

Nuclear Effects for Electron and Neutrino DIS

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Outline

- ▶ Overview of data on nuclear effects in the DIS.
- ▶ Overview of a model of nuclear DIS
 - ▶ Sketch of basic physics mechanisms of nuclear corrections in different kinematic regions
 - ▶ Trying to put those mechanisms together in a model
 - ▶ Discuss performance and predictions of the model
- ▶ Predictions for neutrino cross sections

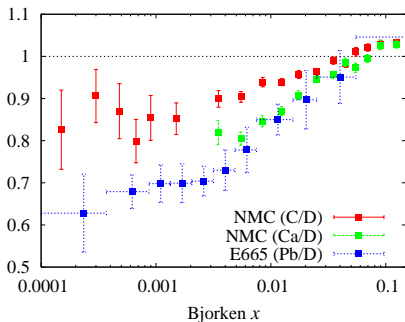
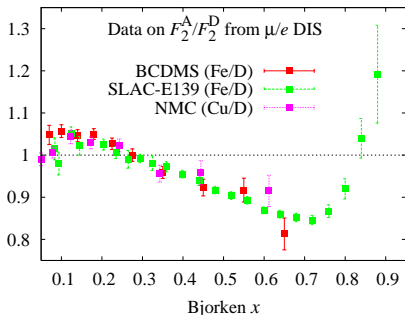
Data on nuclear effects in DIS

- ▶ Data on nuclear effects in DIS are available in the form of the ratio $\mathcal{R}(A/B) = \sigma_A(x, Q^2)/\sigma_B(x, Q^2)$ or F_2^A/F_2^B .
- ▶ Data for nuclear targets from ${}^2\text{H}$ to ${}^{208}\text{Pb}$
- ▶ Fixed-target experiments with e/μ :
 - ▶ Muon beam at CERN (EMC, BCDMS, NMC) and FNAL (E665).
 - ▶ Electron beam at SLAC (E139, E140), HERA (HERMES), JLab (E03-103).
- ▶ Kinematics and statistics:

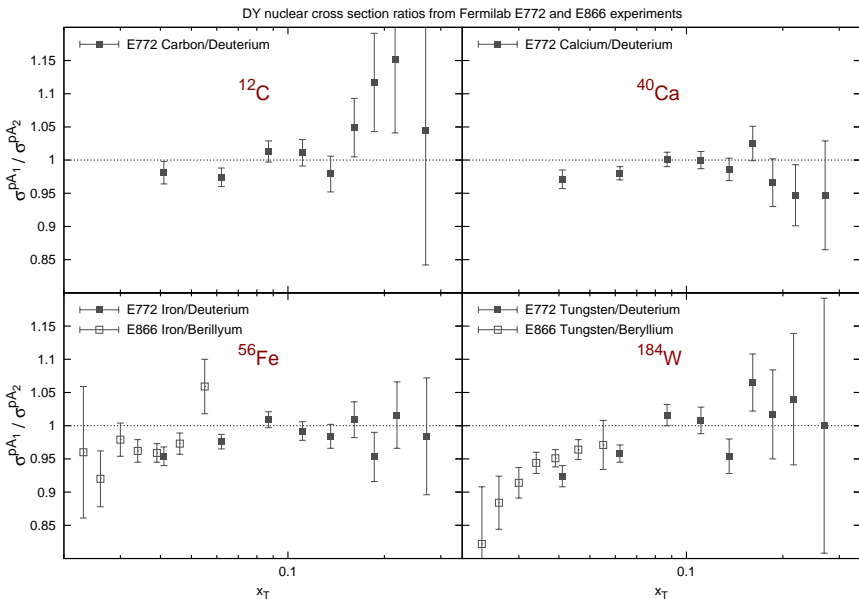
Data covers the region $10^{-4} < x < 1.5$ and $0 < Q^2 < 150 \text{ GeV}^2$. About 800 data points for the nuclear ratios $\mathcal{R}(A/B)$ with $Q^2 > 1 \text{ GeV}^2$.
- ▶ Nuclear effects for antiquarks comes have been probed by Drell-Yan experiments at FNAL (E772, E866).
- ▶ Neutrino data on DIS cross sections on nuclear targets ${}^2\text{H}$, ${}^{20}\text{Ne}$, ${}^{12}\text{C}$, ${}^{56}\text{Fe}$, ${}^{207}\text{Pb}$ from CERN (BEBC, CDHS, CHORUS, NOMAD) and FNAL (CCFR, NuTeV).
- ▶ Recent measurement of the nuclear ratios from MINERvA in the region of low Q^2 .

Data on the nuclear ratios $\mathcal{R}(A/D)$ show a pronounced A dependence and a weak Q^2 dependence. The ratios have *oscillating* shape vs. the Bjorken x

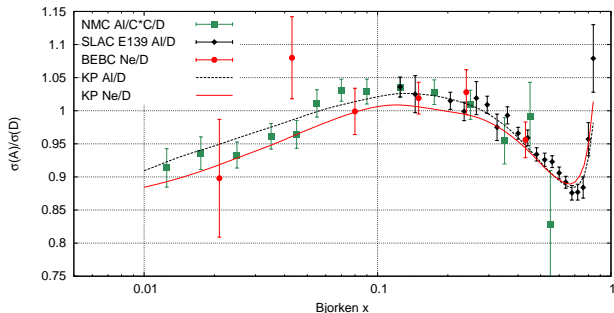
- ▶ Suppression (**shadowing**) at small x ($x < 0.05$).
- ▶ Enhancement (**antishadowing**) at $0.1 < x < 0.25$.
- ▶ A well with a minimum at $x \sim 0.6 \div 0.75$ (**EMC effect**).
- ▶ Enhancement at large values of $x > 0.75 \div 0.8$ (**Fermi motion region**).



Data on nuclear ratios from Drell-Yan experiments



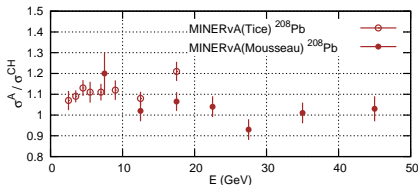
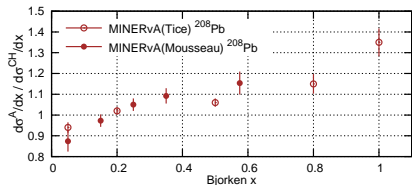
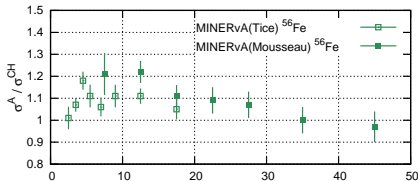
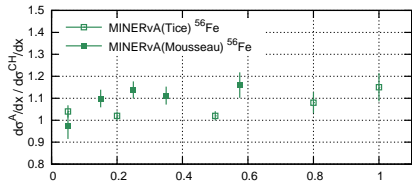
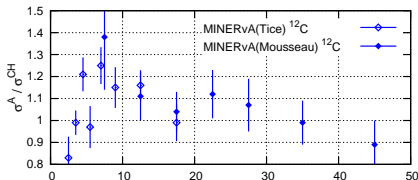
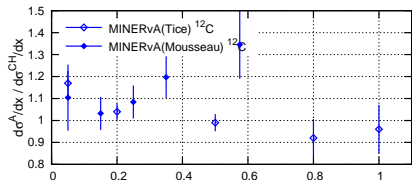
BEBC measurement of nuclear effects with neutrino



- ▶ For a long time was the only DIRECT measurement of nuclear effects in $\nu(\bar{\nu})$ DIS from ratio $^{20}\text{Ne}/\text{D}$ by *BEBC Coll.*, *ZPC 36 (1987) 337*; *PLB 232 (1989) 417*
 - ▶ Consistent with shadowing at small x_{Bj} but large uncertainties;
 - ▶ Consistent with the EMC effect measured from e, μ DIS.
- ▶ Differences with respect to e, μ DIS at small x mainly due to the axial-vector current.

New measurement of nuclear effects with ν from MINERvA

MINERvA(Tice) (QE+RES+DIS) MINERvA(Mousseau) (DIS) cross section ratios. STAT errors only. NO isoscalar correction.

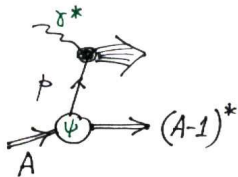


Modelling nuclear effects

A good starting point is approximation of incoherent scattering off bound protons and neutrons

$$F_2^A(x, Q^2) = \int d^4p \mathcal{P}_A(p) \left(1 + \frac{p_z}{M}\right) F_2^N(x', Q^2, p^2),$$

$$x = \frac{Q^2}{2Mq_0}, \quad x' = \frac{Q^2}{2p \cdot q} \approx \frac{Mx}{p_0 + p_z}$$



In this approx the basic corrections are due to the nucleon momentum distribution (Fermi motion) and its energy spectrum. Both effects are driven by nuclear spectral function, which describes probability to find a bound nucleon with momentum \mathbf{p} and energy $p_0 = M + \varepsilon$:

$$\mathcal{P}_A(p) = \sum_n |\psi_n(\mathbf{p})|^2 (2\pi)^4 \delta(\varepsilon + E_n(A-1) - E_0(A)).$$

Nuclear spectral function

- ▶ The nuclear spectral function determines the rate of nucleon removal reactions such as $(e, e'p)$. At low energy and momentum, $|\varepsilon| < 50 \text{ MeV}$, $p < 300 \text{ MeV}/c$, the observed spectrum is described by mean-field model:

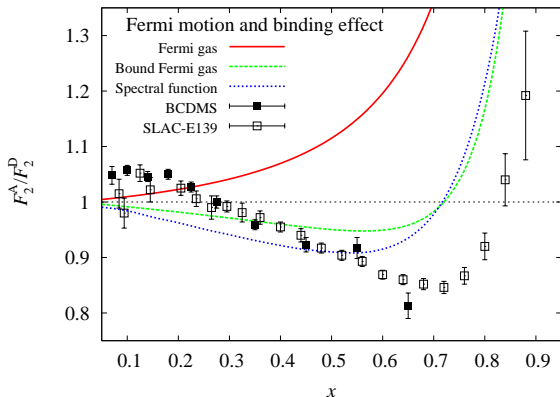
$$\mathcal{P}_{\text{MF}}(\varepsilon, \mathbf{p}) = \sum_{\lambda < \lambda_F} n_\lambda |\phi_\lambda(\mathbf{p})|^2 \delta(\varepsilon - \varepsilon_\lambda)$$

- ▶ At high-energy and momentum $p < 300 \text{ MeV}/c$ the mean field fails. The spectrum is driven by $(A - 1)^*$ excited states with one or more nucleons in the continuum, which are due to correlation effects in nuclear ground state as witnessed by numerous studies.

EMC effect in impulse approximation

Impulse approximation: $F_2(x', Q^2, p^2) = F_2(x', Q^2)$

- ▶ Fermi motion qualitatively describes data at $x > 0.7$
- ▶ Binding correction is important and brings the calculation closer to data in the dip region.
- ▶ However, even realistic nuclear spectral function fails to explain the slope and the position of the minimum.



Nucleon off-shell effect

Bound nucleons are off-mass-shell $p^2 = (M + \varepsilon)^2 - \mathbf{p}^2 < M^2$. In off-shell region nucleon structure functions depend on additional variable $F_2(x, Q^2, p^2)$.

The nucleon virtuality parameter $v = (p^2 - M^2)/M^2$ is small (average virtuality $v \sim -0.15$ for ^{56}Fe). Expand $F_2(x, Q^2, p^2)$ in series in v :

$$F_2^N(x, Q^2, p^2) = F_2^N(x, Q^2)(1 + \delta f(x, Q^2)(p^2 - M^2)/M^2)$$

- ▶ $\delta f(x, Q^2)$ is a new structure function that describes modification of the off-shell nucleon PDFs in the vicinity of the mass shell.
- ▶ Off-shell correction is closely related to modification of the nucleon PDFs in nuclear environment *S.K. & R.Petti, 2004*. In fact this is another way to describe this effect.

Other important corrections include

- ▶ Meson exchange currents – contribution from the fields which mediate nuclear force. Available treatments of MEC in DIS and resonance production are model dependent. Equations of motion together with nuclear light-cone momentum sum rule help to constrain MEC in DIS (*S.K., 1986; S.K. & R.Petti, 2004*). Characteristic region of MEC correction is $x \sim 0.1 - 0.2$ while the magnitude of the correction $\sim 5\%$ for medium range nuclei such as ^{40}Ca .
- ▶ Nuclear shadowing (NS) effect due to propagation and coherent nuclear interaction of intermediate hadronic states. NS is relevant at small $x < 0.05$.

A sketch of a model of the nuclear structure functions

S.K. & R.Petti, Nucl. Phys. A765 (2006) 126.

$$F_i^A = \langle F_i^p \rangle + \langle F_i^n \rangle + \delta_{\text{MEC}} F_i + \delta_{\text{coh}} F_i$$

- ▶ $\langle F_i^p \rangle$ and $\langle F_i^n \rangle$ are the bound proton and neutron structure functions with off-shell effects averaged with nuclear spectral function. As input we use the proton and neutron structure functions computed in NNLO pQCD + TMC + HT using phenomenological PDFs and HTs from fits to DIS data by *S.Alekhin*. The model of nuclear spectral function has two components: mean field and a correlated part responsible for high-momentum component.
- ▶ MEC correction $\delta_{\text{MEC}} F_i$ as a convolution of nuclear meson distribution function with pion SFs (PDFs) extracted from DY process with pions.
- ▶ Coherent term $\delta_{\text{coh}} F_i$ is calculated by evaluating multiple scattering series of an intermediate state with a mass m_{eff} and cross section σ_{eff} .

Analysis of nuclear ratios (EMC effect)

Strategy: Parameterize unknown off-shell function $\delta f(x)$ and effective scattering amplitude a_T . Calculate nuclear structure functions, test with data and extract parameters from data.

- ▶ We study the data from e/μ DIS in the form of ratios $R_2(A/B) = F_2^A/F_2^B$ for a variety of targets. The data are available for $A/{}^2H$ and $A/{}^{12}C$ ratios.
- ▶ We perform a fit to minimize $\chi^2 = \sum_{\text{data}} (\mathcal{R}_2^{\text{exp}} - \mathcal{R}_2^{\text{th}})^2 / \sigma^2(\mathcal{R}_2^{\text{exp}})$ with σ the experimental uncertainty of $\mathcal{R}_2^{\text{exp}}$. We use data with $Q^2 > 1 \text{ GeV}^2$. The nuclear ratios used in our analysis (overall about 560 points available before 1996):

${}^4\text{He}/D$	${}^7\text{Li}/D$	${}^9\text{Be}/D$
${}^{12}\text{C}/D$	${}^{27}\text{Al}/D$	${}^{27}\text{Al}/{}^{12}\text{C}$
${}^{40}\text{Ca}/D$	${}^{40}\text{Ca}/{}^{12}\text{C}$	
${}^{56}\text{Fe}/D$	${}^{63}\text{Cu}/D$	${}^{56}\text{Fe}/{}^{12}\text{C}$
${}^{108}\text{Ag}/D$	${}^{119}\text{Sn}/{}^{12}\text{C}$	
${}^{197}\text{Au}/D$	${}^{207}\text{Pb}/D$	${}^{207}\text{Pb}/{}^{12}\text{C}$

- ▶ Verify the model by comparing the calculations with data not used in analysis.

Parameters of the model

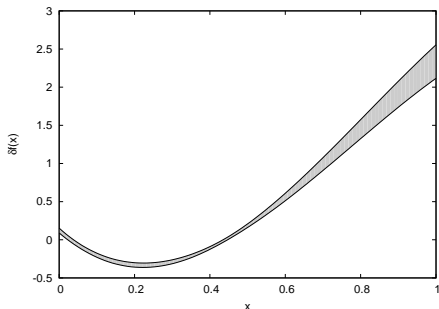
- ▶ Off-shell structure function $\delta f_2(x) = C_N(x - x_1)(x - x_0)(h - x)$
 - ▶ From preliminary studies we observe that h is fully correlated with x_0 , i.e. $h = 1 + x_0$.
 - ▶ C_N , x_0 , x_1 are independent adjustable parameters.
- ▶ Effective amplitude

$$\bar{a}_T = \bar{\sigma}_T(i + \alpha)/2, \quad \bar{\sigma}_T = \sigma_1 + \frac{\sigma_0 - \sigma_1}{1 + Q^2/Q_0^2}$$

- ▶ Parameters $\sigma_0 = 27 \text{ mb}$ and $\alpha = -0.2$ were fixed in order to match the vector meson dominance model predictions at low Q .
- ▶ Parameter $\sigma_1 = 0$ fixed (preferred by preliminary fits and fixed in the final studies).
- ▶ Q_0^2 is adjustable scale parameter controlling transition between low and high Q regimes.

Off-shell function

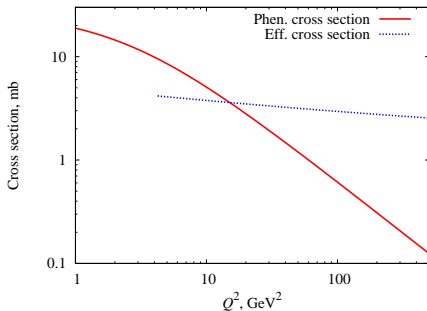
- ▶ The function $\delta f(x)$ provides a measure of the modification of the quark distributions in a bound nucleon.
- ▶ The slope of $\delta f(x)$ in a single-scale nucleon model is related to $d \log \Lambda / d \log p^2$. The observed slope suggests an increase in the bound nucleon radius in Iron by about 10% and in the deuteron by about 2%.



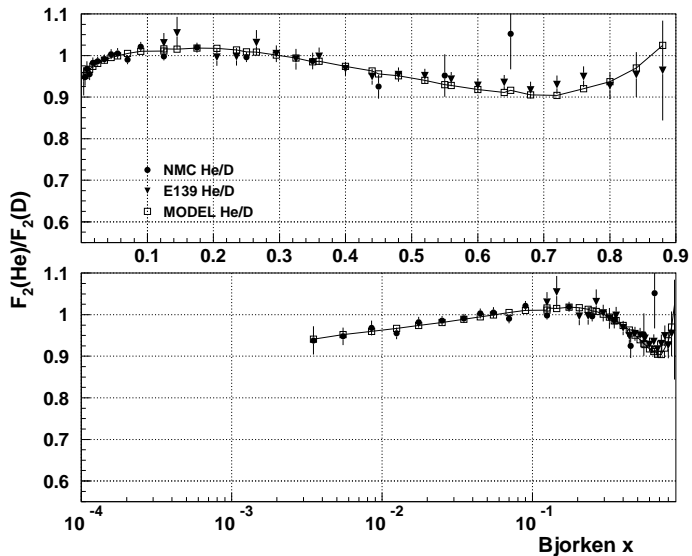
Effective cross section

- ▶ The monopole form $\sigma_T = \sigma_0 / (1 + Q^2/Q_0^2)$ with $\sigma_0 = 27$ mb and $Q_0^2 = 1.43 \pm 0.06 \pm 0.195$ GeV² provides a good fit to existing DIS data on nuclear shadowing for $Q^2 < 20$ GeV².
- ▶ The cross section at high Q^2 is not constrained by data. However, it is possible to evaluate using the results on the off-shell function and normalization condition.

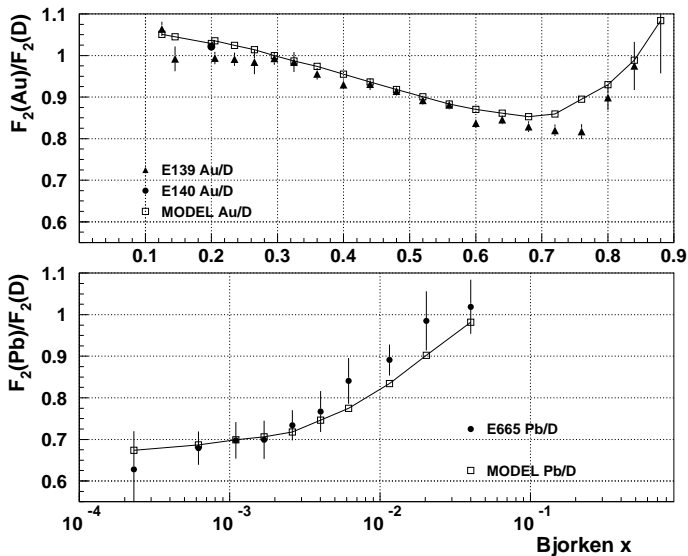
We require exact cancellation between off-shell (OS) and shadowing (NS) contributions to normalization: $\delta N_{\text{val}}^{\text{OS}} + \delta N_{\text{val}}^{\text{NS}} = 0$. Numeric solution to this equation is shown by blue curve.



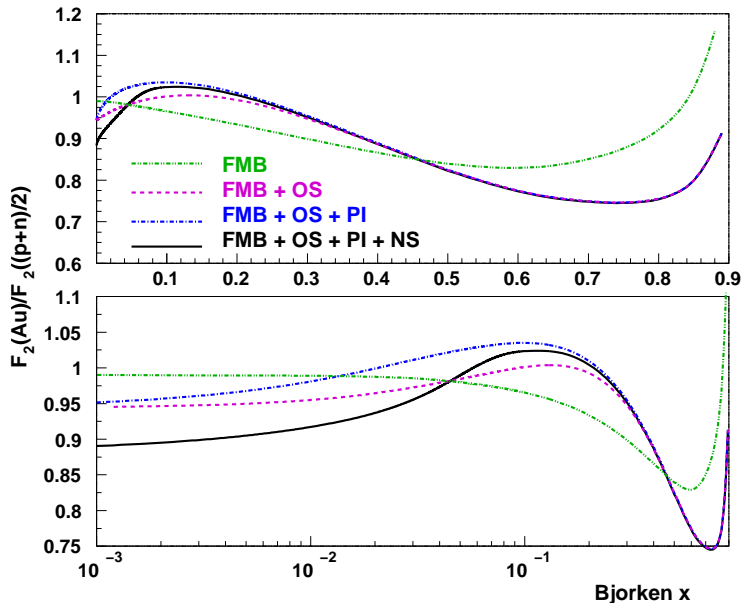
$^4\text{He}/\text{D}$



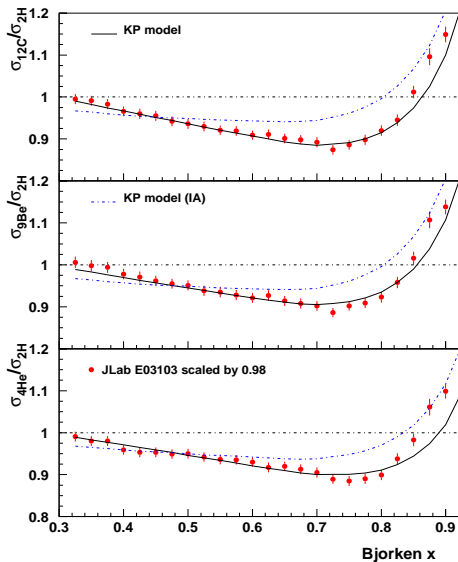
$^{197}\text{Au}/\text{D}$ & $^{207}\text{Pb}/\text{D}$



Different nuclear effects for ^{197}Au at $Q^2 = 10 \text{ GeV}^2$

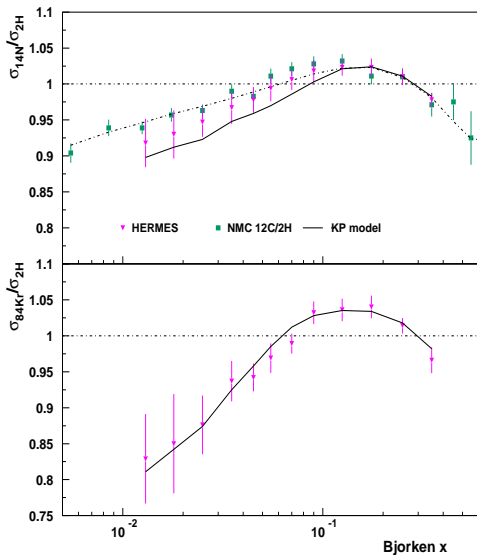


Comparison with E03-103 (not a fit) *S.K. & R.Petti, 2010*



- ▶ Apply overall normalization factor 0.98 to JLab data on ${}^4\text{He}/\text{D}$, ${}^9\text{Be}/\text{D}$ and ${}^{12}\text{C}/\text{D}$
- ▶ Very good agreement of our predictions with JLab E03-103 for all nuclear targets: $\chi^2/d.o.f. = 26.3/60$ for $W^2 > 2 \text{ GeV}^2$
- ▶ Nuclear corrections at large x is driven by nuclear spectral function, the off-shell function $\delta f(x)$ was fixed from previous studies.
- ▶ A comparison with the Impulse Approximation (shown in blue) demonstrates that the off-shell correction is crucial to describe the data leading to both modification of the slope and position of the minimum of the EMC ratios.

Comparison with HERMES (not a fit) *S.K. & R.Petti, 2010*



- ▶ A good agreement of our predictions with HERMES data for $^{14}\text{N}/\text{D}$ and $^{84}\text{Kr}/\text{D}$ with $\chi^2/d.o.f. = 14.7/24$
- ▶ A comparison with NMC data for $^{12}\text{C}/\text{D}$ shows a significant Q^2 dependence at small x in the shadowing region related to the cross-section for scattering of hadronic states off the bound nucleons. The model correctly describes the observed x and Q^2 dependence.

Application to neutrino scattering

Neutrino scattering is affected by both vector (V) and axial-vector (A) currents.

$$VV, AA \implies F_{1,2} \quad (\text{or } F_L, F_T)$$

$$VA \implies F_3 \quad (\text{not present for electromagnetic current})$$

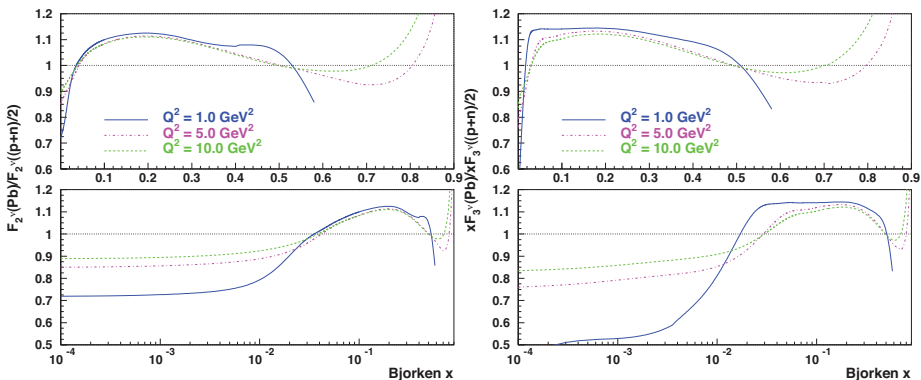
(Anti)neutrino differential cross sections in terms of Bjorken x and inelasticity y :

$$\frac{d^2\sigma_{\text{CC}}^{(\nu, \bar{\nu})}}{dx dy} = \frac{G_F^2 M E}{\pi(1 + Q^2/M_W^2)^2} [Y_+ F_2^{\nu, \bar{\nu}} - y^2 x F_L^{\nu, \bar{\nu}} \pm Y_- x F_3^{\nu, \bar{\nu}}],$$

$$Y_+ = \frac{1}{2} [1 + (1 - y)^2] + M^2 x^2 y^2 / Q^2,$$

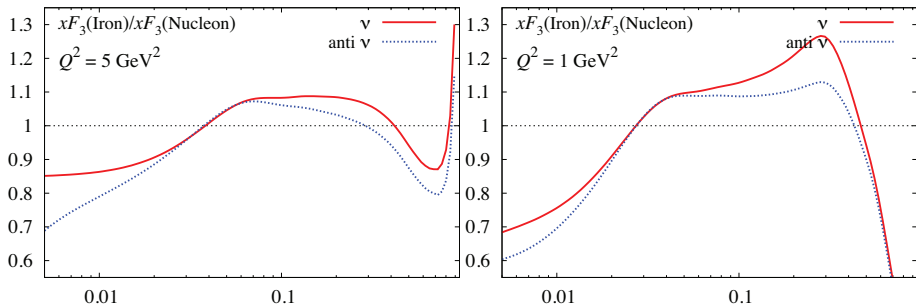
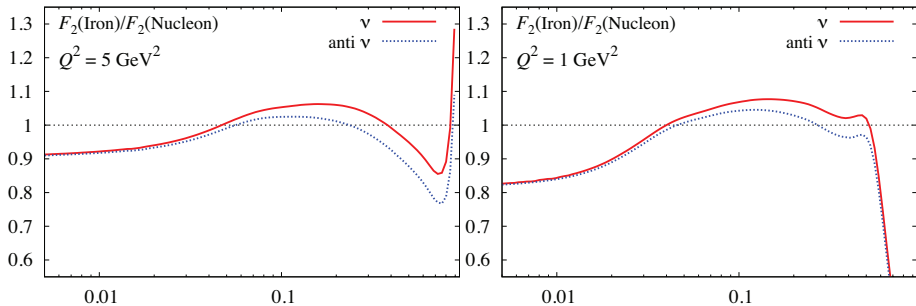
$$Y_- = \frac{1}{2} [1 - (1 - y)^2].$$

Nuclear effects for F_2 vs. xF_3



Ratio of Charged Current structure functions on ^{207}Pb and isoscalar nucleon $(p+n)/2$

Nuclear effects for ν vs. $\bar{\nu}$



Neutrino cross sections

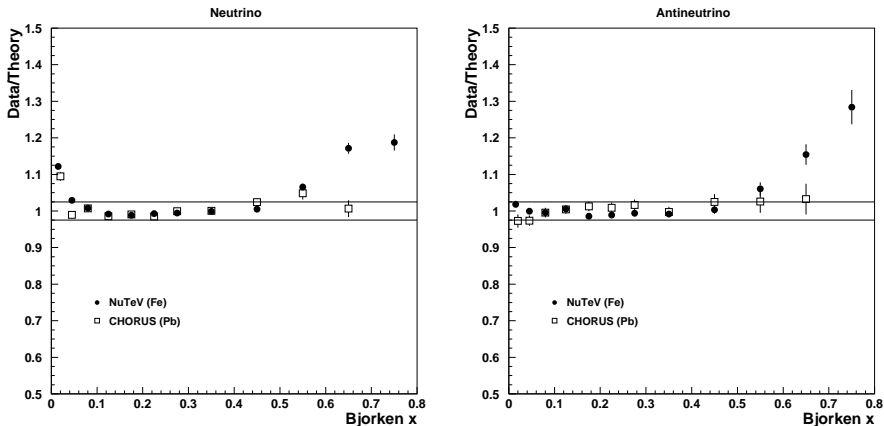
$$\frac{d\sigma}{dx} = \int dy \frac{d^2\sigma}{dx dy} \theta(Q^2 - Q_{cut}^2) \theta(W^2 - W_{cut}^2)$$
$$\sigma_{\text{tot}} = \int dx dy \frac{d^2\sigma}{dx dy} \theta(Q^2 - Q_{cut}^2) \theta(W^2 - W_{cut}^2)$$

Contribution from DIS $\implies Q_{cut}^2 = 1 \text{ GeV}^2$ and $W_{cut}^2 = 4 \text{ GeV}^2$

These cuts restricts the integration in (x, y) plane:

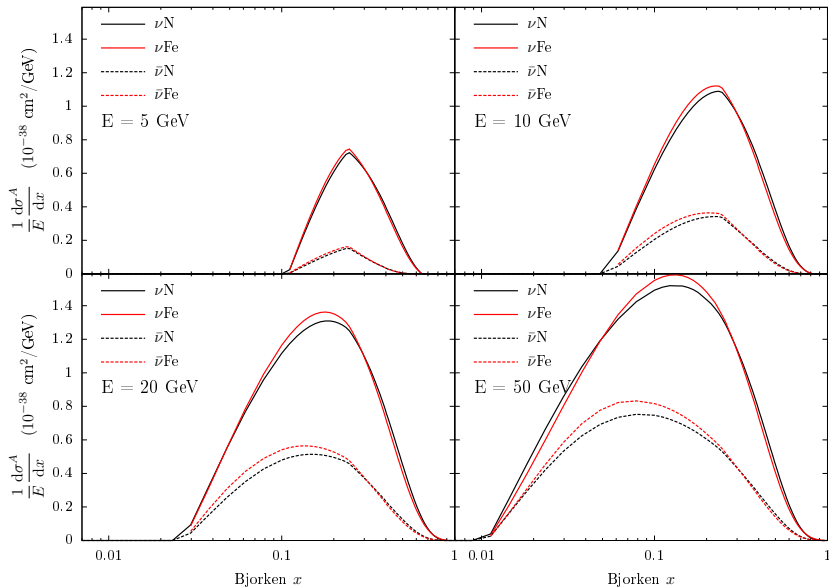
$$\frac{Q_{cut}^2}{2ME} \leq x \leq 1 - \frac{W_{cut}^2 - M^2}{2ME} \max \left[\frac{Q_{cut}^2}{2ME x}, \frac{W_{cut}^2 - M^2}{2ME(1-x)} \right] \leq y \leq 1$$

CHORUS and NuTeV diff cross sections (High E)



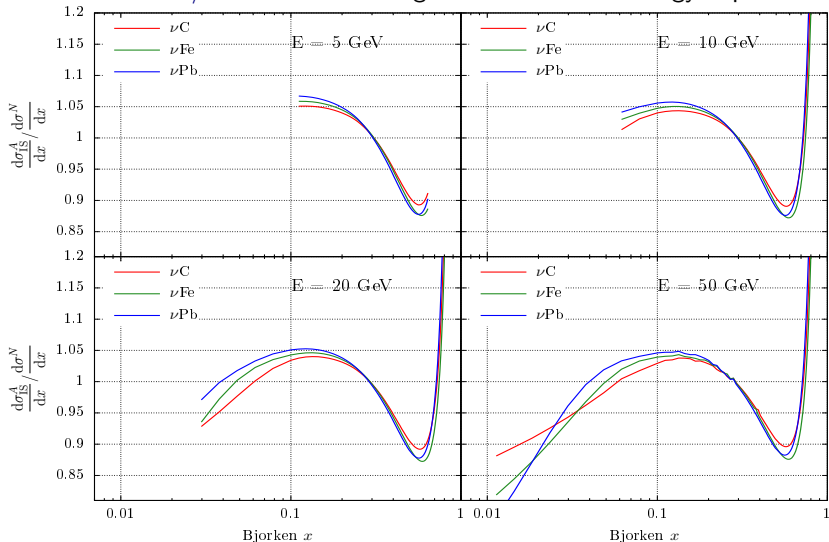
Data/model predictions by [S.K. and R.Petti, NPA 765 \(2006\) 126; PRD 76 \(2007\) 094023](#). The x -point is the weighted average over available E and y . The solid horizontal lines indicate a $\pm 2.5\%$ band.

Energy dependence of $(d\sigma/dx)/E$ (Intermediate E)

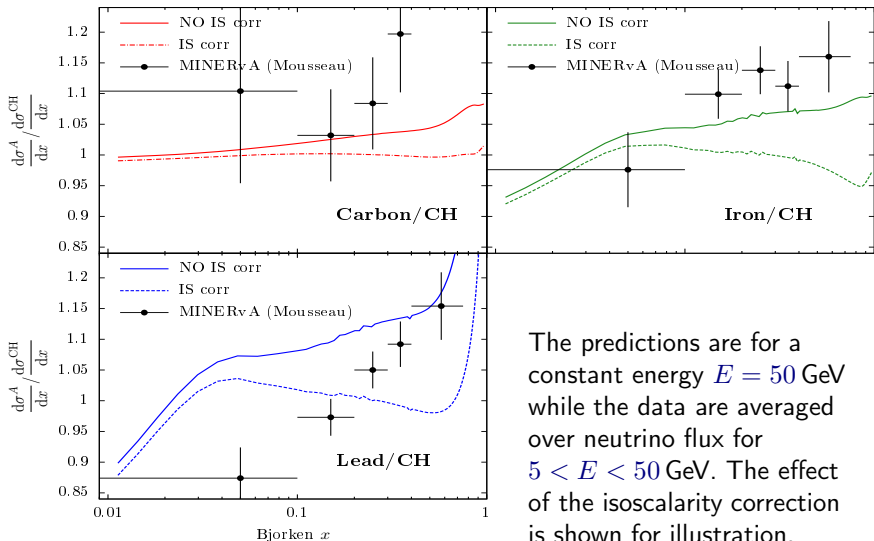


Nuclear ratios of $d\sigma/dx$

The ratios of $d\sigma/dx$ for different targets are much less energy-dependent:

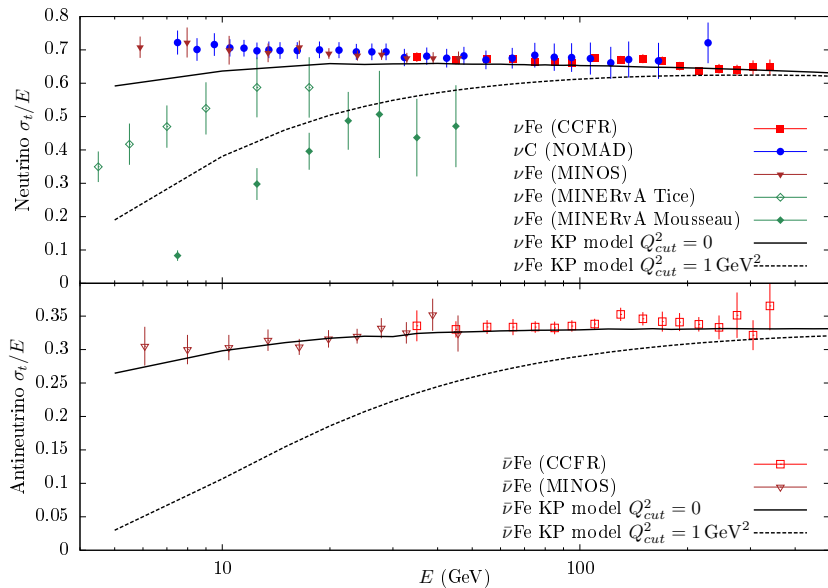


Comparison with MINERvA data on the ratios of $d\sigma/dx$

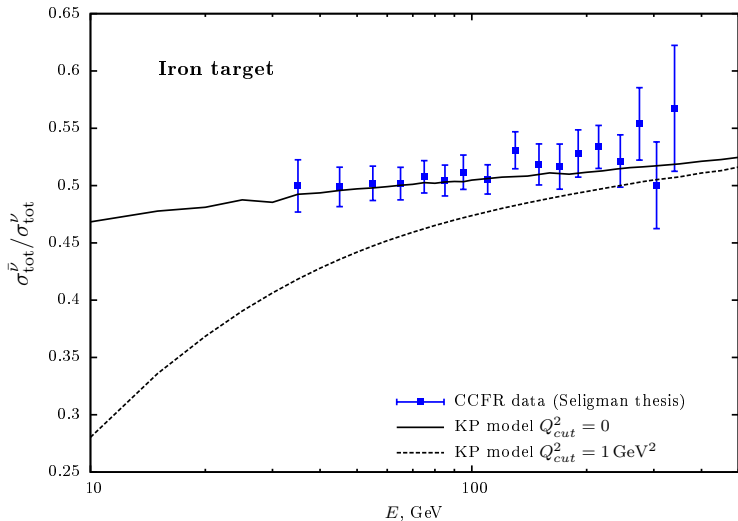


The predictions are for a constant energy $E = 50$ GeV while the data are averaged over neutrino flux for $5 < E < 50$ GeV. The effect of the isoscalarity correction is shown for illustration.

Absolute total cross sections

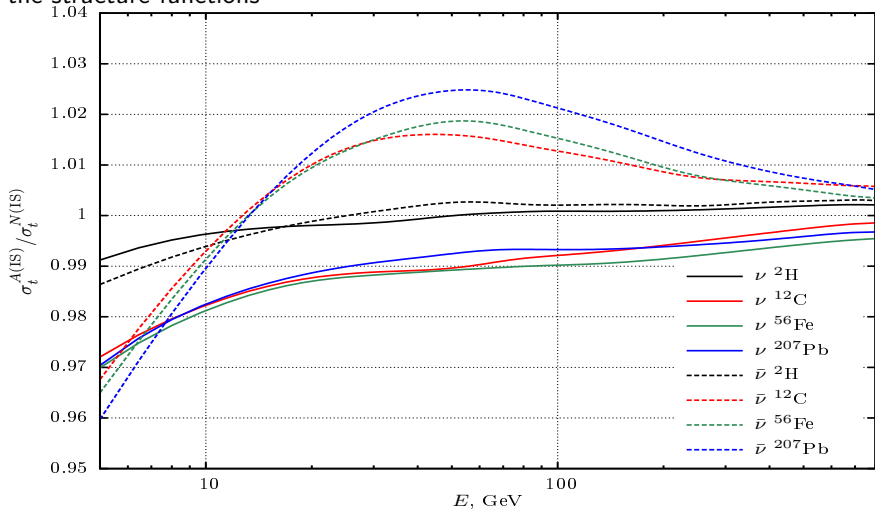


$\bar{\nu}/\nu$ ratio of total cross sections

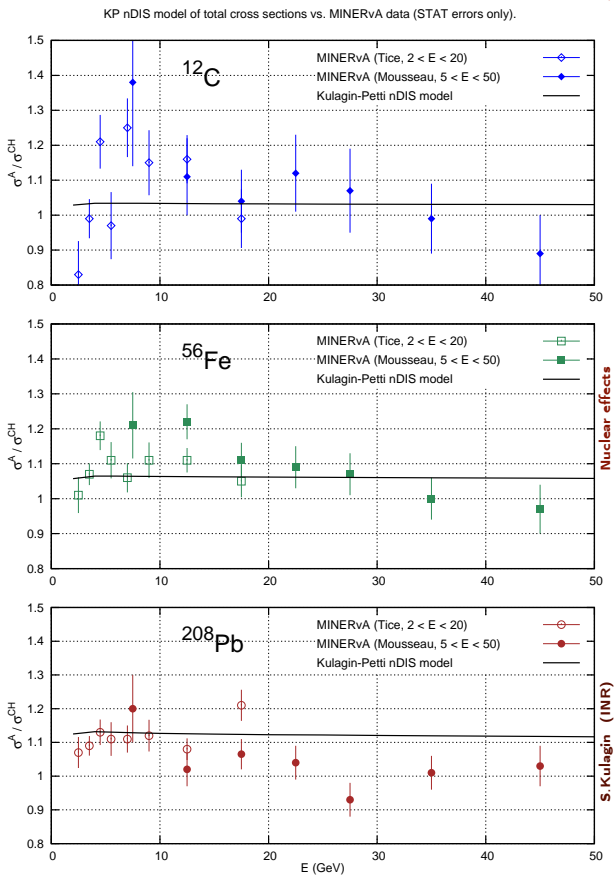


Nuclear ratios of total cross sections

The total cross sections show an order of magnitude less nuclear corrections vs. the structure functions



Comparison with MINERvA total cross sections



Summary

- ▶ We discussed the performance of a semi-microscopic model of nuclear corrections in DIS. Note the importance of nuclear binding along with off-shell corrections. The model provides a good agreement with observed x , A and Q^2 dependencies of nuclear EMC effect.

The EMC effect at large x is explained by nuclear binding together with off-shell correction.

- ▶ We discussed application of the model to study various combinations of ν and $\bar{\nu}$ structure functions. Nuclear corrections depend on the type of the structure functions (F_2 vs. F_3) as well as on the type of the probe (ν vs. $\bar{\nu}$).
- ▶ Neutrino cross section studies.

Good agreement (within $\pm 2.5\%$ band) with the CHORUS ^{208}Pb data on double differential cross sections everywhere except for the smallest available x bin. Good agreement with NuTeV ^{56}Fe data for $0.15 < x < 0.55$. Systematic excess of data/theory for the NuTeV data at large $x > 0.5$ for both the neutrino and antineutrino. Note also about 10% data/theory excess for small $x = 0.015$ for neutrino scattering for both ^{208}Pb and ^{56}Fe data that may indicate smaller shadowing correction for neutrino.

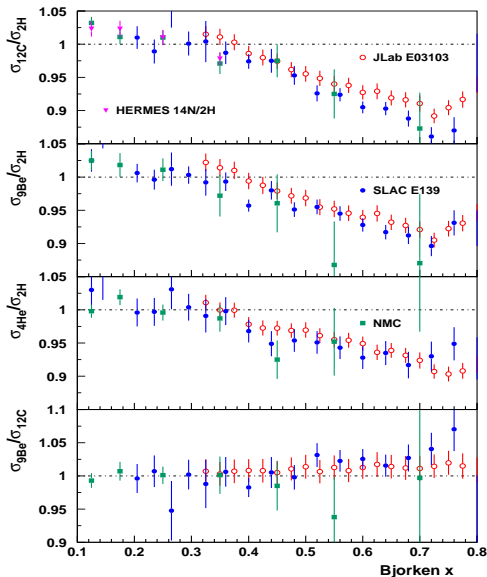
- ▶ Much smaller nuclear corrections on the total cross sections than on structure functions.
- ▶ Reasonable agreement with MINERvA measurement for Iron at intermediate energy. We have less shadowing effect for Lead than measured by MINERvA.

Backup

Targets	χ^2/DOF						
	NMC	EMC	E139	E140	BCDMS	E665	HERMES
$^4\text{He}/^2\text{H}$	10.8/17		6.2/21				
$^7\text{Li}/^2\text{H}$	28.6/17						
$^9\text{Be}/^2\text{H}$			12.3/21				
$^{12}\text{C}/^2\text{H}$	14.6/17		13.0/17				
$^9\text{Be}/^{12}\text{C}$	5.3/15						
$^{12}\text{C}/^7\text{Li}$	41.0/24						
$^{14}\text{N}/^2\text{H}$							9.8/12
$^{27}\text{Al}/^2\text{H}$			14.8/21				
$^{27}\text{Al}/^{12}\text{C}$	5.7/15						
$^{40}\text{Ca}/^2\text{H}$	27.2/16		14.3/17				
$^{40}\text{Ca}/^7\text{Li}$	35.6/24						
$^{40}\text{Ca}/^{12}\text{C}$	31.8/24						
$^{56}\text{Fe}/^2\text{H}$			18.4/23	4.5/8	14.8/10	1.0/5	
$^{56}\text{Fe}/^{12}\text{C}$	10.3/15						
$^{63}\text{Cu}/^2\text{H}$		7.8/10					
$^{84}\text{Kr}/^2\text{H}$							4.9/12
$^{108}\text{Ag}/^2\text{H}$			14.9/17				
$^{119}\text{Sn}/^{12}\text{C}$	94.9/161						
$^{197}\text{Au}/^2\text{H}$			18.2/21	2.4/1			
$^{207}\text{Pb}/^2\text{H}$						5.0/5	
$^{207}\text{Pb}/^{12}\text{C}$	6.1/15					0.2/5	

Values of χ^2/DOF between different data sets with $Q^2 \geq 1 \text{ GeV}^2$ and the predictions of KP model [NPA765\(2006\)126](#); [PRC82\(2010\)054614](#). The sum over all data results in $\chi^2/\text{DOF} = 466.6/586$.

Comment on normalization of different experiments



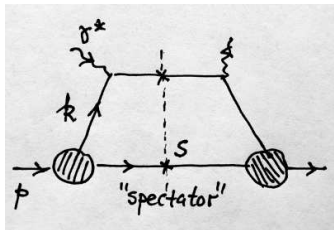
- ▶ Shapes of all nuclear cross-section ratios are consistent
- ▶ Evaluate χ^2 for each pair of experiments in coarse x -bins within the overlap region of the data sets
- ▶ Consistent overall normalization for **SLAC E139**, **NMC** and **HERMES** data sets
- ▶ Recent **JLab E03-103** data is systematically above previous measurements resulting in a $\chi^2/d.o.f. = 42.7/12$ with respect to **SLAC E139** data on the same targets
- ▶ An overall normalization factor 0.98 for all **JLab E03-103** points improves the statistical consistency with **SLAC E139**
S.K. and R. Petti, PRC82 (2010)

Off-shell effect and the bound nucleon radius

The valence quark distribution in (off-shell) nucleon
(see, e.g., *Kulagin, Piller & Weise, PRC50, 1154 (1994)*)

$$q_{\text{val}}(x, p^2) = \int^{k^2_{\text{max}}} dk^2 \Phi(k^2, p^2)$$

$$k^2_{\text{max}} = x (p^2 - s/(1-x))$$



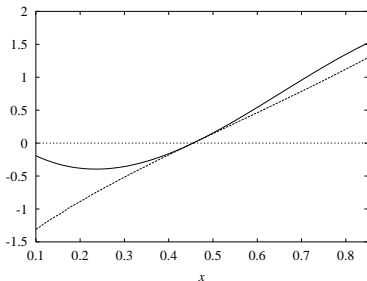
- ▶ A one-scale model of quark k^2 distribution: $\Phi(k^2) = C\phi(k^2/\Lambda^2)/\Lambda^2$, where C and ϕ are dimensionless and Λ is the scale.
- ▶ Off-shell: $C \rightarrow C(p^2)$, $\Lambda \rightarrow \Lambda(p^2)$
- ▶ The derivatives $\partial_x q_{\text{val}}$ and $\partial_{p^2} q_{\text{val}}$ are related

$$\delta f(x) = \frac{\partial \ln q_{\text{val}}}{\partial \ln p^2} = c + \frac{dq_{\text{val}}(x)}{dx} x(1-x)h(x)$$

$$h(x) = \frac{(1-\lambda)(1-x) + \lambda s/M^2}{(1-x)^2 - s/M^2}$$

$$c = \frac{\partial \ln C}{\partial \ln p^2}, \quad \lambda = \frac{\partial \ln \Lambda^2}{\partial \ln p^2}$$

- ▶ A simple pole model $\phi(y) = (1 - y)^{-n}$ (note that $y < 0$ so we do not run into singularity) provides a reasonable description of the nucleon valence distribution for $x > 0.2$ and large Q^2 ($s = 2.1 \text{ GeV}^2$, $\Lambda^2 = 1.2 \text{ GeV}^2$, $n = 4.4$ at $Q^2 = 15 \div 30 \text{ GeV}^2$).
- ▶ The size of the valence quark confinement region $R_c \sim \Lambda^{-1}$ (nucleon core radius).
- ▶ Off-shell corection is independent of specific choice of profile $\phi(y)$ and is given by $(\ln q_{\text{val}}(x))'$.
- ▶ Fix c and λ to reproduce $\delta f_2(x_0) = 0$ and the slope $\delta f_2'(x_0)$. We obtain $\lambda \approx 1$ and $c \approx -2.3$. The positive parameter λ suggests decreasing the scale Λ in nuclear environment (swelling of a bound nucleon)



$$\frac{\delta R_c}{R_c} \sim -\frac{1}{2} \frac{\delta \Lambda^2}{\Lambda^2} = -\frac{1}{2} \lambda \frac{\langle p^2 - M^2 \rangle}{M^2}$$

$${}^{56}\text{Fe} : \delta R_c/R_c \sim 9\%$$

$${}^2\text{H} : \delta R_c/R_c \sim 2\%$$

PCAC and neutrino scattering

Axial current is not conserved and dominates at low Q^2 (*Adler 1966*)

$$\text{PCAC: } \partial A = f_\pi m_\pi^2 \varphi \implies F_L = \frac{f_\pi^2 \sigma_\pi}{\pi} + \mathcal{O}(Q^2)$$

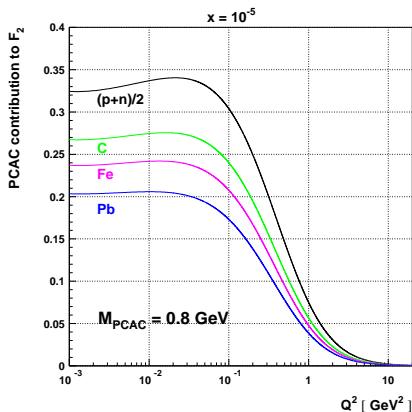
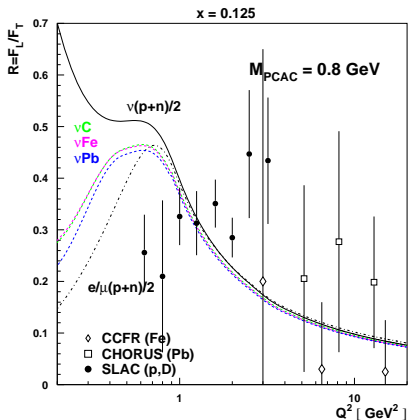
What is relevant Q^2 scale for this relation?

- ▶ It is widely believed that the scale $\sim m_\pi^2$.
- ▶ However the contribution from the pion current $f_\pi \partial_\mu \varphi \propto q_\mu$ cancels out in the cross sections.
- ▶ Therefore the transition scale M_{PCAC} between low and high Q^2 is NOT m_π^2 but rather determined by the mass of meson resonance with relevant quantum numbers, $M_{\text{PCAC}} \sim 1 \text{ GeV}$.

Model that interpolates between low and high Q^2 (*S.K. and R. Petti, PRD76,094023(2007)*):

$$F_L = \frac{f_\pi^2 \sigma_\pi}{\pi} \left(1 + \frac{Q^2}{M_{\text{PCAC}}^2} \right)^{-2} + \tilde{F}_L$$
$$\tilde{F}_L = \begin{cases} F_L^{\text{QCD}} & , Q > 1 \text{ GeV}, \\ \propto Q^4 & , Q \rightarrow 0 \end{cases}$$

The PCAC term in F_L^ν strongly affects the asymptotic behavior of $R = F_L/F_T$ as $Q^2 \rightarrow 0$



Determination for ^{56}Fe target: $F_2^\nu(Q^2 \rightarrow 0) = 0.21 \pm 0.02$ by *CCFR Coll. PRL 86 (2001) 5430*