Theory and phenomenology of a knot particle model in deformed spacetime

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Aim

Present a possible systematic theoretical framework for developing HEP beyond the SM

The approach is as follows:

Replace point particles in Minkowski spacetime by solitons in anti de Sitter spacetime

Outline of Presentation

I. Lie-type deformations: Deforming spacetime symmetries

Minkowski spacetime → anti de Sitter spacetime

II. Hopf-type deformations: From point particles to knots

Point particles → solitons/knots

III: Topological particle phyiscs

Describing particles and their interactions in terms of topology

I. Lie-type deformations: Deforming spacetime symmetries

$$[T_A, T_B] = i f_{AB}^{C} T_C$$

Structure Constants

- In physics, structure constants are often fundamental constants of the theory.

$$[x_i, p_j] = i\hbar \delta_{ij} I$$

- Numerical values of these fundamental constants determined experimentally. Not known without some error.
- Don’t want physical theory to be sensitive to the precise numerical values of the fundamental constants.

A Lie algebra may be deformed:

$$[T_A, T_B]_t = [T_A, T_B]_0 + \sum_{m=1}^{\infty} \phi_m(T_A, T_B) t^m$$

Deformation parameter

Antisymmetric bilinear map

Deformation is trivial if deformed algebra is isomorphic to the original algebra. This means there exists a linear transformation M_t such that

$$M_t\left([T_A, T_B]_t\right) = [M_t T_A, M_t T_B]_0$$

$$[T_A, T_B]_t = i f_{AB}^{(t)C} T_C$$

Deformed structure constants

A Lie algebra is said to be *stable* if all infinitesimal deformations of its structure constants lead to isomorphic Lie algebras.

Gerstenhaber, Murray. "On the deformation of rings and algebras." *Annals of Mathematics* (1964): 59-103.

Theories based on unstable Lie algebras should be deformed until a stable theory is reached. Theories based on stable Lie algebras give rise to robust physics free of fine tuning issues.

Both the transition from classical mechanics to quantum mechanics and from Galilean relativity to special relativity can be understood as Lie-algebraic deformations

Classical Mechanics → Quantum Mechanics

Galilean Relativity → Special Relativity*

Mendes, R. Vilela. "Deformations, stable theories and fundamental constants." *Journal of Physics A: Mathematical and General* 27.24 (1994): 8091.

Ahluwalia, D. V., Gresnigt, N. G., Nielsen, A. B., Schritt, D., & Watson, T. F. (2008). Possible polarization and spin-dependent aspects of quantum gravity. *International Journal of Modern Physics D*, *17*(03n04), 495-504.

Chryssomalakos, C., and E. Okon. "Generalized quantum relativistic kinematics: a stability point of view." *International Journal of Modern Physics D* 13.10 (2004): 2003-2034.

Deforming Poincare Symmetry

$$[J_{\mu\nu}, J_{\rho\sigma}] = -i(\eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\nu\rho} - \eta_{\mu\rho}J_{\nu\sigma} - \eta_{\nu\sigma}J_{\mu\rho})$$

$$[J_{\mu\nu}, P_\rho] = -i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu)$$

$$[P_\mu, P_\nu] = 0$$

$$ISO(1,3) = T_4 \otimes_s SO(1,3)$$

Stable

Unstable!

$$[P_\mu, P_\nu] = 0 \quad \rightarrow \quad [P_\mu, P_\nu] = \frac{i}{\ell^2} J_{\mu\nu}$$

New invariant scale

Anti de Sitter algebra

$$
\begin{aligned}
[J_{\mu\nu}, J_{\rho\sigma}] &= i(\eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\nu\rho} - \eta_{\mu\rho}J_{\nu\sigma} - \eta_{\nu\sigma}J_{\mu\rho}), \\
[J_{\mu\nu}, P_{\rho}] &= i(\eta_{\mu\rho}P_{\nu} - \eta_{\nu\rho}P_{\mu}), \\
[P_{\mu}, P_{\nu}] &= \frac{i}{\ell^2}J_{\mu\nu}
\end{aligned}
$$

Identifying $P_{\mu} = J_{\mu 4}$

$$[J_{AB}, J_{CD}] = i(\eta_{BC}J_{AD} + \eta_{AD}J_{BC} - \eta_{AC}J_{BD} - \eta_{BD}J_{AC})$$

Stable isometry group: $SO(2,3)$

The deformation introduces an invariant length (deformation parameter)

Spacetime at small scales looks anti de Sitter

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 4, NUMBER 7 JULY 1963

A Remarkable Representation of the 3 + 2 de Sitter Group

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(Received 20 February 1963)

Among the infinitesimal operators of the 3 + 2 de Sitter group, there are four independent cyclic ones, one of which is separate from the other three. A representation is obtained for which this one has integral eigenvalues while the other three have half-odd eigenvalues, or vice versa. The representation is of a specially simple kind, with the wavefunctions involving only two variables.

INTRODUCTION

We consider the group of rotations of five real variables x_1, x_2, x_3, x_4, x_5 which leave the quadratic form

$$x_1^2 + x_2^2 + x_3^2 - x_4^2 - x_5^2 \tag{1}$$

invariant. The infinitesimal operators of the group are $m_{ab} = -m_{ba}(a, b = 1, 2, 3, 4, 5)$. There are ten independent ones. They satisfy the following commutation relations, in the notation $[\xi, \eta] = \xi\eta - \eta\xi$, with a, b, c, d all different:

The present paper is concerned with a more primitive representation, for which im_{12}, im_{23}, im_{31} have half-odd integral eigenvalues while im_{45} has integral eigenvalues. There is nothing inconsistent in such a mixing, because the m_{45} rotation is completely detached from the m_{12}, m_{23}, m_{31} rotations. In fact, if one goes over to the covering group of the 3 + 2 de Sitter group, the detachment of the m_{45} rotation allows im_{45} to have any real eigenvalues, independently of what eigenvalues the other rotations have. A general theory of the representations of this

Singleton representations

- Most degenerate representations first discovered by Dirac

$$Di = D(1, \frac{1}{2}) \qquad Rac = D(\frac{1}{2}, 0)$$

- Massless particles are composites of two singletons
- No Poincare counterparts to singletons. At flat space limit, singletons become vacua.
- QED and electroweak can be understood in terms of singletons.
- Locally unobservable.

ONE MASSLESS PARTICLE EQUALS TWO DIRAC SINGLETONS

VI *Elementary Particles in a Curved Space*

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ABSTRACT. The 'remarkable representations of the 3+2 de Sitter group', discovered by Dirac, later called singleton representations and here denoted Di and Rac, are shown to possess the following truly remarkable property: Each of the direct products Di ⊗ Di, Di ⊗ Rac, and Rac ⊗ Rac decomposes into a direct sum of unitary, irreducible representations, each of which admits an extension to a unitary, irreducible representation of the conformal group $SO(4, 2)$. Therefore, in de Sitter space, every state of a free, 'massless' particle may be interpreted as a state of two free singletons – and vice versa. The term 'massless' is associated with a set of particle-like representations of $SO(3, 2)$ that, besides the noted conformal extension, exhibit other phenomena typical of masslessness, especially gauge invariance.

JGP - Vol. 5, n. 1, 1988

Composite Electrodynamics

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This work is dedicated to I.M. Gelfand
upon the occasion of his 75th birthday
with friendship and admiration

Abstract. *This paper solidifies the foundations for a singleton theory of light, first proposed two years ago. This theory is based on a pure gauge coupling of the scalar singleton field to the electromagnetic current. Like quarks, singletons are essentially unobservable. The field operators are not local observables and therefore need not commute for spacelike separation. This opens up possibilities for generalized statistics, just as is the case for quarks. It then turns out that a pure gauge coupling, in which* $\partial_\mu \phi(x)$ *couples to the conserved current* $j^\mu(x)$ *, generates real interactions – the effective theory is precisely ordinary electrodynamics in de Sitter space. Here we improve our theory and explain it in much more detail than before, adding two new results. (1) The concept of normal ordering in a theory with unconventional statistics is worked out in detail. (2) We have discovered the natural way of including both photon helicities. Quantization, it may be noted, is a study in representation theory of certain infinite-dimensional, nilpotent Lie algebras, of which the Heisenberg algebra is the prototype.*

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QUARKS OR SINGLETONS?

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Received 3 March 1986

Singletons and quarks share the most important characteristic of being (locally) unobservable. This opens the door to uncommon statistics. A first stage of deviation from ordinary quantization of singletons leads directly to a formulation of QED as an "effective" theory. A second stage incorporates massive particles. The massless fields are bilinear, the massive ones trilinear, in the "constituent" singleton fields. The scheme strongly suggests a possibility of unifying all interactions.

1. The observation that the 56 most important baryons can be interpreted as completely symmetric states of 3 quarks has had lasting and decisive influence on strong interaction phenomenology. It is a cornerstone of the naive quark model and the main source of inspiration for QCD. Indeed, the puzzle presented by the apparent dominance of a symmetric

tons and that three singletons make a massive particle. Interactions between ordinary particles are induced by elementary interactions of singletons. A gauge principle leads to a minimal interaction that preserves the transparency of singletons in the presence of interactions.

Recap so far

- Spacetime symmetries encoded in Lie algebras,
- We require a stable Lie algebra,
- Lie-algebraic deformation gives anti de Sitter spacetime with stable isometry group,
- Implications in particle physics.

Lie-algebraic deformations

Minkowski spacetime→ anti de Sitter spacetime (at small scales!)

II. Hopf-type deformations: from point particles to knots

- Anti de Sitter Lie algebra is stable and cannot be deformed further within the Lie algebra framework.
- However, universal enveloping algebra (UEA) can be deformed as a Hopf algebra.

$$T_1 T_2 = q T_2 T_1, \quad q \in \mathbb{R}$$

Deformation parameter

- Hopf algebra deforms into a quantum group.

Quantum Groups and Knots

- Deformed gauge theory has more degrees of freedom that classical theory
- The additional degrees of freedom can be interpreted as solitonic sctructure

$$SU_q(2) \text{ - Algebra of oriented knots}$$

- SUq(2) contains enough information to describe electroweak and solitons
- Go over from point particles to solitonic particles.

Elementary particles may be described as solitons carrying the symmetries of oriented knots

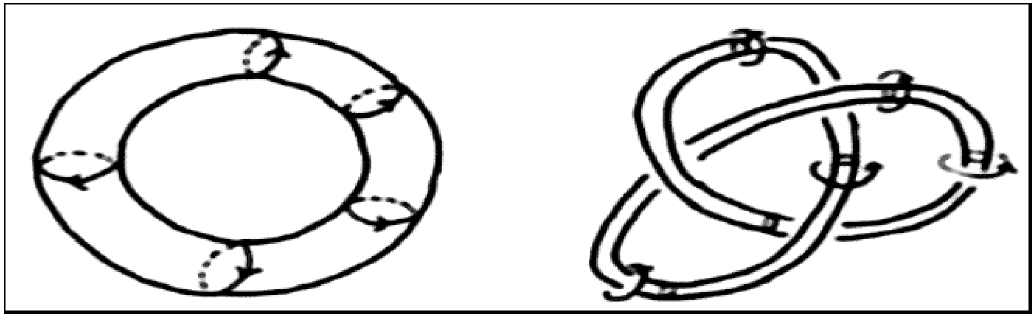
Finkelstein, Robert J., and A. C. Cadavid. "Masses and interactions of q-fermionic knots." *International Journal of Modern Physics A* 21.21 (2006): 4269-4302.

Particles as topological objects

First attempt dates back to Kelvin's (William Thomson) vortex atom model in 1867.

Developed after observing Peter Taits demonstrations of vortex smoke rings and noting that:

1. they behaved as independent solids.
2. They rebounded upon collision
3. Exhibited oscillating vibration modes
4. Were indivisible



- Jehle considered quantized magnetic flux tubes as a model for leptons,
- Stability of flux tubes investigated by Faddeev and Niemi (trefoil stable),
- Finkelstein studied particles as irreducible representations of quantum group SUq(2),
- Ranada showed that there exist knotted solutions to Maxwell’s equations,
- Bilson-Thompson developed a model of SM particles in terms of braids.

Knots have played an increasingly important role in physics.

nature physics

ARTICLES

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Creation and dynamics of knotted vortices

Dustin Kleckner* and William T. M. Irvine*

Knots and links have been conjectured to play a fundamental role in a wide range of physical fields, including plasmas and fluids, both quantum and classical. In fluids, the fundamental knottedness-carrying excitations occur in the form of linked and knotted vortex loops, which have been conjectured to exist for over a century. Although they have been the subject of considerable theoretical study, their creation in the laboratory has remained an outstanding experimental goal. Here we report the creation of isolated trefoil vortex knots and pairs of linked vortex rings in water using a new method of accelerating specially shaped hydrofoils. Using a high-speed scanning tomography apparatus, we measure their three-dimensional topological and geometrical evolution in detail. In both cases we observe that the linked vortices stretch themselves and then deform—as dictated by their geometrically determined energy—towards a series of local vortex reconnections. This work establishes the existence and dynamics of knotted vortices in real fluids.

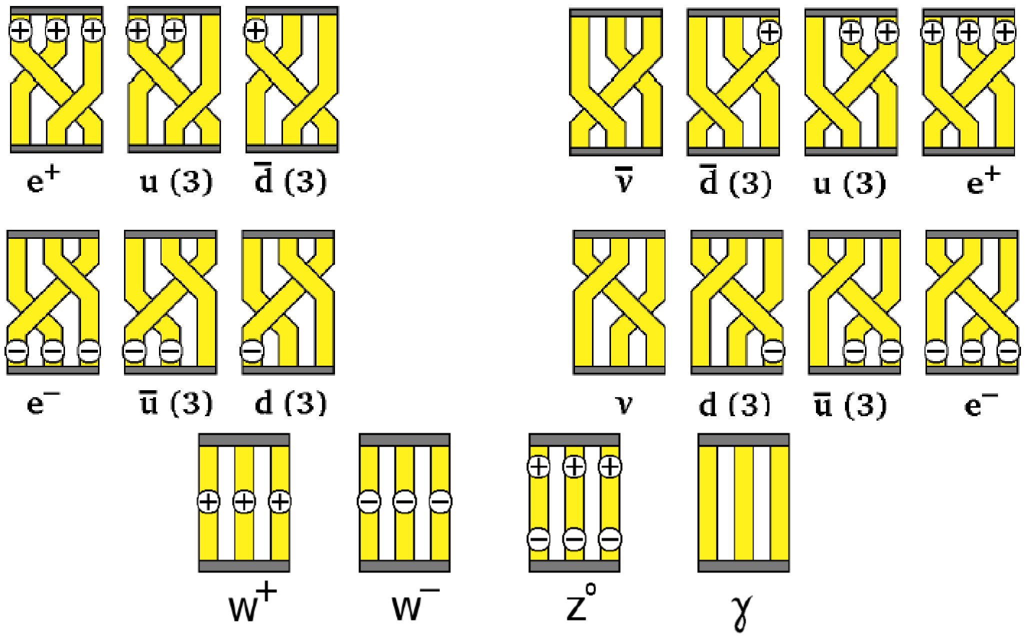
Whereas tying a shoelace into a knot is a relatively simple affair, tying a field, for example a magnetic field, into a knot is a different story: the entire space-filling field must be twisted everywhere to match the knot being tied at the core. This interplay between knots and the space they live in lies at the heart of modern topology; beyond the world of mathematics, there is a growing realization that knots in space-filling fields are an essential part of physical processes spanning classical and quantum field theories[1–3], liquid crystals[4,5], electromagnetism[6–8], plasmas[9–12], and quantum and classical fluids[13–17], with static knotted structures having been demonstrated in the nodal lines of weakly focused laser beams[8] and liquid crystals[4,5,18].

In fluid and fluid-like systems, there is a long-standing basis for suggesting that knottedness is a conserved physical quantity, and that corresponding knotted excitations should exist[9–16,19–23]. The relevant knottedness comes in different forms: for example, in ideas have been hampered by the lack of methods for creating topologically non-trivial vorticity fields on demand.

Here, we report an experimental observation of topological vortices in the form of trefoil knots and linked pairs of rings, generated by the acceleration of specially shaped hydrofoils. We observe rapid vortex stretching for both linked and knotted vortices, which is not present in isolated vortex rings (unknots), even if they are strongly distorted. This stretching is accompanied by a change of vortex geometry to conserve energy, and this process drives the vortices towards a series of local reconnections. Ultimately, this results in a change of the vortex topology to a set of unlinked rings. The present work establishes the existence and dynamics of long-sought-after knotted vortex loops in experiment and offers a glimpse into their topological evolution, paving the way for the experimental study of knotted excitations in hydrodynamic systems, including turbulent flows and quantum fluids.

The Helon Model

- Developed by Sundance Bilson-Thompson
- Considers Standard Model fermions as braids made of three ribbons with two crossings



Bilson-Thompson, Sundance O. "A topological model of composite preons." *arXiv preprint hep-ph/0503213* (2005).

A topological model of composite preons

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(Dated: February 2, 2008)

We describe a simple model, based on the preon model of Shupe and Harari, in which the binding of preons is represented topologically. We then demonstrate a direct correspondence between this model and much of the known phenomenology of the Standard Model. In particular we identify the substructure of quarks, leptons and gauge bosons with elements of the braid group B_3. Importantly, the preonic objects of this model require fewer assumed properties than in the Shupe/Harari model, yet more emergent quantities, such as helicity, hypercharge, and so on, are found. Simple topological processes are identified with electroweak interactions and conservation laws. The objects which play the role of preons in this model may occur as topological structures in a more comprehensive theory, and may themselves be viewed as composite, being formed of truly fundamental sub-components, representing exactly two levels of substructure within quarks and leptons.

PACS numbers: 12.60.Rc, 12.10.Dm

I. INTRODUCTION

The Standard Model (SM) provides an extremely succesful and simple means of classifying and understanding the physical processes which fill the Universe. However the existence of many seemingly arbitrary features hints at a more fundamental physical theory from which the SM arises. Considering the successful series of ideas leading through molecules, to atoms, nuclei, nucleons, and quarks, it was perhaps inevitable that a model based on compositeness of quarks and leptons would be developed. The first such was proposed by Pati and Salam [1] in 1974, however it lacked any real explanatory power. Pati and Salam gave the name *preons* to their hypothetical constituent particles, and this name was gradually adopted to refer to the sub-quark/sub-lepton particles of any model. Other notable preon models were

tric charge and colour charge. Unfortunately, as originally proposed it also had several problems, including the lack of a dynamical framework, and the lack of an explanation as to why the ordering of rishons within triplets should matter. A charge called "hypercolour" was proposed to solve these problems [5]. The introduction of hypercolour implied the existence of "hypergluons" and some QCD-like confinement mechanism for the rishons. Hence, the simplicity of the original model was reduced, and many of the fundamental questions about particles and interactions were simply moved to the realm of rishons, yielding little obvious advantage over the SM. Furthermore preon models were never able to adequately answer several fundamental questions, such as how preons confined at all length scales experimentally probed can form very light composites (see e.g. [6] for an attempt to address this issue).

This article presents an idea based on the original rishon

The Helon Model

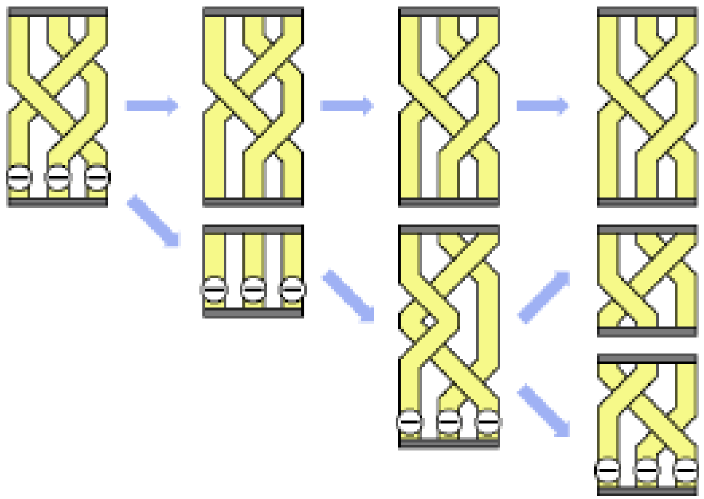
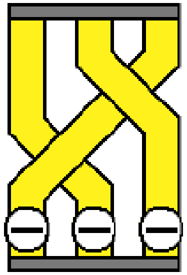


FIG. 2: A representation of the decay $\mu \rightarrow \nu_\mu + e^- + \overline{\nu_e}$, showing how the substructure of fermions and bosons demands that charged leptons decay to neutrinos of the same generation.

The Helon Model

- Describes much of SM phenomenology in terms of ribbon braids.
- Spin not represented. No charge mixing allowed on braids



e^-

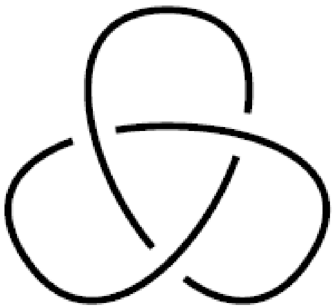
- Alexander's theorem: the closure of a braid is a knot

Consider knots instead of braids

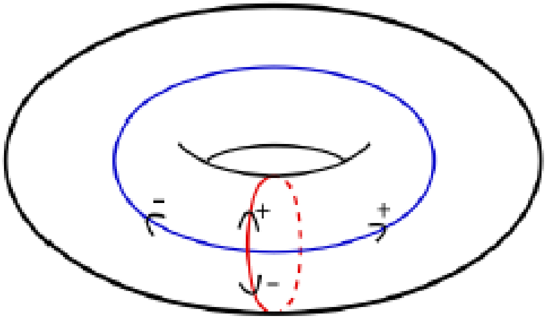
One difference between knots and braids is that although a braid has an associated anti-braid, a knot does not have an anti-knot.

Knot model of elementary particles

Most elementary particles should correspond to the simplest knots. Two basic trefoil knots.



Knots made of (quantized flux) tubes so they have a volume.

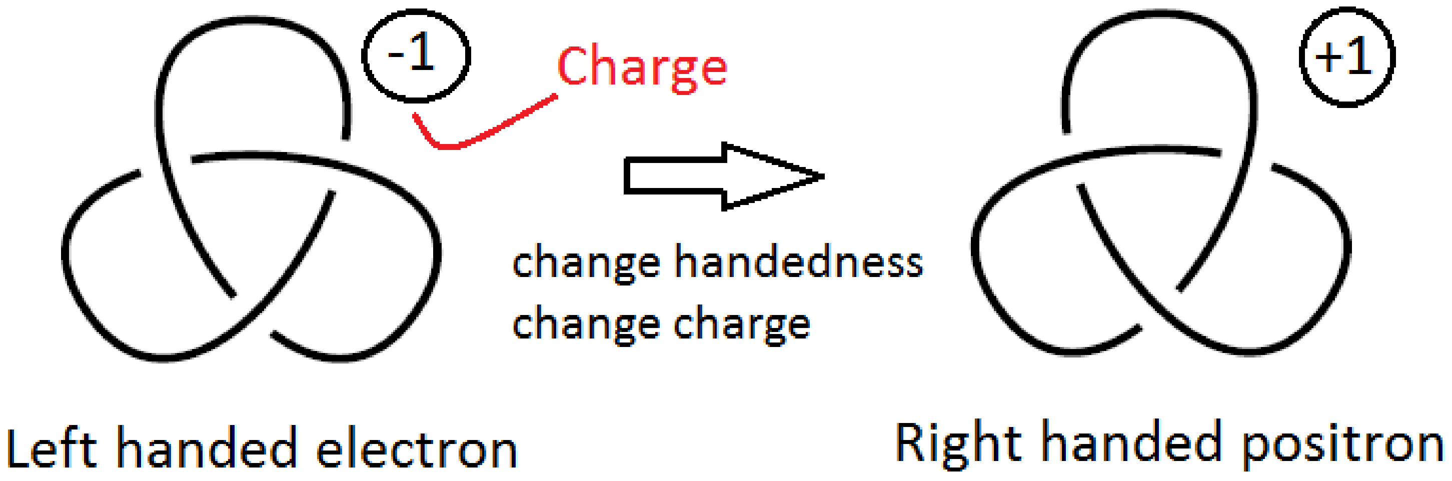


Represent charge by twisting of flux tube

Knot model of particles

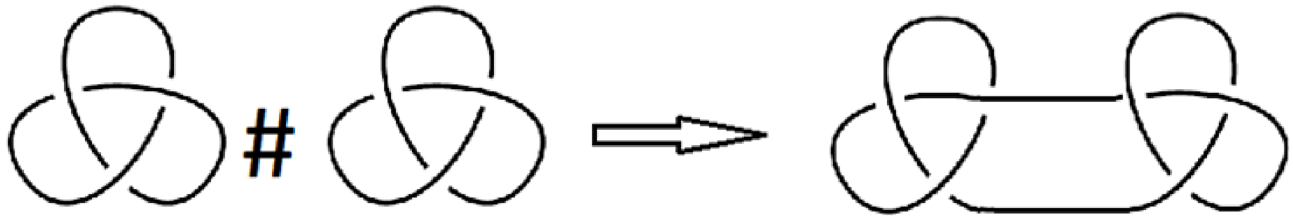
Represent electric charge by twisting of flux tube.

Twist of π represents a charge of 1/3 (-π represents -1/3)

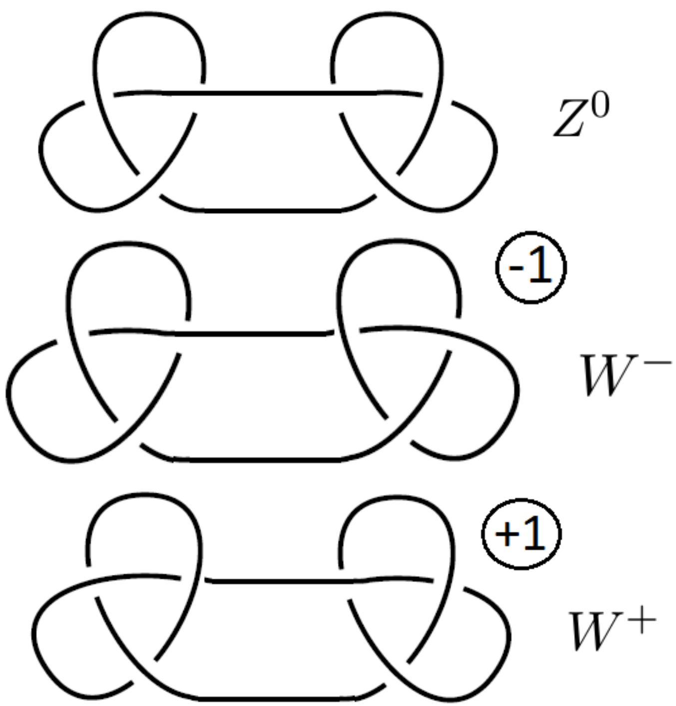


Considering knots naturally explains the no charge mixing assumption of the Helon model.

Knots can be composed via a knot sum*

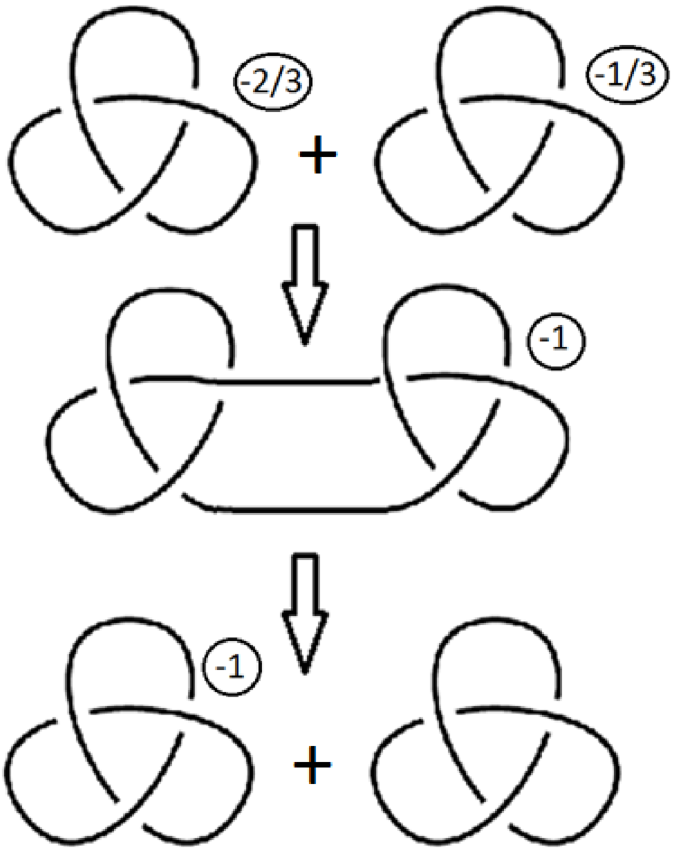


Electroweak gauge bosons are di-trefoil knots (composites of two trefoil knots)

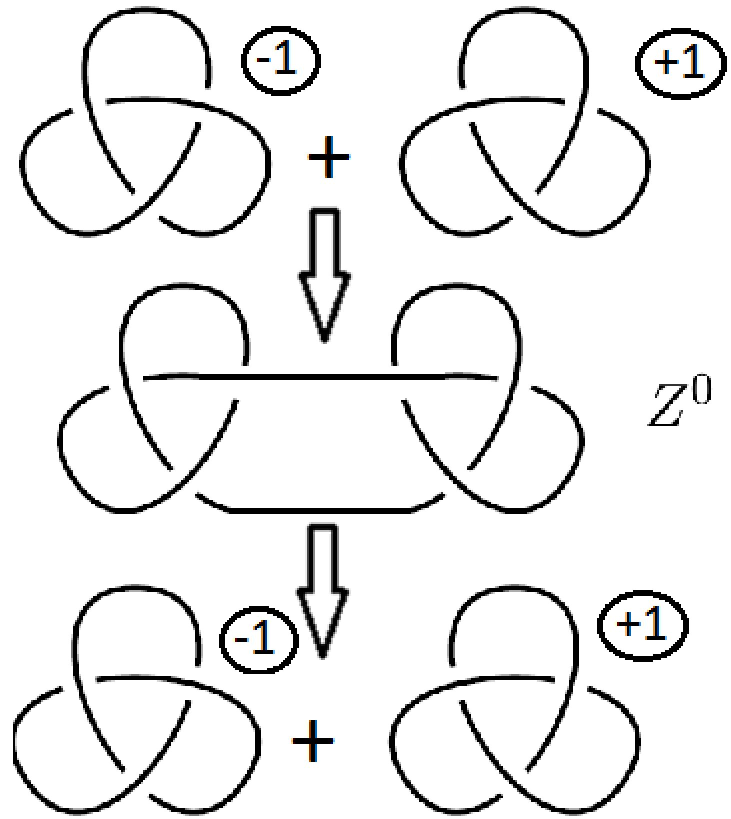


Z^0 is its own anti-particle whereas W^- and W^+ are anti-particles of each other, as required.

Fermionic electroweak interaction:



$$\bar{u} + d \rightarrow e^{-} + \bar{\nu}_e$$



$$e^{-} + e^{+} \rightarrow e^{-} + e^{+}$$

Conclusion

Lie algebra deformations:

Spacetime at small scales looks like anti de Sitter spacetime

Hopf algebra deformations:

Elementary particles may be described as solitons carrying the symmetries of oriented knots

- Deformations provide a systematic approach to developing HEP theory.
- Some initial success in describing electroweak interactions between fermions in terms of knots.
- Some advantages over Helon model.
- Likely of interest in Loop Quantum Gravity
- Many questions remain! Much work still to be done!

Thank you