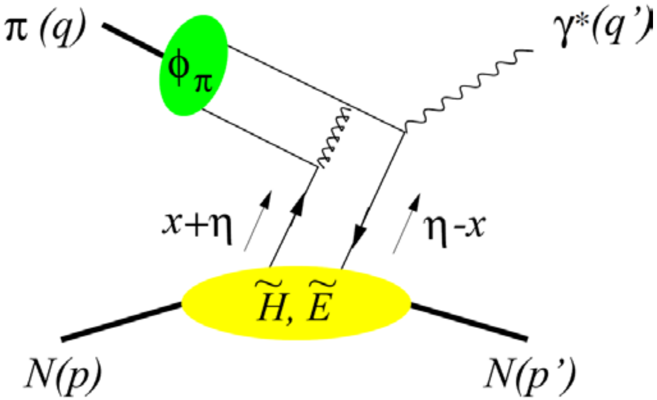

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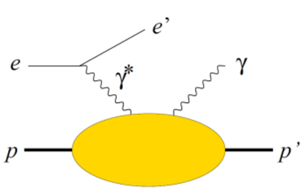
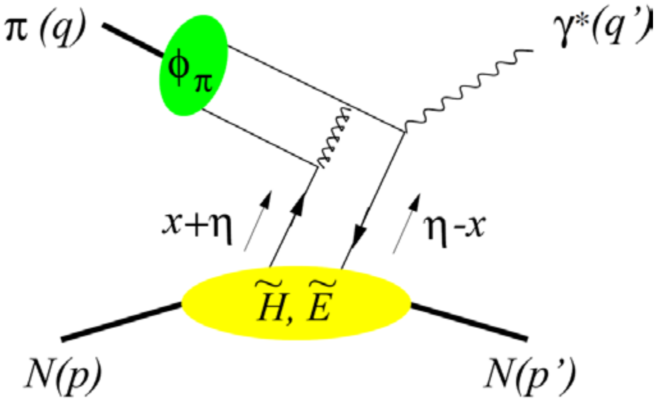


$$\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle p'|\bar{\psi}(-\frac{z^-}{2})\gamma^+\psi(\frac{z^-}{2})|p\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\right]$$

$$J_q = \frac{1}{2}\int_{-1}^{1} dx\, x\left(H^q(x,\eta,0) + E^q(x,\eta,0)\right)$$



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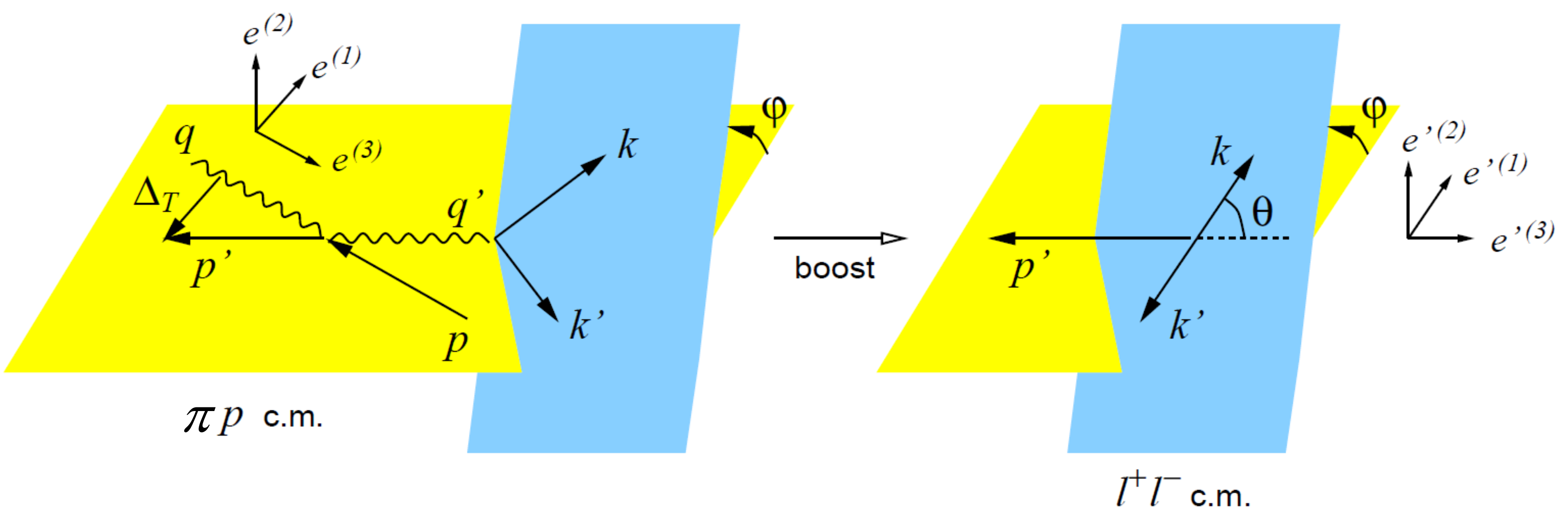

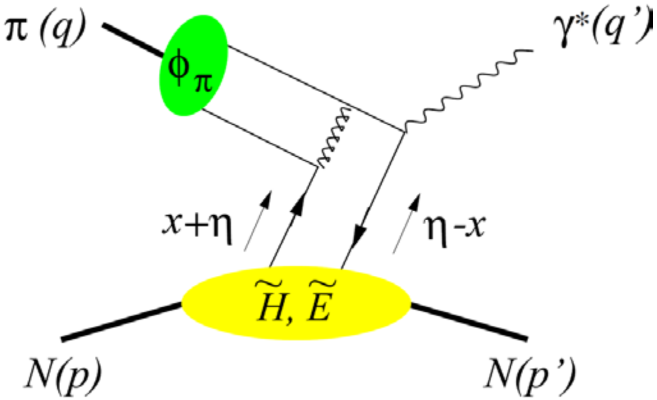

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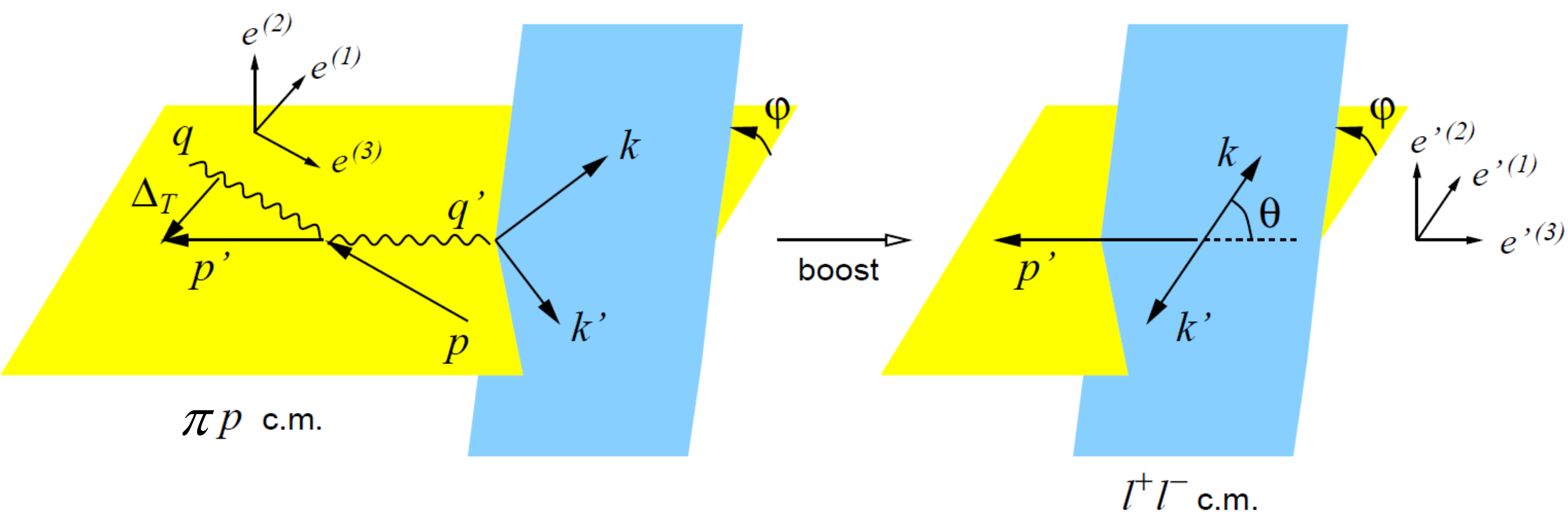

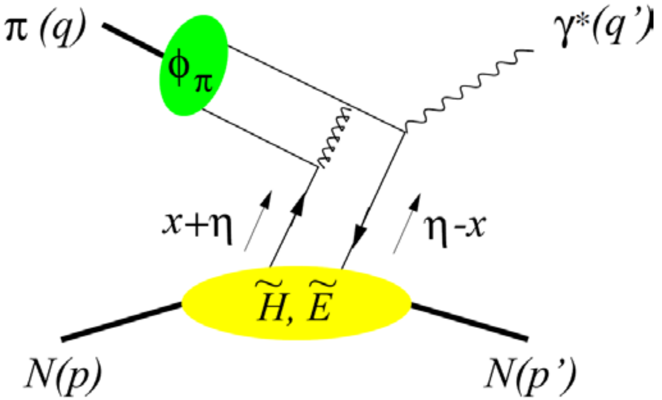

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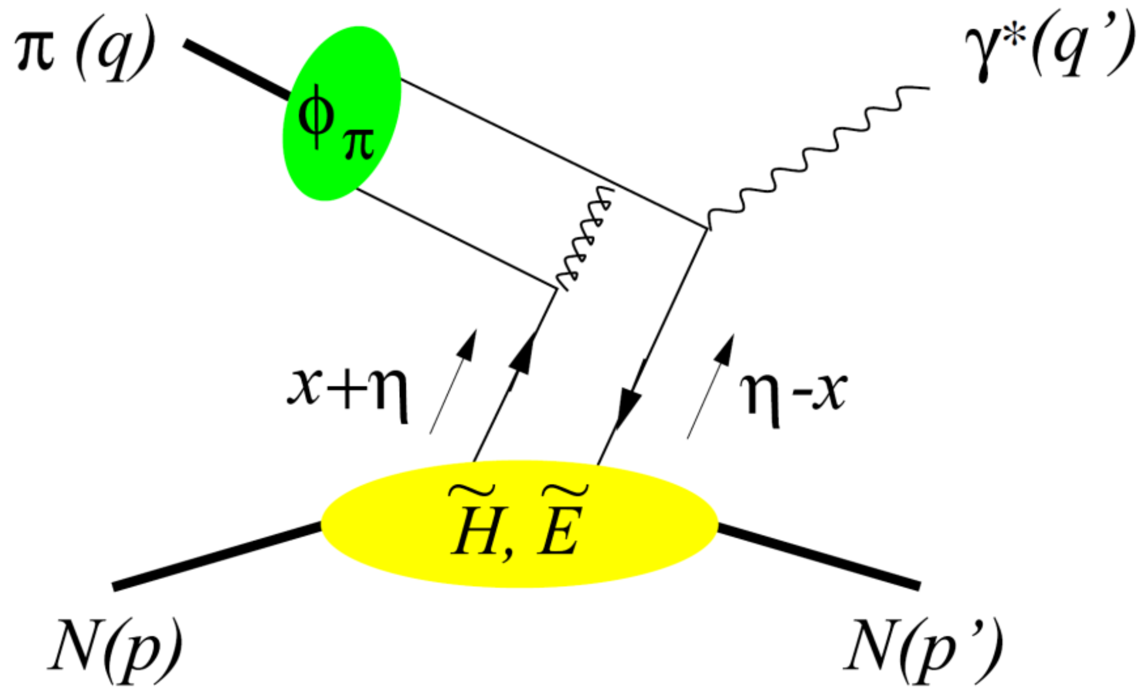
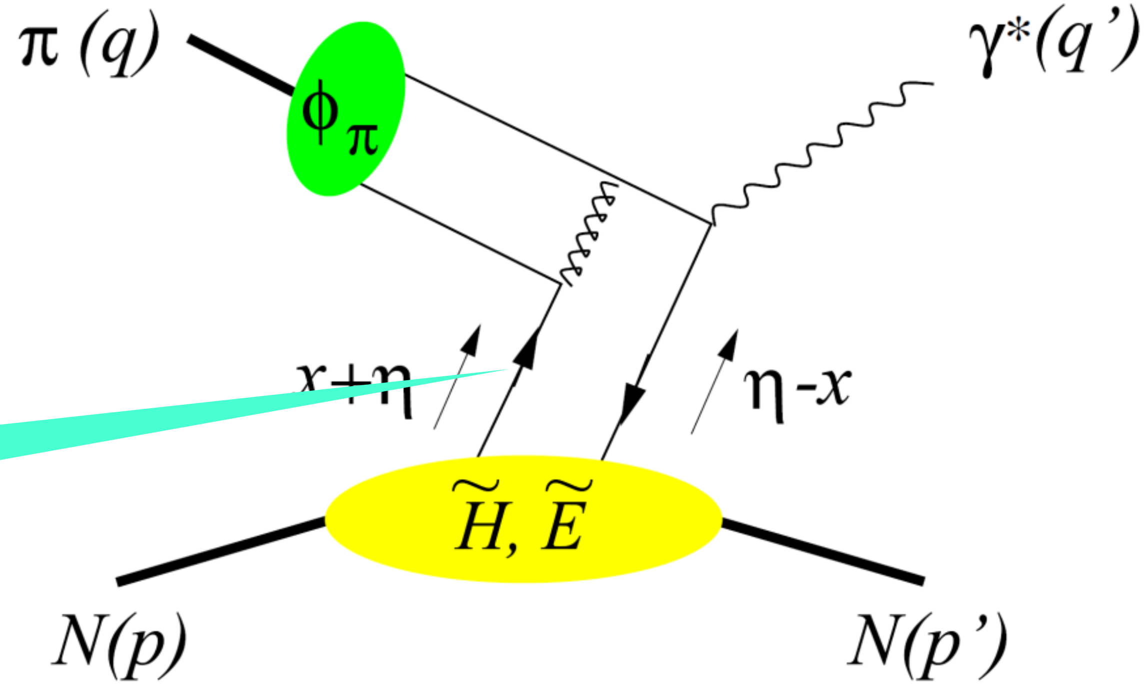


π (q)
φπ
γ*(q')
x+η
η-x
H̃, Ẽ
N(p)
N(p')

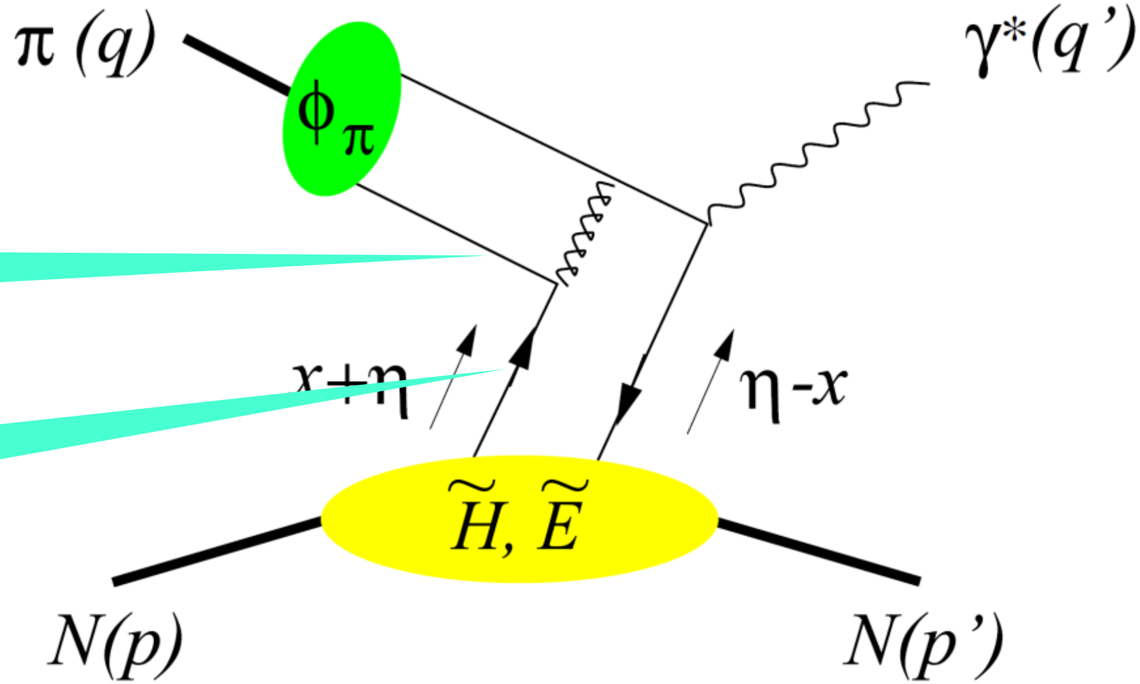


$$\left(x+\eta\right)\bar{P}^{+}$$

$$\bar{P}^{+} = \frac{p^{+} + p'^{+}}{2}$$

uq^{-}

$(x+\eta)\bar{P}^{+}$



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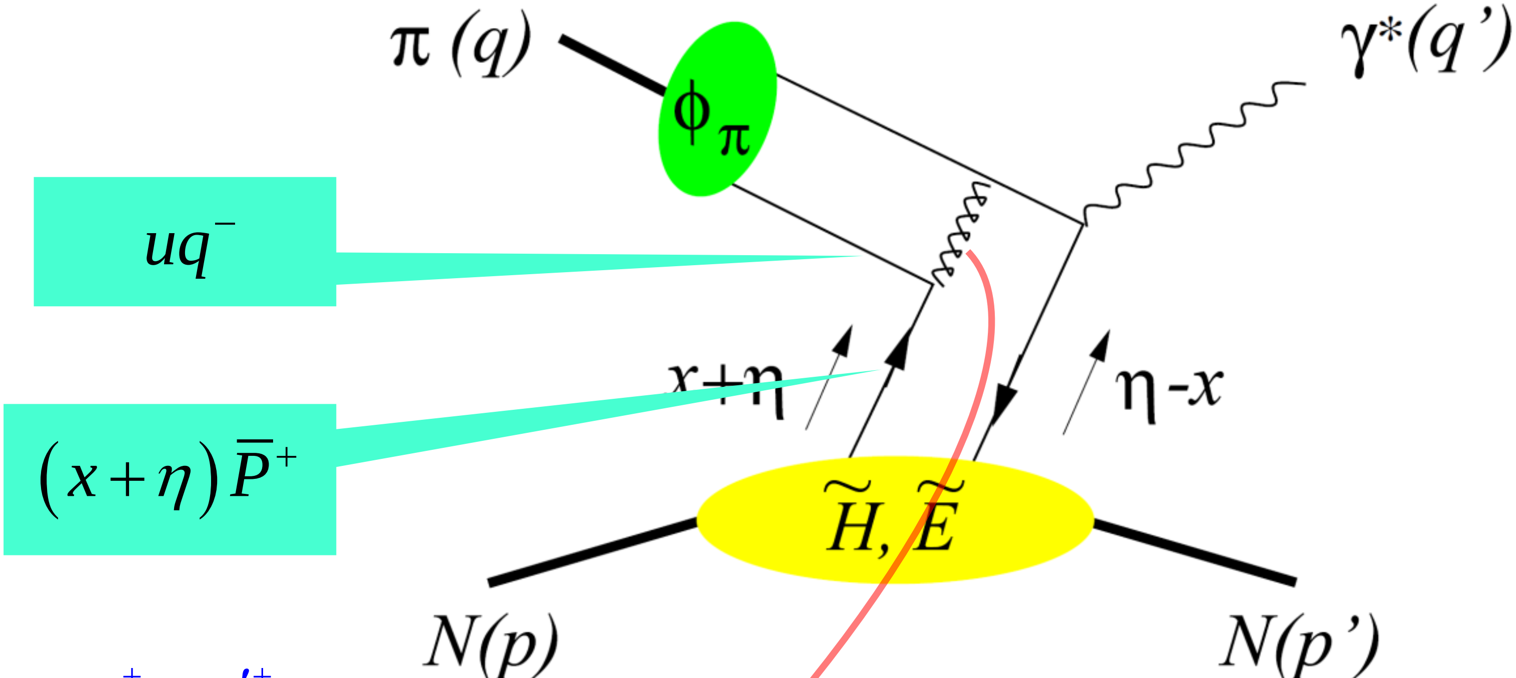
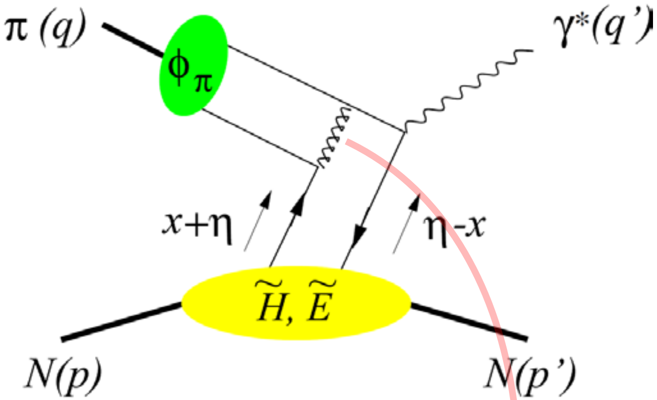


$$\bar{P}^{+} = \frac{p^{+} + p'^{+}}{2}$$

$$\frac{1}{\left[uq + (x+\eta)\bar{P}\right]^{2}} \simeq \frac{1}{u(x+\eta)s}$$



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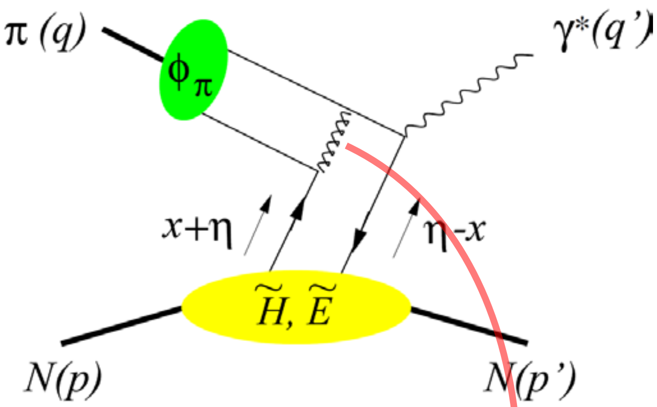

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$$\phi_\pi(u) \sim u(1-u)$$

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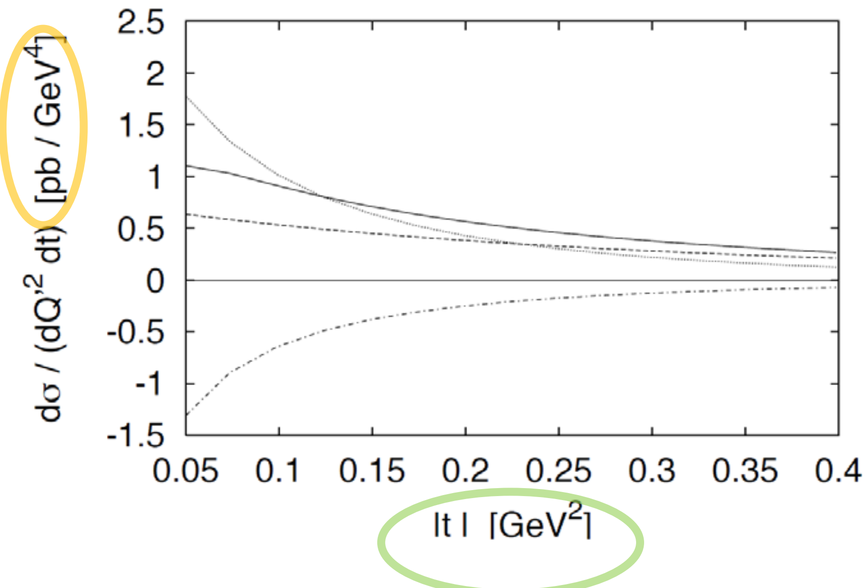
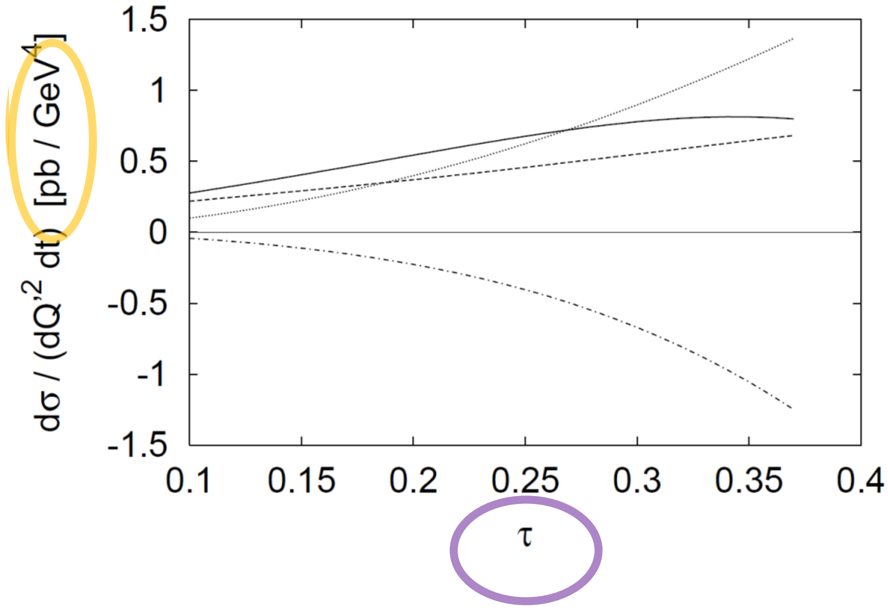
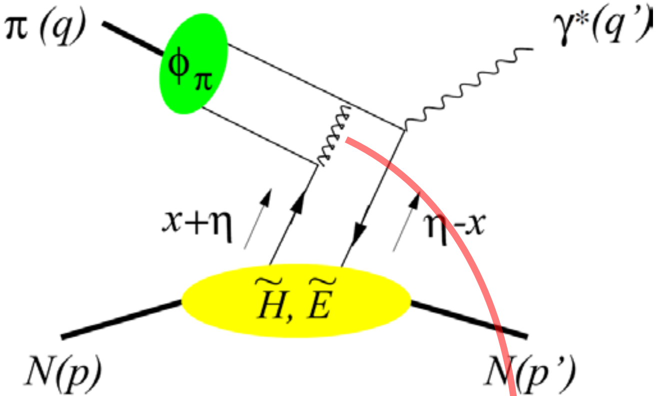


$|t| = 0.2\ \mathrm{GeV}^2$



(dashed) $= |\tilde{\mathcal{H}}|^2$; **(dash-dotted)** $=$ **Re**$(\tilde{\mathcal{H}}^* \tilde{\mathcal{E}})$; **(dotted)** $= |\tilde{\mathcal{E}}|^2$

$$\frac{d\sigma}{dQ'^2\, dt}(\pi^- p \to \gamma^* n) = \frac{4\pi\alpha_{em}^2}{27}\frac{\tau^2}{Q'^8} f_\pi^2 \left[(1-\eta^2)\,|\tilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \,\mathrm{Re}\left(\tilde{\mathcal{H}}^{du*}\tilde{\mathcal{E}}^{du}\right) - \eta^2 \frac{t}{4M^2} |\tilde{\mathcal{E}}^{du}|^2 \right]$$



Bjorken variable: $\tau = \dfrac{Q'^2}{2p\cdot q}$

Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

long. photon

$\left|d^1_{-1\,0}(\theta)\right|^2 + \left|d^1_{1\,0}(\theta)\right|^2$

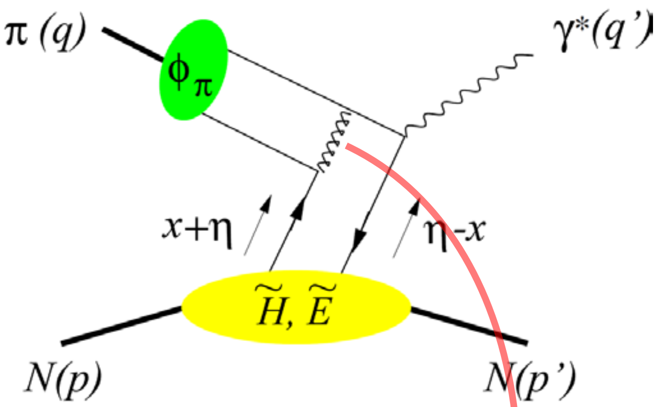
$$\frac{d\sigma}{dQ'^2\, dt\, d(\cos\theta)\, d\varphi} = \frac{\alpha_{\rm em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2 \sin^2\theta$$

$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\,\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^1 dx\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)\right]$$

$$\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle p'|\bar{\psi}(-\frac{z^-}{2})\gamma^+\gamma_5\psi(\frac{z^-}{2})|p\rangle = \frac{1}{\bar{P}^+}\left[\tilde{H}^q(x,\eta,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\eta,t)\bar{u}(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$

$$\phi_\pi(u) \sim u(1-u)$$



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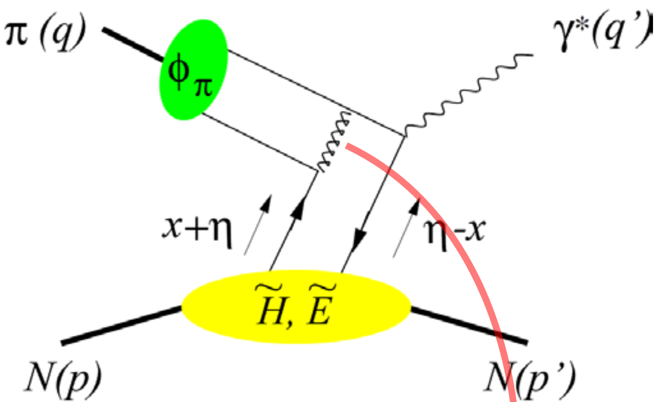

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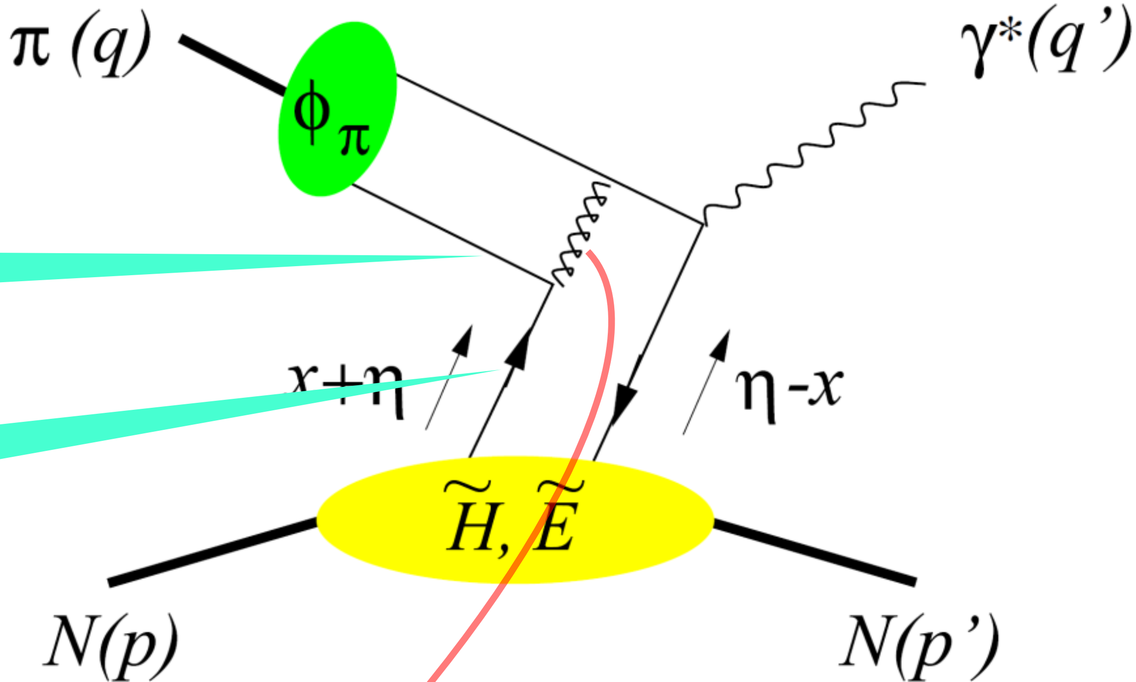

uq^-

$(x+\eta)\bar{P}^+$

$$\bar{P}^+ = \frac{p^+ + p'^+}{2}$$

$$\frac{1}{\left[uq+(x+\eta)\bar{P}\right]^2} \simeq \frac{1}{u(x+\eta)s}$$

Collinear factorization does not work at twist-3:

- quark $k_\perp$ ("k_T-factorization") Goloskokov, Kroll

  with Sudakov resummation

  Li, Sterman

uq^-

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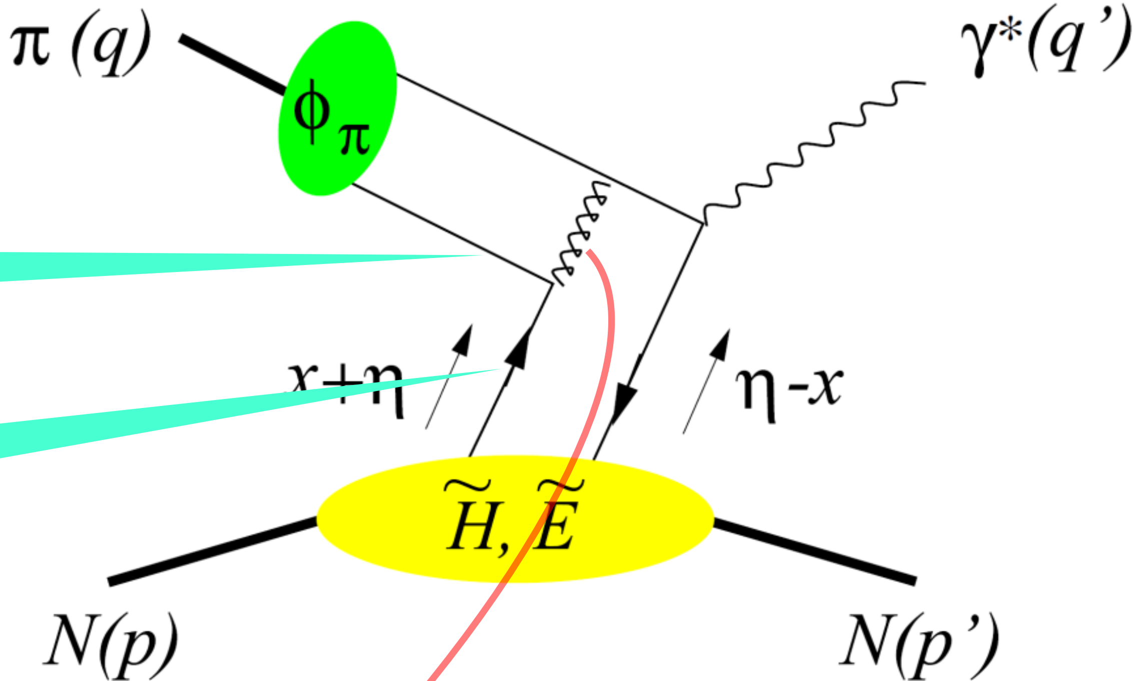

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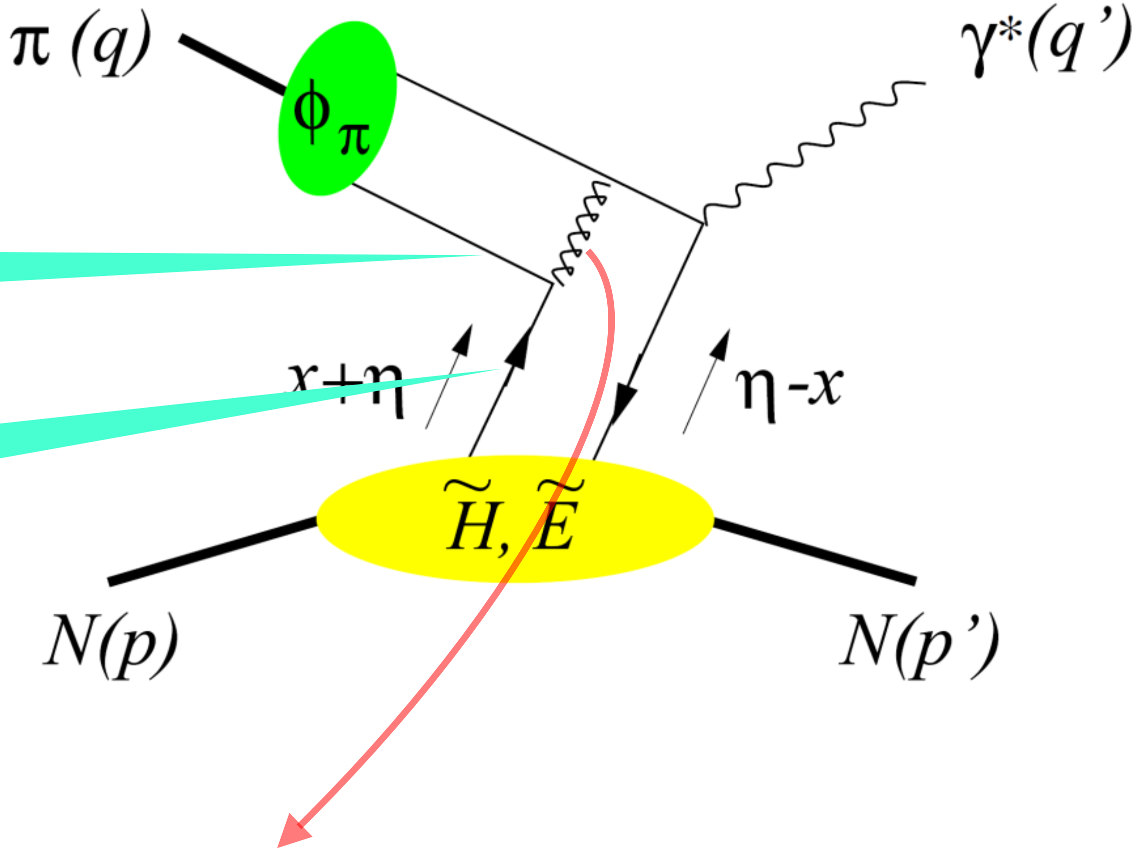

$$\bar{P}^+ = \frac{p^+ + p'^+}{2}$$

$$\frac{1}{\left[uq + (x+\eta)\bar{P}\right]^2 - \boldsymbol{k}_\perp^2} \simeq \frac{1}{u(x+\eta)s - \boldsymbol{k}_\perp^2}$$

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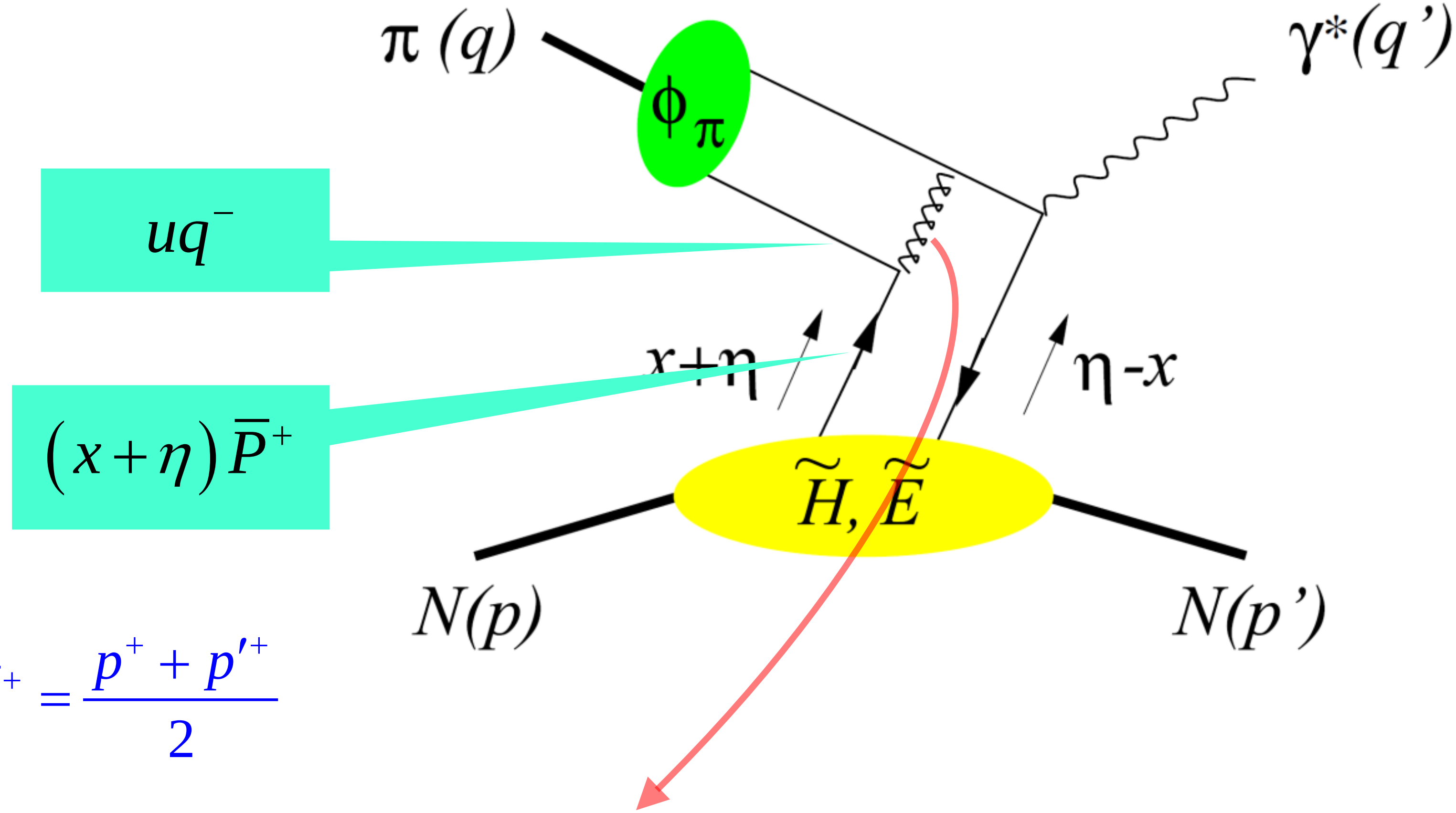

$$\bar{P}^{+} = \frac{p^{+} + p'^{+}}{2}$$

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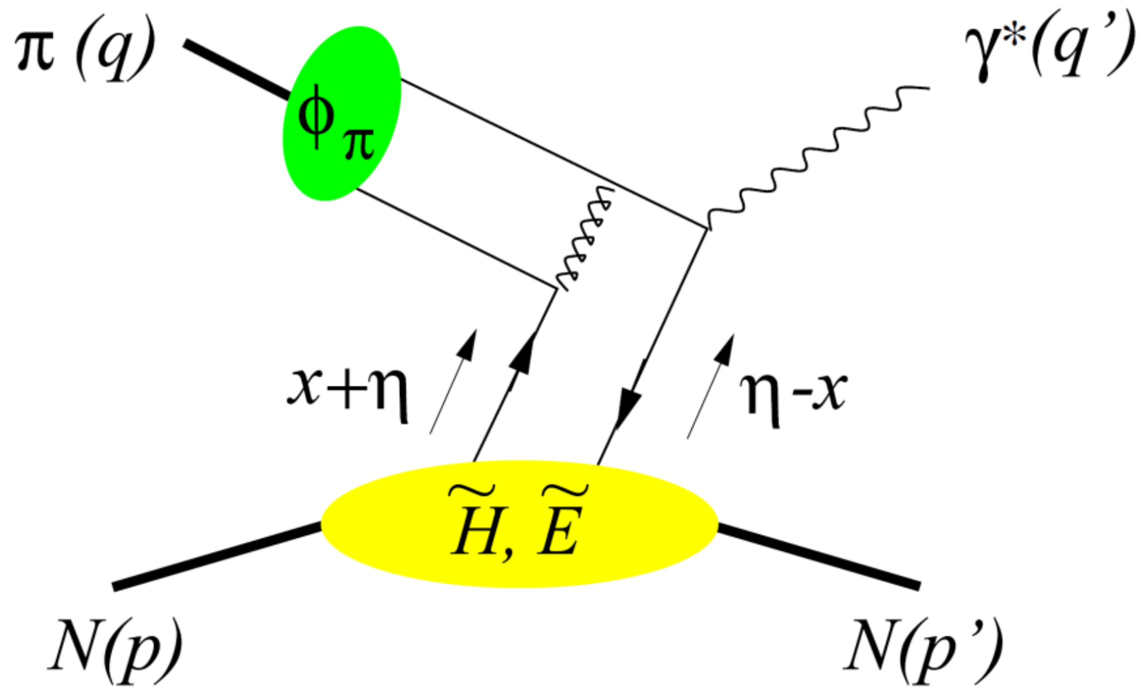
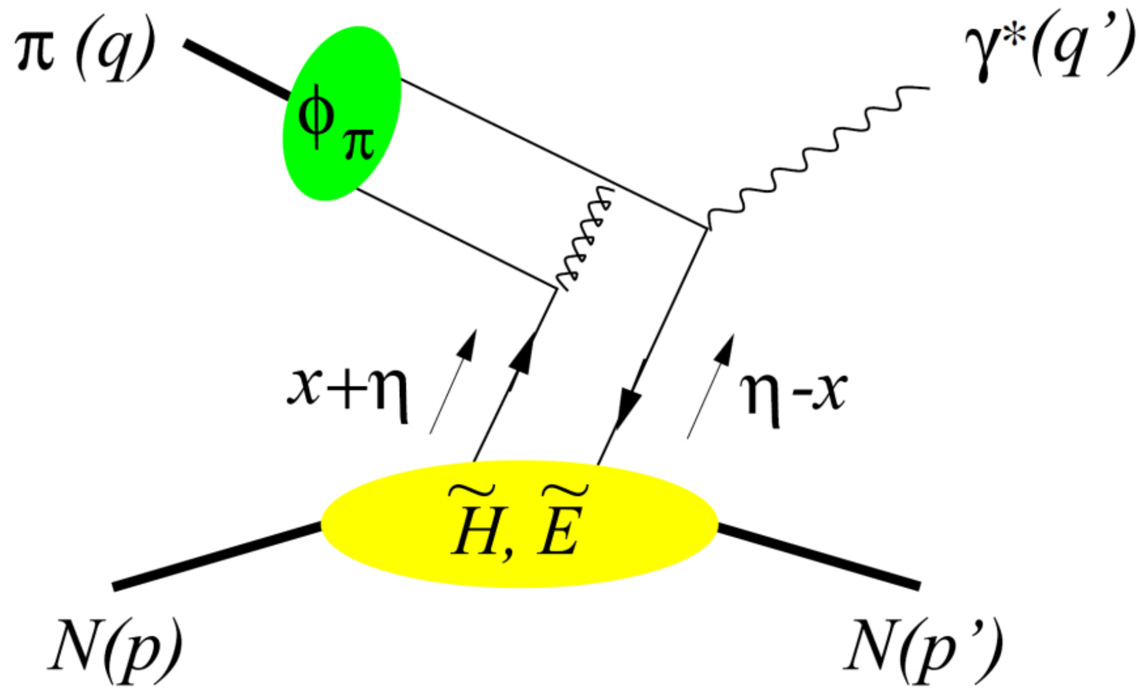
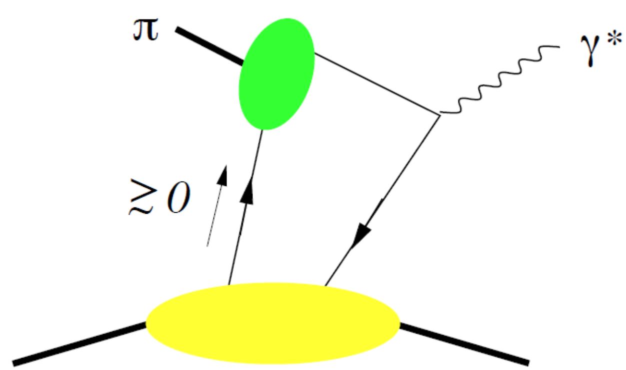


π (q)
γ*(q')
φπ
x+η
η-x
H̃, Ẽ
N(p)
N(p')

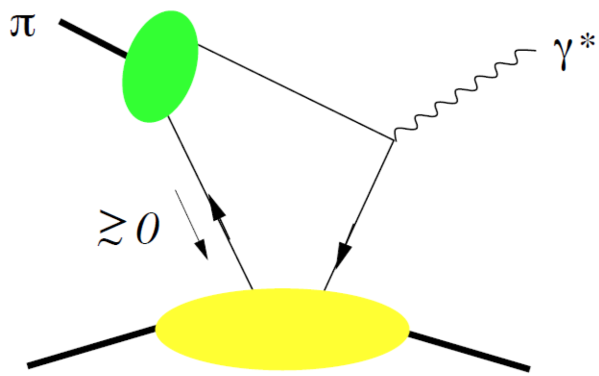


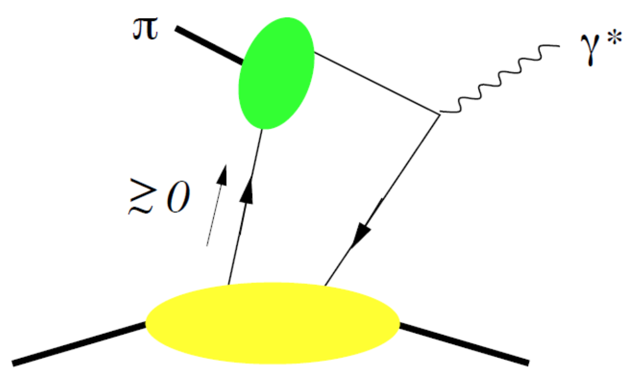
π (q)
γ*(q')
ϕ_π
x+η
η-x
$\widetilde{H}, \widetilde{E}$
N(p)
N(p')



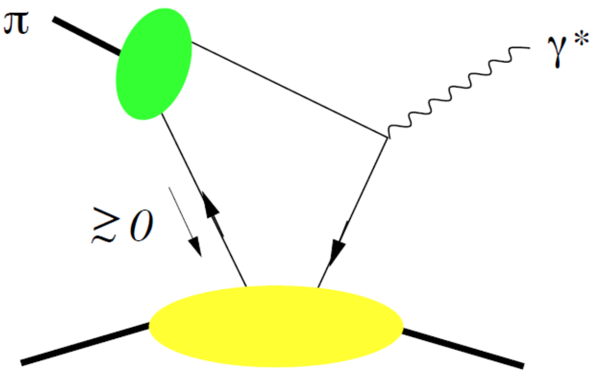
π
γ*
≳ 0

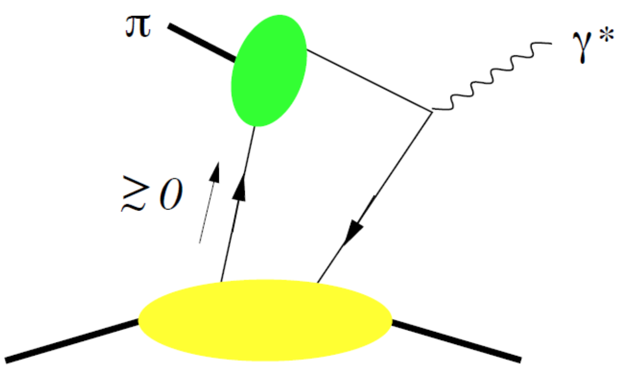
“nonfactorizable” mechanism





“nonfactorizable” mechanism





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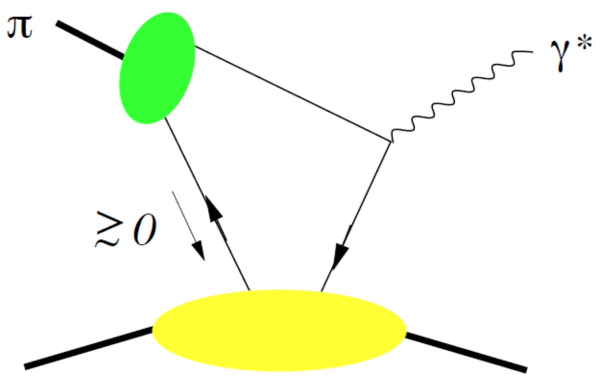
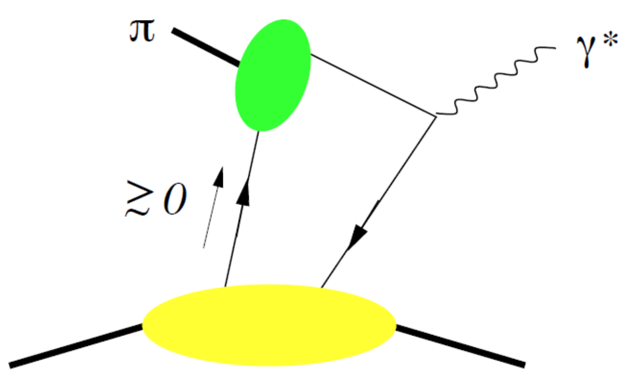
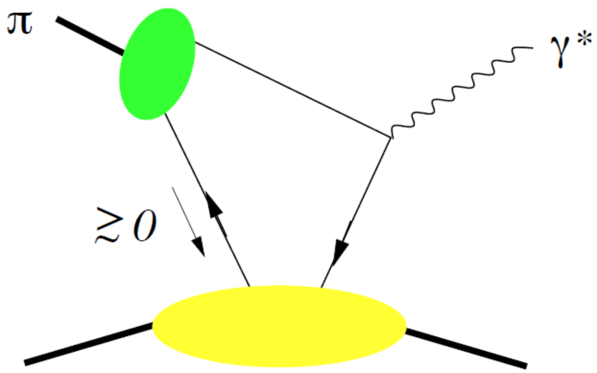


lower order in α_s

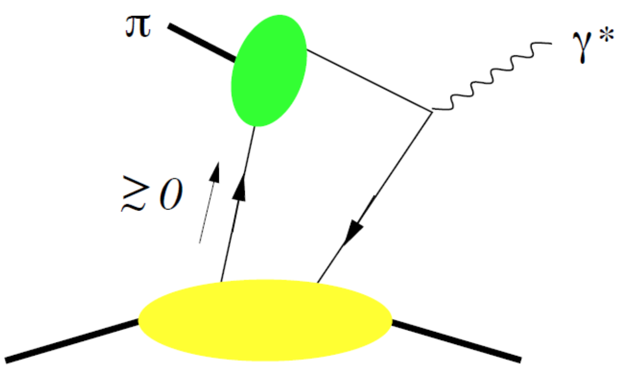


“nonfactorizable” mechanism

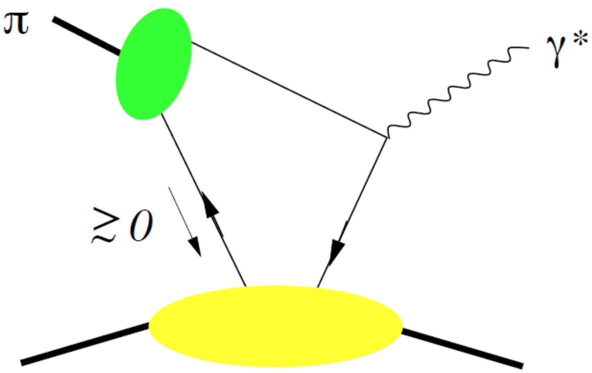


lower order in α_s

“Feynman mechanism”

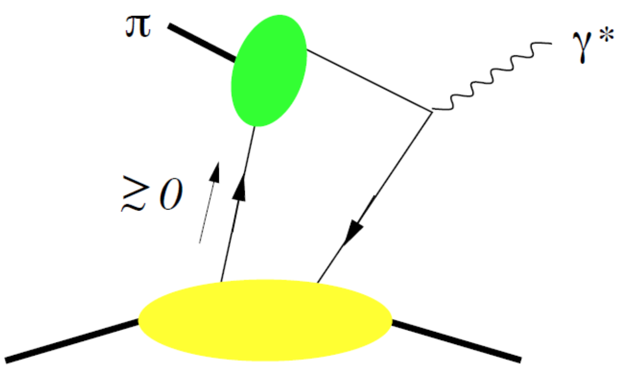


“nonfactorizable” mechanism

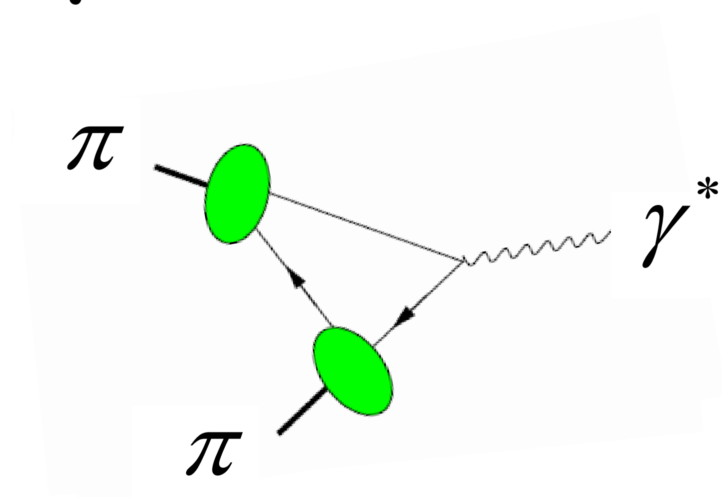


lower order in α_s

“Feynman mechanism”



(d)



Collinear factorization does not work at twist-3:

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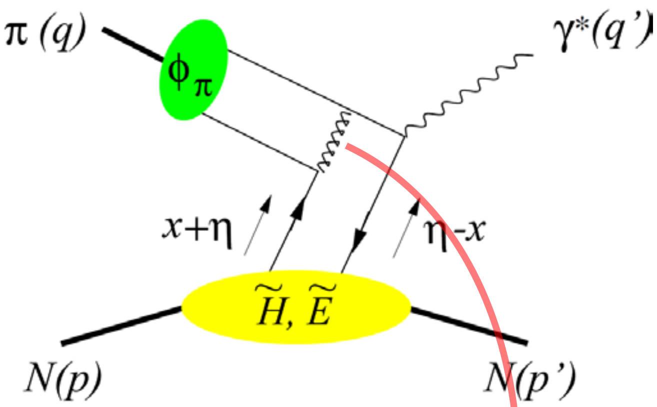
  with Sudakov resummation

  Li, Sterman

- include "soft" propagator in long-distance part

  nonfactorizable "Feynman mechanism" at lower order in α_s



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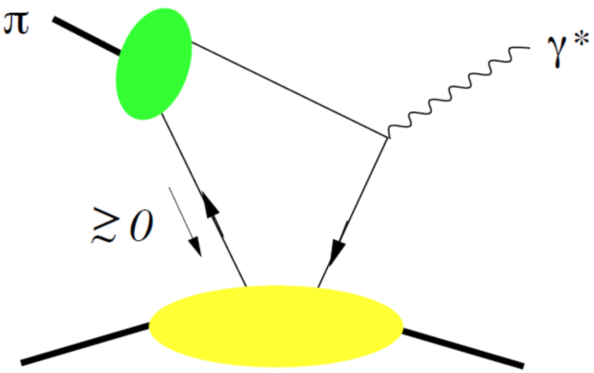
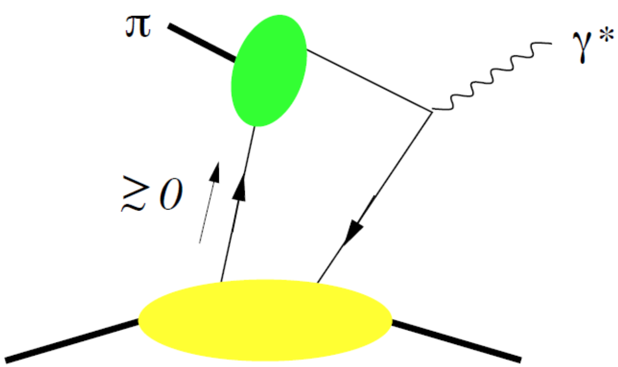
“nonfactorizable” mechanism

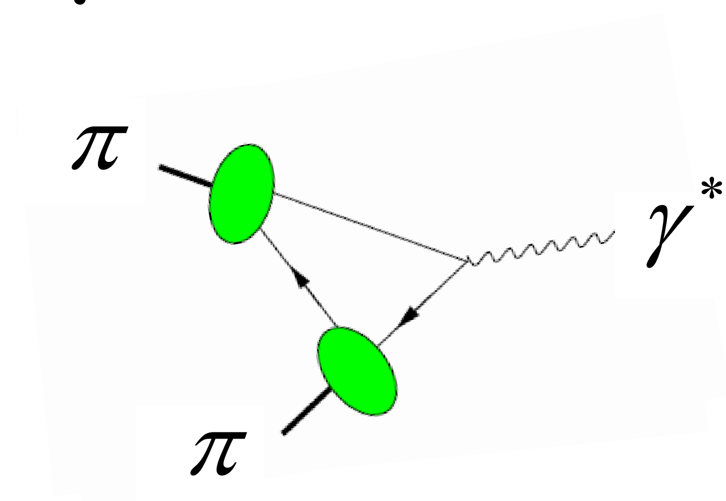


lower order in α_s



(d)

“Feynman mechanism”



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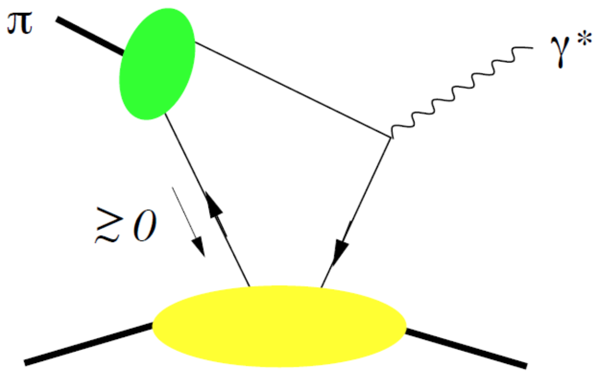
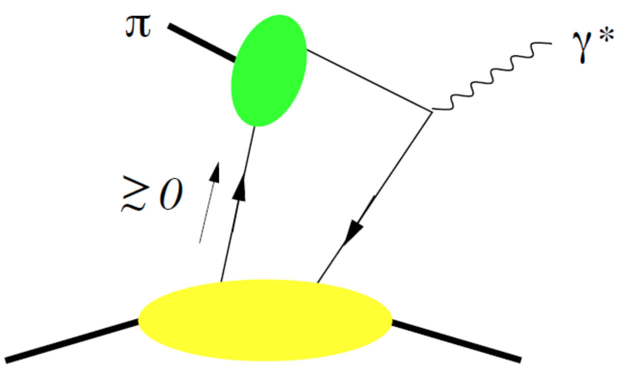
  nonfactorizable "Feynman mechanism" at lower order in α_s

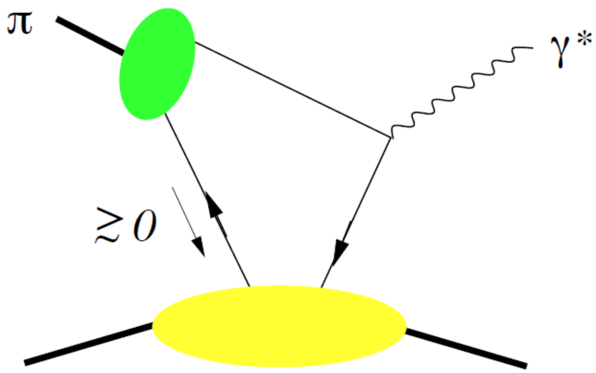
  relevent also for leading twist!

“nonfactorizable” mechanism

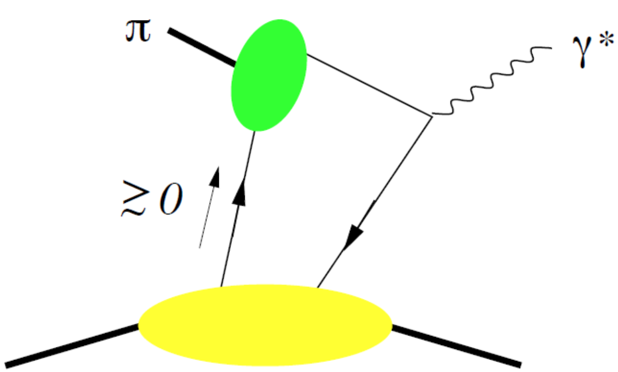




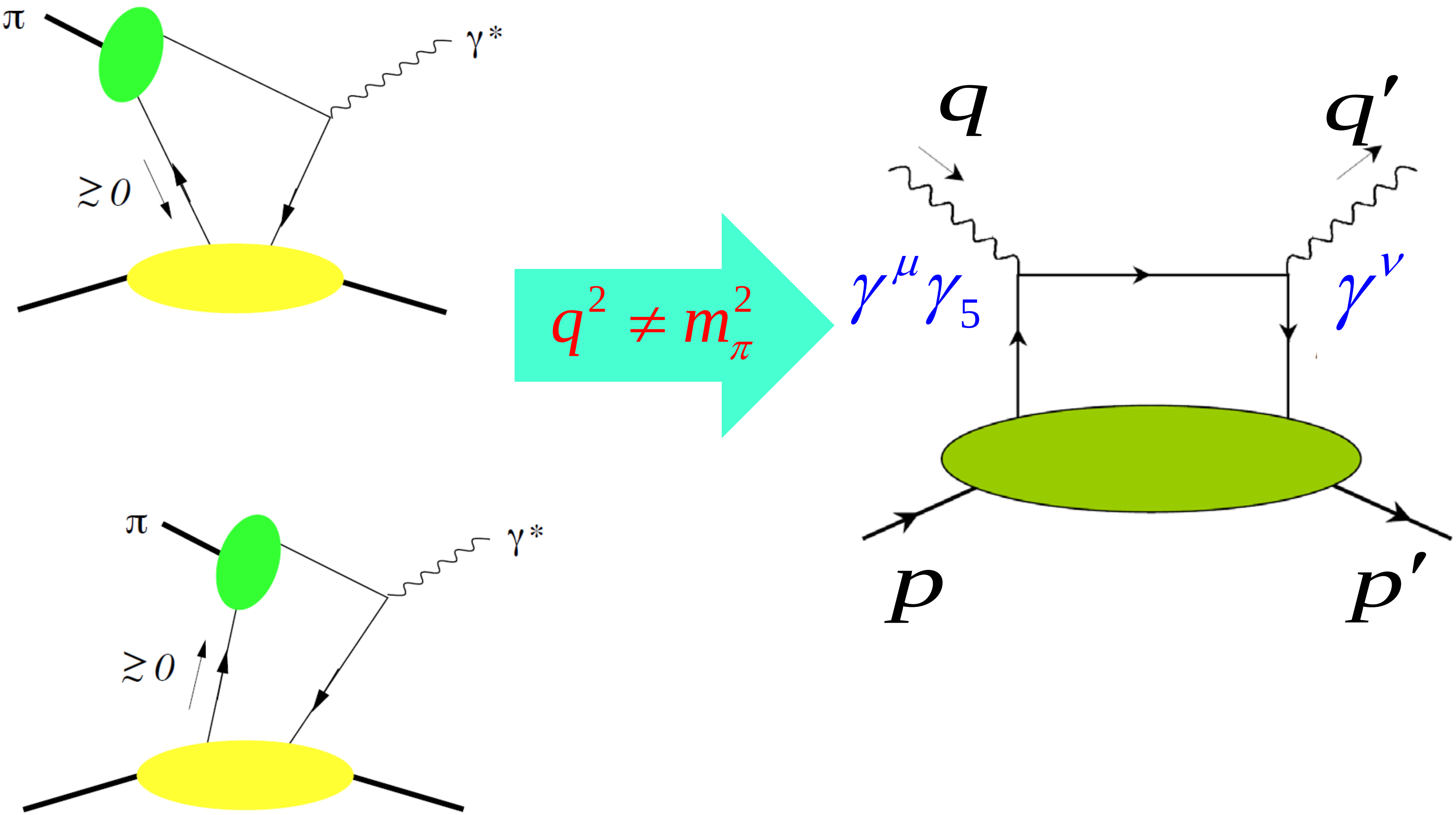
“nonfactorizable” mechanism



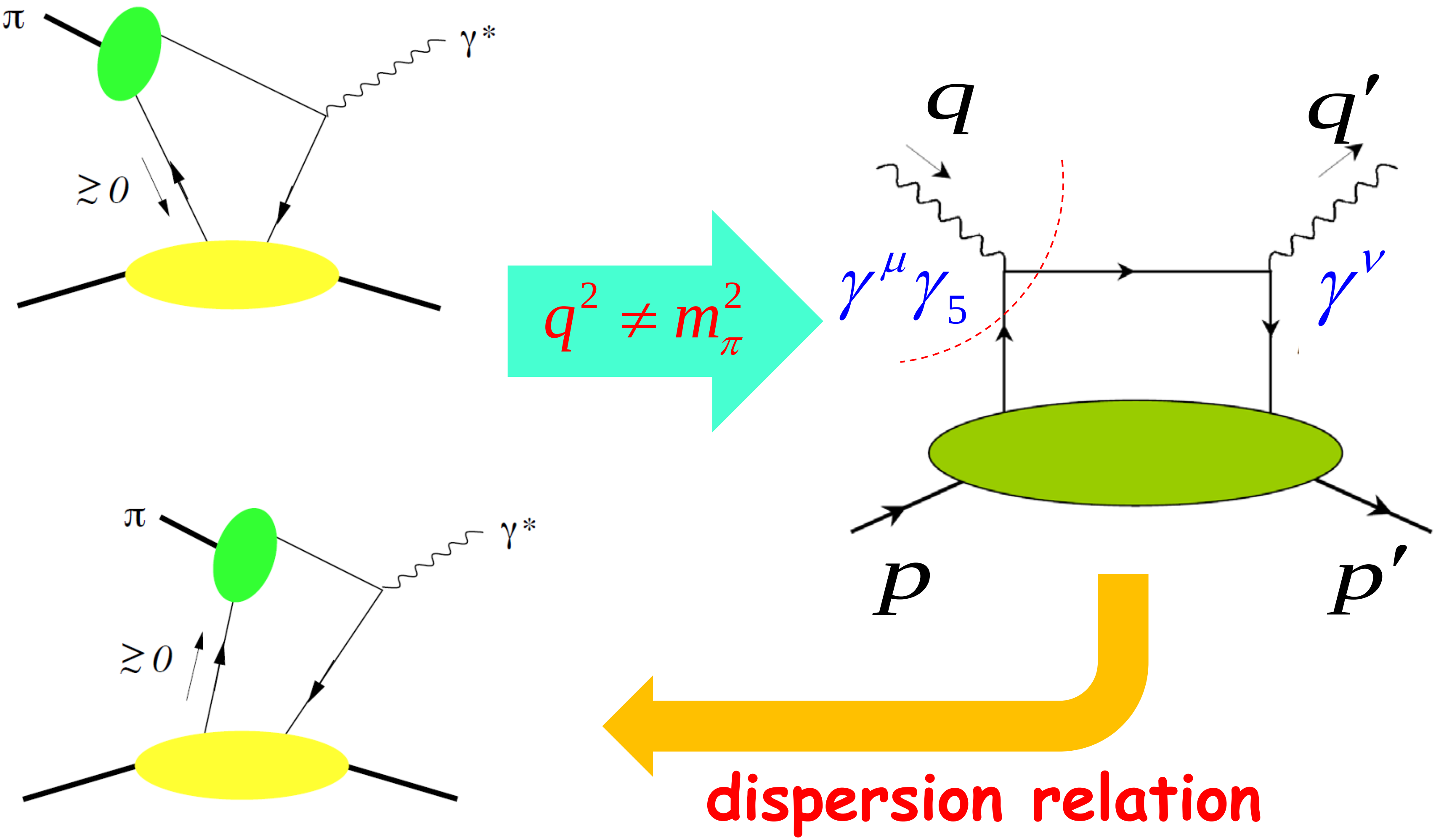
$$q^2 \neq m_\pi^2$$



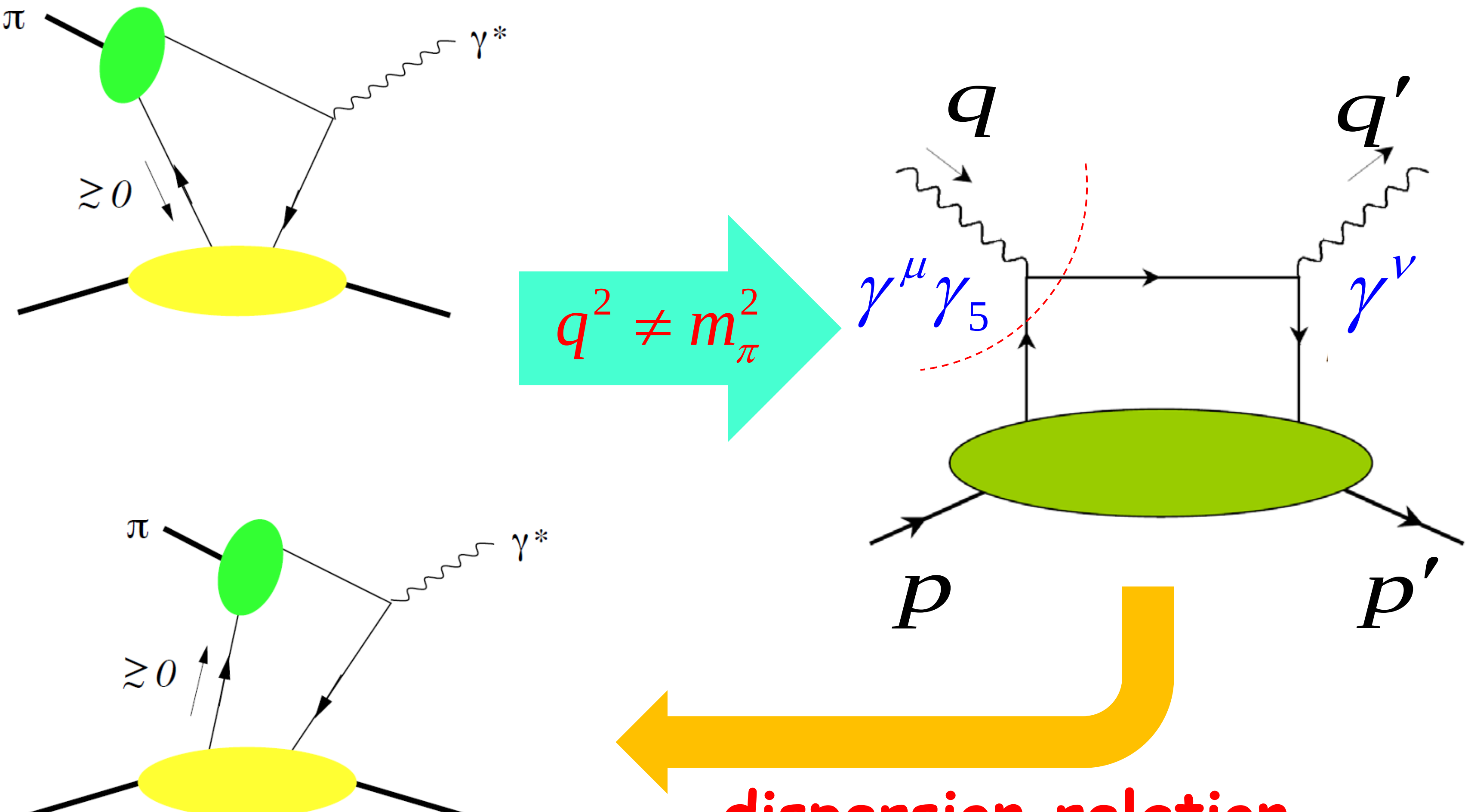
“nonfactorizable” mechanism



“nonfactorizable” mechanism

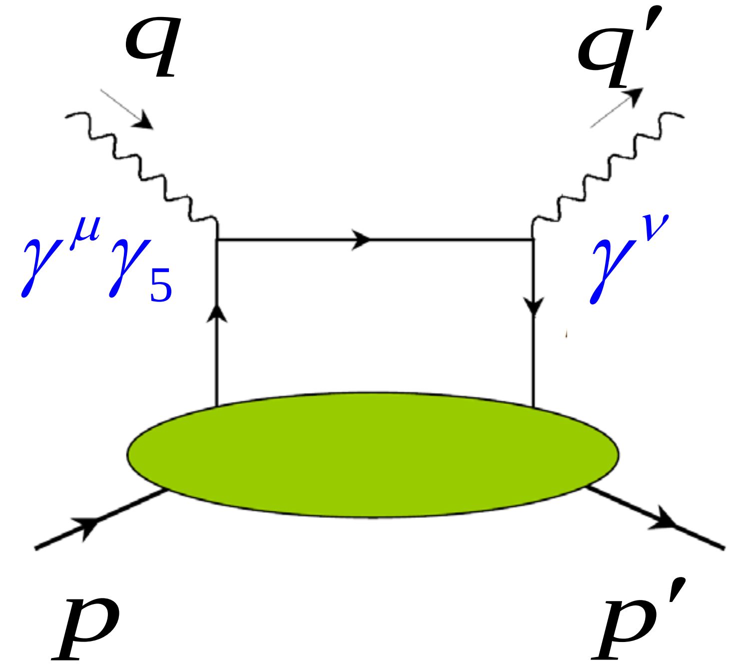


“nonfactorizable” mechanism



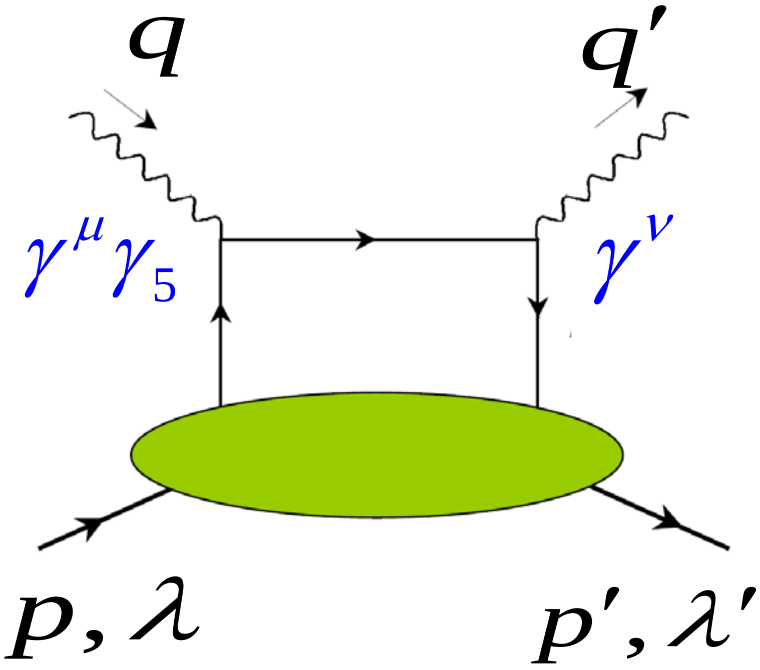
dispersion relation

quark-hadron duality



q
q′
$\gamma^\mu\gamma_5$
γ^ν
p
p′

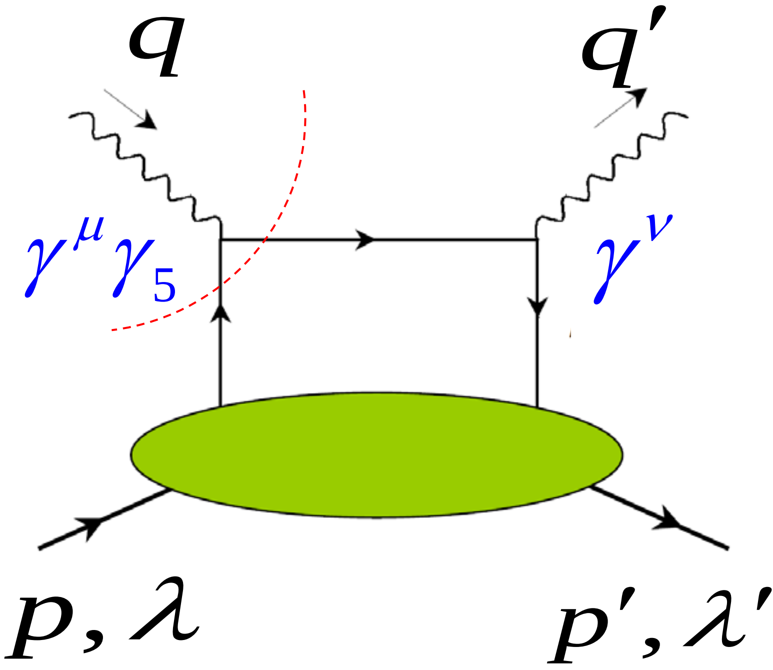
$$\int d^4x e^{iq'\cdot x} \langle p'\lambda' | \mathrm{T}\, j_\mu^5(0)\, j_\nu^{\mathrm{em}}(x) | p\lambda\rangle$$
$$\equiv -iT_{\mu\nu}$$



$$j_\mu^5 = \bar{d}\gamma_\mu\gamma_5 u$$

$$j_\nu^{\mathrm{em}} = e_u \bar{u}\gamma_\nu u + e_d \bar{d}\gamma_\nu d$$

$$\int d^4x e^{iq'\cdot x}\langle p'\lambda' | \mathrm{T}\, j^5_\mu(0)\, j^{\mathrm{em}}_\nu(x) | p\lambda\rangle$$
$$\equiv -iT_{\mu\nu}$$



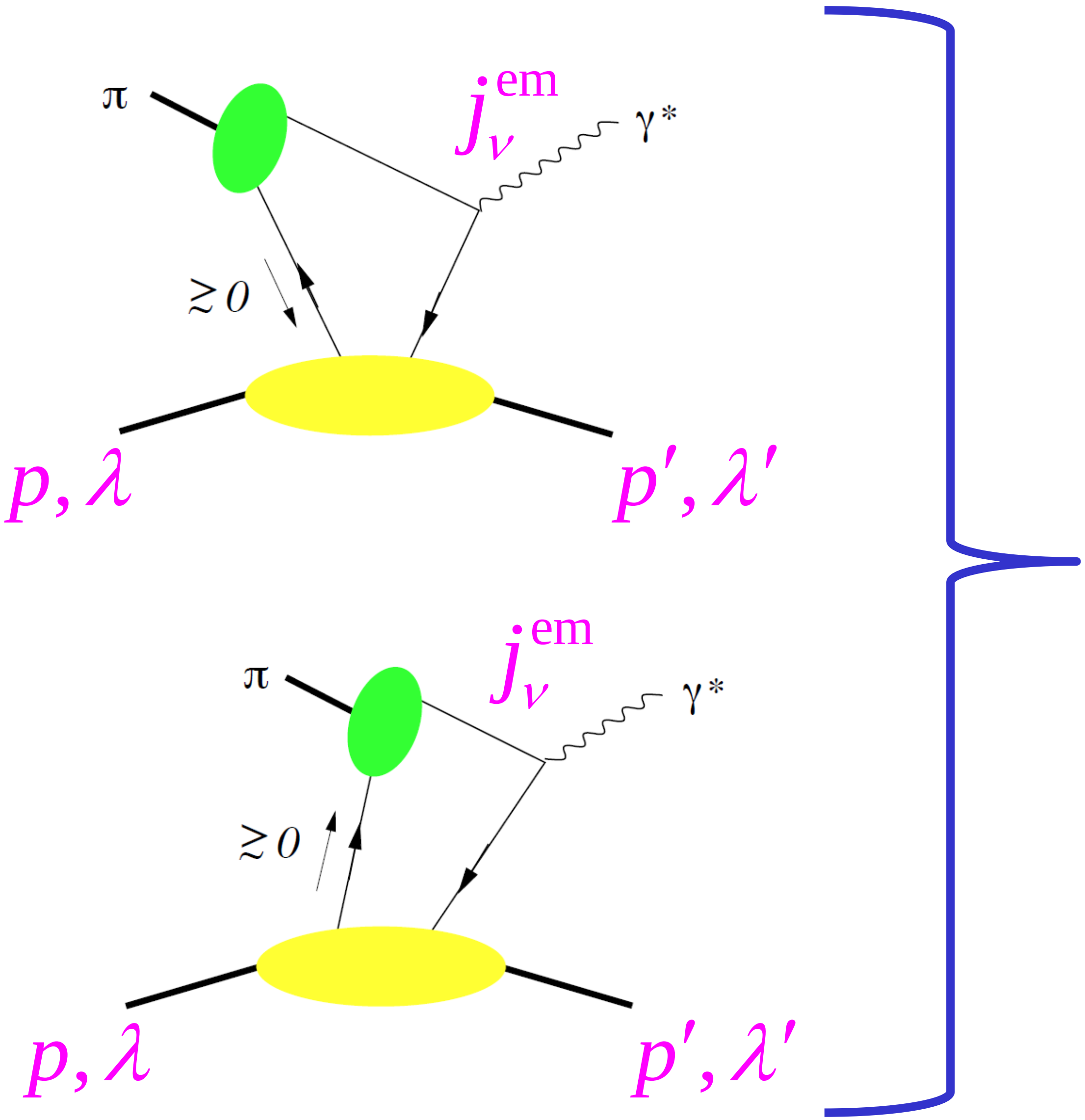
$$j^5_\mu = \bar{d}\gamma_\mu\gamma_5 u$$

$$j^{\mathrm{em}}_\nu = e_u \bar{u}\gamma_\nu u + e_d \bar{d}\gamma_\nu d$$

$$T_{\mu\nu} = iq_\mu f_\pi \frac{1}{q^2 - m_\pi^2} M_\nu^{\lambda',\lambda}(\pi^- p \to \gamma^* n) + \cdots$$

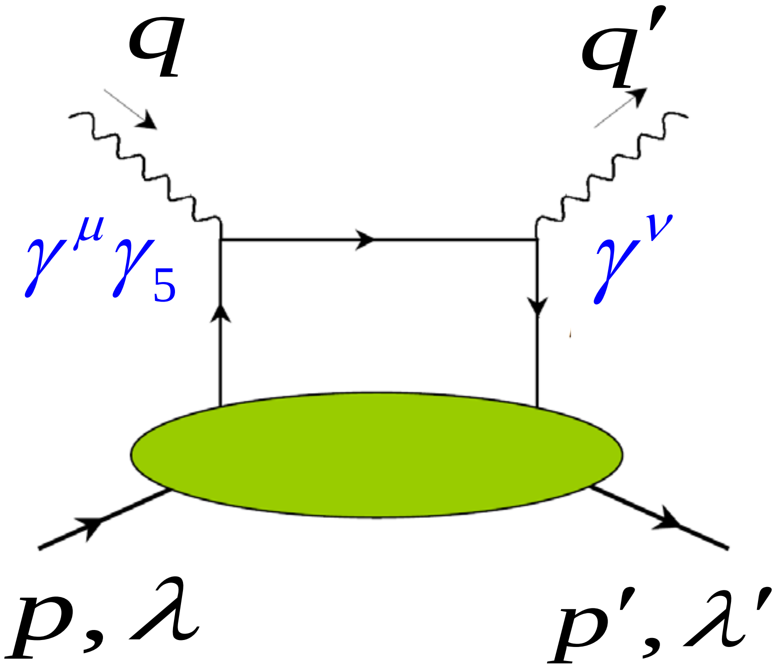
$$\langle 0 | j^5_\mu | \pi^-(k)\rangle = ik_\mu f_\pi$$

“nonfactorizable” mechanism



$$M_\nu^{\lambda',\lambda}(\pi^- p \to \gamma^* n)$$

$$\int d^4x e^{iq'\cdot x}\langle p'\lambda' | \mathrm{T}\, j^5_\mu(0)\, j^{\text{em}}_\nu(x) | p\lambda\rangle$$
$$\equiv -iT_{\mu\nu}$$



$$j^5_\mu = \bar{d}\gamma_\mu\gamma_5 u$$

$$j^{\text{em}}_\nu = e_u\bar{u}\gamma_\nu u + e_d\bar{d}\gamma_\nu d$$

$$\int d^4x e^{iq'\cdot x} \langle p'\lambda' | \mathrm{T}\, j^5_\mu(0)\, j^{\mathrm{em}}_\nu(x) | p\lambda\rangle$$
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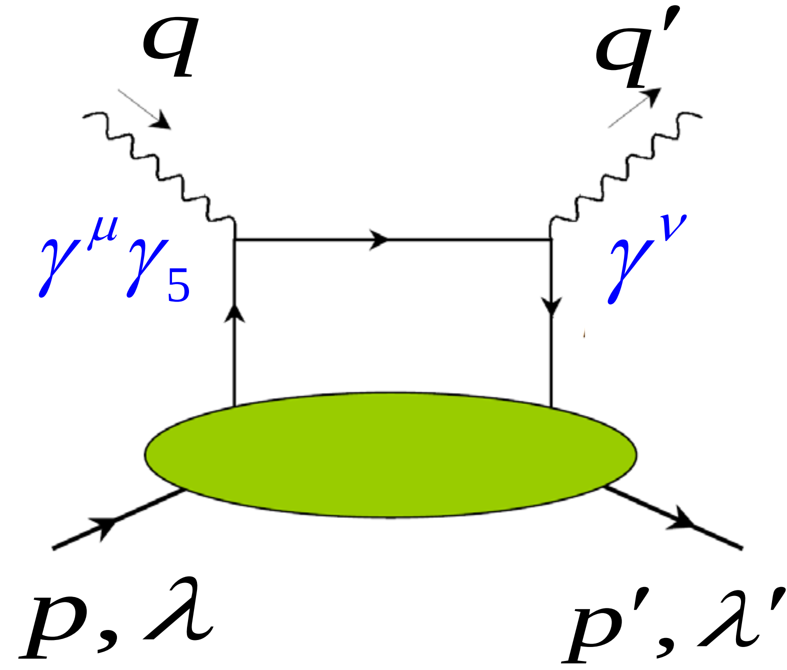

$$j^5_\mu = \bar{d}\gamma_\mu\gamma_5 u$$

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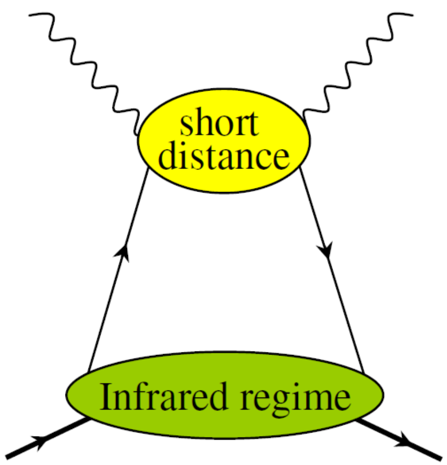
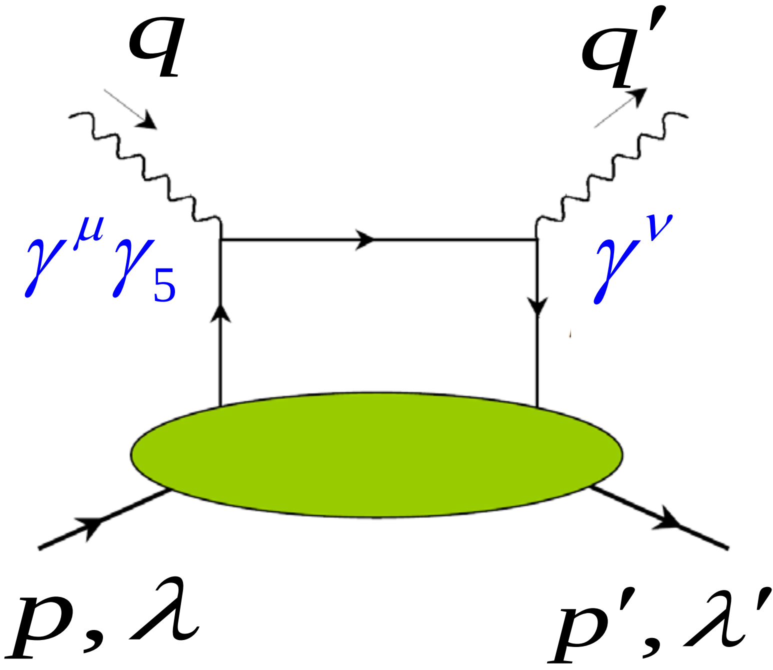
$$|q^2|, |q'^2| \gg \Lambda^2_{\mathrm{QCD}}$$



$$\int d^4x e^{iq'\cdot x}\langle p'\lambda'|\mathrm{T}\, j_\mu^5(0)\, j_\nu^{\mathrm{em}}(x)|p\lambda\rangle$$
$$\equiv -iT_{\mu\nu}$$

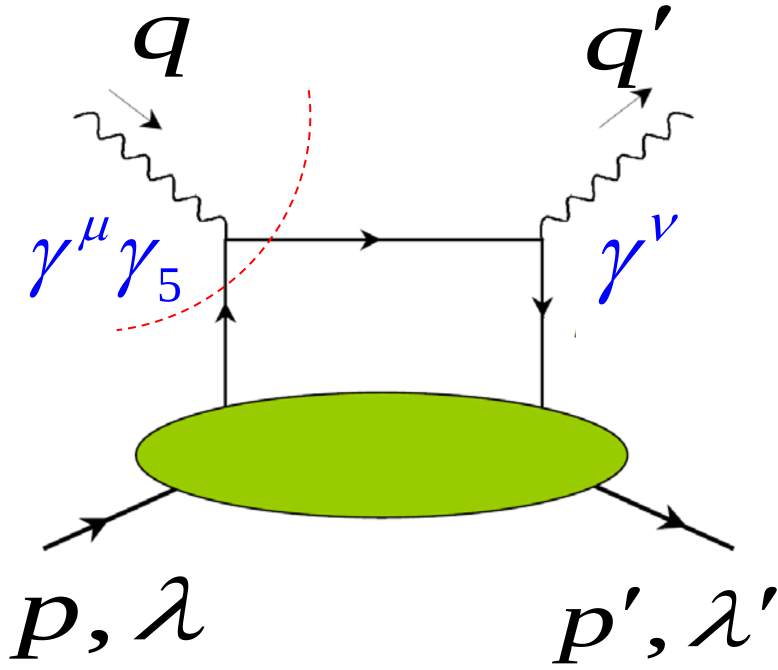


$$|q^2|,|q'^2| \gg \Lambda_{\mathrm{QCD}}^2$$

$$T_{\mu\nu}$$
$$= -q_\mu g_\nu^- \int dx \Big\{ C_H(x,\eta,Q'^2,q^2)\left[e_u \tilde{H}^{du}(x,\eta,t) - e_d \tilde{H}^{du}(-x,\eta,t)\right]\overline{u}(p'\lambda')\gamma^+\gamma_5 u(p\lambda)$$
$$+ C_E(x,\eta,Q'^2,q^2)\left[e_u \tilde{E}^{du}(x,\eta,t) - e_d \tilde{E}^{du}(-x,\eta,t)\right]\overline{u}(p'\lambda')\frac{\gamma_5(p'-p)^+}{2M}u(p\lambda)\Big\} + \cdots$$

$$\int d^4x e^{iq'\cdot x} \langle p'\lambda' | \mathrm{T}\, j^5_\mu(0)\, j^{\mathrm{em}}_\nu(x) | p\lambda \rangle$$

$$\equiv -iT_{\mu\nu}$$



$$j^5_\mu = \bar{d}\gamma_\mu\gamma_5 u$$

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$$T_{\mu\nu} = iq_\mu f_\pi \frac{1}{q^2 - m_\pi^2} M_\nu^{\lambda',\lambda}(\pi^- p \to \gamma^* n) + \cdots$$

$$\int d^4x e^{iq'\cdot x} \langle p'\lambda' | \mathrm{T}\, j_\mu^5(0)\, j_\nu^{\mathrm{em}}(x) | p\lambda \rangle$$
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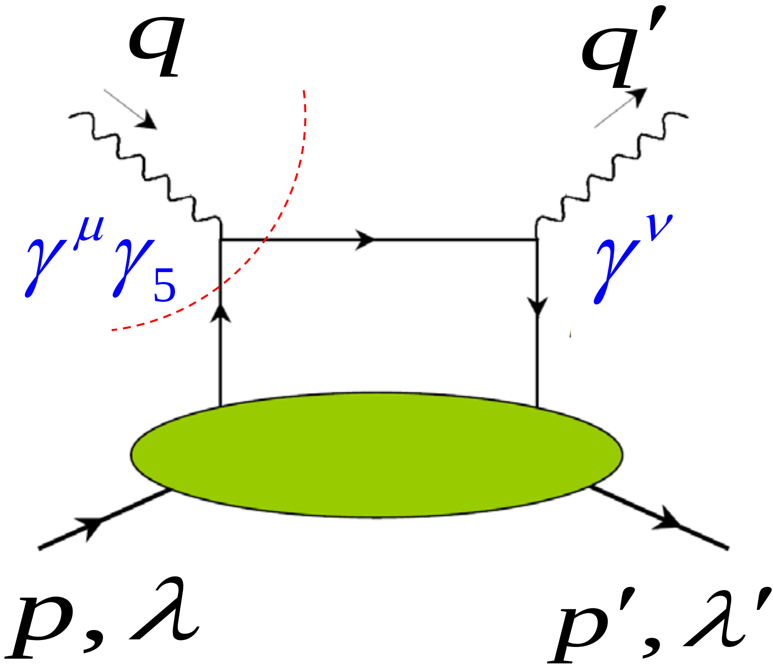

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$$T_{\mu\nu} = iq_\mu \left[f_\pi \frac{1}{q^2 - m_\pi^2} M_\nu^{\lambda',\lambda}(\pi^- p \to \gamma^* n) + \sum_{H'} \frac{a_\nu(m_{H'}^2)}{q^2 - m_{H'}^2} \right] + \cdots$$

$$\int d^4x e^{iq'\cdot x} \langle p'\lambda' | \mathrm{T}\, j^5_\mu(0)\, j^{\mathrm{em}}_\nu(x) | p\lambda \rangle$$

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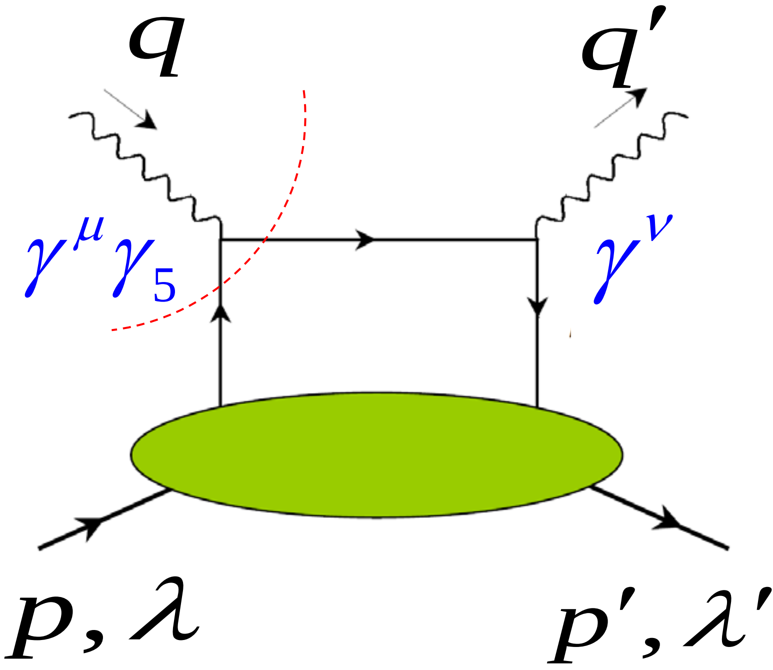

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$$T_{\mu\nu} = iq_\mu \left[f_\pi \frac{1}{q^2 - m_\pi^2} M_\nu^{\lambda',\lambda}(\pi^- p \to \gamma^* n) + \int_{q^2_{\mathrm{th}}}^{\infty} dm^2 \frac{\tilde{a}_\nu(m^2)}{q^2 - m^2} \right] + \cdots$$

$$\int d^4x e^{iq'\cdot x} \langle p'\lambda' | \mathrm{T}\, j^5_\mu(0)\, j^{\mathrm{em}}_\nu(x) | p\lambda \rangle$$

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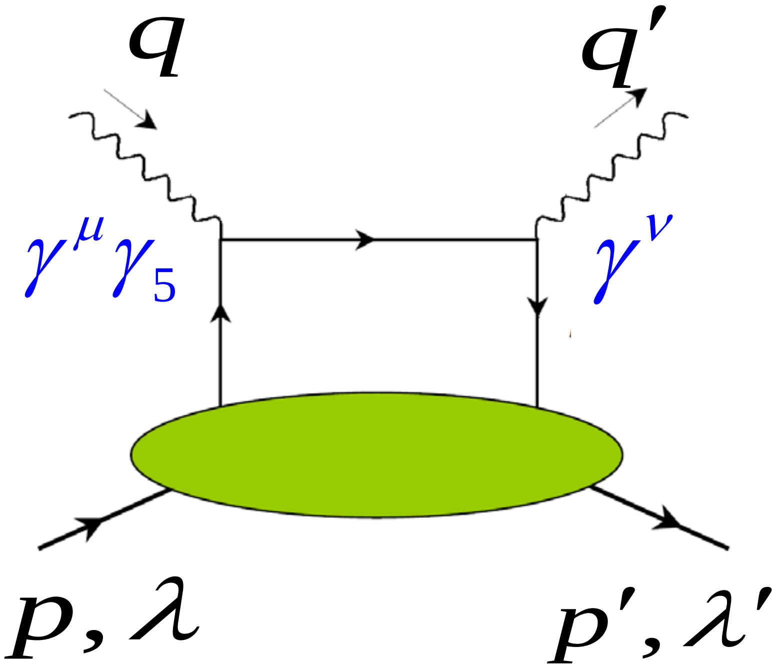

$$\left|q^2\right|, \left|q'^2\right| \gg \Lambda^2_{\mathrm{QCD}}$$

$$T_{\mu\nu}$$

$$= -q_\mu g^-_\nu \int dx \Big\{ C_H(x,\eta,Q'^2,q^2)\Big[e_u \tilde{H}^{du}(x,\eta,t) - e_d \tilde{H}^{du}(-x,\eta,t)\Big]\overline{u}(p'\lambda')\gamma^+\gamma_5 u(p\lambda)$$

$$+ C_E(x,\eta,Q'^2,q^2)\Big[e_u \tilde{E}^{du}(x,\eta,t) - e_d \tilde{E}^{du}(-x,\eta,t)\Big]\overline{u}(p'\lambda')\frac{\gamma_5(p'-p)^+}{2M}u(p\lambda)\Big\} + \cdots$$

$$M_{\nu}^{\lambda',\lambda}(\pi^- p \to \gamma^* n)$$

$$= g_{\nu}^{-} \int_{\eta}^{x_0} dx\, e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B^2}} \Big\{ \tilde{C}_H(x,\eta,Q'^2)\left[e_u \tilde{H}^{du}(x,\eta,t) - e_d \tilde{H}^{du}(-x,\eta,t)\right] \bar{u}(p'\lambda')\gamma^+\gamma_5 u(p\lambda)$$

$$+ \tilde{C}_E(x,\eta,Q'^2)\left[e_u \tilde{E}^{du}(x,\eta,t) - e_d \tilde{E}^{du}(-x,\eta,t)\right] \bar{u}(p'\lambda') \frac{\gamma_5 (p'-p)^+}{2M} u(p\lambda) \Big\}$$

$$\tilde{H}^{du}(x,\eta,t) = \tilde{H}^{u}(x,\eta,t) - \tilde{H}^{d}(x,\eta,t)$$

$$M_{\nu}^{\lambda',\lambda}(\pi^- p \to \gamma^* n)$$

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$$+\tilde{C}_E(x,\eta,Q'^2)\left[e_u \tilde{E}^{du}(x,\eta,t) - e_d \tilde{E}^{du}(-x,\eta,t)\right]\overline{u}(p'\lambda')\frac{\gamma_5 (p'-p)^+}{2M} u(p\lambda)\Big\}$$

$$\tilde{H}^{du}(x,\eta,t) = \tilde{H}^{u}(x,\eta,t) - \tilde{H}^{d}(x,\eta,t)$$

$$x_0 = \eta \frac{Q'^2 + q_{\rm th}^2}{Q'^2 - q_{\rm th}^2}$$ **: quark-hadron duality**

$$M_\nu^{\lambda',\lambda}(\pi^- p \to \gamma^* n)$$

$$= g_\nu^- \int_\eta^{x_0} dx\, e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B^2}} \Big\{ \tilde{C}_H(x,\eta,Q'^2)\left[e_u \tilde{H}^{du}(x,\eta,t) - e_d \tilde{H}^{du}(-x,\eta,t)\right]\overline{u}(p'\lambda')\gamma^+\gamma_5 u(p\lambda)$$

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$$\tilde{H}^{du}(x,\eta,t) = \tilde{H}^u(x,\eta,t) - \tilde{H}^d(x,\eta,t)$$

$$x_0 = \eta \frac{Q'^2 + q_{\text{th}}^2}{Q'^2 - q_{\text{th}}^2}$$: **quark-hadron duality**

Borel trnsf.: $$\hat{L}_{M_B}\left(\frac{1}{m^2 - q^2}\right) = \frac{1}{M_B^2} e^{-\frac{m^2}{M_B^2}}$$

$$M_\nu^{\lambda',\lambda}(\pi^- p \to \gamma^* n)$$

"Light-cone QCD SR (LCSR)"

$$= g_\nu^- \int_\eta^{x_0} dx\, e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B^2}} \Big\{ \tilde{C}_H(x,\eta,Q'^2)\left[e_u \tilde{H}^{du}(x,\eta,t) - e_d \tilde{H}^{du}(-x,\eta,t)\right]\bar{u}(p'\lambda')\gamma^+\gamma_5 u(p\lambda)$$

$$+\tilde{C}_E(x,\eta,Q'^2)\left[e_u \tilde{E}^{du}(x,\eta,t) - e_d \tilde{E}^{du}(-x,\eta,t)\right]\bar{u}(p'\lambda')\frac{\gamma_5 (p'-p)^+}{2M}u(p\lambda)\Big\}$$

$$\tilde{H}^{du}(x,\eta,t) = \tilde{H}^u(x,\eta,t) - \tilde{H}^d(x,\eta,t)$$

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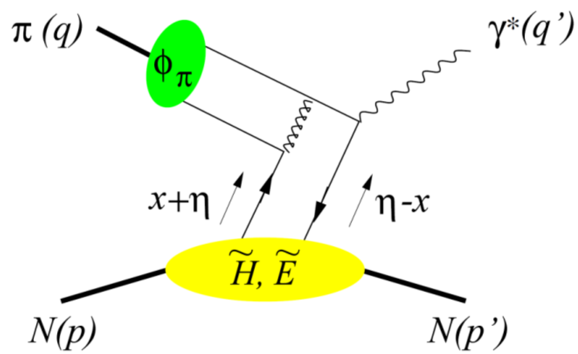
Borel trnsf.: $$\hat{L}_{M_B}\left(\frac{1}{m^2 - q^2}\right) = \frac{1}{M_B^2} e^{-\frac{m^2}{M_B^2}}$$

long. photon

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\sin^2\theta$$

$$\tau = \frac{Q'^2}{2\,p\cdot q} \qquad \eta = \frac{p^+ - p'^+}{p^+ + p'^+}$$



factorization

$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\,\frac{4\pi}{3}\,\frac{f_\pi}{Q'}\,\frac{1}{(p+p')^+}\,\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^1 dx\,\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)\right]$$

long. photon

$$\frac{d\sigma}{dQ'^2\, dt\, d(\cos\theta)\, d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda} |M^{0\lambda',\lambda}|^2 \sin^2\theta$$

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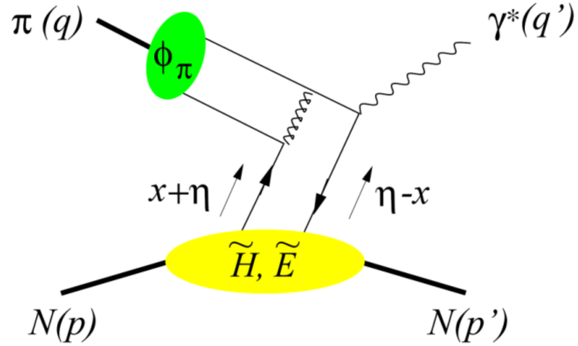
factorization



$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\,\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\,\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

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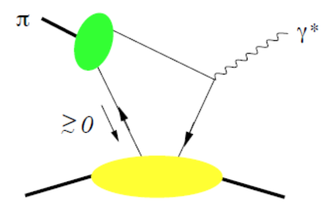
LCSR for nonfactorizable amp.



$$M_{LCSR}^{0\lambda',\lambda}(\pi^- p \to \gamma^* n)$$
$$= -ie\frac{1}{(p+p')^+}\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\tilde{\mathscr{H}}_{LCSR}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\tilde{\mathscr{E}}_{LCSR}^{du}(\eta,t)\right]u(p,\lambda)$$

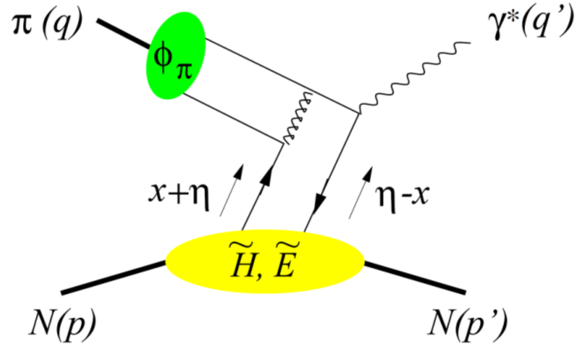
$$\tilde{\mathscr{H}}_{LCSR}^{du}(\eta,t) = \int_\eta^{x_0} dx\; e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B^2}}\,\bar{C}_H(x,\eta,Q'^2)\left\{e_d\left[\tilde{H}^d(-x,\eta,t) - \tilde{H}^u(-x,\eta,t)\right] - e_u\left[x\to -x\right]\right\}$$

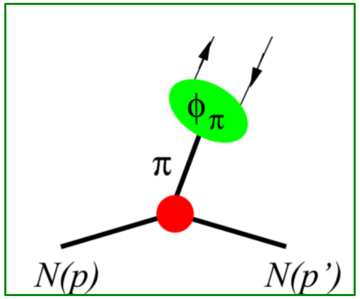
long. photon

$$\frac{d\sigma}{dQ'^2\, dt\, d(\cos\theta)\, d\varphi} = \frac{\alpha_{em}}{256\,\pi^3} \frac{\tau^2}{Q'^6} \sum_{\lambda',\lambda} |M^{0\lambda',\lambda}|^2 \sin^2\theta$$

$$\tau = \frac{Q'^2}{2p\cdot q} \qquad \eta = \frac{p^+ - p'^+}{p^+ + p'^+}$$

factorization

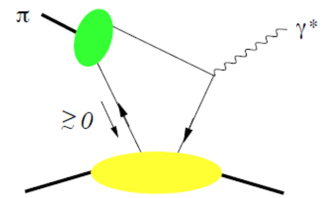




$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\,\frac{4\pi}{3}\,\frac{f_\pi}{Q'}\,\frac{1}{(p+p')^+}\,\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right] u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^{1} dx\,\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)\right]$$

LCSR for nonfactorizable amp.



$$M^{0\lambda',\lambda}_{LCSR}(\pi^- p \to \gamma^* n)$$

$$= -ie\frac{1}{(p+p')^+}\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\tilde{\mathcal{H}}^{du}_{LCSR}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\tilde{\mathcal{E}}^{du}_{LCSR}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}_{LCSR}(\eta,t) = \int_\eta^{x_0} dx\; e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B^2}}\,\bar{C}_H(x,\eta,Q'^2)\left\{e_d\left[\tilde{H}^d(-x,\eta,t) - \tilde{H}^u(-x,\eta,t)\right] - e_u\left[x \to -x\right]\right\}$$

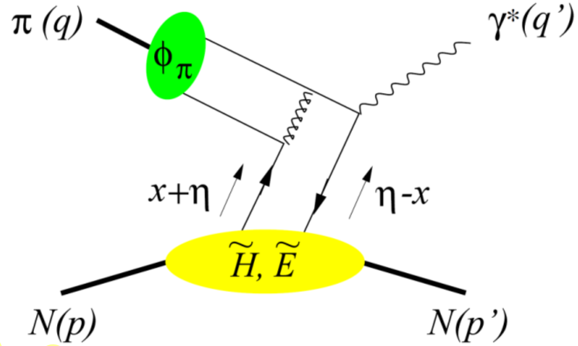
long. photon

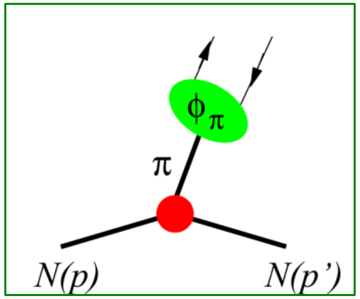
$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\sin^2\theta$$

$$\tau = \frac{Q'^2}{2p\cdot q} \qquad \eta = \frac{p^+ - p'^+}{p^+ + p'^+}$$

factorization

ERBL

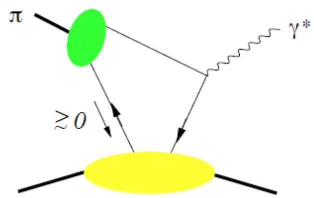




$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\,\frac{\cdot}{3}\,\frac{\cdot}{Q'}\,\frac{1}{(p+p')^+}\,\bar u(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^1 dx\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde H^d(x,\eta,t) - \tilde H^u(x,\eta,t)\right]$$

LCSR for nonfactorizable amp.



$$M^{0\lambda',\lambda}_{LCSR}(\pi^- p \to \gamma^* n)$$

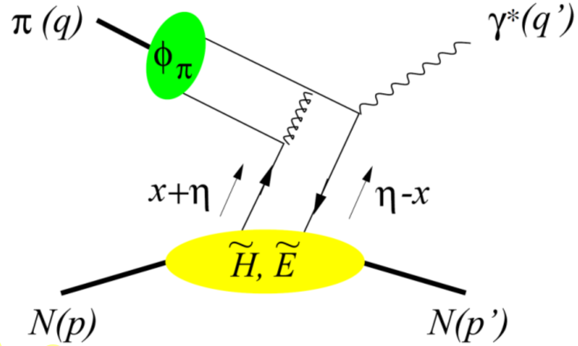
$$= -ie\frac{1}{(p+p')^+}\bar u(p',\lambda')\left[\gamma^+\gamma_5\tilde{\mathcal{H}}^{du}_{LCSR}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\tilde{\mathcal{E}}^{du}_{LCSR}(\eta,t)\right]u(p,\lambda)$$

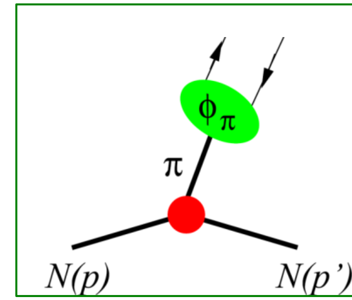
$$\tilde{\mathcal{H}}^{du}_{LCSR}(\eta,t) = \int_\eta^{x_0} dx\, e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B^2}}\,\bar C_H(x,\eta,Q'^2)\left\{e_d\left[\tilde H^d(-x,\eta,t) - \tilde H^u(-x,\eta,t)\right] - e_u\left[x\to -x\right]\right\}$$

long. photon

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\sin^2\theta$$

$$\tau = \frac{Q'^2}{2p\cdot q} \qquad \eta = \frac{p^+ - p'^+}{p^+ + p'^+}$$





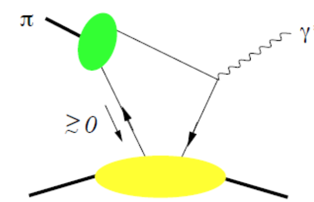
factorization

ERBL

$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\bar u(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^{1}dx\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde H^d(x,\eta,t) - \tilde H^u(x,\eta,t)\right]$$

LCSR for nonfactorizable amp.



$$M^{0\lambda',\lambda}_{LCSR}(\pi^- p \to \gamma^* n)$$

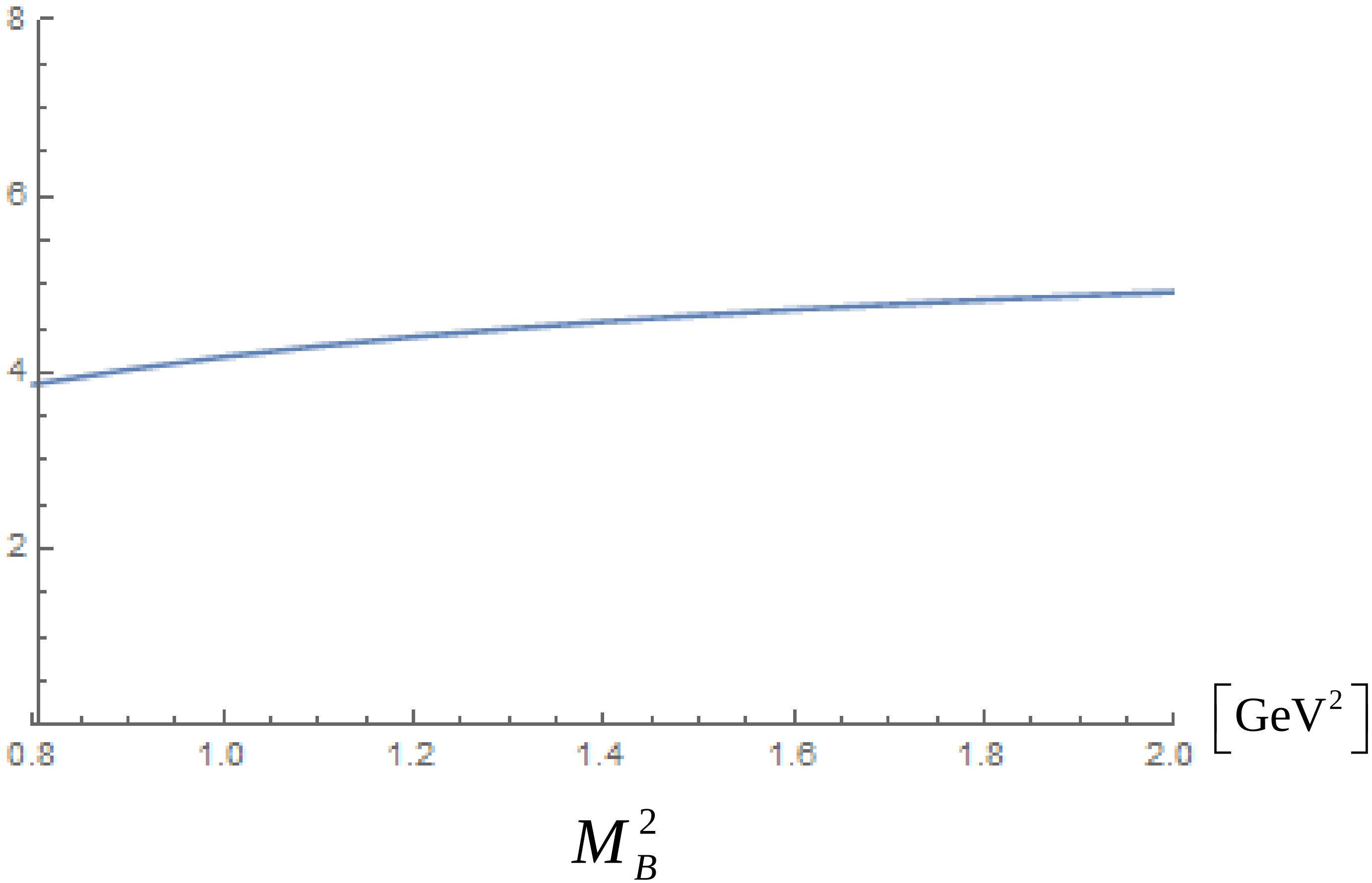
$$= -ie\frac{1}{(p+p')^+}\bar u(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}_{LCSR}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}_{LCSR}(\eta,t)\right]u(p,\lambda)$$

DGLAP

$$\tilde{\mathcal{H}}^{du}_{LCSR}(\eta,t) = \int_\eta^{x_0} dx\; e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M^2}}\,\bar C(x,\eta,Q'^2)\left\{e_d\left[\tilde H^d(-x,\eta,t) - \tilde H^u(-x,\eta,t)\right] - e_u\left[x\to -x\right]\right\}$$

$\tilde{\mathscr{H}}^{du}_{LCSR}(\eta, t)$

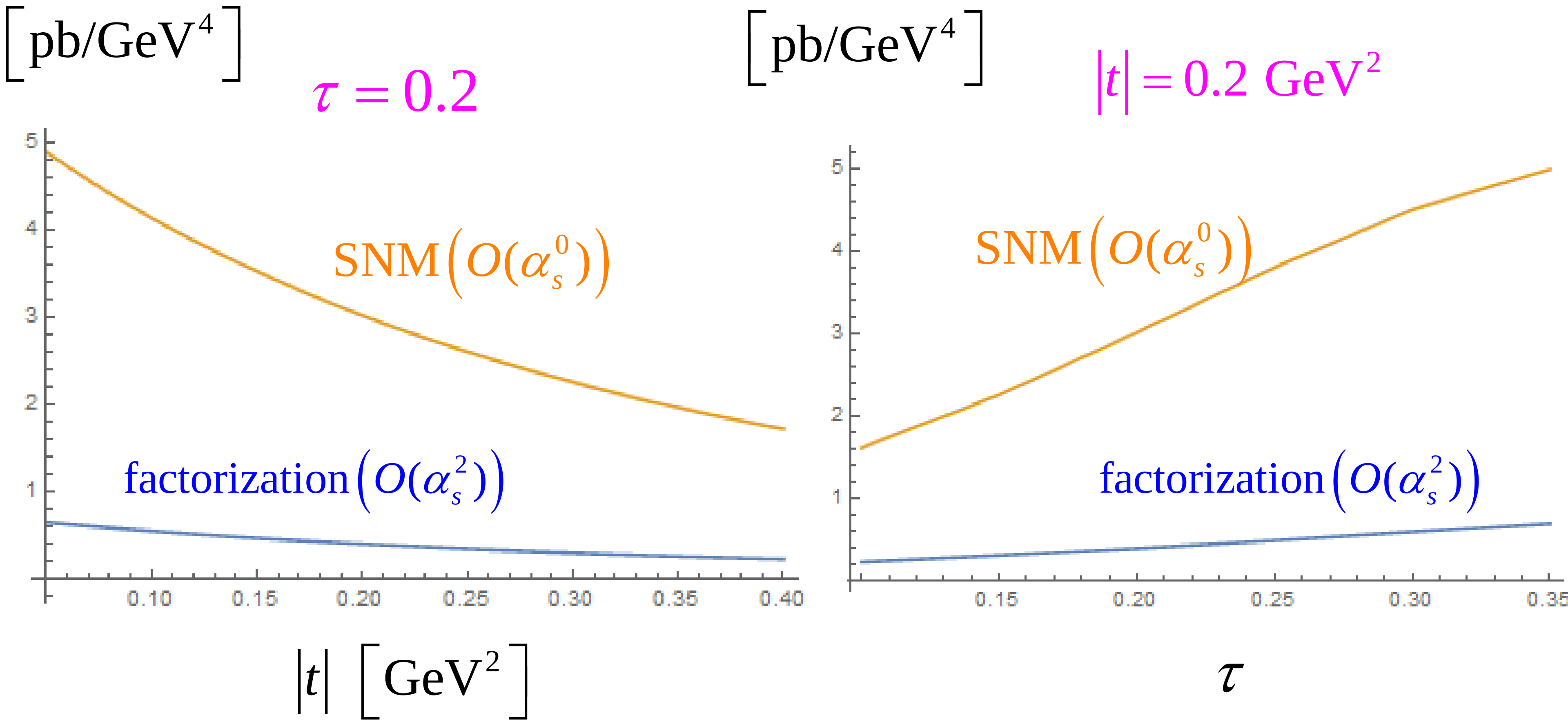
"Light-cone QCD SR (LCSR)"



Borel trnsf.: $\hat{L}_{M_B}\left(\frac{1}{m^2 - q^2}\right) = \frac{1}{M_B^2} e^{-\frac{m^2}{M_B^2}}$

contribution to $\frac{d\sigma}{dQ'^2\,dt}(\pi^- p \to \gamma^* n)$

$Q'^2 = 5\ \mathrm{GeV}^2$



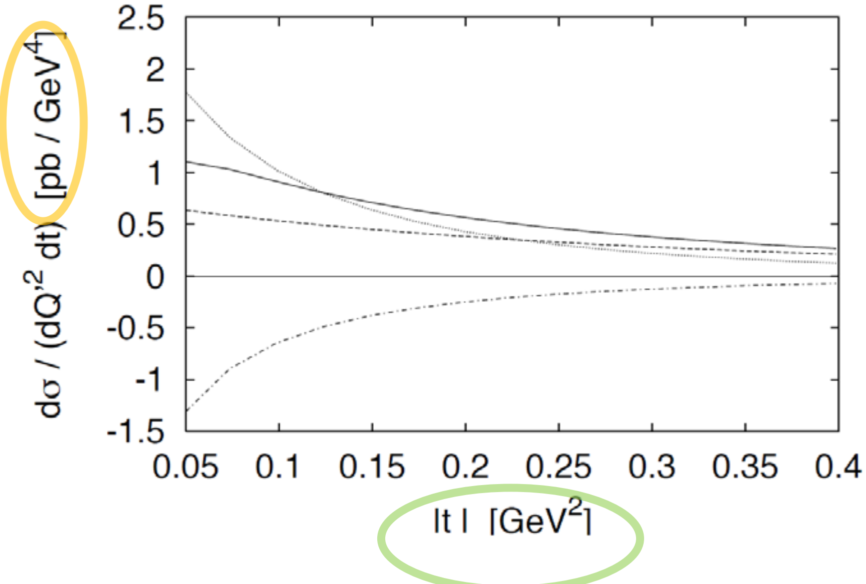
LO Estimates

Bjorken variable $\tau = \frac{Q'^2}{s - M^2}$

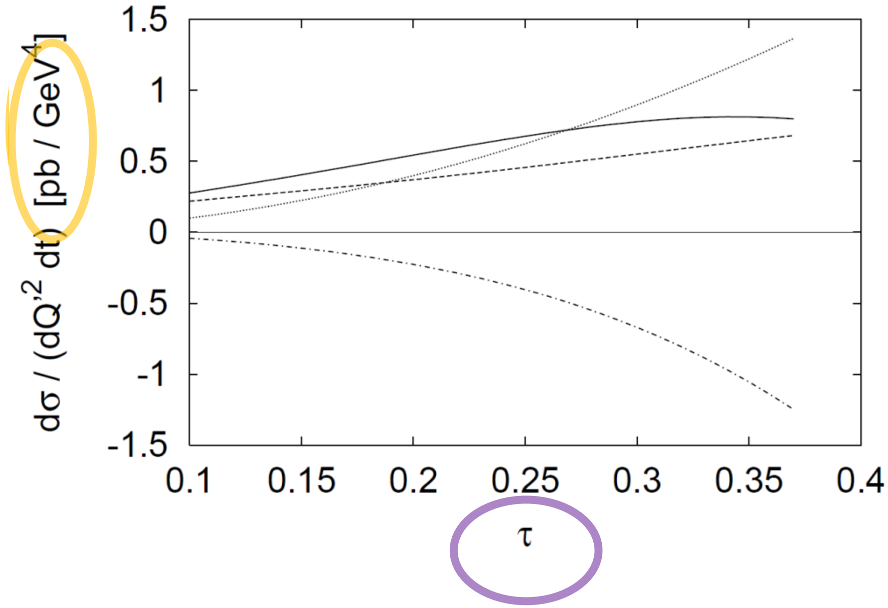
Berger, Diehl, Pire, PLB523(2001)265

$$Q'^2 = 5\ \mathrm{GeV}^2$$

$\tau = 0.2$



$|t| = 0.2\ \mathrm{GeV}^2$



(dashed) $= |\tilde{\mathcal{H}}|^2$; **(dash-dotted)** $=$ **Re**$(\tilde{\mathcal{H}}^* \tilde{\mathcal{E}})$; **(dotted)** $= |\tilde{\mathcal{E}}|^2$

$$\frac{d\sigma}{dQ'^2\,dt}(\pi^- p \to \gamma^* n) = \frac{4\pi\alpha_{\mathrm{em}}^2}{27}\frac{\tau^2}{Q'^8} f_\pi^2 \left[(1-\eta^2)\,|\tilde{\mathcal{H}}^{du}|^2 - 2\eta^2\,\mathrm{Re}\left(\tilde{\mathcal{H}}^{du*}\tilde{\mathcal{E}}^{du}\right) - \eta^2 \frac{t}{4M^2}|\tilde{\mathcal{E}}^{du}|^2 \right]$$

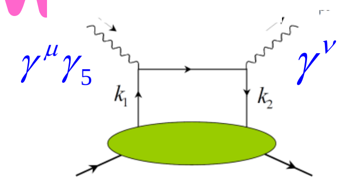
Summary

$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$ at J-PARC GPDs

LO $\left(O(\alpha_s^2)\right)$ factorization formula is known, but it misses soft nonfactorizable mechanism (SNM)

LCSR at LO $\left(O(\alpha_s^0)\right)$ is derived for largely model-independent estimate for SNM

$\tilde{H},\ \tilde{E},\ q_{\mathrm{th}}^2 (\sim 0.7\ \mathrm{GeV}^2)$



- numerical estimate: SNM > factorization
- NLO LCSR ⟷ quark $k_\perp$, pion pole contri.
- twist-3 LCSR ⟶ $M_{LCSR}^{\pm 1 \lambda', \lambda}(\pi^- p \to \gamma^* n)$

interplay of soft/hard QCD mechanism

Exclusive lepton pair production in πN scattering

$$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$$

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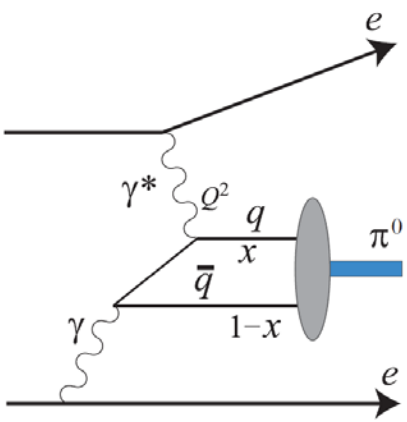
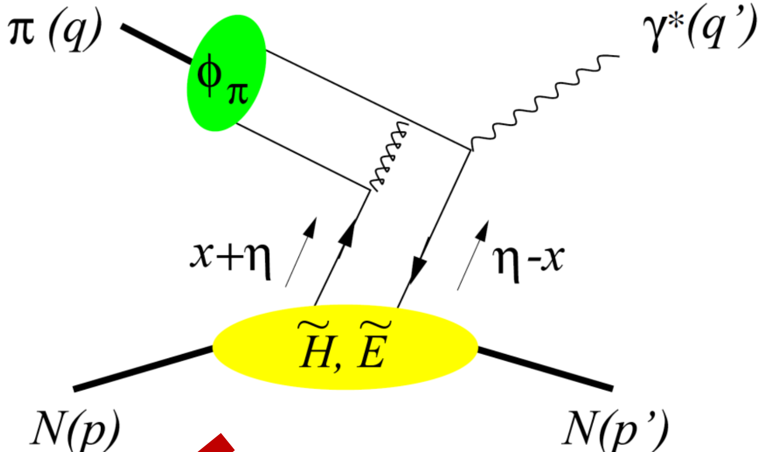


@Belle, Babar

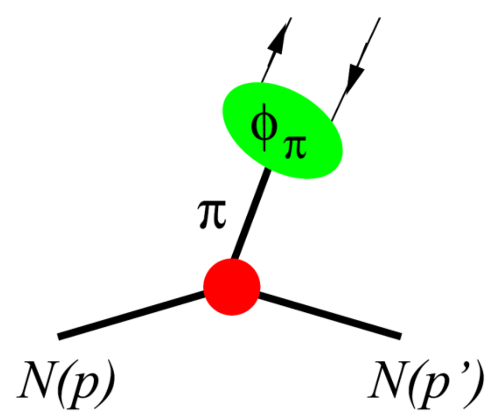
"exclusive limit of DY"

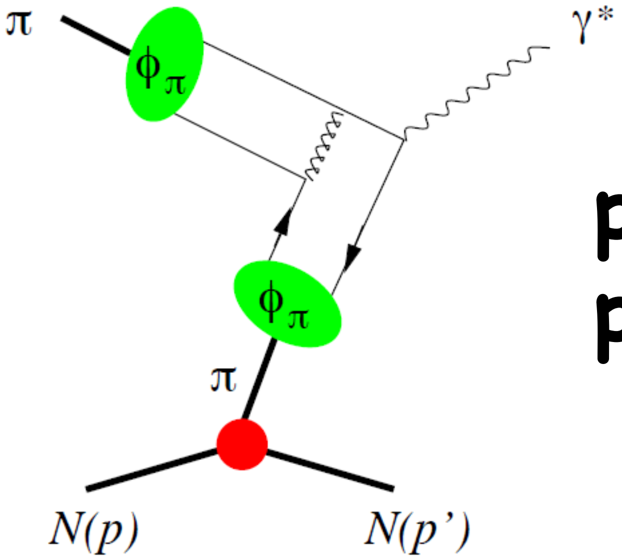
small $t = (q - q')^2$



$t \to 0$

$\Delta q(x)$





pion-pole contribution using pion form factor $F_\pi(Q'^2)$

Goloskokov, Kroll

$F_\pi(Q^2)$: important soft nonfactorizable contr. was shown with LCSR

Braun, Khodjamirian, Maul