(toward) NLO di-jet production in DIS at small x

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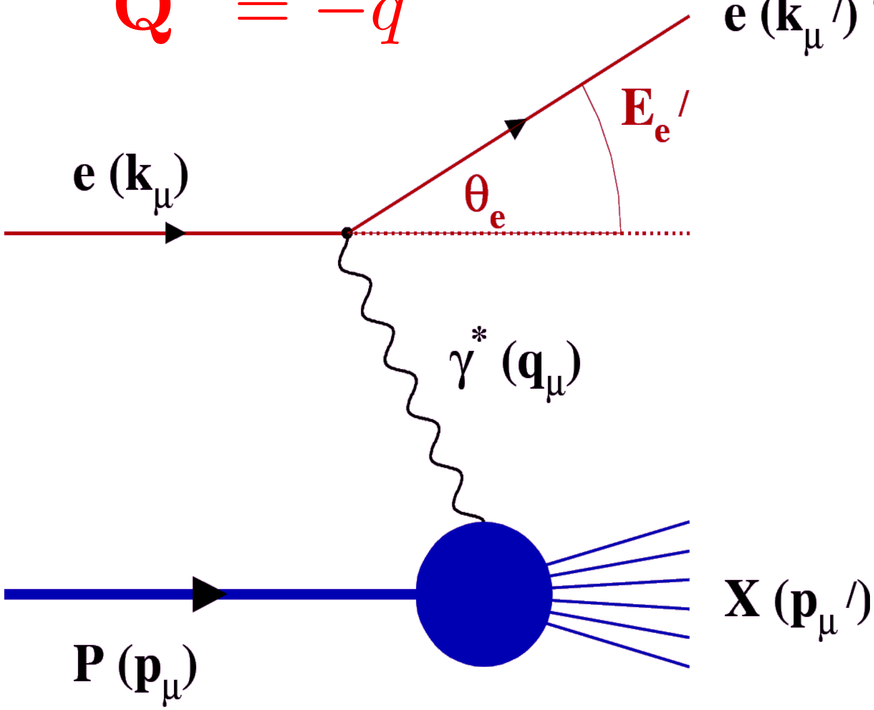
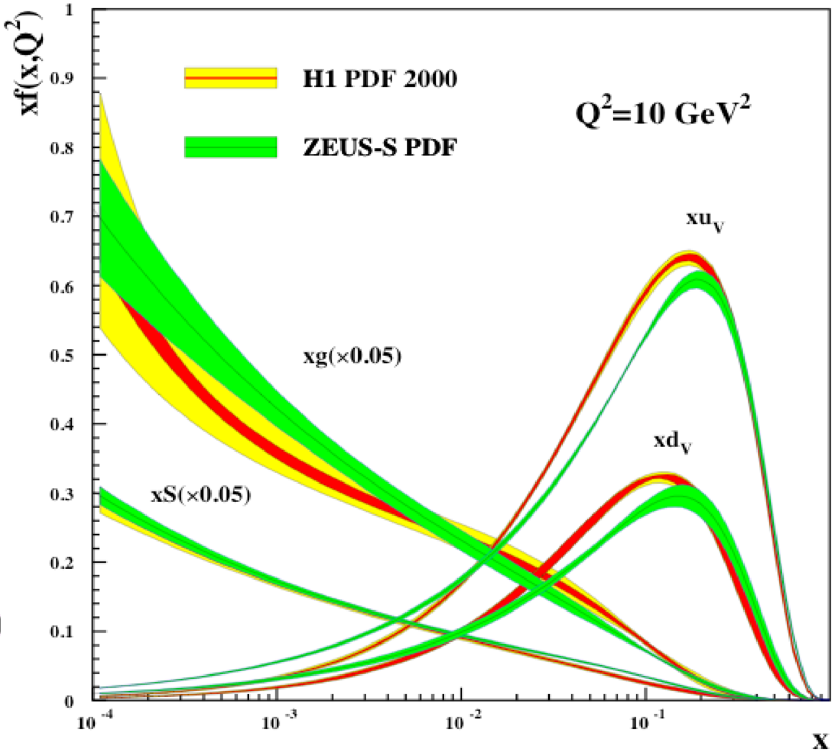
DIS at HERA: parton distributions

$\mathbf{Q^2} = -q^2$



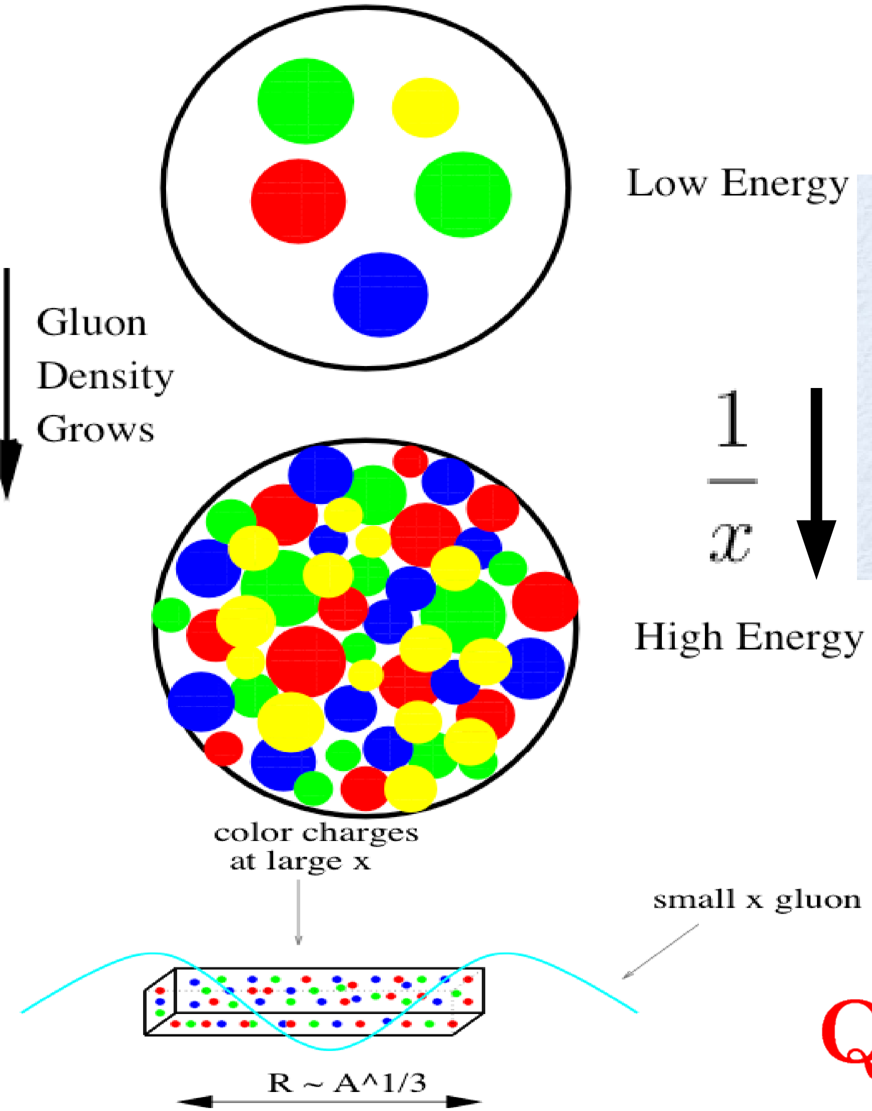


power-like growth of gluon and sea quark distributions with x

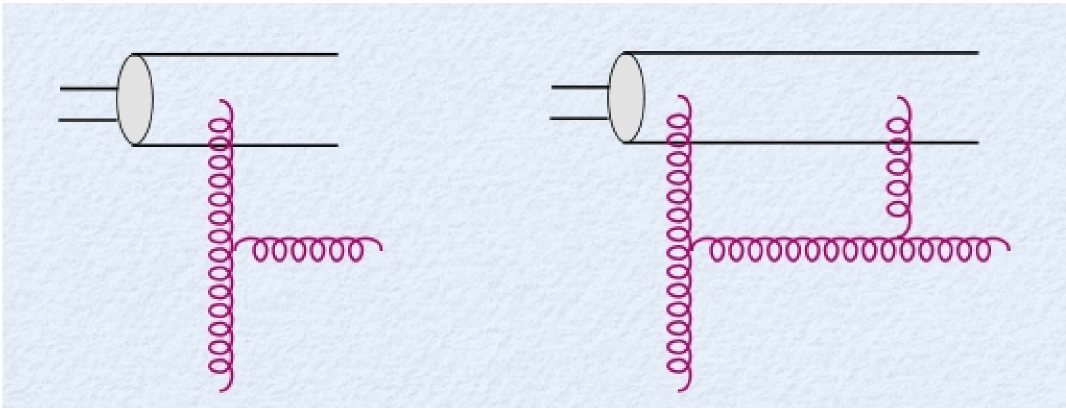
new QCD dynamics at small x?

Gluon saturation

Gribov-Levin-Ryskin
Mueller-Qiu



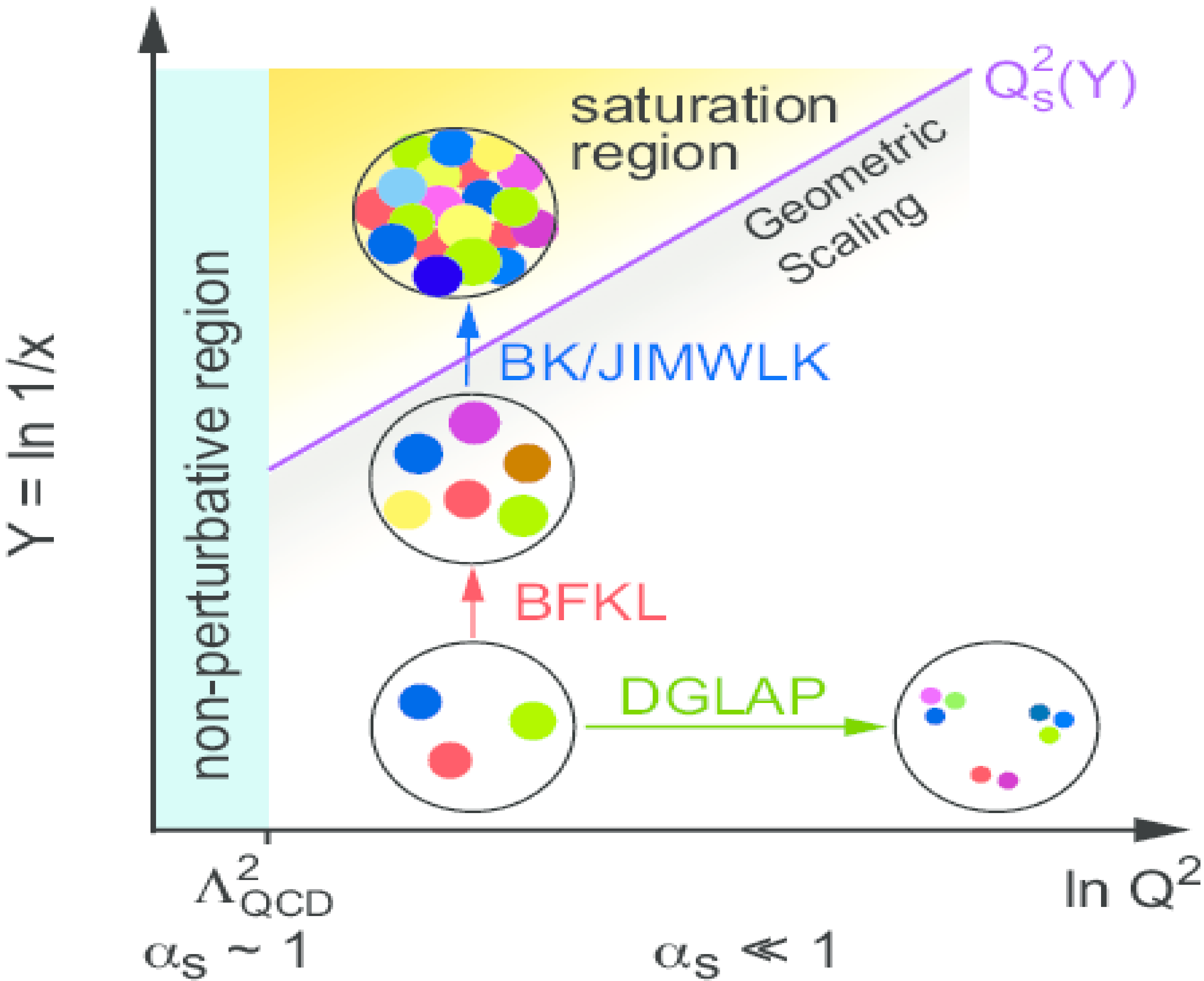
"attractive" bremsstrahlung vs.
"repulsive" recombination



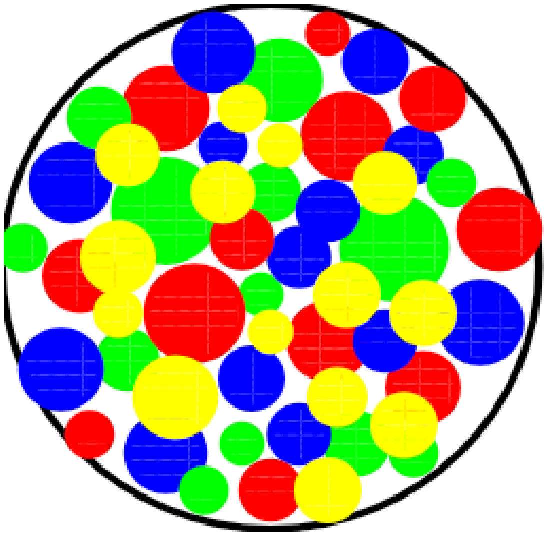
$$\frac{\alpha_s}{Q^2}\frac{xG(x, b_t, Q^2)}{S_\perp} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

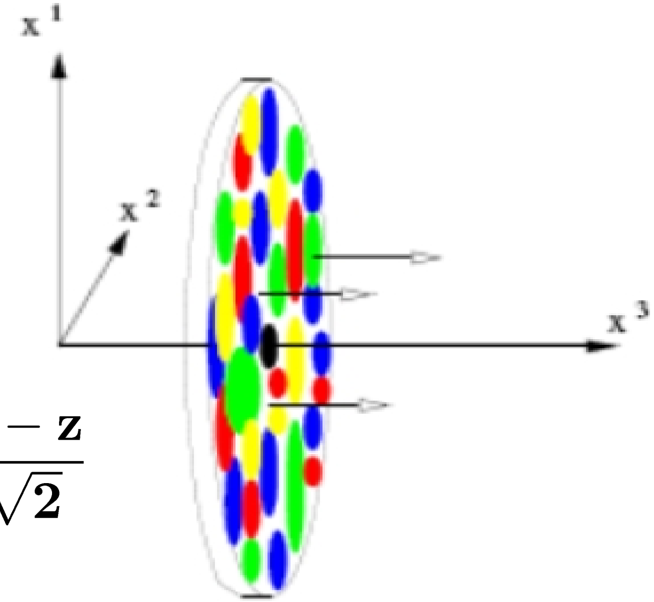
A proton at high energy: saturation



Large A/high energy → saturation



boost →



$$\mathbf{x}^{+} \equiv \frac{\mathbf{t}+\mathbf{z}}{\sqrt{2}} \qquad \mathbf{x}^{-} \equiv \frac{\mathbf{t}-\mathbf{z}}{\sqrt{2}}$$

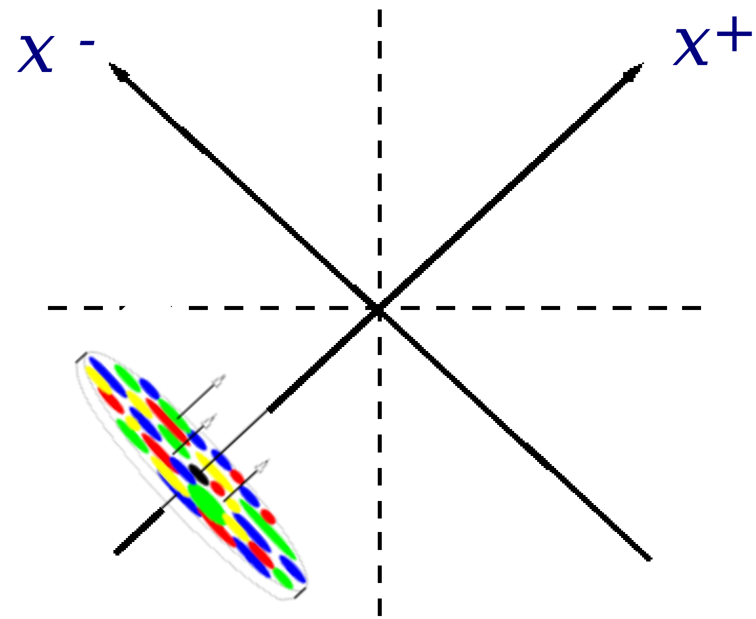
sheet of color charge moving along x^+ and sitting at $x^- = 0$

$$\boxed{\mathbf{J}^{\mu}_{\mathbf{a}}(\mathbf{x}) \equiv \delta^{\mu\,+}\,\delta(\mathbf{x}^{-})\,\rho_{\mathbf{a}}(\mathbf{x_t})}$$

color current — *color charge*

$$\mathbf{A}^{+}_{\mathbf{a}}(\mathbf{z}^{-},\mathbf{z_t}) = \delta(\mathbf{z}^{-})\,\alpha_{\mathbf{a}}(\mathbf{z_t})$$

with $\partial_t^2\,\alpha_a(z_t) = g\rho_a(z_t)$



low x QCD in a background field: CGC

(a high gluon density environment)

two main effects:

*"multiple scatterings" encoded in classical field (**p_t broadening**)*

*evolution with ln (1/x) a la BK/JIMWLK equation (**suppression**)*

LT pQCD with collinear factorization:

single scattering

evolution with ln Q^2

Signatures

dense-dense (AA, pA, pp) collisions

multiplicities, spectra

long range rapidity correlations

dilute-dense (pA, forward pp) collisions

multiplicities

p_t *spectra*

angular correlations

DIS

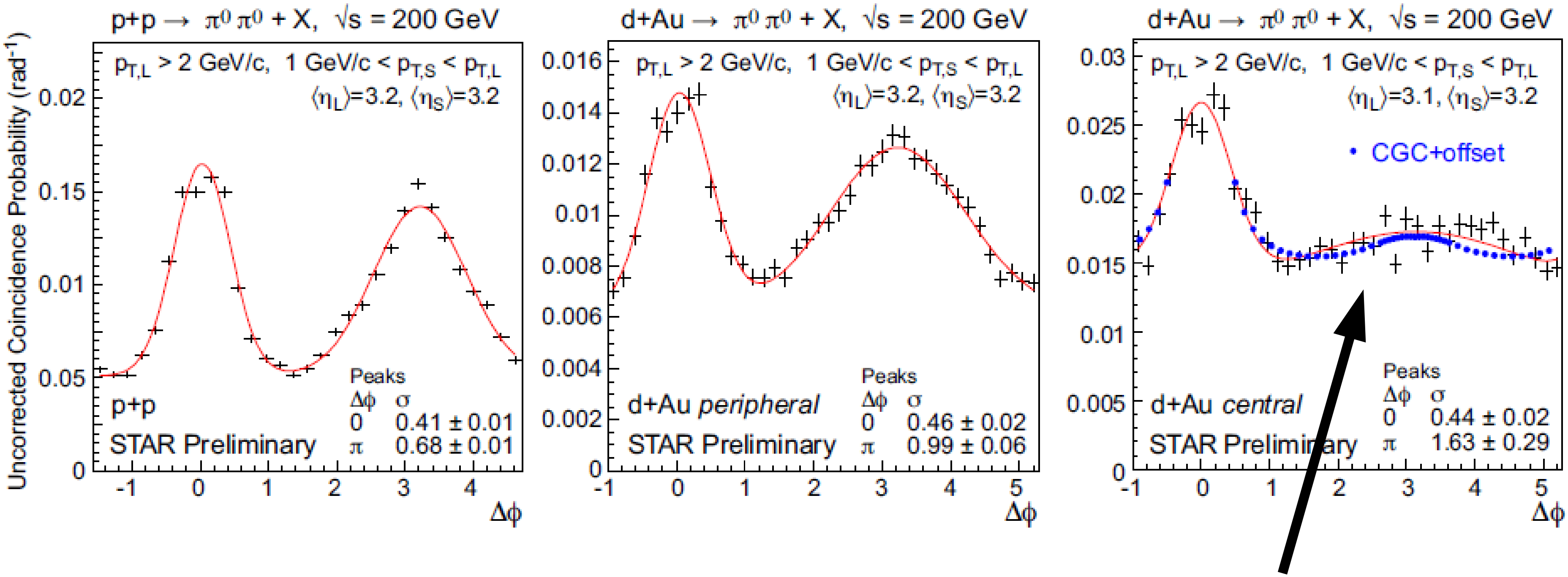
structure functions (diffraction)

di-hadron correlations

spin asymmetries

di-hadron correlations are a sensitive probe of CGC

Recent STAR measurement (arXiv:1008.3989v1):



Marquet, NPA (2007), Albacete + Marquet, PRL (2010)
Tuchin, NPA846 (2010)
A. Stasto + B-W. Xiao + F. Yuan, PLB716 (2012)
T. Lappi + H. Mantysaari, NPA908 (2013)

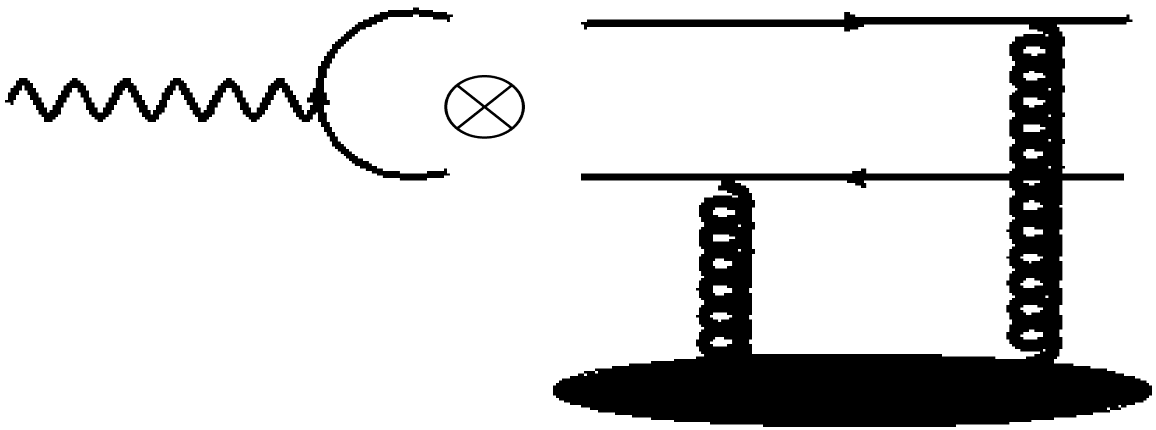
saturation effects de-correlate the hadrons

shadowing+energy loss: Z. Kang, I. Vitev, H. Xing, PRD85 (2012) 054024

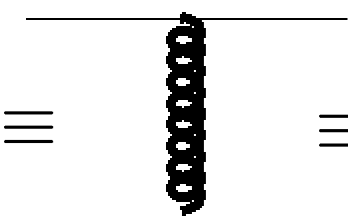
DIS total cross section

$$\sigma^{\mathrm{total}}_{\mathrm{DIS}} = 2\int_0^1 dz \int d^2x_t\, d^2y_t \left|\Psi(k^{\pm}, k_t | z, x_t, y_t)\right|^2 T(x_t, y_t)$$

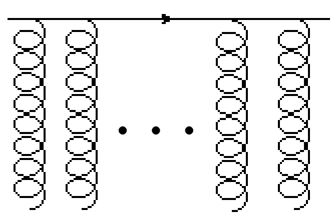
$$T(x_t, y_t) \equiv \frac{1}{N_c} \mathrm{Tr} \left\langle 1 - V(x_t) V^{\dagger}(y_t) \right\rangle$$



$$V \equiv$$



$$\equiv$$



$$\sim 1 + O(g\, A) + O(g^2\, A^2)$$

Wilson line encodes multiple scatterings from the color field of the target

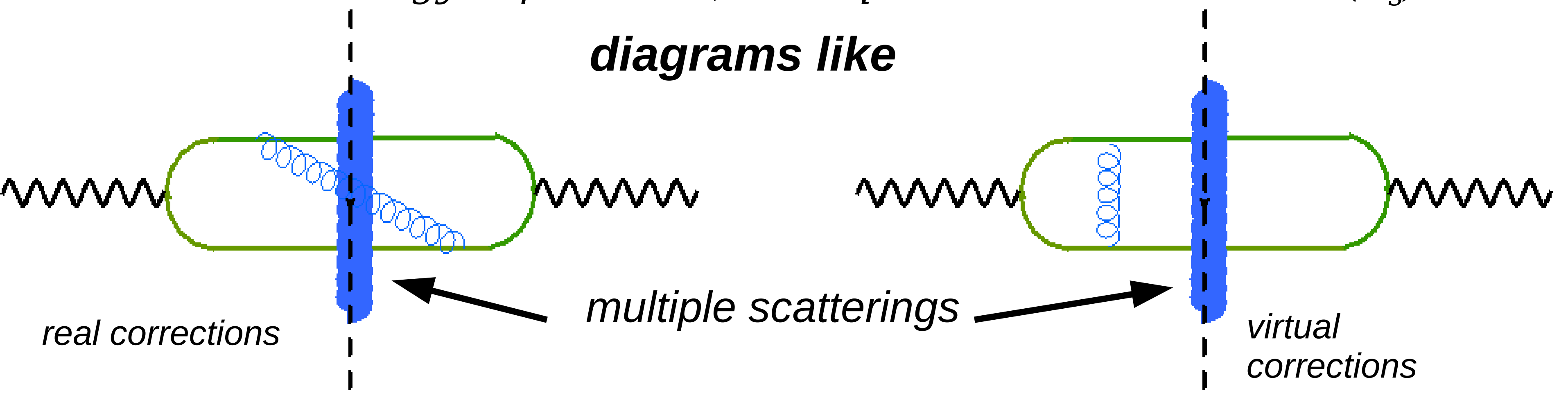
DIS total cross section: energy (x) dependence

recall the parton model was scale invariant, scaling violation (dependence on Q^2) came after quantum corrections - $O(\alpha_s)$

what we have done so far is to include high gluon density effects but no energy dependence yet

to include the energy dependence, need quantum corrections - $O(\alpha_s)$

diagrams like

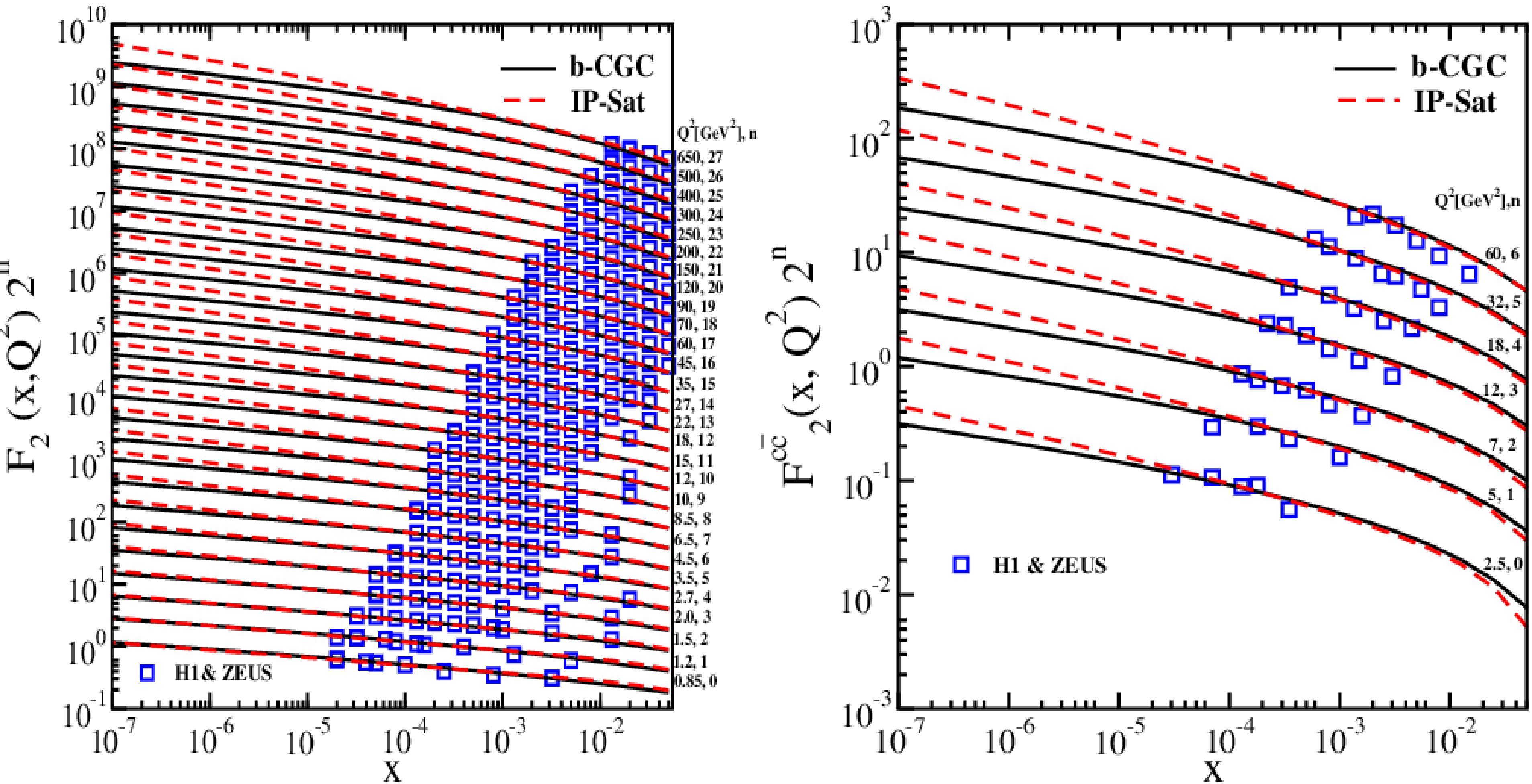


x dependence of dipole cross section: BK/JIMWLK evolution equation

NLO corrections recently computed

Extensive phenomenology at HERA

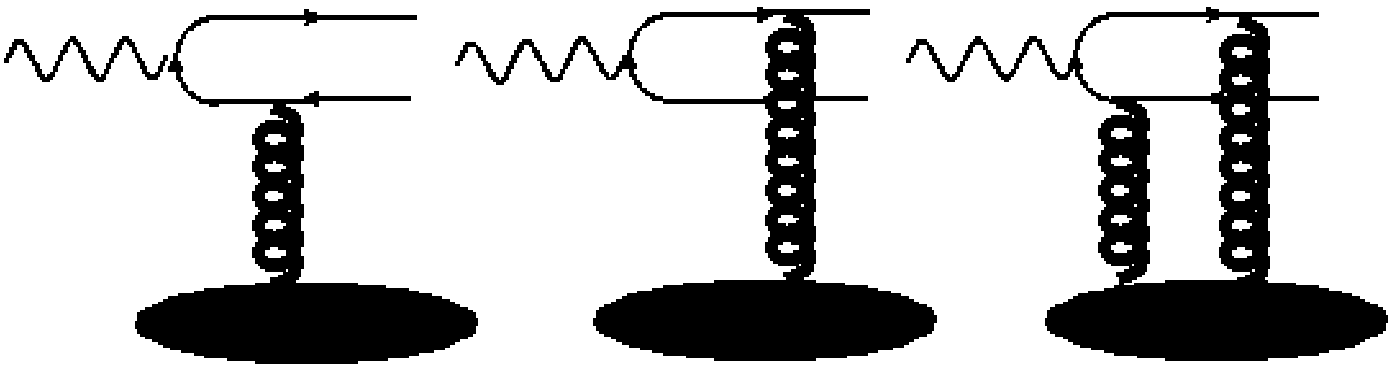
HERA



A. Rezaeian and I. Schmidt, PRD88 (2013) 074016 something with more discriminating power:

di-hadron correlations in DIS

LO: $\gamma^\star(\mathbf{k})\,\mathbf{p} \rightarrow \mathbf{q}(\mathbf{p})\,\bar{\mathbf{q}}(\mathbf{q})\,\mathbf{X}$



quark propagator in the background color field

$$\mathrm{S}_F(q,p) \equiv (2\pi)^4\delta^4(p-q)\,S_F^0(p) + S_F^0(q)\,\tau_f(q,p)\,S_F^0(p)$$

$$\tau_f(q,p) \equiv (2\pi)\delta(p^- - q^-)\,\gamma^- \int d^2x_t\, e^{i(q_t-p_t)\cdot x_t} \left\{\theta(p^-)[V(x_t)-1] - \theta(-p^-)[V^\dagger(x_t)-1]\right\}$$

di-hadron production in DIS

$$\gamma^\star(\mathbf{k})\,\mathbf{p} \to \mathbf{q}(\mathbf{p})\,\bar{\mathbf{q}}(\mathbf{q})\,\mathbf{X}$$

$$\mathcal{A}^\mu(k,q,p) = \frac{i}{2}\int \frac{d^2l_\perp}{(2\pi)^2} d^2x_\perp d^2y_\perp\, e^{i(p_\perp+q_\perp-k_\perp-l_\perp)\cdot y_\perp}$$
$$e^{il_\perp\cdot x_\perp}\,\overline{u}(q)\,\Gamma^\mu(k^\pm, k_\perp, q^-, p^-, q_\perp - l_\perp)\,v(p)$$
$$\left[V(x_\perp)V^\dagger(y_\perp) - 1\right]$$

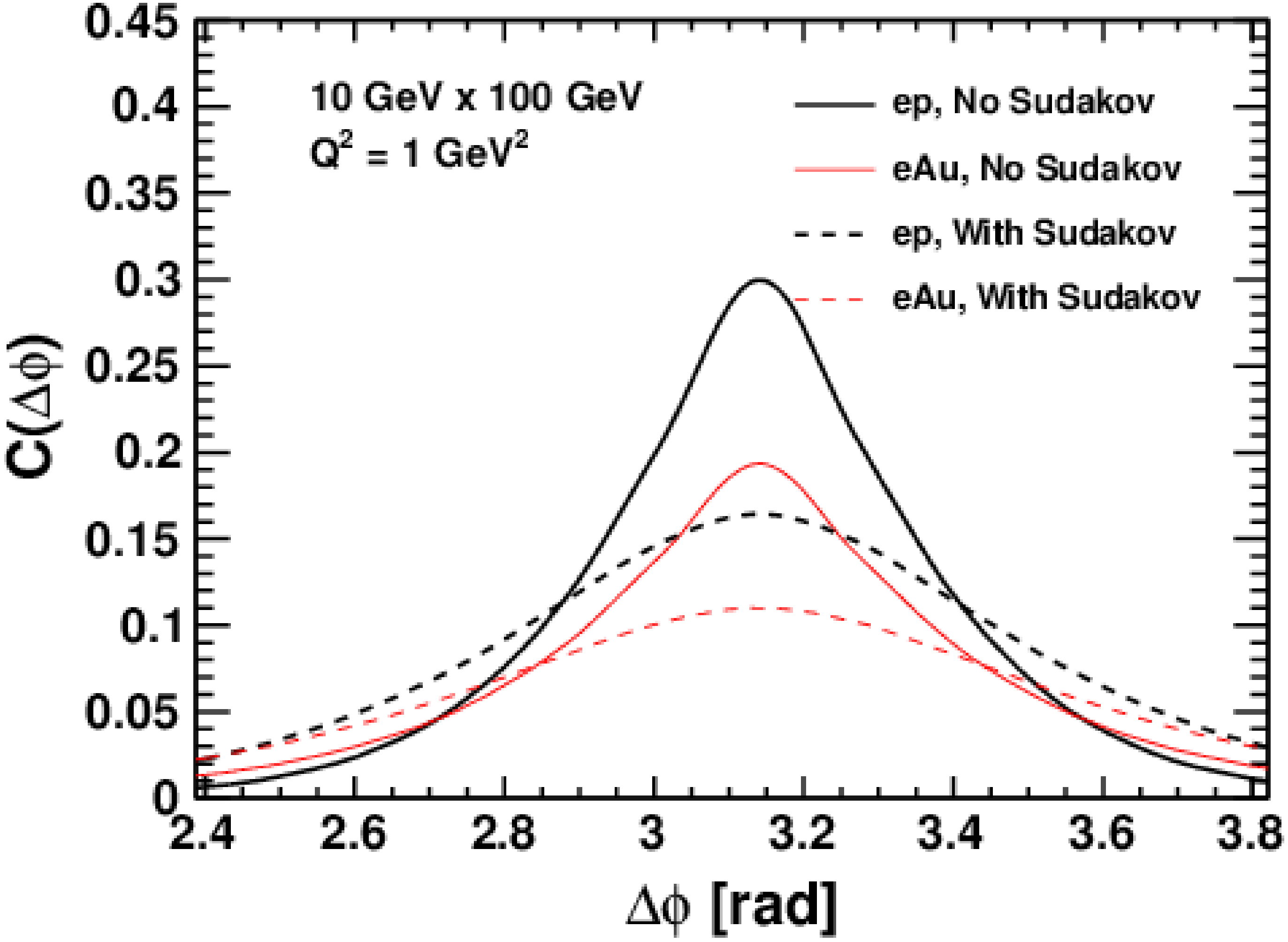
with

$$\Gamma^\mu \equiv$$
$$\frac{\gamma^-(\not{q} - \not{l} + m)\gamma^\mu(\not{q} - \not{k} - \not{l} + m)\gamma^-}{p^-[(q_\perp - l_\perp)^2 + m^2 - 2q^-k^+] + q^-[(q_\perp - k_\perp - l_\perp)^2 + m^2]}$$

F. Gelis and J. Jalilian-Marian, PRD67 (2003) 074019

Zheng + Aschenauer + Lee + Xiao, PRD89 (2014)7, 074037

Azimuthal correlations in DIS



Zheng + Aschenauer + Lee + Xiao, PRD89 (2014)7, 074037

Precision CGC: *NLO corrections*

DIS total cross section:

photon impact factor
evolution equations

pA collisions:

Single inclusive particle production

NLO di-jet production in DIS

LO 3-jet production

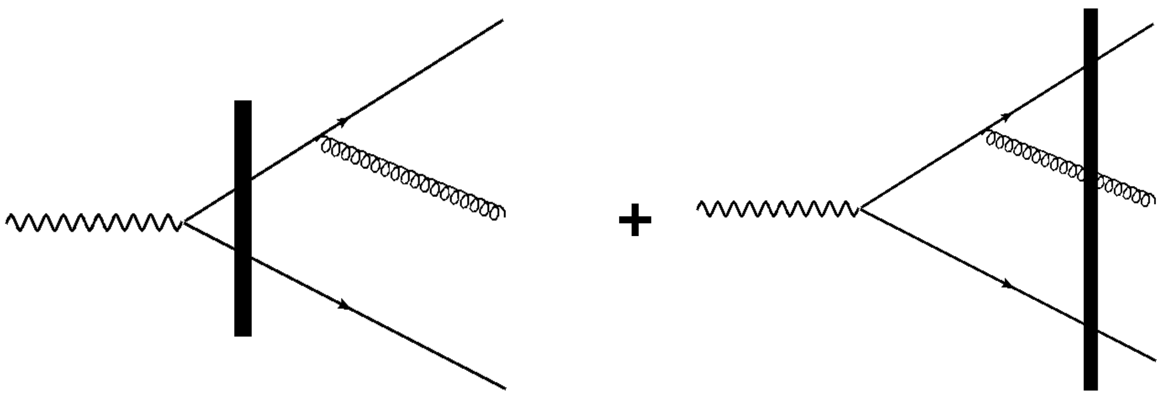
two away side hadrons: additional knob

Azimuthal correlations in DIS

*di-jet production in DIS: **NLO***

real contributions: $\gamma^{\star}\mathbf{T} \rightarrow \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{g}\,\mathbf{X}$

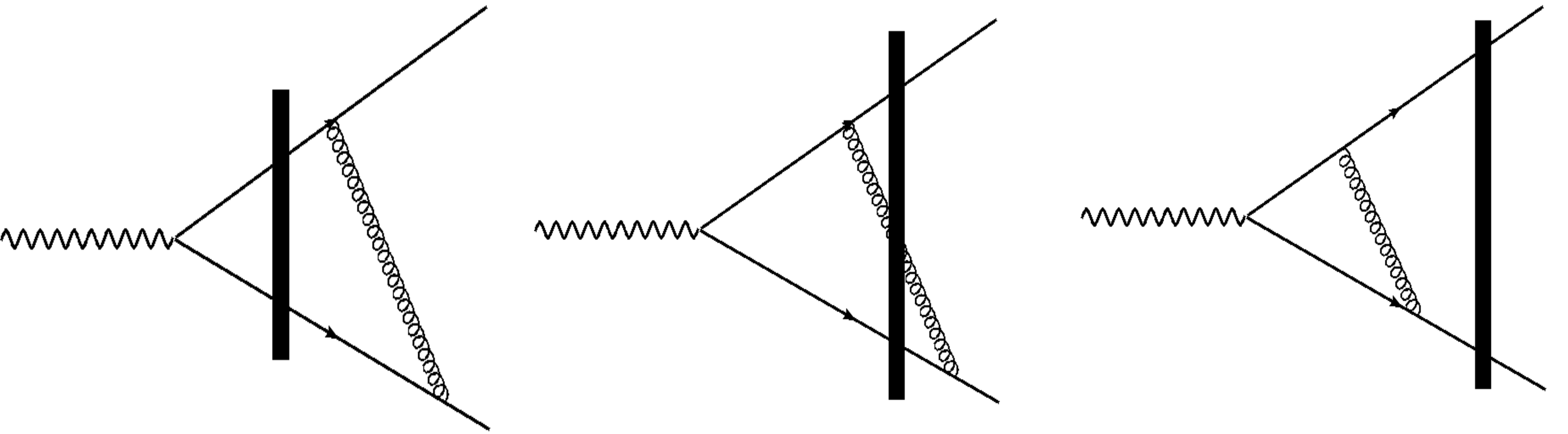
integrate out one of the produced partons



work in progress: Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans

di-jet azimuthal correlations in DIS

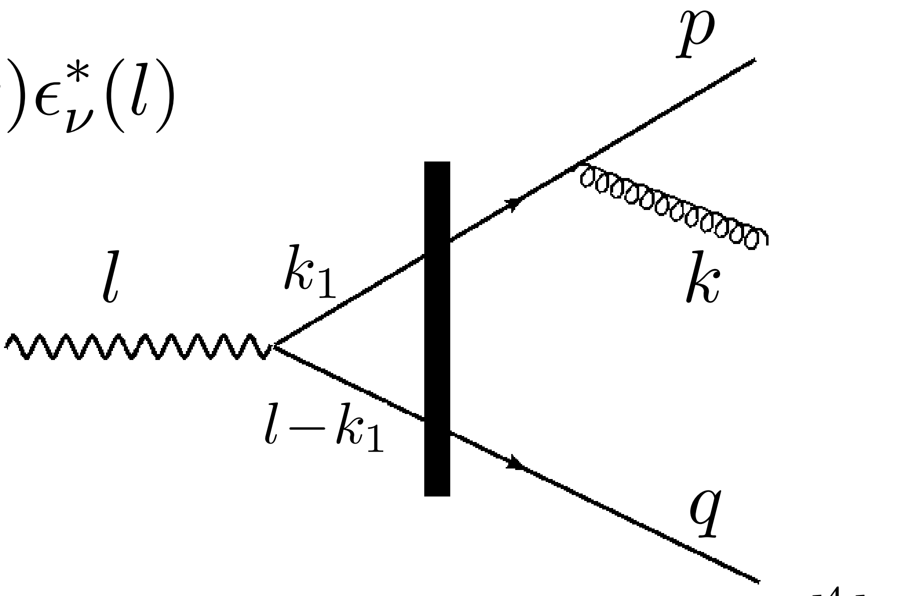
virtual contributions: $\gamma^{\star}\mathbf{T} \rightarrow \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{X}$



+ "self-energy" diagrams

real contributions:

$$\mathcal{A} \equiv -eg\,\bar{u}(p)\,[A]^{\mu\nu}\,v(q)\,\epsilon_\mu\,(k)\epsilon^*_\nu(l)$$



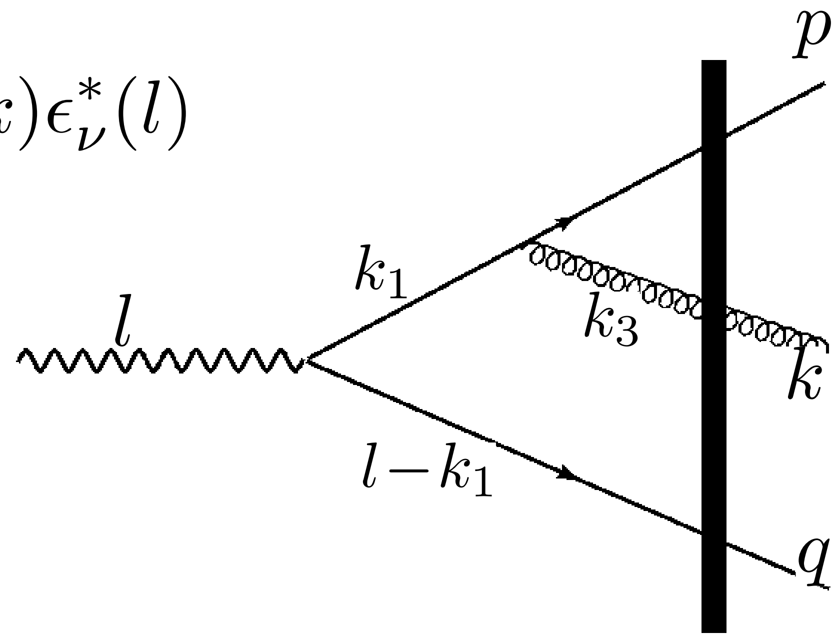
$$A_1^{\mu\nu} = \gamma^\mu\,t^a\,S_F^0(p+k)\,\tau_F(p+k,k_1)\,S_F^0(k_1)\,\gamma^\nu\,S_F^0(l-k_1)\,\tau_F(l-k_1,q)\,\frac{d^4k_1}{(2\pi)^4}$$

$$= \frac{i}{2\,l^-}\frac{\delta(l^- - p^- - q^- - k^-)}{(p+k)^2}\int d^2x_t\,d^2y_t\,e^{-i(p_t+k_t)\cdot x_t}\,e^{-iq_t\cdot y_t}$$
$$\gamma^\mu\,t^a\,i(\not{p}+\not{k})\gamma^-\,i\not{k}_1\,\gamma^\nu\,i(\not{l}-\not{k}_1)\,\gamma^-\,K_0\left[L\,(x_t-y_t)\right]$$
$$V(x_t)\,V^\dagger(y_t)$$

with

$$L^2 = \frac{q^-(p^-+k^-)}{l^-\,l^-}Q^2 \qquad k_1^- = p^- - k^- \qquad k_1^+ = \frac{k_{1t}^2 - i\epsilon}{2(p^-+k^-)} \qquad k_{1t} = -i\,\partial_{x_t-y_t}$$

$$\mathcal{A} \equiv -eg\,\bar{u}(p)\,[A]^{\mu\nu}\,v(q)\,\epsilon_\mu\,(k)\epsilon^*_\nu(l)$$



$$A_2^{\mu\nu} = \tau_F(p, k_1 - k_3)\,S_F^0(k_1 - k_3)\gamma^\mu\,t^a\,S_F^0(k_1)\,\gamma^\nu\,S_F^0(l - k_1)\,\tau_F(l - k_1, q)$$
$$G^{0\,\lambda}_\mu(k_3)\,\tau_g^{ac}(k_3, k)\frac{d^4k_1}{(2\pi)^4}\,\frac{d^4k_3}{(2\pi)^4}$$

Two different ways of evaluating these:

1: Use the 1-d delta function, do k^+ integration using contour integration, reduced to 2-d transverse integration over k_t

2: promote all to 4-d, Schwinger parameterization,...

$$\left[\frac{i}{k^2 - m^2 + i\epsilon}\right]^\lambda = \frac{1}{\Gamma(\lambda)}\int_0^\infty d\alpha\,\alpha^{\lambda-1}\,e^{i\alpha(k^2-m^2+i\epsilon)}$$

scalar part of $A_2^{\mu\nu}$:can be reduced to a 1-d integral

$$\int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_3}{(2\pi)^4} \frac{\delta(k_1^- - k_3^- - p_1^-)\delta(l^- - k_1^- - p_2^-)\delta(k_3^- - p_3^-)}{[k_1^2][(l-k_1)^2][(k_1-k_3)^2][k_3^2]}$$

$$e^{ix_{1,t}\cdot(k_{1,t}-k_{3,t}-p_{1,t})} e^{ix_{2,t}\cdot(l_t-k_{1,t}-p_{2,t})} e^{ix_{3,t}\cdot(k_{3,t}-p_{3,t})}$$

$$= \delta(l^- - \sum_i^3 p_i^-) e^{-ix_{1,t}\cdot p_{1,t}} e^{-ix_{2,t}\cdot p_{2,t}} e^{-ix_{3,t}\cdot p_{3,t}} \frac{(-i)^2}{(1-\beta_2)(l^-)^2(4\pi)^4}$$

$$\int_0^{\frac{\beta_2(1-\beta_2)^2}{\beta_1\beta_3}} \frac{d\rho_3}{\rho_3} K_0\left[\sqrt{\beta_2(1-\beta_2)Q^2\left[\frac{(x-y)^2}{\rho_3} + \frac{(\beta_1x_1+\beta_3x_3-(\beta_1+\beta_3)x_2)^2}{(1-\beta_2)^2}\right]}\right]$$

similarly for the other diagrams

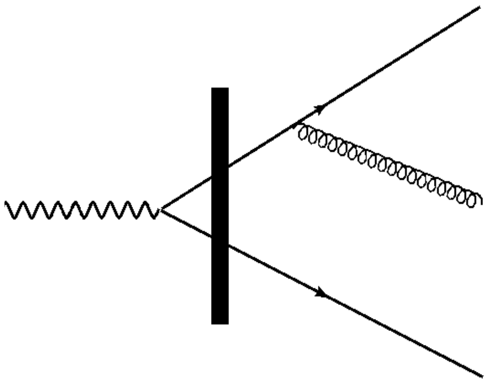
cross section: square the amplitude,....

Traces: ~ 23 pages long!

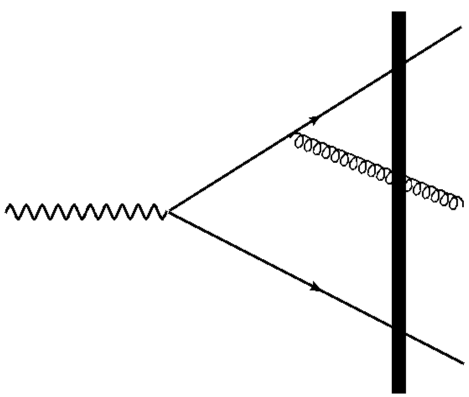
```
A1squared =
   + qminus * ( DENn(k)*dot(p,k)*IntR1(nminus,nminus,nminus,1,1,1,p)*
    IntR1c(muc1,muc1,nminus,1,1,1,p) + DENn(k)*dot(p,k)*IntR1(nminus,
    nminus,nminus,1,1,1,p)*IntR1c(muc2,muc2,nminus,1,1,1,p) - DENn(k)*
    dot(p,k)*IntR1(nminus,mu2,nminus,1,1,1,p)*IntR1c(nminus,mu2,nminus,1,
    1,1,p) - DENn(k)*dot(p,k)*IntR1(nminus,muc2,nminus,1,1,1,p)*IntR1c(
    nminus,muc2,nminus,1,1,1,p) - DENn(k)*dot(p,k)*IntR1(mu1,nminus,
    nminus,1,1,1,p)*IntR1c(mu1,nminus,nminus,1,1,1,p) + DENn(k)*dot(p,k)*
    IntR1(mu1,mu1,nminus,1,1,1,p)*IntR1c(nminus,nminus,nminus,1,1,1,p) +
    DENn(k)*dot(p,k)*IntR1(mu2,mu2,nminus,1,1,1,p)*IntR1c(nminus,nminus,
    nminus,1,1,1,p) - DENn(k)*dot(p,k)*IntR1(muc1,nminus,nminus,1,1,1,p)*
    IntR1c(muc1,nminus,nminus,1,1,1,p) - IntR1(nminus,nminus,p,1,1,1,p)*
    IntR1c(muc1,muc1,nminus,1,1,1,p) + IntR1(nminus,nminus,p,1,1,1,p)*
    IntR1c(muc2,muc2,nminus,1,1,1,p) + IntR1(nminus,mu2,p,1,1,1,p)*
    IntR1c(nminus,mu2,nminus,1,1,1,p) - IntR1(nminus,muc2,p,1,1,1,p)*
    IntR1c(nminus,muc2,nminus,1,1,1,p) + IntR1(mu1,p,mu1,1,1,1,p)*IntR1c(
    nminus,nminus,nminus,1,1,1,p) - IntR1(mu1,nminus,p,1,1,1,p)*IntR1c(
    mu1,nminus,nminus,1,1,1,p) - IntR1(mu1,nminus,mu1,1,1,1,p)*IntR1c(p,
    nminus,nminus,1,1,1,p) + IntR1(mu1,mu1,p,1,1,1,p)*IntR1c(nminus,
    nminus,nminus,1,1,1,p) - IntR1(mu2,mu2,p,1,1,1,p)*IntR1c(nminus,
    nminus,nminus,1,1,1,p) - IntR1(mu3,p,mu3,1,1,1,p)*IntR1c(nminus,
    nminus,nminus,1,1,1,p) + IntR1(mu3,nminus,mu3,1,1,1,p)*IntR1c(p,
    nminus,nminus,1,1,1,p) + IntR1(muc1,nminus,p,1,1,1,p)*IntR1c(muc1,
    nminus,nminus,1,1,1,p) )

   + pminus*qminus * (  - DENn(k)*IntR1(k,nminus,nminus,1,1,1,p)*IntR1c(
    muc3,nminus,muc3,1,1,1,p) + DENn(k)*IntR1(k,nminus,mu3,1,1,1,p)*
    IntR1c(mu3,nminus,nminus,1,1,1,p) - DENn(k)*IntR1(k,mu3,mu3,1,1,1,p)*
    IntR1c(nminus,nminus,nminus,1,1,1,p) + DENn(k)*IntR1(k,muc3,nminus,1,
    1,1,p)*IntR1c(nminus,nminus,muc3,1,1,1,p) + DENn(k)*IntR1(nminus,k,
    nminus,1,1,1,p)*IntR1c(nminus,muc3,muc3,1,1,1,p) - DENn(k)*IntR1(
    nminus,k,mu3,1,1,1,p)*IntR1c(nminus,mu3,nminus,1,1,1,p) + DENn(k)*
    IntR1(nminus,nminus,k,1,1,1,p)*IntR1c(muc1,muc1,nminus,1,1,1,p) -
    DENn(k)*IntR1(nminus,nminus,nminus,1,1,1,p)*IntR1c(k,muc3,muc3,1,1,1,
    p) + DENn(k)*IntR1(nminus,nminus,nminus,1,1,1,p)*IntR1c(muc2,muc2,k,1
    ,1,1,p) + DENn(k)*IntR1(nminus,nminus,nminus,1,1,1,p)*IntR1c(muc3,k,
    muc3,1,1,1,p) + DENn(k)*IntR1(nminus,nminus,mu3,1,1,1,p)*IntR1c(k,mu3
    ,nminus,1,1,1,p) - DENn(k)*IntR1(nminus,nminus,mu3,1,1,1,p)*IntR1c(
    mu3,k,nminus,1,1,1,p) - DENn(k)*IntR1(nminus,mu2,k,1,1,1,p)*IntR1c(
    nminus,mu2,nminus,1,1,1,p) + DENn(k)*IntR1(nminus,mu3,mu3,1,1,1,p)*
    IntR1c(nminus,k,nminus,1,1,1,p) - DENn(k)*IntR1(nminus,muc2,nminus,1,
    1,1,p)*IntR1c(nminus,muc2,k,1,1,1,p) - DENn(k)*IntR1(nminus,muc3,
    nminus,1,1,1,p)*IntR1c(nminus,k,muc3,1,1,1,p) - DENn(k)*IntR1(mu1,
    nminus,nminus,1,1,1,p)*IntR1c(mu1,nminus,k,1,1,1,p) + DENn(k)*IntR1(
```

structure of Wilson lines: amplitude



$$\left(\cdots \right)_{ij} = \left[V^\dagger(y_t)\, V(x_t)\, t^a\right]_{ij}$$



$$\left(\cdots \right)_{ij} = \left[V^\dagger(y_t)\, V(x_t)\, t^b\right]_{ij}\; U^{ba}(z_t)$$

structure of Wilson lines: cross section

$$
\begin{aligned}
\mathrm{tr}\left[W_1 W_1^*\right] &= \frac{\left(N_c^2-1\right) S_Q(x_t,x'_t,y'_t,y_t)}{2N_c} \\
\mathrm{tr}\left[W_1 W_2^*\right] &= \frac{1}{4}\left(S_D(z'_t,x'_t) S_Q(x_t,z'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_1 W_3^*\right] &= \frac{1}{2}\left(S_D(x_t,y) S_D(y'_t,x'_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_1 W_4^*\right] &= \frac{1}{4}\left(S_D(z'_t,x'_t) S_Q(x_t,z'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_2 W_1^*\right] &= \frac{1}{4}\left(S_D(x_t,z) S_Q(z_t,x'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_2 W_2^*\right] &= \frac{1}{8}\left(S_Q(x_t,x'_t,z'_t,z_t) S_Q(z,z'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_2 W_3^*\right] &= \frac{1}{4}\left(S_D(z,y_t) S_Q(x_t,x'_t,y'_t,z) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_2 W_4^*\right] &= \frac{1}{8}\left(S_Q(x_t,x'_t,z'_t,z) S_Q(z_t,z'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_3 W_1^*\right] &= \frac{1}{2}\left(S_D(x_t,y_t) S_D(y'_t,x'_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_3 W_2^*\right] &= \frac{1}{4}\left(S_D(y'_t,z'_t) S_Q(x_t,x'_t,z'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_3 W_3^*\right] &= \frac{\left(N_c^2-1\right) S_Q(x_t,x'_t,y'_t,y_t)}{2N_c} \\
\mathrm{tr}\left[W_3 W_4^*\right] &= \frac{1}{4}\left(S_D(y'_t,z'_t) S_Q(x_t,x'_t,z'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_4 W_1^*\right] &= \frac{1}{4}\left(S_D(x_t,z_t) S_Q(z,x'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_4 W_2^*\right] &= \frac{1}{8}\left(S_Q(x_t,x'_t,z'_t,z_t) S_Q(z,z'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_4 W_3^*\right] &= \frac{1}{4}\left(S_D(z,y_t) S_Q(x_t,x'_t,y'_t,z) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_4 W_4^*\right] &= \frac{1}{8}\left(S_Q(x_t,x'_t,z'_t,z_t) S_Q(z_t,z'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)
\end{aligned}
$$

$$\gamma^\star \mathbf{p} \to \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{g}\,\mathbf{X}$$

we are developing a Mathematica package to put all this together

di-jet azimuthal correlations in DIS

NLO: $\gamma^\star \mathbf{p} \to \mathbf{h}\,\mathbf{h}\,\mathbf{X}$

integrate out one of the produced partons - there are divergences:

rapidity divergences: JIMWLK evolution of n-point functions

collinear divergences: DGLAP evolution of fragmentation functions

infrared divergences cancel

the finite pieces are written as a factorized cross section

work in progress: Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans

related work by:

Boussarie, Grabovsky, Szymanowski, Wallon, JHEP1409, 026 (2014)
Balitsky, Chirilli, PRD83 (2011) 031502, PRD88 (2013) 111501
Beuf, PRD85, (2012) 034039

SUMMARY

CGC is a systematic approach to high energy collisions

it has been used to fit a wealth of data; ep, eA, pp, pA, AA

Leading Log CGC works (too) well for a qualitative/semi-quantitative description of data, NLO is needed

Need to eliminate/minimize late time/hadronization effects

Di-jet angular correlations offer a unique probe of CGC

3-hadron/jet correlations should be even more discriminatory

an EIC is needed for precision CGC studies