Phenomenology of the heavy quarkonium electric dipole transitions

Héctor Martínez

TU München

January 9, 2016

HEP in the LHC era.

Universidad Tecnica Federico Santa Maria

Valparaiso, Chile.

Outline

1 Motivation

2 The $Q\bar{Q}$ potential

3 Counting and parameter fitting

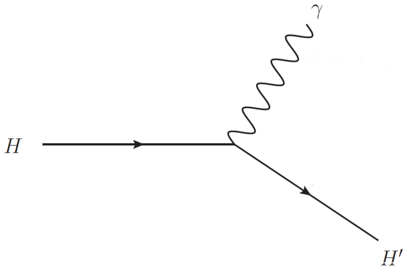
4 Evaluation method and results

5 Summary and Outlook

Motivation

Electric dipole transitions (E1) of heavy quarkonium:

- $|\Delta L| = 1$
- $|\Delta S| = 0$
- Example transition $1^3P_J \to 1^3S_1\gamma$:
  - in charmonium: $\chi_c \to J/\psi\gamma$
  - in bottomonium: $\chi_b \to \Upsilon\gamma$



Experimental results available:

- $h_c \to \eta_c\gamma$ first measured at BESIII in 2010
- $h_b \to \eta_b\gamma$ first measured in 2011 at BaBar and BELLE
- First measurement of the EM branching ratios of the χ_b states by CLEO and BaBar in 2011

Branching fraction of E1 transition significant for some states (from PDG):

- $\mathcal{BR}(\chi_{b1}(1P) \to \Upsilon(1S)\gamma) \approx 34 \pm 2\%$,
- $\mathcal{BR}(h_b(1P) \to \eta_b(1S)\gamma) \approx 49 \pm 8\%$

Potential non-relativistic QCD (pNRQCD)

- An effective field theory that takes full advantage of the hierarchy of scales that appear in the quarkonium system:

$$m \gg mv \gg mv^2$$

- Obtained from QCD:

$$QCD \mapsto NRQCD \mapsto pNRQCD$$

$$\mathcal{L}_{\mathrm{pNRQCD}} = S^{\dagger}(i\partial_0 - h_s)S + \ldots$$

$$h_s = \frac{\mathbf{p}^2}{m} + V^{(0)} + \ldots$$

Brambilla, Pineda, Soto and Vairo, *Nucl. Phys. B566, 275 (2000)*

E1 transitions in pNRQCD:

- Define a counting
- Write down the relevant NRQCD Lagrangian up to operators of relative order v^2

$$\mathcal{L}_{\rm NRQCD} = \psi^\dagger\left(iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3}\right)\psi + \frac{c_F}{2m}\psi^\dagger\sigma\cdot g\mathbf{B}\psi + \frac{c_F}{2m}\psi^\dagger\sigma\cdot ee_Q\mathbf{B}^{\rm em}\psi$$
$$+ \cdots + \left[\psi \to i\sigma^2\chi^*, A_\mu \to -A_\mu^T, A_\mu^{\rm em} \to -A_\mu^{\rm em}\right]$$

- Write down the pNRQCD up to the same order

$$\mathcal{L}_{\rm pNRQCD} = \cdots + \int d^3r{\rm Tr}\left\{V^{r\cdot E}S^\dagger\mathbf{r}\cdot ee_Q\mathbf{E}^{\rm em}S\right.$$
$$\left. + \frac{1}{24}V^{(r\nabla)^2 r\cdot E}S^\dagger\mathbf{r}\cdot\left[(\mathbf{r}\cdot\nabla)^2 ee_Q\mathbf{E}^{\rm em}\right]S + \dots\right\} + \Delta\mathcal{L}_{\rm pNRQCD}$$

- Match both theories (Green functions)

The formula for the E1 transition rates including NLO relativistic corrections has the following structure

$$\Gamma^{\rm E1}_{H\to H'\gamma} = \Gamma^{\rm LO}_{H\to H'\gamma}(1 + R_{H\to H'\gamma} + \delta\Gamma_{H\to H'\gamma})$$

Brambilla, Pietrulewicz and Vairo, *PRD 85, 094005 (2012)*

where the $R_{H\to H'\gamma} + \delta\Gamma_{H\to H'\gamma}$ are v^2 suppressed.
For instance, in the case of the $n^3P_J \to n'^3S_1\gamma$ the terms in this formula read:

$$\Gamma^{\rm LO}_{n^3P_J\to n'^3S_1\gamma} = \frac{4}{9}\alpha_{em}e_Q^2k_\gamma^3I_3^2,$$

$$\delta\Gamma_{n^3P_J\to n'^3S_1\gamma} = -\frac{k_\gamma^2}{60}\frac{I_5}{I_3} - \frac{k_\gamma}{6m} + \left(\frac{J(J+1)}{2} - 2\right)\left(\frac{1}{m^2}\frac{I_2^{(\prime)} + 2I_1}{I_3} - \frac{k_\gamma}{2m}\right),$$

k_γ is the energy of the emitted photon. $R_{H\to H'\gamma}$ accounts for the relativistic corrections to the quarkonium wavefunction, it has the general structure

$$R_{H\to H'\gamma} = \frac{1}{A^{(0)}_{H\to H'\gamma}}\left(-^{(0)}\langle H'\gamma|\int d^3R\mathcal{L}^{(0)}_{E_1}|H\rangle^{(1)} -^{(1)}\langle H'\gamma|\int d^3R\mathcal{L}^{(0)}_{E_1}|H\rangle^{(0)} + \ldots\right)$$

where

$$I_N^{(k)} = \int_0^\infty dr r^N R_{n'0}(r) \frac{d^k}{dr^k} R_{n1}(r),$$

$$\left(-\frac{\nabla_r^2}{m} + V^{(0)}\right) \phi_H^{(0)} = E_H^{(0)} \phi_H^{(0)},$$

with the quarkonium wavefunctions given by

$$|H(P)\rangle^{(0)} = \int d^3R \int d^3r\, e^{iP\cdot R} \mathrm{Tr}\left\{\phi_H^{(0)} S^\dagger(r,R)|US\rangle\right\},$$

$$|H(P)\rangle^{(1)} = \sum_{H'\neq H} {}^{(0)}\langle H'(P)| \int d^3R \int d^3r \mathrm{Tr}\{S^\dagger \delta h S\} |H(P)\rangle^{(0)} \frac{|H'(P)\rangle^{(0)}}{E_{H'}^{(0)} - E_H^{(0)}}.$$

δh accounts for the relativistic corrections to the quark-antiquark potential

$$\delta h = -\frac{\mathbf{p}^4}{4m^3} + \delta V.$$

In order to evaluate the formula of the rates we need to include the relativistic corrections to the potential $\delta V \sim \mathcal{O}(v^2)$

- To evaluate transitions between lower quarkonium states (weakly-coupled) we can use the perturbative expressions for these corrections.
- For transitions that involve higher states (strongly-coupled) we need to include non-perturbative terms in the potential.

The $Q\bar{Q}$ potential

In the equal mass case the relativistic corrections to the quark-antiquark potential can be organized in powers of $1/m$:

$$V = V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \cdots$$

where

$$\begin{aligned} V^{(2)} &= V_{SD}^{(2)} + V_{SI}^{(2)}, \\ V_{SI}^{(2)} &= \frac{1}{2}\left\{\mathbf{p}^2, V_{p^2}^{(2)}(r)\right\} + \frac{V_{L^2}^{(2)}(r)}{r^2}\mathbf{L}^2 + V_r^{(2)}(r), \\ V_{SD}^{(2)} &= V_{LS}^{(2)}(r)\,\mathbf{L}\cdot\mathbf{S} + V_{S^2}^{(2)}(r)\left(\frac{\mathbf{S}^2}{2} - \frac{3}{4}\right) + V_{S_{12}}^{(2)}(r)\mathbf{S}_{12}, \end{aligned}$$

The potentials $V_{\hat{O}}^{(i)}$ were obtained in terms of operator insertions in the expectation value of the rectangular Wilson loop. For instance for the $1/m$ correction we have

$$V^{(1)}(r) = -\lim_{T\to\infty}\int_0^T dt\, t\, \langle\langle g\mathbf{E}_1(t)\cdot g\mathbf{E}_1(0)\rangle\rangle_c,$$

where $\langle\langle \ldots \rangle\rangle \equiv \langle \ldots W_\Box \rangle / \langle W_\Box \rangle$ and

$$\langle\langle O_1(t_1) O_2(t_2) \rangle\rangle_c = \langle\langle O_1(t_1) O_2(t_2) \rangle\rangle - \langle\langle O_1(t_1) \rangle\rangle \langle\langle O_2(t_2) \rangle\rangle$$

Brambilla, Pineda, Soto and Vairo, *PRD 63, 014023 (2001)*

The other potentials follow the same way

Pineda and Vairo, *PRD 63, 054007 (2001)*

- In the short-distance regime, $r\Lambda_{QCD} \ll 1$, these correlators can be computed in perturbation theory.
- In the long-distance regime, $r\Lambda_{QCD} \sim 1$, these correlators should be computed in Lattice QCD.

Not all these correlators have been calculated in the Lattice. To evaluate the rates we take the intermediate approach of computing the correlators in the **effective string theory (EST)**:
The static quark-antiquark potential is given by

$$V^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \ln\langle W_\Box \rangle$$

where

$$W_{\Box} \equiv \mathrm{P}\exp\left\{-ig\oint_{r\times T} dz^{\mu}A_{\mu}(z)\right\},$$

is the rectangular Wilson loop. In the long distance limit this leads to a static potential with a linear, string-like, dependence in r:

$$V^{(0)} = \kappa r$$

where κ can be identified as the string tension.
The EST hypothesis states that at long distances ($r\Lambda_{QCD} \gg 1$)

$$\lim_{T\to\infty}\langle 0|W_{\Box}(T,r)|0\rangle = Z\int \mathcal{D}\xi^{1}\mathcal{D}\xi^{2}\, e^{iS_{string}(\xi^{1},\xi^{2})}$$

where

$$S_{string} = \int dtdz\, \mathcal{L}(\partial^{\mu}\xi^{l}) = -\kappa\int dtdz\, \left(1-\frac{1}{2}\partial_{\mu}\xi^{l}\partial^{\mu}\xi^{l}\right),$$

Since both are expected to be valid at long distances, one could expect to have a mapping relating the operator insertions in the Wilson loop and degrees of freedom of the EST.

Using symmetry considerations the following mapping between the transverse string coordinates and the operator insertions in the correlators appearing in the relativistic corrections to the quark-antiquark potential has been obtained:

$$\langle\langle \dots \mathbf{E}_1^l(t) \dots \rangle\rangle \to \langle \dots \Lambda^2 \partial_z \xi^l(t, r/2) \dots \rangle\,,$$
$$\langle\langle \dots \mathbf{E}_2^l(t) \dots \rangle\rangle \to \langle \dots \Lambda^2 \partial_z \xi^l(t, -r/2) \dots \rangle\,,$$
$$\langle\langle \dots \mathbf{B}_1^l(t) \dots \rangle\rangle \to \langle \dots \Lambda' \epsilon^{lm} \partial_t \partial_z \xi^m(t, r/2) \dots \rangle\,,$$
$$\langle\langle \dots \mathbf{B}_2^l(t) \dots \rangle\rangle \to \langle \dots - \Lambda' \epsilon^{lm} \partial_t \partial_z \xi^l(t, -r/2) \dots \rangle\,,$$
$$\langle\langle \dots \mathbf{E}_1^3(t) \dots \rangle\rangle \to \langle \dots \Lambda''^2 \dots \rangle\,,$$
$$\langle\langle \dots \mathbf{E}_2^3(t) \dots \rangle\rangle \to \langle \dots \Lambda''^2 \dots \rangle\,,$$
$$\langle\langle \dots \mathbf{B}_1^3(t) \dots \rangle\rangle \to \langle \dots \Lambda''' \epsilon^{lm} \partial_t \partial_z \xi^l(t, r/2) \partial_z \xi^m(t, r/2) \dots \rangle\,,$$
$$\langle\langle \dots \mathbf{B}_2^3(t) \dots \rangle\rangle \to \langle \dots - \Lambda''' \epsilon^{lm} \partial_t \partial_z \xi^l(t, -r/2) \partial_z \xi^m(t, -r/2) \dots \rangle$$

Perez-Nadal and Soto, *PRD79, 114002 (2009)* & **Kogut and Parisi** *PRL47, 1089 (1981)*

The potentials in the EST read

$$V^{(0)} = \kappa r,$$

$$V^{(1)}(r) = \frac{2g^2\Lambda^4}{\pi\kappa}\ln(\sqrt{\kappa}r) + \mu_1\,,$$

$$V^{(2)}_{p^2}(r) = 0\,,$$

$$V^{(2)}_{L^2}(r) = -\frac{g^2\Lambda^4 r}{6\sigma}\,,$$

$$V^{(2)}_{LS}(r) = -\frac{\mu_2}{r} - \frac{2c_F^{(1)}g^2\Lambda^2\Lambda'}{\kappa\, r^2}\,,$$

$$V^{(2)}_{S^2}(r) = \frac{2\pi^3 c_F^{(1)}c_F^{(2)}g^2\Lambda'''^2}{45\kappa^2 r^5}$$

$$V^{(2)}_{\mathbf{S}_{12}}(r) = \frac{\pi^3 c_F^{(1)}c_F^{(2)}g^2\Lambda'''^2}{90\kappa^2 r^5}\,,$$

$$V^{(2)}_{r}(r) = -\frac{9\,\zeta_3\,g^4\Lambda^8 r}{2\pi^3\kappa^2} + \mu_3 + \frac{\mu_4}{r^2} + \frac{\mu_5}{r^4} + \frac{\pi^3 c_F^{(1)\,2}g^2\Lambda'''^2}{30\kappa^2 r^5}\,.$$

Keeping the LO terms in r, dropping the renormalization constants and applying further known constraints to the potentials (i.e. Poincaré invariance) the equal mass long-range $Q\bar{Q}$ potential can be reduced to

$$V(r)^{\text{long}-\text{range}} \approx \kappa r + \frac{1}{m}\left[\frac{2\kappa}{\pi}\ln(\sqrt{\kappa}r)\right] + \frac{1}{m^2}\left(-\frac{\kappa}{6r}\mathbf{L}^2 - \frac{\kappa}{2r}\mathbf{L}\cdot\mathbf{S} - \frac{9\,\zeta_3\,\kappa^2 r}{2\pi^3}\right)$$

N. Brambilla, M. Groher, H.M. and A. Vairo, *PRD 90, 114032 (2014)*

We need to add the short-distance part of the potential:

$$V^{(0)} = -C_F \frac{\alpha_s(r)}{r}$$

$$V^{(1)} = -\frac{C_F C_A \alpha_s(r)^2}{2r^2}$$

$$V^{(2)}_{p^2} = -\frac{C_F \alpha_s(r)}{r}$$

$$V^{(2)}_{L^2} = \frac{C_F \alpha_s(r)}{2r}$$

$$V^{(2)}_{r} = \pi C_F \alpha_s(r) \delta^{(3)}(\mathbf{r})$$

$$V^{(2)}_{S^2} = \frac{4\pi C_F \alpha_s(r)}{3} \delta^{(3)}(\mathbf{r})$$

$$V^{(2)}_{LS} = \frac{3 C_F \alpha_s(r)}{2r^3}$$

$$V^{(2)}_{S_{12}} = \frac{C_F \alpha_s(r)}{4r^3}$$

Buchmüller et al., *PRD 24, 3003 (1981)*
Gupata and Radford , *PRD 24, 2307 (1981)*
Gupata and Radford , *PRD 25, 3430 (1982)*
Pantaleone et al. , *PRD 33, 777 (1986)*
Titard and Yndurain, *PRD 49, 6007 (1994)*

In the fashion of the Cornell potential, we construct the full-range potential adding both contributions:

$$V(r) = V^{\rm short-range}(r) + V^{\rm long-range}(r)$$

We take the further simplification of freezing $\alpha_s(r) \mapsto a$ where a will be a parameter that we will fix later.
In this approach the quark-antiquark potential up to $1/m^2$ corrections is given by

$$
\begin{aligned}
V(a,\kappa,m,r) \quad = \quad & -\frac{C_F a}{r} + \kappa r + \frac{1}{m}\left\{-\frac{C_F C_A a^2}{2r^2} + \frac{2\kappa}{\pi}\log(\sqrt{\kappa}r)\right\} \\
+ \quad & \frac{1}{m^2}\left\{\frac{1}{2}\left\{\mathbf{p}^2, -\frac{C_F a}{r}\right\} + \left(\frac{C_F a}{2r} - \frac{\kappa}{6r}\right)\mathbf{L}^2 + \left(\frac{3C_F a}{2r^3} - \frac{\kappa}{2r}\right)\mathbf{L}\cdot\mathbf{S}\right. \\
+ \quad & \left.\frac{4\pi C_F a}{3}\delta^{(3)}(\mathbf{r})\mathbf{S}^2 + \frac{C_F a}{4r^3}\mathbf{S}_{12}(\hat{\mathbf{r}}) + \pi C_F a \delta^{(3)}(\mathbf{r}) - \frac{9\,\zeta_3\,\kappa^2 r}{2\pi^3}\right\}
\end{aligned}
$$

Counting and parameter fitting

Parametric size of the relativistic corrections:

$$\Gamma^{\mathrm{E1}}_{H\to H'\gamma} = \Gamma^{\mathrm{LO}}_{H\to H'\gamma}\Big(1 + \overbrace{R_{H\to H'\gamma}}^{\sim ?} + \overbrace{\delta\Gamma_{H\to H'\gamma}}^{\sim v^2}\Big)$$

The counting adopted to obtain these corrections was

Counting E1

$$\begin{aligned} k_\gamma &\sim mv^2, \\ \mathbf{p} &\sim mv, \\ r &\sim 1/mv, \\ \mathbf{E}^{em}, \mathbf{B}^{em} &\sim k_\gamma^2, \end{aligned}$$

Brambilla, Pietrulewicz and Vairo, *PRD 85, 094005 (2012)*

Bottomonium Counting

$$
\begin{aligned}
a &\sim v, \\
\kappa &\sim m^2 v^3, \\
p &\sim mv, \\
r &\sim 1/mv,
\end{aligned}
$$

Considering this counting we can order the potential

$$
\begin{aligned}
V^{\mathrm{LO}} &= -C_F \frac{a}{r} + \kappa r \sim mv^2, \\
V^{\mathrm{NLO}} &\equiv \frac{2\kappa}{m\pi} \log(\sqrt{\kappa} r) \sim LO \times v, \\
V^{\mathrm{NNLO}} &\equiv -\frac{C_F C_A a^2}{2mr^2} + \frac{1}{m^2} \left\{ \frac{1}{2} \left\{ \mathbf{p}^2, -\frac{C_F a}{r} \right\} + \left(\frac{C_F a}{2r^3} - \frac{\kappa}{6r} \right) \mathbf{L}^2 + \left(\frac{3 C_F a}{2r^3} - \frac{\kappa}{2r} \right) \mathbf{L} \cdot \mathbf{S} \right. \\
&+ \left. \frac{4\pi C_F a}{3} \delta^{(3)}(\mathbf{r}) \mathbf{S}^2 + \frac{C_F a}{4r^3} \mathbf{S}_{12}(\hat{\mathbf{r}}) + \pi C_F a \delta^{(3)}(\mathbf{r}) \right\} - \frac{\mathbf{p}^4}{4m^3} \sim LO \times v^2
\end{aligned}
$$

Charmonium counting

(Motivated by the fact that for charmonium states $a \sim v^2$)

$$\begin{aligned} a &\sim v^2, \\ \kappa &\sim m^2 v^3, \\ p &\sim mv, \\ r &\sim 1/mv, \end{aligned}$$

The potential is ordered as

$$\begin{aligned} V^{\mathrm{LO}} &= \kappa r, \sim mv^2 \\ V^{\mathrm{NLO}} &= -C_F \frac{a}{r} + \frac{2\kappa}{m\pi} \log(\sqrt{\kappa} r) \sim LO \times v \\ V^{\mathrm{NNLO}} &= -\frac{\mathbf{p}^4}{4m^3} + \frac{1}{m^2} \left\{ -\frac{\kappa}{6r} \mathbf{L}^2 - \frac{\kappa}{2r} \mathbf{L} \cdot \mathbf{S} \right\} \sim LO \times v^2 \end{aligned}$$

Parameter fixing:

We fix the parameters of the potential requiring them to reproduce the masses of quarkonium states:

$$M_{\text{theo}}(n^{2s+1}L_J) = \underbrace{2m_{c,b} + E_{nl}^{(0)}}_{LO} + \underbrace{\frac{\langle nl|V^{\text{NLO}}(r)|nl\rangle}{m_{c,b}}}_{\sim LO\times v} + \underbrace{\frac{1}{m_{c,b}^2}\sum_{m\neq n}^{\infty}\frac{|\langle nl|V^{\text{NLO}}(r)|ml\rangle|^2}{E_{nl}^{(0)} - E_{ml}^{(0)}} + \frac{\langle nljs|V^{\text{NNLO}}(r)|nljs\rangle}{m_{c,b}^2}}_{\sim LO\times v^2}$$

Phenomenology input:

We take the values of the quarkonium masses below threshold from the PDG and solve:

$$M_{\text{theo}}(n^{2S+1}L_J; a, \kappa, m) - M_{\text{PDG}}(n^{2S+1}L_J) = 0.$$

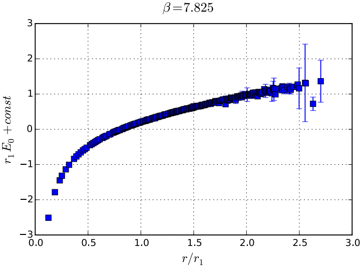
Using sixteen different quarkonium states we obtain sixteen sets of parameters.

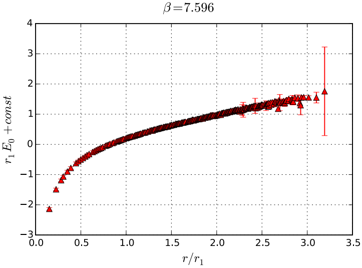
Parameter fixing: Lattice input

- We fix the parameters a and κ of the $1/m^0$ potential using the Lattice data provided by the Hot QCD collaboration.
- For each quarkonium state, $n^{2S+1}L_J$, and each lattice dataset we solve

$$M_{\rm theo}(n^{2S+1}L_J; a = a_{{\rm latt}\, i}, \kappa = \kappa_{{\rm latt}\, i}, m_i') - M_{\rm exp}(n^{2S+1}L_J) = 0,$$

- Our final values of the quark masses are obtained averaging the eight values we get for $m_c(n^{2S+1}L_J)$ and the fourteen values we get for $m_b(n^{2S+1}L_J)$





Bazavov et al., *PRD 90 094503 (2014)*

Parameter fixing: Summary

Input	$a\,(\sigma_a)$	$\kappa\,(\sigma_\kappa)\,[GeV^2]$	$m_c\,(\sigma_{m_c})\,[GeV]$	$m_b\,(\sigma_{m_b})\,[GeV]$
Phen.	0.246 (0.074)	0.210 (0.041)	1.123 (0.074)	4.704 (0.086)
Latt.	0.309 (0.018)	0.211 (0.006)	1.143 (0.094)	4.743 (0.164)

The counting implies:

$$\Gamma^{\mathrm{E1}}_{H\to H'\gamma} = \Gamma^{\mathrm{LO}}_{H\to H'\gamma}(1 + \overbrace{R_{H\to H'\gamma}}^{\sim v} + \overbrace{\delta\Gamma_{H\to H'\gamma}}^{\sim v^2})$$

Evaluation method and results

Evaluation method and error

- For each of the parameters in each of the approaches we generate sixteen values that distribute normally around the mean.
- In each approach we evaluate

$$\Gamma^{\mathrm{E1}}_{H\to H'\gamma}(a,\kappa,m) = \Gamma^{\mathrm{LO}}_{H\to H'\gamma}(1 + R_{H\to H'\gamma} + \delta\Gamma_{H\to H'\gamma})$$

  using the sixteen parameter sets.
- The central value of each approach corresponds to the mean of these sixteen values.
- The error is calculated as

$$\epsilon_{\mathrm{phen,\,latt}} \equiv \sqrt{\epsilon^2_{\mathrm{par}} + \epsilon^2_{\mathrm{rel}}}.$$

Evaluation method and error

- ϵ_{par} is the standard deviation of the sixteen values
- For the LO rate ϵ_{rel} is the mean of the sixteen points obtained after evaluating

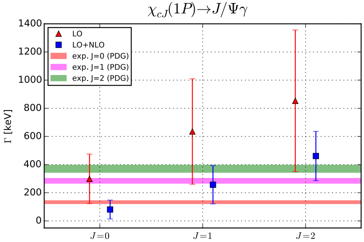
$$\Gamma' = \Gamma^{LO}_{H \to H'\gamma} \times K_{\text{LO}}\, v(a, \kappa, m),$$

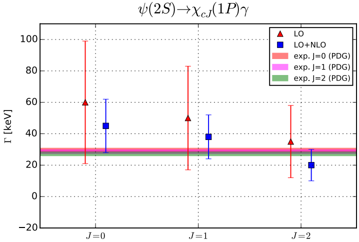
- and for the LO + NLO rates ϵ_{rel} is the mean of the sixteen points obtained after evaluating

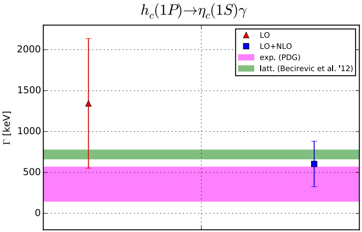
$$\Gamma'' = \Gamma^{LO}_{H \to H'\gamma} \times K_{\text{NLO}}\, v(a, \kappa, m)^3,$$

- In the evaluations presented here we have set $K_{\text{LO}} = K_{\text{NLO}} = 1$.
- For our final results we take the average of the transitions we get with phenomenology and lattice inputs.
- The error corresponds to $\text{Max}(\epsilon_{\text{phen}}, \epsilon_{\text{latt}})$

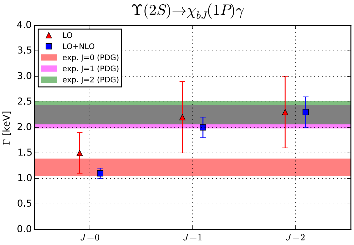
Results: Charmonium

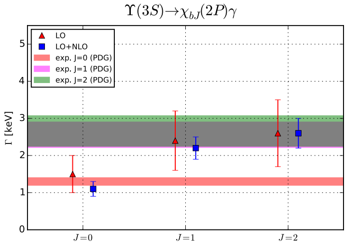




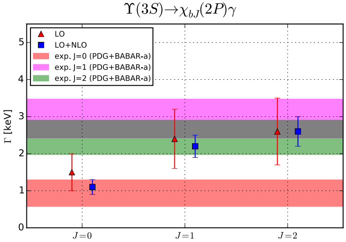


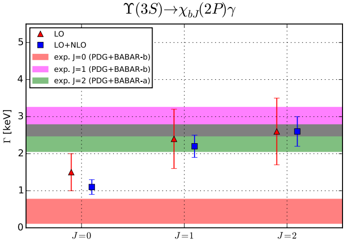
Results: Bottomonium (PDG)

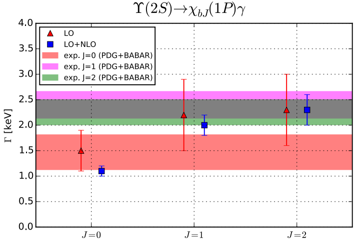




Results: Bottomonium (BABAR)







Results: Predictions for the total width

State (Process)	Γ_H [keV]
$h_b(1P)$ $(1P \to 1S)$	35 ± 35
$h_b(2P)$ $(2P \to 1S)$	95 ± 35
$h_b(2P)$ $(2P \to 2S)$	15 ± 15
$\chi_{b0}(1P)$ $(1P \to 1S)$	1023 ± 498
$\chi_{b1}(1P)$ $(1P \to 1S)$	65 ± 33
$\chi_{b2}(1P)$ $(1P \to 1S)$	136 ± 63
$\chi_{b0}(2P)$ $(2P \to 1S)$	667 ± 497
$\chi_{b0}(2P)$ $(2P \to 2S)$	196 ± 99
$\chi_{b1}(2P)$ $(2P \to 1S)$	109 ± 34
$\chi_{b1}(2P)$ $(2P \to 2S)$	50 ± 16
$\chi_{b2}(2P)$ $(2P \to 1S)$	229 ± 62
$\chi_{b2}(2P)$ $(2P \to 2S)$	104 ± 46

Summary and Outlook

- Our results compare favourable to experiment.
- Based on our evaluation of the E1 transitions we are able to made predictions for the total width of at least one state ($\chi_{b0}(1P)$) that has the prospect of being measured soon (BELLE).
- (Possible) Next step: elaborate more on the perturbative potential, reduce theoretical uncertainty.
- A pure perturbative potential may be suitable to describe the $1P \to 1S\gamma$ process.

THANKS FOR YOUR ATTENTION!