Toward a purely affine theory of gravity

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Outline

Motivations and Considerations

- GR is (so far) the most successful theory of gravity.
- GR interprets geometry as interaction.
- In GR the main character is curvature.
- Curvature is built by a connection, $\mathrm{Riem} = \mathrm{Riem}(\Gamma)$.
- The connection might not be the one of Levì-Civita.

But, Where is the metric?

In the contraction to obtain the Ricci scalar!

$$S = \int *R$$

Is it possible to build a non-metric model of gravity?

Eddington (1923): $S = \int \sqrt{\mathrm{Ric}(\Gamma)}$.

Building a gravitational affine theory is similar to building a Chern–Simons gauge theory.

Do we need to change the gravitational theory?

Excellent classical behaviour, but not at quantum level:

- The metric is an unbounded operator, with multiple signature states
- The Wheeler–DeWitt equation is (generally) ill-defined
  $\rightarrow$ Non-polynomial dependence on ADM variables
- Solutions (WDW eqs.) and interpretations, evolution, Dirac observable, etc.
- It is non-renormalizable (avoidable problem!)
  Could address some problems:
  - Vacuum energy (Micro scale)
  - Rotational curves (Galactic scale)
  - Dark sector (Cosmological scale)

Possible solution???

A new quantizable model of gravity

A glance to the gravity Zoo

- Palatini: Non Renormalizable
- Super Gravity: SUSY, Non Renormalizable
- String Theory: 10 dim, SUSY, Landscape problem
- Loop Quantum Gravity: Semiclassic limit, Continuous.
- Spin foams: Lorentz group anomalies
- Conformal gravity: Newtonian Limit
- Hořava–Lifshitz Gravity: Difeomorphisms
- Massive gravity: Ghosts , 2 metrics,

  1410.2289 S. Desser et al.: non unique evolution, superluminal signals, inconsistencies with matter couplings, micro-acausality (propagation through closed timelike curves)

- Brans-Dicke, Teleparallel gravity, Modified Newtonian dynamics (MOND), Causal sets, Regge Calculus, Dynamical Triangulations, Penrose Twistor theory, Penrose spin networks, Connes Non–commutative geometry, Causal Fermion Systems

Our bet: A new affine theory of gravity

- An affine connection as solely dynamical field
- The action is polynomial in the fields
- Diffeomorphism invariance
- The invariant tensors (density) are δ and ϵ

Affine connection

- Defines a covariant linear derivative
- Defines a parallel transport
- It can be decomposed into irreps

$$\hat{\Gamma}^{\mu}{}_{\rho\sigma} = \Gamma^{\mu}{}_{\rho\sigma} + \epsilon_{\rho\sigma\lambda_1\dots\lambda_{d-2}} T^{\mu,\lambda_1\dots\lambda_{d-2}} + A_{[\rho}\delta^{\mu}_{\nu]}$$

with $\epsilon_{\rho\mu\lambda_1\dots\lambda_{d-2}} T^{\mu,\lambda_1\dots\lambda_{d-2}} = 0.$

Cooking with the finest ingredients

- Fields: Γ, T and A.
- Γ do not enter directly, but $\mathrm{Riem}(\Gamma)$.
- Seasonings: ∇, δ and ϵ
- Lagrangian: scalar density
- $W(T) = W(\epsilon^{\bullet}) = 1$
- $N(\Phi)$ number of indices

a	b	c	ℓ
1	3	0	0
1	2	1	0
1	1	2	0
1	0	3	0
2	2	0	-1
2	1	1	-1
2	0	2	-1
3	1	0	-2
3	0	1	-2
4	0	0	-3
0	4	0	1
0	3	1	1
0	2	2	1
0	1	3	1
0	0	4	1

The restrictions

$$N(T^a A^b \nabla^c \epsilon_{\dots}{}^{d} \epsilon^{\dots e}) = n \qquad n = 3a - b - c + 4\ell$$

$$W(T^a A^b \nabla^c \epsilon_{\dots}{}^{d} \epsilon^{\dots e}) = w \qquad w = a + \ell.$$

The MOST general action

$$
\begin{aligned}
S[\Gamma, T, A] = \int \mathrm{d}^4x \, \Big[& B_1 \, R_{\mu\nu}{}^{\mu}{}_{\rho} T^{\nu,\alpha\beta} T^{\rho,\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} + B_2 \, R_{\mu\nu}{}^{\sigma}{}_{\rho} T^{\beta,\mu\nu} T^{\rho,\gamma\delta} \epsilon_{\sigma\beta\gamma\delta} \\
& + B_3 \, R_{\mu\nu}{}^{\mu}{}_{\rho} T^{\nu,\rho\sigma} A_\sigma + B_4 \, R_{\mu\nu}{}^{\sigma}{}_{\rho} T^{\rho,\mu\nu} A_\sigma \\
& + B_5 \, R_{\mu\nu}{}^{\rho}{}_{\rho} T^{\sigma,\mu\nu} A_\sigma + C_1 \, R_{\mu\rho}{}^{\mu}{}_{\nu} \nabla_\sigma T^{\nu,\rho\sigma} \\
& + C_2 \, R_{\mu\nu}{}^{\rho}{}_{\rho} \nabla_\sigma T^{\sigma,\mu\nu} + D_1 \, T^{\alpha,\mu\nu} T^{\beta,\rho\sigma} \nabla_\gamma T^{(\lambda,\kappa)\gamma} \epsilon_{\beta\mu\nu\lambda} \epsilon_{\alpha\rho\sigma\kappa} \\
& + D_2 \, T^{\alpha,\mu\nu} T^{\lambda,\beta\gamma} \nabla_\lambda T^{\delta,\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\mu\nu\rho\sigma} \\
& + D_3 \, T^{\mu,\alpha\beta} T^{\lambda,\nu\gamma} \nabla_\lambda T^{\delta,\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\mu\nu\rho\sigma} + D_4 \, T^{\lambda,\mu\nu} T^{\kappa,\rho\sigma} \nabla_{(\lambda} A_{\kappa)} \epsilon_{\mu\nu\rho\sigma} \\
& + D_5 \, T^{\lambda,\mu\nu} \nabla_{[\lambda} T^{\kappa,\rho\sigma} A_{\kappa]} \epsilon_{\mu\nu\rho\sigma} + D_6 \, T^{\lambda,\mu\nu} A_\nu \nabla_{(\lambda} A_{\mu)} \\
& + D_7 \, T^{\lambda,\mu\nu} A_\lambda \nabla_{[\mu} A_{\nu]} + E_1 \, \nabla_{(\rho} T^{\rho,\mu\nu} \nabla_{\sigma)} T^{\sigma,\lambda\kappa} \epsilon_{\mu\nu\lambda\kappa} \\
& + E_2 \, \nabla_{(\lambda} T^{\lambda,\mu\nu} \nabla_{\mu)} A_\nu + T^{\alpha,\beta\gamma} T^{\delta,\eta\kappa} T^{\lambda,\mu\nu} T^{\rho,\sigma\tau} \Big(F_1 \, \epsilon_{\beta\gamma\eta\kappa} \epsilon_{\alpha\rho\mu\nu} \epsilon_{\delta\lambda\sigma\tau} \\
& + F_2 \, \epsilon_{\beta\lambda\eta\kappa} \epsilon_{\gamma\rho\mu\nu} \epsilon_{\alpha\delta\sigma\tau} \Big) + F_3 \, T^{\rho,\alpha\beta} T^{\gamma,\mu\nu} T^{\lambda,\sigma\tau} A_\tau \epsilon_{\alpha\beta\gamma\lambda} \epsilon_{\mu\nu\rho\sigma} \\
& + F_4 \, T^{\eta,\alpha\beta} T^{\kappa,\gamma\delta} A_\eta A_\kappa \epsilon_{\alpha\beta\gamma\delta} \Big].
\end{aligned}
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\begin{aligned}
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& + B_3\, R_{\mu\nu}{}^{\mu}{}_{\rho} T^{\nu,\rho\sigma} A_\sigma + B_4\, R_{\mu\nu}{}^{\sigma}{}_{\rho} T^{\rho,\mu\nu} A_\sigma \\
& + B_5\, R_{\mu\nu}{}^{\rho}{}_{\rho} T^{\sigma,\mu\nu} A_\sigma + C_1\, R_{\mu\rho}{}^{\mu}{}_{\nu} \nabla_\sigma T^{\nu,\rho\sigma} \\
& + C_2\, R_{\mu\nu}{}^{\rho}{}_{\rho} \nabla_\sigma T^{\sigma,\mu\nu} + D_1\, T^{\alpha,\mu\nu} T^{\beta,\rho\sigma} \nabla_\gamma T^{(\lambda,\kappa)\gamma} \epsilon_{\beta\mu\nu\lambda} \epsilon_{\alpha\rho\sigma\kappa} \\
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& + E_2\, \nabla_{(\lambda} T^{\lambda,\mu\nu} \nabla_{\mu)} A_\nu + T^{\alpha,\beta\gamma} T^{\delta,\eta\kappa} T^{\lambda,\mu\nu} T^{\rho,\sigma\tau} \Big(F_1\, \epsilon_{\beta\gamma\eta\kappa} \epsilon_{\alpha\rho\mu\nu} \epsilon_{\delta\lambda\sigma\tau} \\
& + F_2\, \epsilon_{\beta\lambda\eta\kappa} \epsilon_{\gamma\rho\mu\nu} \epsilon_{\alpha\delta\sigma\tau} \Big) + F_3\, T^{\rho,\alpha\beta} T^{\gamma,\mu\nu} T^{\lambda,\sigma\tau} A_\tau \epsilon_{\alpha\beta\gamma\lambda} \epsilon_{\mu\nu\rho\sigma} \\
& + F_4\, T^{\eta,\alpha\beta} T^{\kappa,\gamma\delta} A_\eta A_\kappa \epsilon_{\alpha\beta\gamma\delta} \Bigg].
\end{aligned}
$$

A Yang–Mills aside

- Field: A_μ a connection
- Strength: $F_{\mu\nu} = 2D_{[\mu}A_{\nu]}$
- Action:

$$S \propto \int \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

- EOM and Bianchi:

$$D_\mu F^{\mu\nu} = 0$$
$$D_{[\lambda}F_{\mu\nu]} = 0$$

... Or in differential forms

- $\boldsymbol{A}_{(1)} = A_\mu \, \mathrm{d}x^\mu$
- $\boldsymbol{F}_{(2)} = \boldsymbol{\mathcal{D}}\boldsymbol{A}_{(1)}$
- Action:

$$S \propto \int \mathrm{Tr}(\boldsymbol{F}_{(2)} \star \boldsymbol{F}_{(2)})$$

- EOM and Bianchi:

$$\boldsymbol{\mathcal{D}} \star \boldsymbol{F}_{(2)} = 0$$
$$\boldsymbol{\mathcal{D}}\boldsymbol{F}_{(2)} = 0$$

And now, the Relativistic limit!

We shall focus on a sector of a torsion-free connection, $T^{\lambda,\mu\nu} \to 0$ and $A_\mu \to 0$.

- It cannot be taken at the action level.
- It should be taken in the field equations.
- It can be consistently *truncated*.

Moral

Only the terms of the action linear in these fields will be relevant, one can consider the effective action linear in the torsion's fields

Effective action

$$S_{\text{eff}} = \int \mathrm{d}^4x \, \Big(C_1 \, R_{\lambda\mu}{}^{\lambda}{}_{\nu} \nabla_\rho + C_2 \, R_{\mu\rho}{}^{\lambda}{}_{\lambda} \nabla_\nu \Big) T^{\nu,\mu\rho},$$

The equations of motion...

- General:

$$\nabla_{[\rho}R_{\mu]\nu} + \kappa\nabla_{\nu}R_{\mu\rho}{}^{\lambda}{}_{\lambda} = 0.$$

- Volume compatible connection

$$\nabla_{[\rho}R_{\mu]\nu} = 0.$$

- Equivalent to:

$$\nabla_{\lambda}R_{\mu\nu}{}^{\lambda}{}_{\rho} = 0.$$

Some known solutions

Solutions:

- Einstein Manifolds
- Riemannian products
- Conformally flat manifolds
- (Special) Warped products

Stephenson–Kilmister–Yang action

$$S_{\text{YM}} = \int \text{Tr}\left(\boldsymbol{\mathcal{R}} \star \boldsymbol{\mathcal{R}}\right) = \int \left(\boldsymbol{\mathcal{R}}^{a}{}_{b} \star \boldsymbol{\mathcal{R}}^{b}{}_{a}\right).$$

Remarks about quantization

- The polynomial affine gravity proposed has no explicit terms leading to three-point graviton vertices
- All graviton self-interaction is mediated by the torsion.
- Bypass the general postulates supporting the no-go theorems[1]
  - it is proved that generic three-point graviton interactions are highly constrained by causality and analyticity of the S-matrix, and the only *acceptable* structure of the three-point graviton vertices is the one coming from General Relativity.

[1] Phys.Rev.D90,084048 & 1407.5597

Conclusions

- We built a polynomial model of an affine theory of gravity.
- Its first order perturbations yield a Newtonian limit.
- In a torsion-free sector, the effective e.o.m. accept Einstein manifolds as a solution.
- No-go on renormalizability might be bypassed.

Conclusions

- We built a polynomial model of an affine theory of gravity.
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Future projections

- Coupling matter (scalar so far)
- Counting the degrees of freedom (Linearized)
- Formal study of "Eddington inverse metric density"
- Birkhoff-like theorem.