Could Reggeon Field theory serve as effective theory for QCD at high energies?

(Project and first results)

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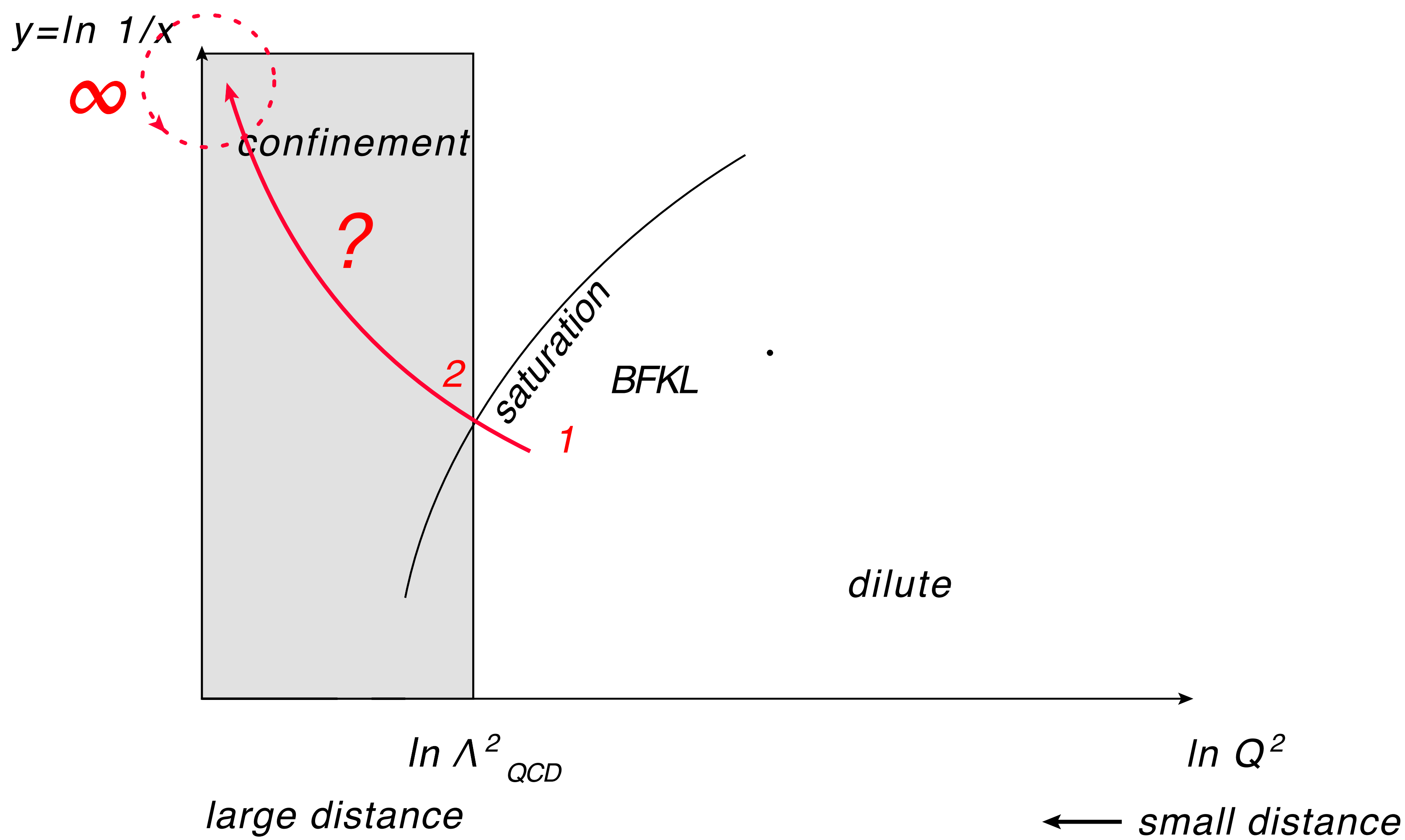
- Introduction
- Flow equations, search for fixed points
- First results of fixed point analysis
- First glimpse at physics
- Conclusions

Introduction

Question: how to continue small-x physics from pQCD to the nonperturbative region?



Regge description: 2+1-dimensional field theory

- (1) short distance region: BFKL, reggeon field theory of reggeized gluons
- (2) saturation: nonlinear BK-equation (fan diagrams)
- (∞) soft region (large distances): Regge poles (DL, Kaidalov, Tel Aviv, Durham)

Attractive idea:

use reggeon field theory and renormalization group,
construct a flow from UV scale to IR scale

$$S = \int dy d^2x \mathcal{L}(\psi, \psi^\dagger)$$

e.g. local approximation: $\mathcal{L} = (\frac{1}{2}\psi^\dagger \overset{\leftrightarrow}{\partial_y} \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V(\psi, \psi^\dagger)$

$$V(\psi, \psi^\dagger) = -\mu \psi^\dagger \psi + i\lambda \psi^\dagger (\psi^\dagger + \psi)\psi$$
$$+ g(\psi^\dagger \psi)^2 + g' \psi^\dagger ({\psi^\dagger}^2 + \psi^2)\psi + \cdots$$

Study the flow as function of IR cutoff k in transverse momentum,
all fields and parameters become k-dependent,
IR limit: infinite transverse momenta, infinite energies

The formalism: functional renormalization, flow equations

Reminder: Wilson approach

The standard Wilsonian action is defined by an iterative change in the UV-cutoff induced by a partial integration of quantum fluctuations:

$$\Lambda \to \Lambda' < \Lambda$$

$$\int [\mathrm{d}\varphi]^{\Lambda} e^{-S^{\Lambda}[\varphi]} = \int [\mathrm{d}\varphi]^{\Lambda'} e^{-S^{\Lambda'}[\varphi]} \qquad k < \Lambda$$

Alternatively: FRG-approach (Wetterich), sequence of theories, IR cutoff

(successful use in statistical mechanics and in gravity)

define a bare theory at scale Λ.
The integration of the modes in the interval $[k, \Lambda]$ defines a k-dependent average functional.
Letting k flowing down to 0 defines a flow for the functional which leads to full theory. k-dependent effective action:

regulator

$$e^{-\Gamma_k[\phi]} = \int [d\varphi] \mu_k e^{-S[\varphi] + \int_x (\varphi - \phi)_x \frac{\delta \Gamma_k[\phi]}{\delta \phi_x} - \Delta S_k[\varphi - \phi]}$$

Taking a derivative with respect the RG time t=log (k/k_0)
one obtains flow equation:

$$\partial_t \Gamma_k = \frac{1}{2} Tr\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k\right] - \frac{\dot{\mu}_k}{\mu_k}$$

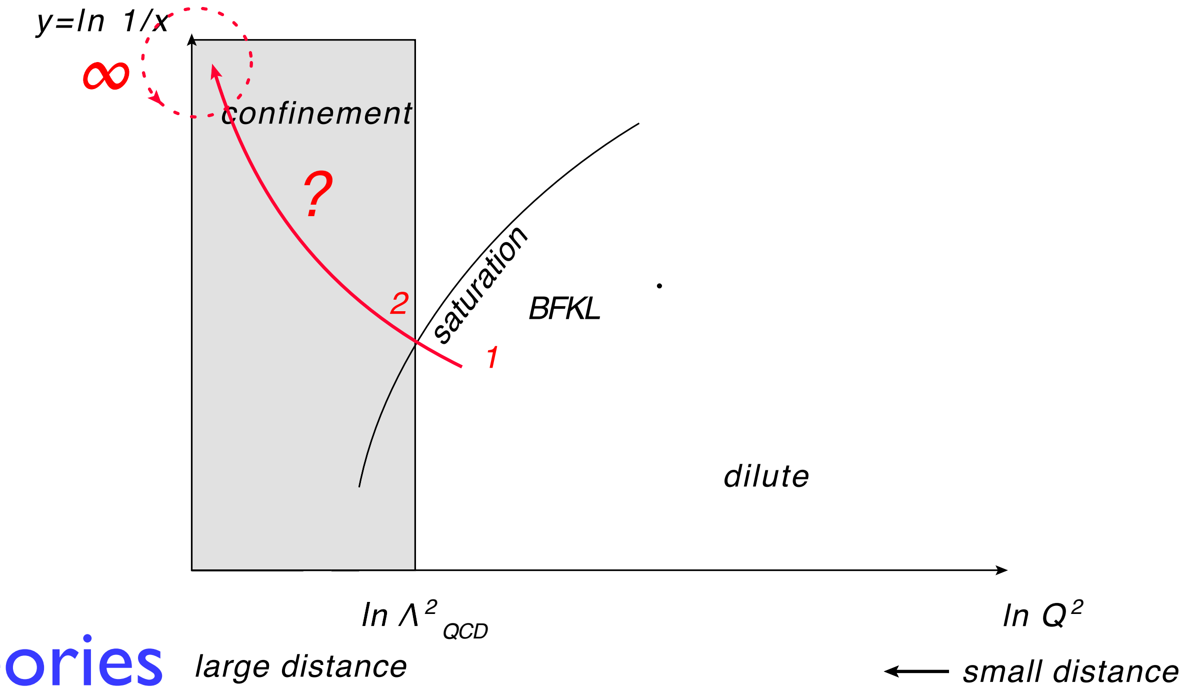
$$\mathcal{R} = \text{regulator operator}$$

which is UV and IR finite
From this derive coupled differential equations for Green's and vertex functions (see below)

A comment on the role of transverse distances and cutoff in transverse momentum:

1) pp scattering at present energies: transverse extension grows with s
2) growth of total cross section varies with transverse size of projectiles
BFKL in $\gamma^*\gamma^*$, $\gamma^* p$ in DIS, pp
Trend: transverse size grows with energy,
intercept decreases with size
$\rightarrow$ IR cutoff in transverse momentum is not unphysical

This talk:
only the first steps



1) Existence of a theory in the IR limit:
fixed point in the space of reggeon field theories
Properties of the fixed point theory

2) How to approach the fixed point

Serious approximation: local approximation

in the UV, BFKL -Pomeron is composite field, nonlocal kernels

Solve flow equations, search for fixed points

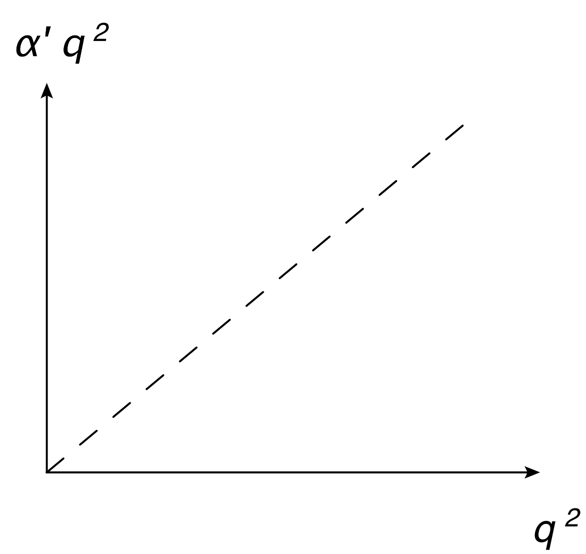
$$\Gamma[\psi^\dagger,\psi] = \int \mathrm{d}^2x\,\mathrm{d}\tau \left(Z(\frac{1}{2}\psi^\dagger \overleftrightarrow{\partial_\tau}\psi - \alpha'\psi^\dagger\nabla^2\psi) + V[\psi^\dagger,\psi] \right), \qquad \mu = \alpha(0) - 1$$

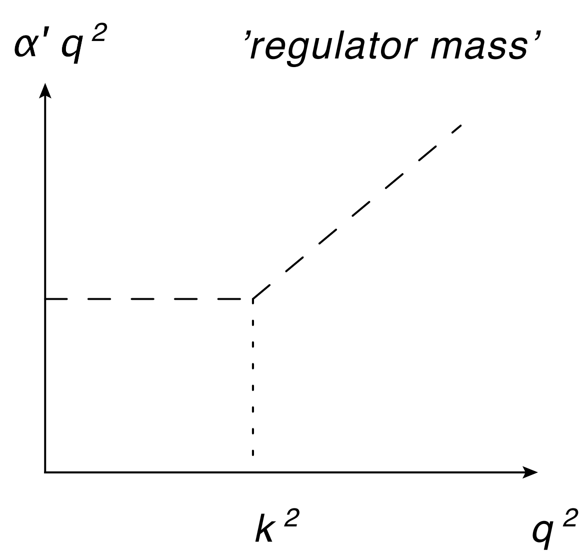
$$V[\psi^\dagger,\psi] = -\mu\psi^\dagger\psi + i\lambda\psi^\dagger(\psi^\dagger+\psi)\psi + g(\psi^\dagger\psi)^2 + g'\psi^\dagger({\psi^\dagger}^2+\psi^2)\psi$$
$$+ i\lambda_5{\psi^\dagger}^2\left(\psi^\dagger+\psi\right)\psi^2 + i\lambda_5'\psi^\dagger\left({\psi^\dagger}^3+\psi^3\right)\psi + ...$$

After introducing a regulator: all parameters become k-dependent

$$\Gamma_k[\psi^\dagger,\psi] = \int d^2x\,d\tau \left(Z_k(\frac{1}{2}\psi^\dagger \overleftrightarrow{\partial_\tau}\psi - \alpha_k'\psi^\dagger\nabla^2\psi) + \psi^\dagger R_k\psi + V_k[\psi,\psi^\dagger] \right)$$

There is freedom in choosing a regulator, for example:





Concretely: partial differential equation for potential $V(\psi, \psi^\dagger)$:

$$\dot{\tilde{V}}_k[\tilde{\psi}^\dagger, \tilde{\psi}] = (-(D+2) + \zeta_k)\tilde{V}_k[\tilde{\psi}^\dagger, \tilde{\psi}] + (D/2 + \eta_k/2)(\tilde{\psi}\frac{\partial \tilde{V}_k}{\partial \tilde{\psi}}|_t + \tilde{\psi}^\dagger \frac{\partial \tilde{V}_k}{\partial \tilde{\psi}^\dagger}|_t) + \frac{\dot{V}_k}{\alpha' k^{D+2}}.$$

$$\dot{V}_k = N_D A_D(\eta_k, \zeta_k)\alpha'_k k^{2+D} \frac{1 + \tilde{V}_{k\tilde{\psi}\tilde{\psi}^\dagger}}{\sqrt{1 + 2\tilde{V}_{k\tilde{\psi}\tilde{\psi}^\dagger} + \tilde{V}^2_{k\tilde{\psi}\tilde{\psi}^\dagger} - \tilde{V}_{k\tilde{\psi}\tilde{\psi}}\tilde{V}_{k\tilde{\psi}^\dagger\tilde{\psi}^\dagger}}}.$$

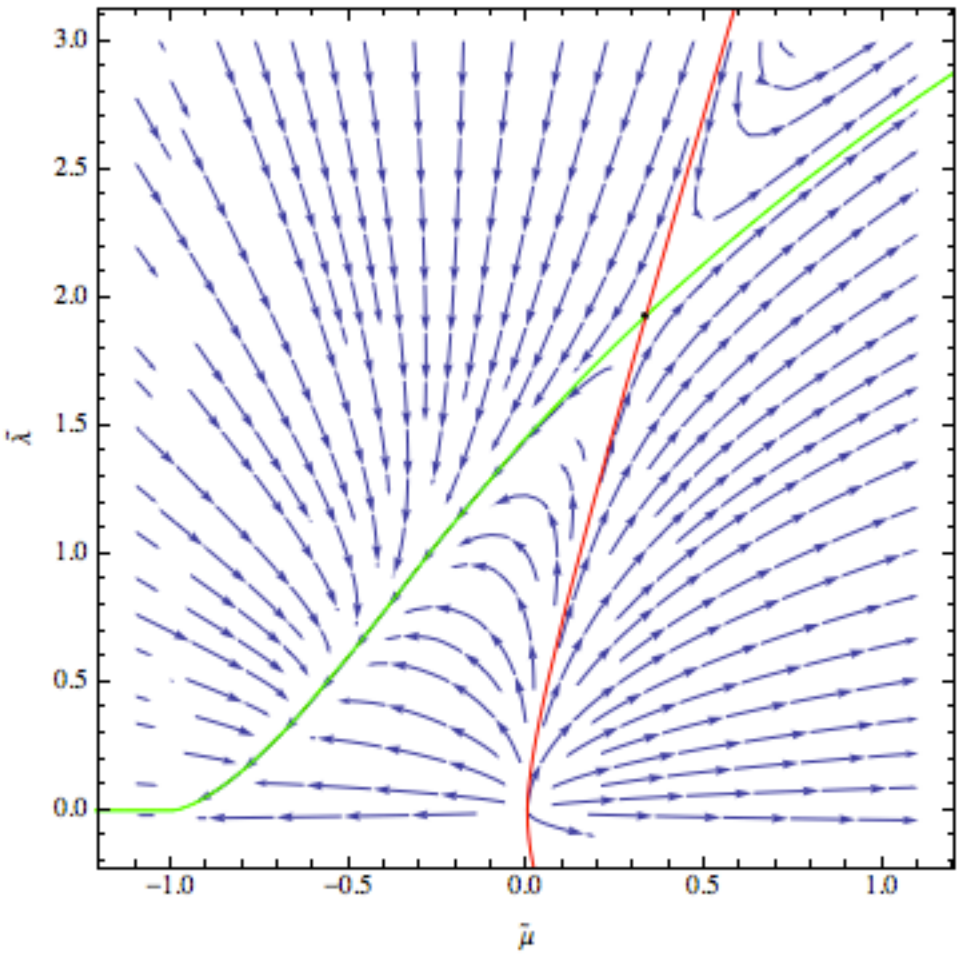
Fixed point: put rhs =0

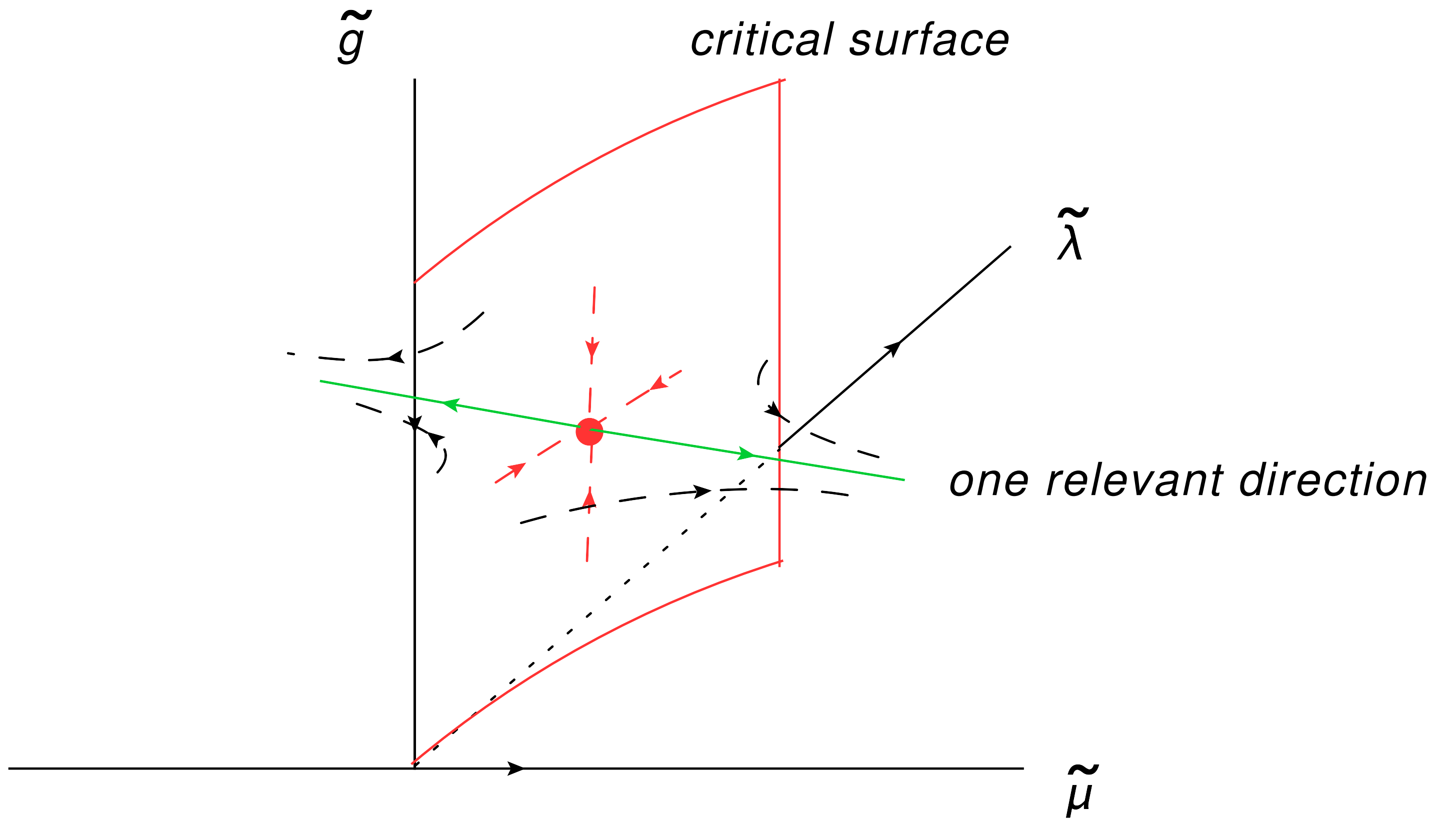
Possible ways to solve (for constant fields, approximately):

- polynomial expansion in fields around zero (beta-functions)
- polynomial expansion in fields around stationary point
- solve differential equations in the region of large fields

Results of fixed point analysis

1) Existence of a fixed point with one relevant direction





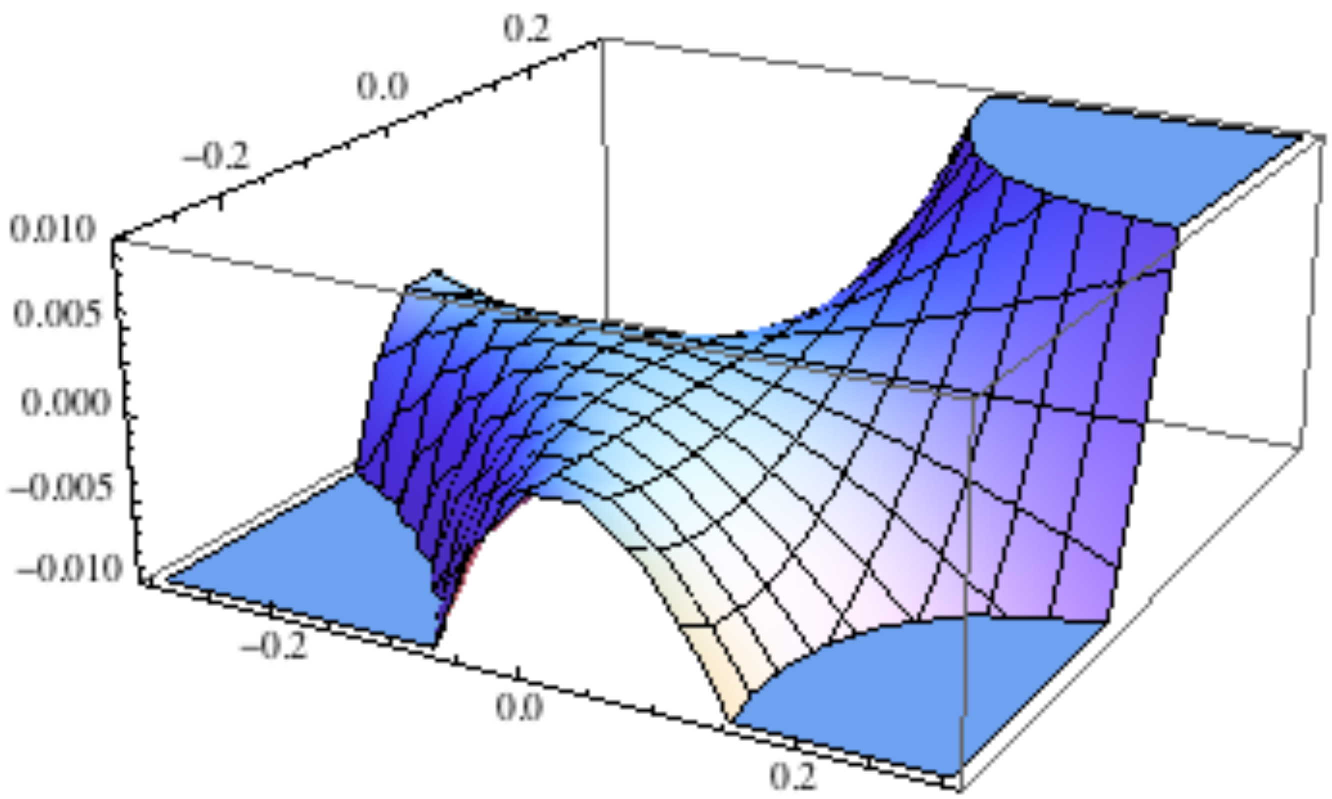
Flow in the space of parameters of the potential (couplings) :
reggeon mass (intercept) $\tilde{\mu} = \alpha(0) - 1$, triple coupling $\tilde{\lambda}$
fixed point IR attractive inside critical surface (red),
repulsive along relevant direction (green)

Convergence for higher truncations (expansion around nonzero stationary point) :

truncation	3	4	5	6	7	8
exponent ν	0.74	0.75	0.73	0.73	0.73	0.73
mass $\tilde{\mu}_{eff}$	0.33	0.362	0.384	0.383	0.397	0.397
$i\psi_{0,diag}$	0.058	0.072	0.074	0.074	0.0.074	0.074
iu_0	0.173	0.213	0.218	0.218	0.218	0.218

Compare with Monte Carlo result for Directed Percolation
(same universality class): $\nu = 0.73$

Shape of the effective potential (in the subspace of imaginary fields):



Extrema, location at lowest truncation:

$$(\tilde{\psi}_0, \tilde{\psi}_0^\dagger) = (0,0), \qquad (\frac{\tilde{\mu}}{i\tilde{\lambda}}, 0), \qquad (0, \frac{\tilde{\mu}}{i\tilde{\lambda}}), \qquad (\frac{\tilde{\mu}}{3i\tilde{\lambda}}, \frac{\tilde{\mu}}{3i\tilde{\lambda}}).$$

No further structure for larger fields

Main result of this part:

- found a candidate for fixed point (IR stable except for one relevant direction
- robust when changing truncations
- know the effective potential

First glimpse at physics

Need to find out: on which trajectory is real physics?

Look at flow of physical physical observable: Pomeron intercept $\mu = \alpha(0) - 1$:

So far: fixed point analysis was done in terms of dimensionless variables: reggeon energy and momentum have different dimensions

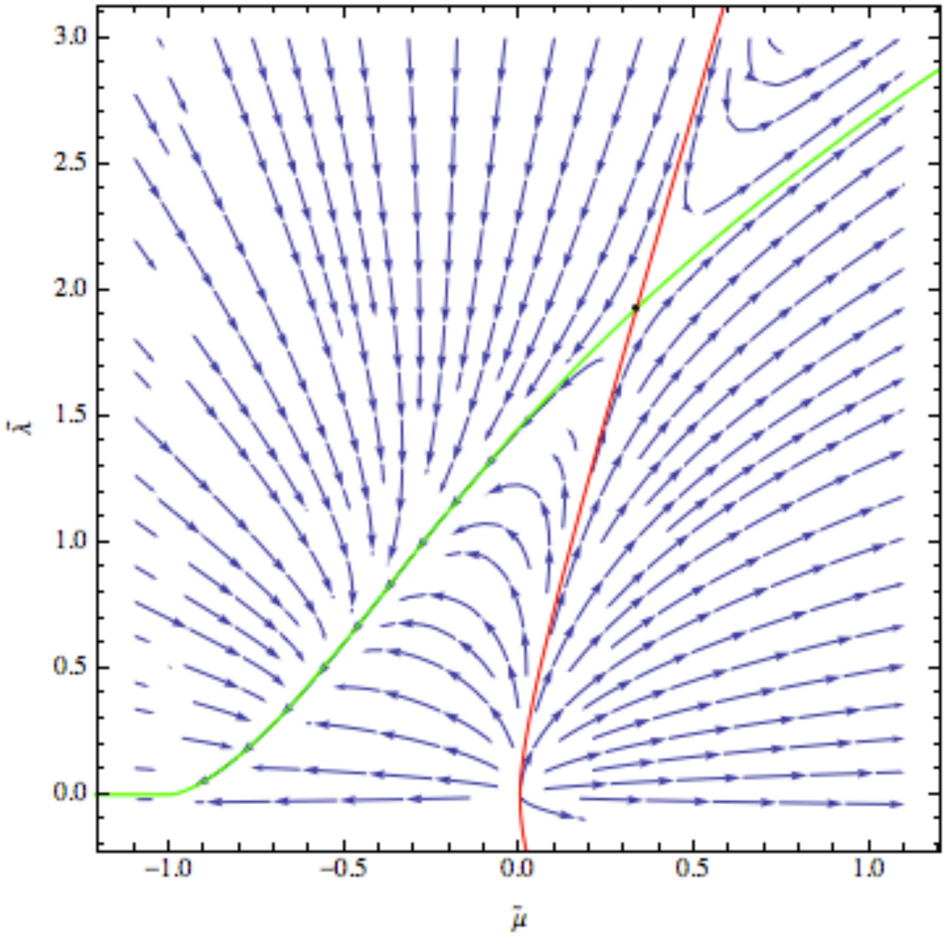
$$S = \int d^2x\, d\tau \left(Z(\frac{1}{2}\psi^\dagger \overleftrightarrow{\partial_\tau} \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V[\psi^\dagger, \psi] \right), \qquad [\psi] = [\psi^\dagger] = k^{D/2}, \qquad [\alpha'] = E k^{-2}.$$

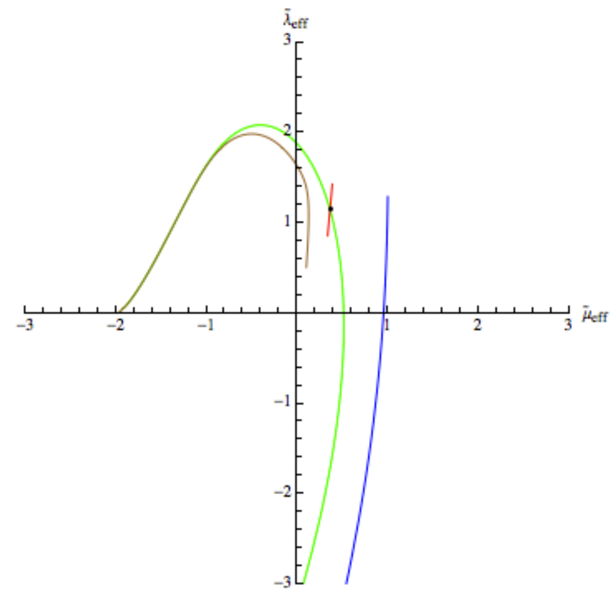
$$\tilde{\mu}_k = \frac{\mu_k}{Z_k \alpha'_k k^2}$$

$$\tilde{\lambda}_k = \frac{\lambda_k}{Z_k^{\frac{3}{2}} \alpha'_k k^2} k^{D/2}$$

Evolution of physical (=dimensionful) parameters $\mu_k, \lambda_k, ...$ looks quite different from dimensionless ones $\tilde{\mu}_k, \tilde{\lambda}_k, ...$

dimensionless parameters

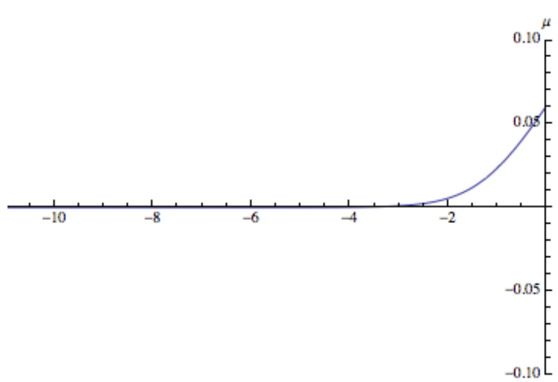


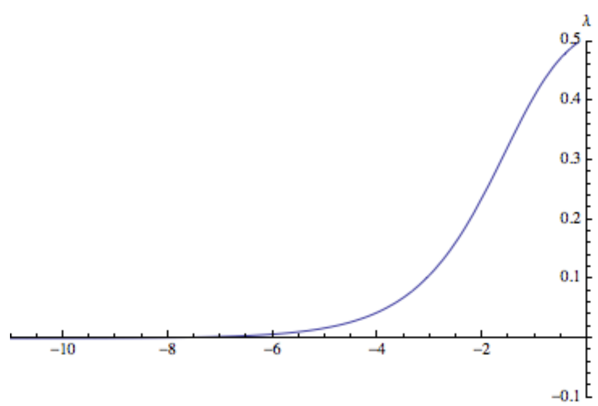


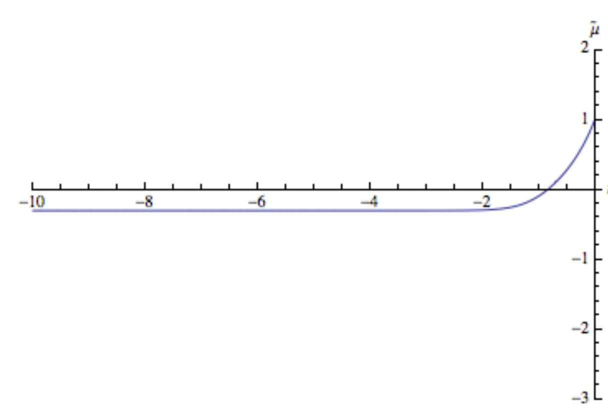
physical parameters :

Critical subspace (red):

Near critical subspace (blue)







$\alpha(0) \to 1$

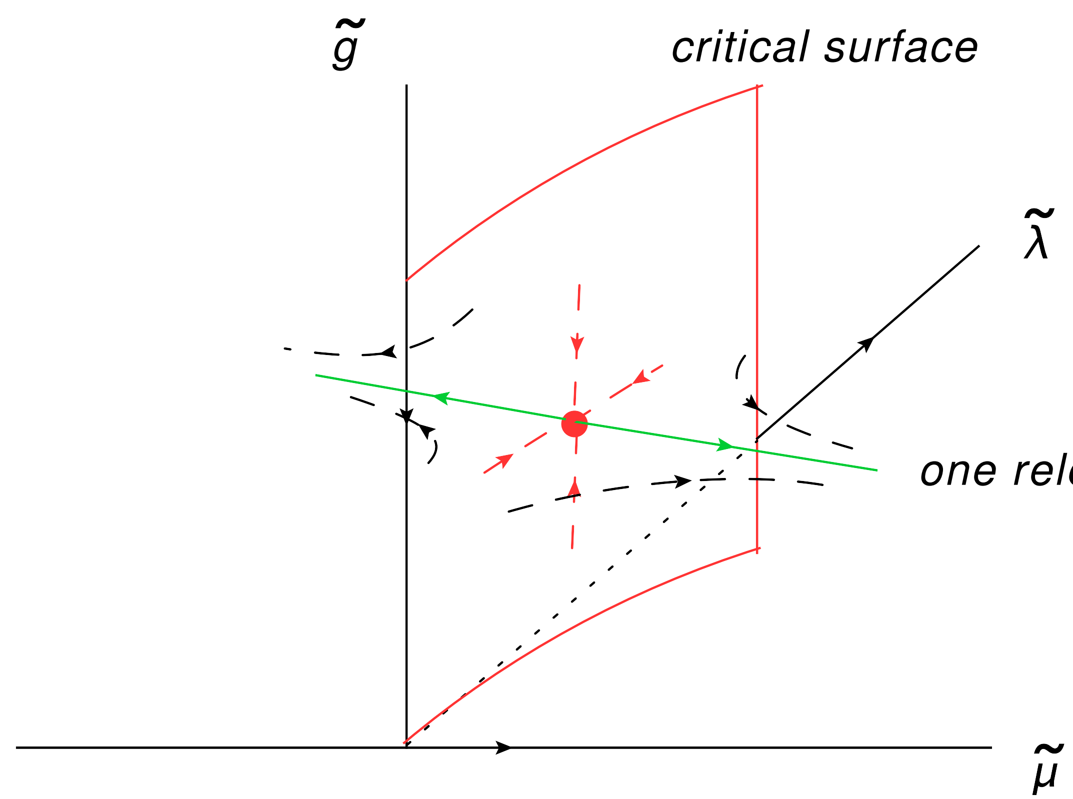
$\lambda_{triple} \to 0$

$\alpha_k(0) \to \alpha_{k=0} < 1$

But: theory not free!

Tentative interpretation: different phases:

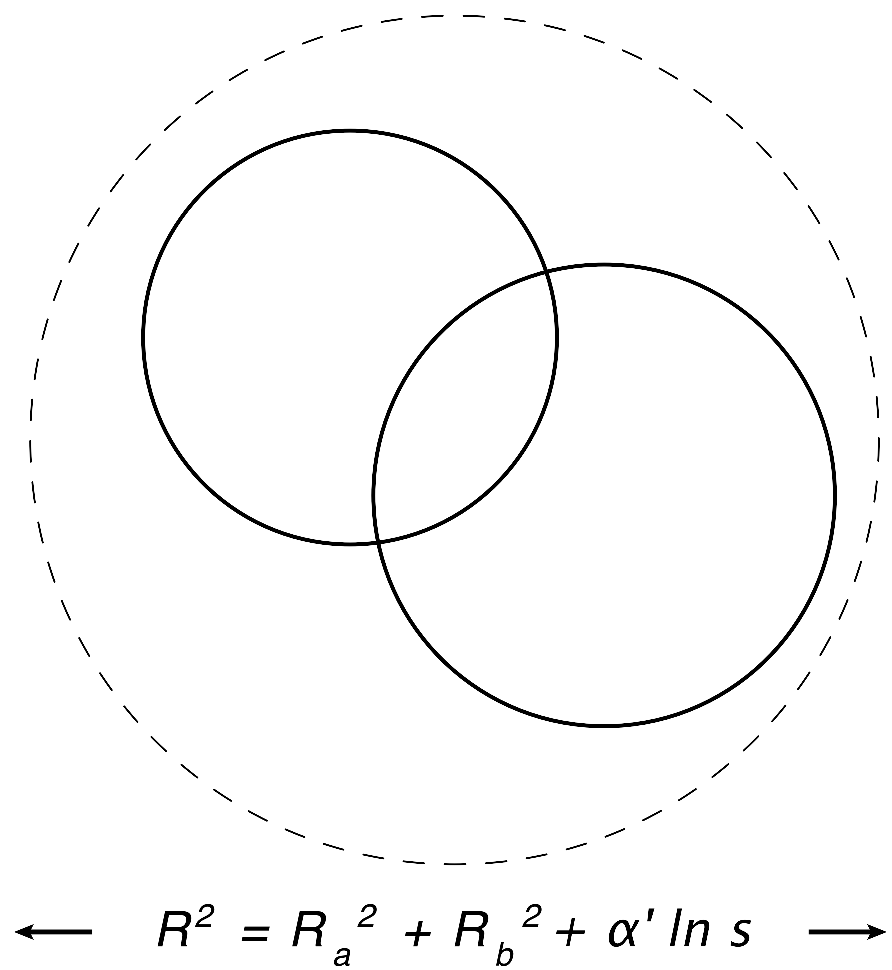
n-1 dim. critical subspace: massless
divides the n-dimensional space into
two (subcritical, supercritical) half spaces



Which phase: depends upon starting point at k=0 (UV)

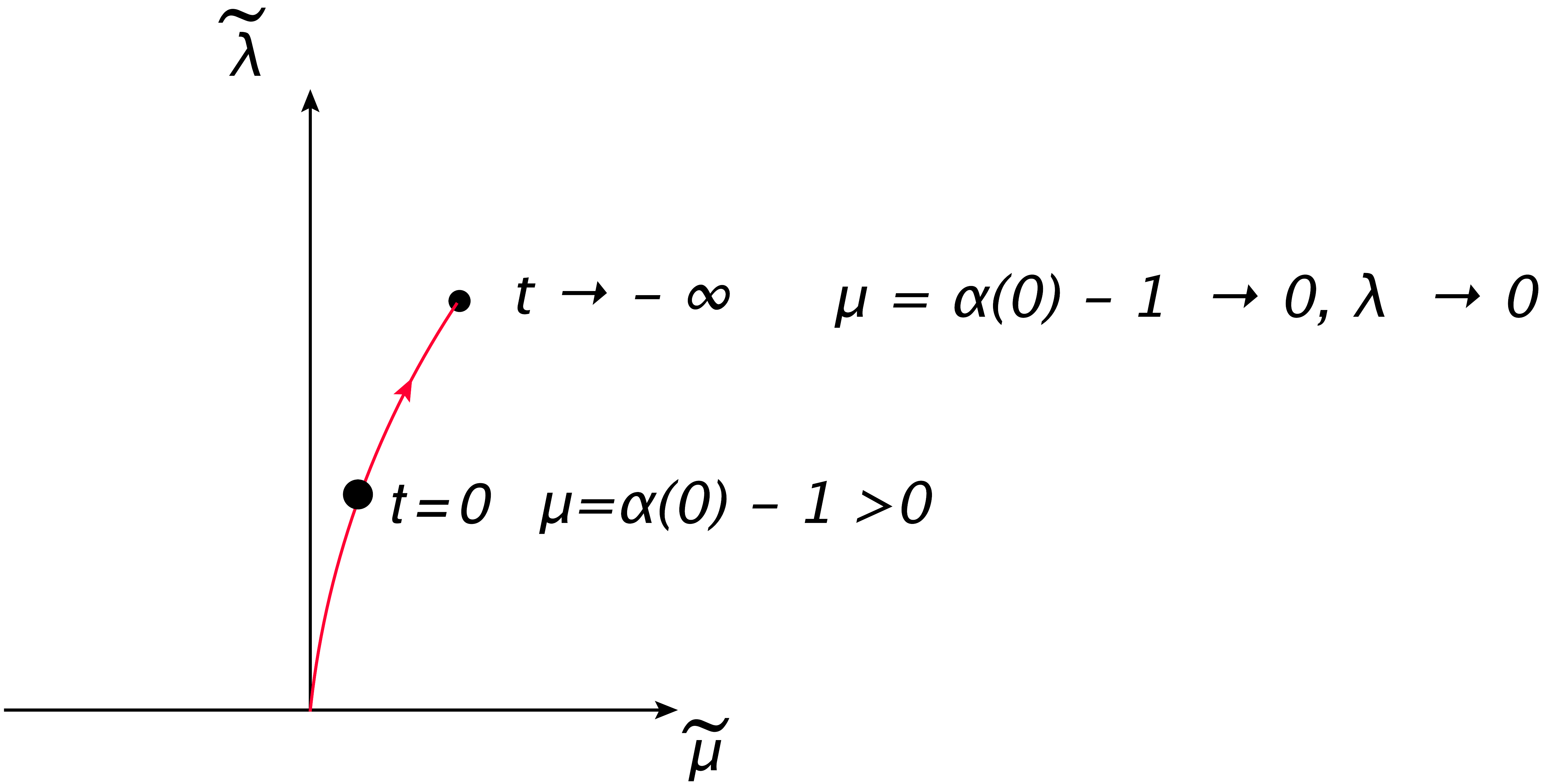
Possible interpretation of IR cutoff, evolution time $\tau = \ln k/k_0$:

IR-cutoff: $k^2 \sim 1/\text{transverse distance}^2 \sim 1/\ln s$



transverse plane

Possible physical scenario:



Conclusions

Achieved so far:

Investigated the IR-limit of a possible connection: IR - UV

- found a fixed point in the space of (local) reggeon field theories
- subspace of critical (massless) theories, but also subcritical and supercritical theories are possible

- possible scenario:
  IR - (infinite energies and transverse distances): critical theory with intercept 1
  UV - BFKL etc.

Next steps:

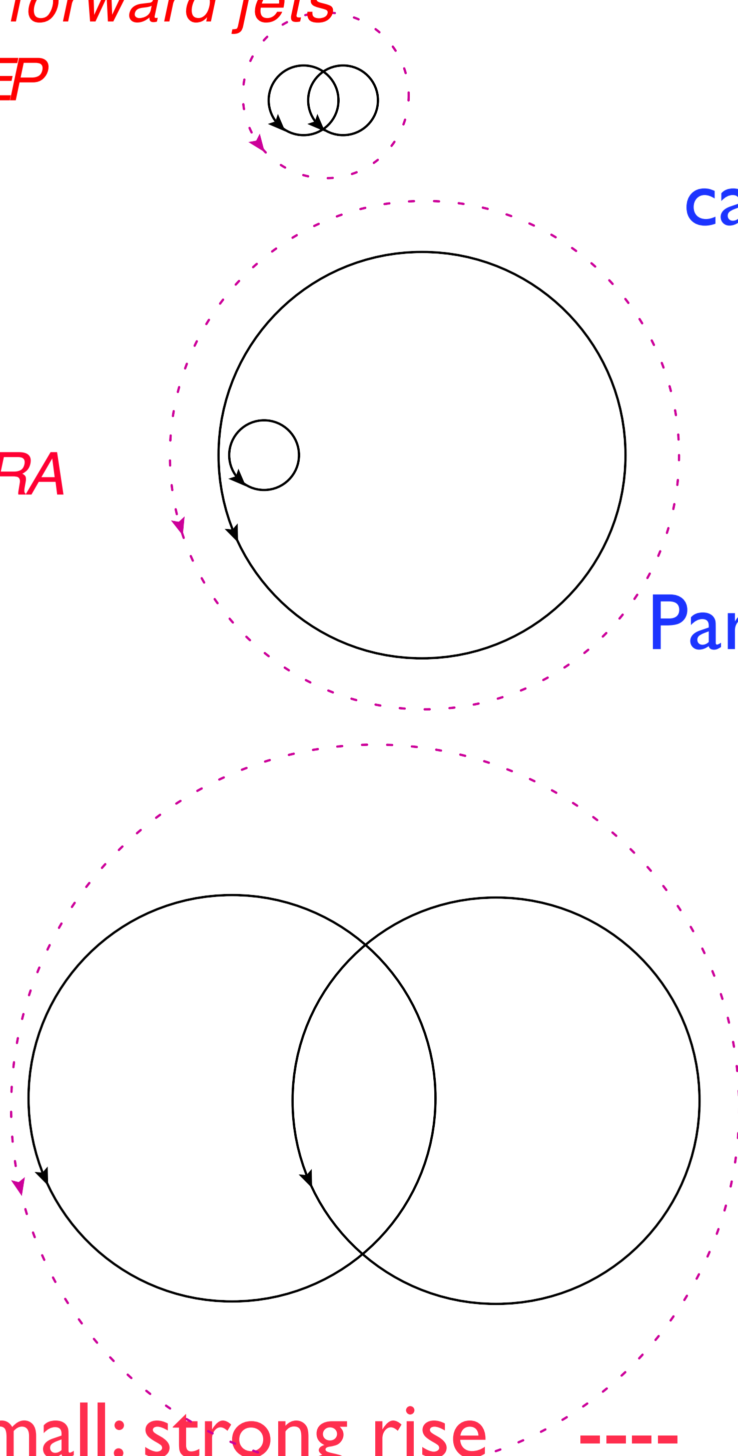
- Phenomenology of fixed point theory: does it work?
- Connection with 'old' critical RFT
- extend to nonlocal theory (composite fields)
- construct sequence: BFKL $\rightarrow$ IR-limit

1. Motivation and project

Energy dependence of total cross sections varies with transverse size:

HERA forward jets

LEP



$\gamma^*\gamma^* \quad \sigma_{tot} \approx s^{\omega_{BFKL}}$

calculable in pQCD

HERA

$\gamma^* p \quad \sigma_{tot} \approx (W^2)^{\lambda}$

Partly calculable in pQD

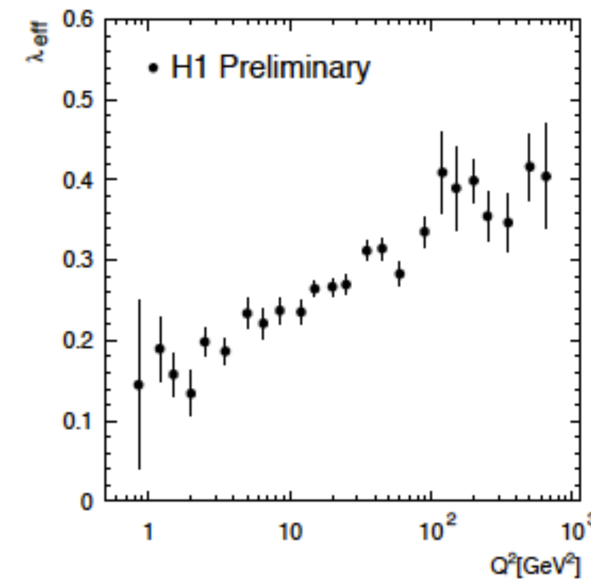


Figure 6: The slope $\lambda_{\rm eff}$ of F_2 as a function of Q^2 .

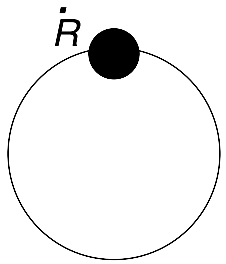
LHC

$p\,p \quad \sigma_{tot} \approx s^{0.08}$

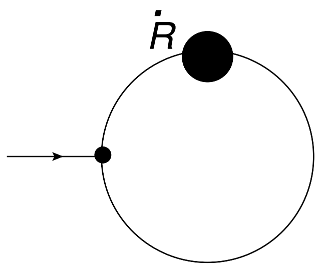
nonperturbative

Small: strong rise ---- large: slow rise

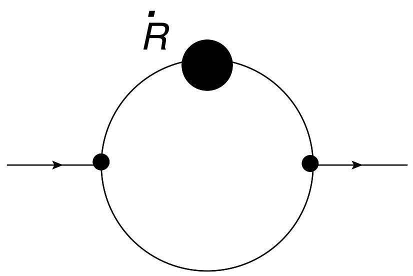
Vertex functions, Green's functions, physical observables:
take functional derivatives w.r.t. the fields:



$$\partial_t \Gamma_k = \frac{1}{2} G_{k;AB}\, \partial_t \mathcal{R}_{k;BA}$$



$$\partial_t \Gamma^{(1)}_{k;A_1} = -\frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_1 BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA}$$



$$\begin{aligned}\partial_t \Gamma^{(2)}_{k;A_1 A_2} = & \frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_1 BC} G_{k;CD} \Gamma^{(3)}_{k;A_2 DE} G_{k;EF}\, \partial_t \mathcal{R}_{k;FA} \\ & + \frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_2 BC} G_{k;AB} \Gamma^{(3)}_{k;A_1 BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA} \\ & - \frac{1}{2} G_{k;AB} \Gamma^{(4)}_{k;A_1 A_2 BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA}\end{aligned}$$

coupled partial differential equations

First step:

Expand the potential in powers of fields, derive beta-functions for parameters of the potential (coupling constants):

$$\dot{\tilde{\mu}} = \tilde{\mu}(-2+\zeta+\eta) + 2N_D A_D(\eta_k,\zeta_k)\frac{\tilde{\lambda}^2}{(1-\tilde{\mu})^2},$$

$$\dot{\tilde{\lambda}} = \tilde{\lambda}\left((-2+\zeta+\frac{D}{2}+\frac{3\eta}{2}) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{4\tilde{\lambda}^2}{(1-\tilde{\mu})^3}+\frac{(\tilde{g}+3\tilde{g}')}{(1-\tilde{\mu})^2}\right)\right),$$

$$\dot{\tilde{g}} = \tilde{g}(-2+D+\zeta+2\eta) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{27\tilde{\lambda}^4}{(1-\tilde{\mu})^4}+\frac{(16\tilde{g}+24\tilde{g}')\tilde{\lambda}^2}{(1-\tilde{\mu})^3}+\frac{\left(\tilde{g}^2+9\tilde{g}'^2\right)}{(1-\tilde{\mu})^2}\right)$$

$$\dot{\tilde{g}}' = \tilde{g}'(-2+D+\zeta+2\eta) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{12\tilde{\lambda}^4}{(1-\tilde{\mu})^4}+\frac{(4\tilde{g}+18\tilde{g}')\tilde{\lambda}^2}{(1-\tilde{\mu})^3}+\frac{3\tilde{g}\tilde{g}'}{(1-\tilde{\mu})^2}\right)$$

Fixed points: zeroes of the beta-functions

First results: fixed points

Local reggeon field theory:

$\mu = \alpha(0) - 1$

$$\mathcal{L} = (\tfrac{1}{2}\psi^\dagger \overleftrightarrow{\partial}_y \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V(\psi, \psi^\dagger)$$

$$V(\psi, \psi^\dagger) = -\mu\psi^\dagger\psi + i\lambda\psi^\dagger(\psi^\dagger + \psi)\psi$$
$$+ g(\psi^\dagger\psi)^2 + g'\psi^\dagger({\psi^\dagger}^2 + \psi^2)\psi + \cdots$$

some universal symmetry properties

Some history:

Gribov, Migdal; Abarbanel, Bronzan; Migdal, Polyakov, Ter-Martirosyan

In early seventies : first studies of RFT with triple couplings, expansion near D=4 (ϵ- expansion). IR-fixed point.

In 1980: J. Cardy and R. Sugar noticed that the RFT is in the same universality class of a Markov process known as Directed Percolation (DP). Critical exponents can then be accessed also with numerical montecarlo computations.

This attempt:
search in the full space of theories, no restriction to D=4

Effective action with local potential:

$$\Gamma_k = \int \mathrm{d}y\, \mathrm{d}^D x \left[Z_k\left(\frac{1}{2}\psi^{dagger} \overleftrightarrow{\partial_y} \psi - \alpha'_k \psi^{dagger} \nabla^2 \psi\right) + V_k(\psi, \psi^\dagger) \right]$$

Propagator of flow equations:

$$\Gamma_k^{(2)} + \mathbb{R} = \begin{pmatrix} V_{k\psi\psi} & -iZ_k\omega + Z_k\alpha'_k q^2 + R_k + V_{k\psi\psi^{dagger}} \\ iZ_k\omega + Z_k\alpha'_k q^2 + R_k + V_{k\psi^{dagger}\psi} & V_{k\psi^{dagger}\psi^{dagger}} \end{pmatrix}$$

Flow equation for potential:

$$\dot{V}_k(\psi, \psi^\dagger) = \frac{1}{2}\mathrm{tr}\left\{ \int \frac{\mathrm{d}\omega\, \mathrm{d}^D q}{(2\pi)^{D+1}} \left[\left(\Gamma_k^{(2)} + \mathbb{R} \right)^{-1} \dot{\mathcal{R}}_k \right] \right\}$$

Coarse graining in momentum space:

$$R_k(q) = Z_k \alpha'_k (k^2 - q^2)\Theta(k^2 - q^2)$$