High Energy Physics in the LHC era

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Tackling the flavour problem via spontaneous breaking of flavour $\mathrm{SU}(3)^3$

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[Work done in part with C.S. Fong and J.R. Espinosa]

The SM fermions gauge invariant kinetic term:

$$\sum_{f=Q,\ell,u,d,e} \bar{\Psi}_f \, \not{D}_f \Psi_f$$

Only five $\not{D}_f$ for 15 fermions.

Formally: $\mathcal{G}_F = U(3)^5$ invariance.

Fermions replicate in triplets.

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No multiplet structure in the spectrum: $\Rightarrow SSB$

Restricting to quarks and the broken subgroup $SU(3)^3$

$$\mathcal{G}_{\mathcal{F}} = SU(3)_Q \times SU(3)_u \times SU(3)_d$$

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Symmt. breaking ansatz: Interpret the SM *explicit* breaking as *spontaneous*, driven by a set of scalar "Yukawa fields" :

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Plan of the talk: the vacuum structure of $V(Y_u, Y_d)$

1. Yukawa hierarchy: **A single Yukawa field** $(\bar{Q}_L Y q_R)$

Flavour symmetry: $\mathcal{G}_{LR} = SU(3)_L \times SU(3)_R$.

See if the the most general $\mathcal{G}_{LR}$ invariant $V(Y)$ admits minima $\langle Y_{aa} \rangle \sim (y_1, y_2, y_3)$ with a hierarchical structure $y_1 \ll y_2 \ll y_3$.

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– **The breaking $\mathcal{G}_{LR} \to U(1)^2$ must occur already at tree level!**

– This can be achieved adding reducible scalar representations: $Y + (Z_L, Z_R)$.

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3. By augmenting $V(Y_u, Y_d) \longrightarrow V(Y_u, Y_d; Z_{Q_1}, Z_{Q_2}, Z_u, Z_d)$

- 🙂 The whole set $\{Y_u, Y_d, V_{CKM}, \delta_{CP}\}$ can be reproduced *without introducing any hierarchical parameter*

$Y_{u,d}$ invariants and the $T,\ A,\ D$ parametrization

Singular value decomposition for the non-Abelian multiplet Y:

$$Y = \mathcal{V}^\dagger \, \chi \, \mathcal{U}$$

$\mathcal{V}, \mathcal{U}$ unitary field matrices, $\chi = \chi_u = \mathrm{diag}\,(u_1,\, u_2,\, u_3)$; [with vevs $\langle u_i \rangle \geq 0$]

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$$T = \mathrm{Tr}(YY^\dagger) = \sum_i u_i^2;$$

$$A = \mathrm{Tr}\left[\mathrm{Adj}\left(YY^\dagger\right)\right] = \tfrac{1}{2}\sum_{i\neq j} u_i^2 u_j^2$$

$$\mathcal{D} = \mathrm{Det}(Y) \equiv e^{i\delta} D = e^{i\delta} \prod_i u_i; \quad [\delta = \mathrm{Arg\,Det}\,(\mathcal{V}^\dagger \mathcal{U})]$$

[Characteristic eqn.: $\mathcal{P}(\xi) = \det\left(\xi I - YY^\dagger\right) = \xi^3 - T\,\xi^2 + A\,\xi - D^2 = 0$]

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The $\mathcal{G_F}$ invariant potential: $V(Y) = V[T(\chi), A(\chi), \mathcal{D}(\chi)]$

Scalar potential and classification of the vacua

$$V = \frac{1}{\Lambda^4}\hat{V} = \lambda\left[T - \frac{m^2}{2\lambda}\right]^2 + \lambda_A A + \underbrace{\tilde{\mu}\,\mathcal{D} + \tilde{\mu}^*\,\mathcal{D}^\dagger}_{2\,\mu\,\cos\tilde{\delta}\cdot\,D}$$

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V admits hierarchical vacua $\langle\chi\rangle_h = (0,0,u)$! $\left[SU(3)\times SU(3)\to SU(2)\times SU(2)\times U(1)\right]$

Can the vanishing entries be lifted $(0,0,1) \to (\epsilon', \epsilon, 1)$?

Ref.[1]: $V \to V^{eff} = V_0 + V_1$; if $V_1 \supset c_A \cdot A \log A; \quad c_D \cdot D \log D$ then:

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Expand the vev vector $v = v_0 + \varepsilon v_1 + \ldots$
Stepwise breaking means: $\mathcal{T} \cdot v_0 = 0 \quad \& \quad \mathcal{T} \cdot v_1 \neq 0$
From Goldstone Theorem: $0 = M^2 \, \mathcal{T} \, v = M^2 \, (\mathcal{T} \cdot v_0) + \varepsilon M_0^2 \, (\mathcal{T} \cdot v_1)$

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Examples of theories with non-NGB massless scalars at tree level:

$V_{CW} = \lambda \phi^4$ (all states are massless at tree level)

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$$H_\epsilon = SU(2)\times U(1) \;\Leftrightarrow\; \langle Y\rangle = (a,a,b) \quad \text{unless} \quad \frac{\partial V}{\partial T} = \frac{\partial V}{\partial A} = \frac{\partial V}{\partial D} = 0$$

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Other impediments to reproduce the hierarchy

In Ref.[2] [J.R. Espinosa, C.S. Fong, EN] it was shown that:

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CONCLUSION: $\mathcal{G}_\mathcal{F} \to H_\epsilon$ breaking should occur already at the tree level!
[$V(Y)$ potential is too simple. We need additional scalar reps.]

A hierarchy $\langle \chi\rangle_\epsilon = (\epsilon', \epsilon, 1)$ and $\mathcal{G}_\mathcal{F} \to \mathcal{H}_\epsilon$ can be obtained by adding scalars in the fundamental of each $SU(3)_Q \times SU(3)_q$ factors: $Z_Q = (\mathbf{3}, \mathbf{1})$, $Z_u = (\mathbf{1}, \mathbf{3})$.

[Michel-Radicati theorem ($\mathcal{G}_\mathcal{F} \to \mathcal{H}_{\rm maximal}$) only applyes to irreducible $SU(3)\times SU(3)$ representations (Y)]

Step 2: quark mixings $\mathcal{G}_{\mathcal{F}} = SU(3)_Q \times SU(3)_u \times SU(3)_d$

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Only one term is relevant when coupling the *u* and *d* sectors

$V \supset \lambda_{ud} T_{ud}$ with $T_{ud} = \mathrm{Tr}(Y_u Y_u^\dagger Y_d Y_d^\dagger) = \mathrm{Tr}\left(\mathscr{V}^\dagger \chi_u^2 \mathscr{V} \chi_d^2\right)$

and $\mathscr{V} = \mathcal{V}_u \mathcal{V}_d^\dagger$ a unitary matrix of fields with vev: $\langle \mathscr{V} \rangle = V_{CKM}$

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With only two "directions" Y_u and Y_d in $SU(3)_Q$ flavour space there is only one relative "angle". Then, if $\lambda_{ud} < 0$, $V(Y_u, Y_d)$ is minimized for $\chi_{u,d}$ aligned $(\uparrow\uparrow)$

and $V_{CKM} \propto I$ [if $(\lambda_{ud} > 0)$ min. occurs for $\chi_{u,d}$ anti-aligned $(\uparrow\downarrow)$].

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This suggests a way towards the construction of a viable scalar potential:

"INTUITIVELY": We need at least four "directions" in $SU(3)_Q$ flavour space to 'define' three relative "angles". [i.e.: add more scalar reps.]

Complete quark hierarchy & mixing pattern from SFSB

Ref [3] [C.S.Fong, EN] program: Search for $V(Y_{u,d},\{Z\})$ that can break at the tree level $\mathcal{G}_{\mathcal{F}} = SU(3)_Q \times SU(3)_u \times SU(3)_d$ generating hierarchies, mixings, and $\not{CP}$.

Previous results suggest: $\{Z\} = \{Z_{Q_{1,2}} \sim (\mathbf{3},\mathbf{1},\mathbf{1}), Z_u \sim (\mathbf{1},\mathbf{3},\mathbf{1}), Z_d \sim (\mathbf{1},\mathbf{1},\mathbf{3})\}$

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– Flavour irrelevant: carry larger symmetries: $T \sim [SO(18){:}\langle\chi\rangle_h \to \langle\,\chi\rangle_s]$, $|Z|^2 \sim [SO(6)]$
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2. Divide $V(Y_{u,d},\{Z\}) = V_{\mathcal{I}} + V_{\mathcal{AR}} + V_{\mathcal{A}}$ and study the *flavour relevant* parts $V_{\mathcal{AR}}$ and

$$V_{\mathcal{A}} \supset \left(\mu_q \mathcal{D}_q + \nu_{iq} Z_{Qi}^\dagger Y_q Z_q\right) + H.c.$$

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CP-violation: $V_{\mathcal{A}}$ contains 4 physical complex phases. At the minimum, one nonvanishing phase $\delta_{\not{C}\not{P}}$ remains in $\langle\mathcal{V}\rangle = V_{CKM}$.

Recap and conclusions

Assuming that the SM fermions belong to triplets of a fundamental flavour symmetry $\mathcal{G}_{\mathcal{F}} = SU(3) \times SU(3) \times \dots$ spontns. broken by "Yukawa fields":

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4. All hierarchical suppressions are dynamical (as opposite to parametric), and do not require any particularly small number in $V(Y_q, \{Z\})$.

One numerical example

With these inputs:

$$\mu_q = \nu_{1q} = \nu_{2q} = v/10, \qquad m_{12}^2 = 0.15\, v^2,$$
$$\gamma_{ud} = 0.81, \qquad \eta_{12} = 0.1, \qquad \lambda_{12} = 1.27,$$
$$\phi_{\gamma_{ud}} = 0.98\pi, \qquad \phi_{\eta_{12}} = 0.92\pi, \qquad \phi_{\nu_{2q}} = 0.95\pi.$$

and all other parameters set to 1 (or to -1), we obtain:

$$|\hat{Y}_u| = v\,\mathrm{diag}\,(0.0003, 0.009, 1.4)\,,$$
$$|\hat{Y}_d| = v\,\mathrm{diag}\,(0.0007, 0.02, 1.2)\,,$$
$$K = V_{CKM} = \begin{pmatrix} 0.974 & 0.223 & 0.027 \\ 0.224 & 0.974 & 0.042 \\ 0.017 & 0.046 & 0.999 \end{pmatrix},$$
$$J = \mathrm{Im}\left(K_{jk}K_{lm}K^*_{jm}K^*_{kl}\right) = 2.9 \times 10^{-5}.$$