Discrete Flavor Symmetries: Higgs and neutrino physics

H. N. Long in collaboration with V. V. Vien
Institute of Physics, Hanoi

Contents:

I. Motivation

The tri-bimaximal form for the lepton mixing scheme proposed by Harrison-Perkins-Scott (HPS), which apart from the phase redefinitions, is given by

$$U_{\rm HPS}=\begin{pmatrix}\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\end{pmatrix}, \quad (1)$$

can be considered as a good approximation for the recent neutrino experimental data.

The most recent data are a clear sign of rather large value θ_{13}. The data imply:

$$\sin^2(2\theta_{12})=0.857\pm0.024,\ \ \sin^2(2\theta_{13})=0.098\pm0.013,\ \sin^2(2\theta_{23})>0.95,$$
$$\Delta m^2_{21}=(7.50\pm0.20)\times10^{-5}{\rm eV}^2,\ \Delta m^2_{32}=(2.32^{+0.12}_{-0.08})\times10^{-3}{\rm eV}^2. \quad (2)$$

The most elegant way for this purpose is discrete symmetries.

We should look for symmetry groups containing 1, 2 - and 3- dimensional irreducible representations, or more than two 1- dimensional irreducible representations.

- A_4 : $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$',$\underline{\mathbf{1}}$", $\underline{\mathbf{3}}$.
- S_3 : $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$', $\underline{\mathbf{2}}$.
- S_4 : $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$', $\underline{\mathbf{2}}$,$\underline{\mathbf{3}}$ and $\underline{\mathbf{3}}$'.
- D_4 : $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$',$\underline{\mathbf{1}}$", $\underline{\mathbf{1}}$"' , $\underline{\mathbf{2}}$.
- T_7 : $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$',$\underline{\mathbf{1}}$" , $\underline{\mathbf{3}}$, $\underline{\mathbf{3}}^*$.
- T_{13} : $\underline{1}_0$, $\underline{1}_1$, $\underline{1}_2$, $\underline{3}_1$, $\underline{\bar{3}}_1$, $\underline{3}_2$, $\underline{\bar{3}}_2$.

In the SM, there is the generation replication, and why number of fermion generation = 3 ? So, the discrete symmetry can give answer on horizontal symmetry.

Now I turn to the concrete model: the 3-3-1 model with T_7 flavor symmetry. T_7 is Frobenius group with 21 elements.

Motivation for studying 3-3-1 model

1. Generation problem: Why $n_f = 3$
2. Electric charge quantization
3. Neutrino masses, Dark matter,...
4. One family of quarks behavior differently from other two $\rightarrow$ why top quark is so heavy,...

I. The 3-3-1 with RH neutrinos

Foot, HNL, Tuan A. Tran, PRD (1994); HNL, PR D53, PR D54 (1996)

0.1 Particle content

-Leptons in triplet

$$f_L^a = (\nu_L^a, l_L^a, \nu_L^{ca})^T \sim (1, 3, -1/3), l_R^a \sim (1, 1, -1), \quad (3)$$

where $a = 1, 2, 3$

- Two quark generations in antitriplets and one in triplet

$$Q_{\alpha L} = (d_{\alpha L}, -u_{\alpha L}, D_{\alpha L},)^T \sim (3, \bar{3}, 0), \ \alpha = 1, 2, \quad (4)$$

$$Q_{3L} = (u_{3L}, d_{3L}, T_L)^T \sim (3, 3, 1/3), T_R \sim (3, 1, 2/3), \quad (5)$$

$$D_{\alpha R} \sim (3, 1, -1/3). \quad (6)$$

0.2 Higgs sector

$$\rho \ = \ (\rho_1^+, \ \rho_2^0, \ \rho_3^+)^T \sim (1, 3, 2/3), \quad \eta = (\eta_1^0, \ \eta_2^-, \ \eta_3^0)^T \sim (1, 3, -1/3), \quad (7)$$
$$\chi \ = \ (\chi_1^0, \ \chi_2^-, \ \chi_3^0)^T \sim (1, 3, -1/3).$$

- Number of generations $N_g = n \times N_c$= 3, 6, 9,...

- Asymptotic freedom $N_g \leq 5 \quad \Rightarrow N_g = 3$!!!

- 3rd generation of quarks transforms differently from two others ⇒ top quark is very heavy.

- Higgs boson production at LHC $pp \to hZ$ was considered [Le Duc Ninh, HNL, PRD (2005)]

- Neutrinos get masses and have pseudo-Dirac inverted hierarchy mass pattern [D. Chang & HNL, PRD (2006)].

Question: Does the 3-3-1 model have disadvantages?

Answer: Higss sector is complicated (with 3 triplet)

- In 2006, the 3-3-1 model with two Higgs triplets has been constructed, and we call it economical [P. V. Dong et al, 2 PR D73, D74, D75].

II. The 3-3-1 model with T_7 symmetry

The T_7 flavor symmetry in 3-3-1 model with neutral leptons
V. V. Vien, H. N. Long, JHEP ***04*** *(2014) 133*

This version is slightly different from paper: *A. Carcamo Hernandez and R. Martinez, arXiv:1501.07261*

Electric charge operator

$$Q = T_3 + \beta T_8 + X$$

$\beta = \sqrt{3}$ for minimal model

A fundamental relation holds among some of the generators of the group:

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X, \tag{8}$$

The ordinary lepton number by diagonal matrices [D. Chang & H. N. Long, PRD 2006]

$$L = \frac{2}{\sqrt{3}} T_8 + \mathcal{L}. \tag{9}$$

Note on connection among operator of discrete number L with gauge operator T_8.

Under the $[\mathrm{SU}(3)_L, \mathrm{U}(1)_X, \mathrm{U}(1)_{\mathcal{L}}, \underline{T}_7]$ symmetries, the fermions transform as follows

$$\begin{aligned}
\psi_L &\equiv \psi_{1,2,3L} = (\nu_L \quad l_L \quad N_R^c)^T \sim [3, -1/3, 2/3, \underline{3}], \\
l_{1R} &\sim [1, -1, 1, \underline{1}], \quad l_{2R} \sim [1, -1, 1, \underline{1}'], \quad l_{3R} \sim [1, -1, 1, \underline{1}''], \\
Q_{1L} &\equiv (d_{1L} \quad -u_{1L} \quad D_{1L})^T \sim [3^*, 0, 1/3, \underline{1}'], \\
Q_{2L} &\equiv (d_{2L} \quad -u_{2L} \quad D_{2L})^T \sim [3^*, 0, 1/3, \underline{1}''], \\
Q_{3L} &= (u_{3L} \quad d_{3L} \quad U_L)^T \sim [3, 1/3, -1/3, \underline{1}], \\
u_R &\sim u_{1,2,3R} = [1, 2/3, 0, \underline{3}], \quad d_R \sim [1, -1/3, 0, \underline{3}^*], \\
U_R &\sim [1, 2/3, -1, \underline{1}], \quad D_{1R} \sim [1, -1/3, 1, \underline{1}''], \quad D_{2R} \sim [1, -1/3, 1, \underline{1}'].
\end{aligned} \tag{10}$$

where subscript numbers on field indicate to respective families which also in order define components of their T_7 multiplets. U and $D_{1,2}$ are exotic quarks carrying lepton numbers $L(U) = -1$ and $L(D_{1,2}) = 1$, known as leptoquarks. The scalar multiplets are also introduced.

A. Charged lepton masses

$\bar{\psi}_L l_{iR}\,(i = 1,2,3)$ **transforms as** 3^* **under** $\mathrm{SU}(3)_L$ **and** $\underline{3}^*$ **under** T_7. **To generate masses for charged leptons, we need a** $SU(3)_L$ **Higgs triplets that lying in** $\underline{3}$ **under** T_7 **and transforms as** 3 **under** $\mathrm{SU}(3)_L$,

$$\phi_i = \begin{pmatrix} \phi^+_{i1} \\ \phi^0_{i2} \\ \phi^+_{i3} \end{pmatrix} \sim [3, 2/3, -1/3, \underline{3}] \quad (i = 1,2,3). \tag{11}$$

The Yukawa interactions are

$$\begin{aligned} -\mathcal{L}_l &= h_1(\bar{\psi}_L\phi)_{\underline{1}} l_{1R} + h_2(\bar{\psi}_L\phi)_{\underline{1}''} l_{2R} + h_3(\bar{\psi}_{iL}\phi)_{\underline{1}'} l_{3R} + H.c \\ &= h_1(\bar{\psi}_{1L}\phi_1 + \bar{\psi}_{2L}\phi_2 + \bar{\psi}_{3L}\phi_3) l_{1R} \\ &+ h_2(\bar{\psi}_{1L}\phi_1 + \omega^2\bar{\psi}_{2L}\phi_2 + \omega\bar{\psi}_{3L}\phi_3) l_{2R} \\ &+ h_3(\bar{\psi}_{1L}\phi_1 + \omega\bar{\psi}_{2L}\phi_2 + \omega^2\bar{\psi}_{3L}\phi_3) l_{3R} + H.c. \end{aligned} \tag{12}$$

The mass Lagrangian for the charged leptons reads

$$-\mathcal{L}_l^{\rm mass} = (\bar{l}_{1L}, \bar{l}_{2L}, \bar{l}_{3L}) M_l (l_{1R}, l_{2R}, l_{3R})^T + H.c, \quad (13)$$

where

$$M_l = \begin{pmatrix} h_1 v & h_2 v & h_3 v \\ h_1 v & h_2 v\omega^2 & h_3 v\omega \\ h_1 v & h_2 v\omega & h_3 v\omega^2 \end{pmatrix}. \quad (14)$$

This matrix can be diagonalized as,

$$U_L^\dagger M_l U_R = \begin{pmatrix} \sqrt{3}h_1 v & 0 & 0 \\ 0 & \sqrt{3}h_2 v & 0 \\ 0 & 0 & \sqrt{3}h_3 v \end{pmatrix} \equiv \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad (15)$$

where

$$U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad U_R = 1. \quad (16)$$

B. Quark masses

To generate masses for quarks with a minimal Higgs content, we additionally introduce the following Higgs triplets:

$$\eta_i = \begin{pmatrix} \eta^0_{i1} \\ \eta^-_{i2} \\ \eta^0_{i3} \end{pmatrix} \sim [3, -1/3, -1/3, \underline{3}] \quad (i = 1, 2, 3), \tag{17}$$

$$\chi = \begin{pmatrix} \chi^0_1 \\ \chi^-_2 \\ \chi^0_3 \end{pmatrix} \sim [3, -1/3, 2/3, \underline{1}] \,. \tag{18}$$

The CKM matrix

$$U_{\mathrm{CKM}} = U^{d\dagger}_L U^u_L = 1. \tag{19}$$

C. Neutrino mass and mixing

$\bar{\psi}^c_L \psi_L$ transforms as $3^* \oplus 6$ under $\mathrm{SU}(3)_L$ and $\underline{3} \oplus \underline{3}^* \oplus \underline{3}^*$ under T_7. With the T_7 group, $\underline{3} \times \underline{3} \times \underline{3}$ has two invariants and $\underline{3} \times \underline{3} \times \underline{3}^*$ has one invariant. For the known scalar triplets (ϕ, χ, η), there is no available interaction because of the $\mathcal{L}$-symmetry. To obtain a realistic neutrino spectrum, the antisextets transform as follows

$$\sigma_i = \begin{pmatrix} \sigma^0_{11} & \sigma^+_{12} & \sigma^0_{13} \\ \sigma^+_{12} & \sigma^{++}_{22} & \sigma^+_{23} \\ \sigma^0_{13} & \sigma^+_{23} & \sigma^0_{33} \end{pmatrix}_i \sim [6^*, 2/3, -4/3, \underline{3}^*] \quad (i = 1, 2, 3), \tag{20}$$

The $T_7 \to Z_3$ can be achieved by a $SU(3)_L$ anti-sextet σ given in (20) with the VEVs is set as $\langle\sigma\rangle = (\langle\sigma_1\rangle, \langle\sigma_1\rangle, \langle\sigma_1\rangle)$ under T_7, where

$$\langle\sigma_1\rangle = \begin{pmatrix} \lambda_\sigma & 0 & v_\sigma \\ 0 & 0 & 0 \\ v_\sigma & 0 & \Lambda_\sigma \end{pmatrix}. \quad (21)$$

To achieve the second direction of the breakings $T_7 \to \{\mathrm{Identity}\}$ (equivalently to $Z_3 \to \{\mathrm{Identity}\}$), we additionally introduce another $SU(3)_L$ anti-sextet Higgs scalar which is either put in $\underline{3}$ or $\underline{3}^*$ under T_7. This is equivalent to breaking the subgroup Z_3 of the first direction into $\{\mathrm{Identity}\}$, and it can be achieved within each case below.

1. A new $SU(3)_L$ anti-sextet s which is put in the $\underline{3}$ under T_7,

$$s_i = \begin{pmatrix} s^0_{11} & s^+_{12} & s^0_{13} \\ s^+_{12} & s^{++}_{22} & s^+_{23} \\ s^0_{13} & s^+_{23} & s^0_{33} \end{pmatrix}_i \sim [6^*, 2/3, -4/3, \underline{3}], \quad (22)$$

with the VEVs given by $\langle s \rangle = (\langle s_1 \rangle, 0, 0)^T$, where

$$\langle s_1 \rangle = \begin{pmatrix} \lambda_s & 0 & v_s \\ 0 & 0 & 0 \\ v_s & 0 & \Lambda_s \end{pmatrix}. \quad (23)$$

2. Another $\mathrm{SU}(3)_L$ anti-sextet σ' is put in the $\underline{3}^*$ under T_7, with the VEVs chosen by

$$\sigma' = \begin{pmatrix} \sigma'^0_{11} & \sigma'^+_{12} & \sigma'^0_{13} \\ \sigma'^+_{12} & \sigma'^{++}_{22} & \sigma'^+_{23} \\ \sigma'^0_{13} & \sigma'^+_{23} & \sigma'^0_{33} \end{pmatrix} \sim [6^*, 2/3, -4/3, \underline{3}^*],$$

$$\langle \sigma'_1 \rangle = \begin{pmatrix} \lambda'_\sigma & 0 & v'_\sigma \\ 0 & 0 & 0 \\ v'_\sigma & 0 & \Lambda'_\sigma \end{pmatrix}, \ \langle \sigma'_2 \rangle = \langle \sigma'_3 \rangle = 0. \quad (24)$$

Note that σ' differs from σ only in the VEVs alignment.

The Yukawa interactions:

$$\begin{aligned}
-\mathcal{L}_\nu &= \frac{1}{2}x(\bar{\psi}^c_L\sigma)_{\underline{3}^*}\psi_L + y(\bar{\psi}^c_L s)_{\underline{3}^*}\psi_L + \frac{z}{2}(\bar{\psi}^c_L\sigma')_{\underline{3}^*}\psi_L + H.c. \\
&= \frac{1}{2}x(\bar{\psi}^c_{1L}\sigma_2\psi_{1L} + \bar{\psi}^c_{2L}\sigma_3\psi_{2L} + \bar{\psi}^c_{3L}\sigma_1\psi_{3L}) \\
&+ y(\bar{\psi}^c_{2L}s_3\psi_{1L} + \bar{\psi}^c_{3L}s_1\psi_{2L} + \bar{\psi}^c_{1L}s_2\psi_{3L}) \\
&+ \frac{z}{2}(\bar{\psi}^c_{1L}\sigma'_2\psi_{1L} + \bar{\psi}^c_{2L}\sigma'_3\psi_{2L} + \bar{\psi}^c_{3L}\sigma'_1\psi_{3L}) + H.c. \qquad \textbf{(25)}
\end{aligned}$$

The neutrino mass Lagrangian can be written in matrix form as follows

$$-\mathcal{L}^{\mathrm{mass}}_\nu = \frac{1}{2}\bar{\chi}^c_L M_\nu \chi_L + h.c., \qquad \textbf{(26)}$$

where

$$\begin{aligned}
\chi_L &\equiv (\nu_L \quad N^c_R)^T, \qquad M_\nu \equiv \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix}, \\
\nu_L &= (\nu_{1L}, \nu_{2L}, \nu_{3L})^T, \;\; N_R = (N_{1R}, N_{2R}, N_{3R})^T, \qquad \textbf{(27)}
\end{aligned}$$

and the mass matrices are then obtained by

$$M_{L,R,D} = \begin{pmatrix} a_{L,R,D} & 0 & 0 \\ 0 & a_{L,R,D} & b_{L,R,D} \\ 0 & b_{L,R,D} & a_{L,R,D} + c_{L,R,D} \end{pmatrix}, \quad (28)$$

with

$$\begin{aligned} a_L &= \lambda_\sigma x, \quad a_D = v_\sigma x, \quad a_R = \Lambda_\sigma x, \\ b_L &= \lambda_s y, \quad b_D = v_s y, \quad b_R = \Lambda_s y, \\ c_L &= \lambda'_\sigma z, \quad c_D = v'_\sigma z, \quad c_R = \Lambda'_\sigma z. \end{aligned} \quad (29)$$

Three observed neutrinos gain masses via a combination of type I and type II seesaw mechanisms derived from (26) and (28) as

$$M_{\mathrm{eff}} = M_L - M_D^T M_R^{-1} M_D = \begin{pmatrix} A & 0 & 0 \\ 0 & B_1 & C \\ 0 & C & B_2 \end{pmatrix}, \quad (30)$$

where

$$
\begin{aligned}
A &= a_L - \frac{a_D^2}{a_R}, \\
B_1 &= a_L - \frac{a_R b_D^2 - 2a_D b_D b_R + a_D^2(a_R + d_R)}{a_R^2 - b_R^2 + a_R d_R} \\
B_2 &= B_1 + d_L + \frac{2(b_D b_R - a_D a_R)d_D + (a_D^2 - b_D^2)d_R - a_R d_D^2}{a_R^2 - b_R^2 + a_R d_R}, \\
C &= b_L - \frac{(a_D^2 + b_D^2)b_R - (2a_D a_R + a_D d_R)b_D + (a_D b_R - a_R b_D)d_D}{a_R^2 - b_R^2 + a_R d_R}.
\end{aligned} \tag{31}
$$

We get the lepton mixing matrix:

$$
U_L^\dagger U_\nu = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1-K}{\sqrt{K^2+1}} & 1 & \frac{1+K}{\sqrt{K^2+1}} \\ \frac{\omega(1-K\omega)}{\sqrt{K^2+1}} & 1 & \frac{\omega(\omega+K)}{\sqrt{K^2+1}} \\ \frac{\omega(\omega-K)}{\sqrt{K^2+1}} & 1 & \frac{\omega(K\omega+1)}{\sqrt{K^2+1}} \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix}. \tag{32}
$$

The lepton mixing matrix (U_{PMNS}) can be parametrized as

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{13} & -s_{13}e^{-i\delta} \\ s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -s_{23}c_{13} \\ s_{12}s_{23} + c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} . \times P. \quad (33)$$

where $P = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$, and $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ with θ_{12}, θ_{23} and θ_{13} being the solar angle, atmospheric angle and the reactor angle respectively. δ is the Dirac CP violating phase while α and β are the two Majorana CP violating phases.

From the (32) and (33) we rule out $\alpha = 0, \beta = \frac{\pi}{2}$, and the lepton mixing matrix in (32) can be parameterized in three Euler's angles θ_{ij} as follows:

$$s_{13}e^{-i\delta} = \frac{-1-K}{\sqrt{3}\sqrt{K^2+1}}, \quad (34)$$

$$t_{12} = \frac{\sqrt{K^2+1}}{K-1}, \quad (35)$$

$$t_{23} = -\frac{\omega+K}{1+K\omega}. \quad (36)$$

Substituting $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ into (36) yields:

$$\begin{aligned} K &= k_1 + ik_2, \\ k_1 &= \frac{t_{23}^2 - 4t_{23} + 1}{2(t_{23}^2 - t_{23} + 1)}, \quad k_2 = \frac{\sqrt{3}(t_{23}^2 - 1)}{2(t_{23}^2 - t_{23} + 1)}. \end{aligned} \tag{37}$$

The expression (37) tells us that $k_1^2 + k_2^2 \equiv |K|^2 = 1$. Combining (34) and (35) yields:

$$\begin{aligned} e^{-i\delta} &= \frac{1}{\sqrt{3}s_{13}t_{12}}\frac{1+K}{1-K} = \frac{1}{\sqrt{3}s_{13}t_{12}}\left[\frac{1-k_1^2-k_2^2}{[(1-k_1)^2+k_2^2]} + \frac{2k_2}{[(1-k_1)^2+k_2^2]}i\right] \\ &= -\frac{i(1-t_{23})}{s_{13}t_{12}(1+t_{23})} \equiv \cos\delta - i\sin\delta \end{aligned}$$

or

$$\cos\delta = 0, \quad \sin\delta = \frac{1-t_{23}}{s_{13}t_{12}(t_{23}+1)}. \tag{38}$$

Since $\cos\delta = 0$ so that $\sin\delta$ must be equal to ± 1, it is then $\delta = \frac{\pi}{2}$ or $\delta = \frac{3\pi}{2}$.

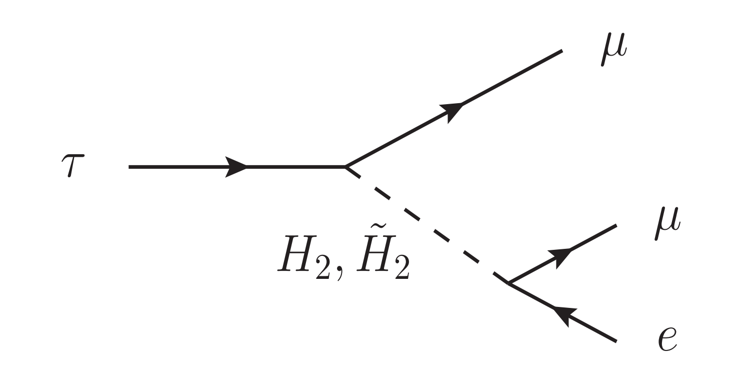
Thus, our model predicts the maximal Dirac CP violating phase which is the same as in [C. H. Albright, W. Rodejohann, Eur. Phys. J. C 62 (2009) 599; D. Marzocca, S. T. Petcov, A. Romanino, M. C. Sevilla, JHEP 05 (2013) 073] , and this is one of the most striking prediction of the model under consideration.

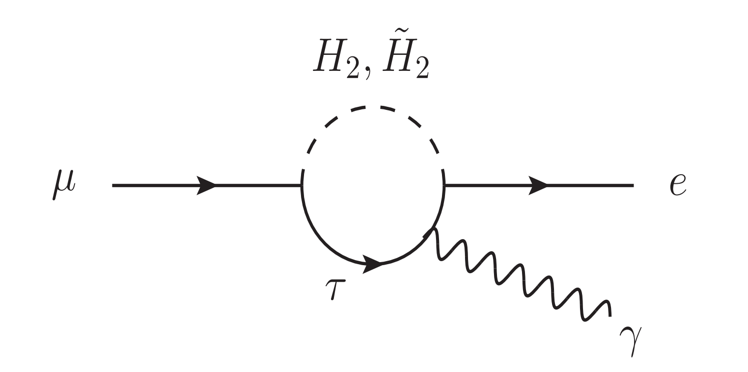
III. Higgs phenomenology

The numbers

Lepton number (L) and lepton parity (P_l) particles:

Particles	L	P_l
N_R, u, d, ϕ_1^+,$\phi_1'^+$, ϕ_2^0,$\phi_2'^0$, η_1^0,$\eta_1'^0$, η_2^-,$\eta_2'^-$ χ_3^0, σ_{33}^0, s_{33}^0	**0**	**1**
ν_L, l, U, D^*, ϕ_3^+,$\phi_3'^+$, η_3^0, $\eta_3'^0$, χ_1^{0*}, χ_2^+, σ_{13}^0, σ_{23}^+, s_{13}^0, s_{23}^+	-1	-1
σ_{11}^0, σ_{12}^+, σ_{22}^{++}, s_{11}^0, s_{12}^+, s_{22}^{++}	-2	**1**


τ
μ
μ
e
$H_2, \tilde{H}_2$


$H_2, \tilde{H}_2$
μ
e
τ
γ

According I. M. Varzielas, O.Fisher, V. Maurer, JHEP **08** (2015) 080

$$R_{\tau^- \to e^- \mu^+ \mu^-} = \frac{2m_\mu^4 \left(m_{\tilde{H}_2}^2 + m_{H_2}^2\right)^2 + m_\tau^2 m_\mu^2 \left(m_{H_2}^2 - m_{\tilde{H}_2}^2\right)^2}{128 m_{H_2}^4 m_{\tilde{H}_2}^4}, \quad (39)$$

$$R_{\tau^- \to e^+ \mu^- \mu^-} = \frac{m_\mu^4 \left(m_{H_2}^2 - m_{\tilde{H}_2}^2\right)^2 + 4m_\mu^2 m_\tau^2 \left(m_{\tilde{H}_2}^2 + m_{H_2}^2\right)^2}{128 m_{H_2}^4 m_{\tilde{H}_2}^4} \quad (40)$$

where we defined $R_{\tau \to X}$ = BR($\tau^- \to X$)/BR($\tau^- \to e^- \bar{\nu}_e \nu_\tau$) and the lepton masses $m_\mu = 0.105$ GeV and $m_\tau = 1.77$ GeV. The process $\tau^- \to \mu^- e^+ e^-$ with the experimental upper bound being $\mathcal{O}(10^{-8})$ results in the very mild constraint on $m_{\tilde{H}_2}, m_{H_2} > 12$ GeV.

For diphoton excess, the Higgs of the first symmetry breaking plays an important role (according A. Carcamo, Boucena, Dong works)

Conclusions

1. The discrete symmetry can play a role of horizontal symmetry among generations.
2. The discrete symmetries can give tri-bimaximal form in elegant way
3. The are lepton violating Yukawa interactions which lead to new physics
4. For diphoton excess, the Higgs scalar of the first step in SSB plays an important role

Thanks for your attention