A family of dilatons for AdS / QCD models

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Outline

Introduction

Scalar Hadrons in AdS / QCD

A Family of Dilatons for AdS / QCD Models

Final Comments and Conclusions

Introduction

- Within the phenomenological models used recently in hadronic physics, some are based on the gauge/gravity duality.
- They suppose the existence of a gravity theory dual to QCD, and are divided into two classes: the Top-Down approach and the Bottom-Up models.
- The Bottom-Up models have proven to be quite useful because they are simple, and they have been used in different examples.

QCD

AdS / QCD

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QCD

AdS / QCD

◇ **Brief (and incomplete) list of uses of Bottom - Up models in hadron physics.**

- **DIS at high x** [Polchinski and Strassler; Ballon - Ballona, Boschi and Braga; Braga and A. V].
- **DIS at low x** [Brower, Polchinski, Strassler and Tan; Watanabe and Suzuki].
- **GPDs** [A.V, Schmidt, Gutsche and Lyubovitskij].
- **Hadronic wave functions** [Brodsky and de Teramond; Gutsche, Lyubovitskij, Schmidt and A.V].
- **Hadronic spectrum** [Brodsky and de Teramond; A.V and Schmidt; Gutsche, Lyubovitskij, Schmidt and A.V; Braga and Boschi; Forkel, Beyer and Frederico].
- **Transition form factors** [Brodsky, Cao and de Teramond; Gutsche, Lyubovitskij, Schmidt and A.V].

⋆ **Dictionary.**

This tell us how are related elements involved in both sides of Gauge / Gravity duality.

Table: Summary of dictionary considered here.

QCD (4d)	Gravity (5d)
Operator ($\mathcal{O}$)	Normalizable Modes (Φ)
Hadron Mass (M)	Eigenvalues of Φ
Twist Dimension ([$\mathcal{O}$] - S)	Conformal Dimension (Δ)

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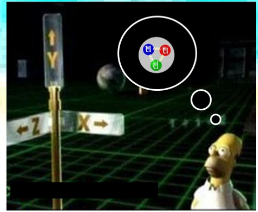
The basic ingredients in AdS / QCD models are

$$S = \int d^{d+1}x \sqrt{g}\, e^{-\Phi(z)} \left(\mathcal{L}_{Had} + \mathcal{L}_{V} + \mathcal{L}_{int} \right),$$

where the traditional choice is

$$ds^2 = \frac{1}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

- ⋆ Hard Wall case: $\Phi(z) = Cte$ y z between 0 and z_0.
- ⋆ Soft Wall case: $\Phi(z) = \kappa^2 z^2$ and z between 0 and ∞.



Scalar Hadrons in AdS / QCD [1]

[1] A. V and I. Schmidt, Phys. Rev. D **78**, 017703 (2008); P. Colangelo, F. De Fazio, F. Giannuzzi, F. Jugeau and S. Nicotri, Phys. Rev. D **78**, 055009 (2008).

Scalar hadrons in AdS side of Soft Wall (SW) models are described by

$$S = \int d^5x \sqrt{g} e^{-\phi(z)} \frac{1}{2} [g^{MN} \partial_M \Phi \partial_N \Phi + m_5^2 \Phi^2],$$

where $M, N = 0, 1, 2, 3, 4, z$.

$$ds^2 = e^{-2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

⋆ Hard Wall case: $\Phi(z) = Cte$ y z between 0 and z_0.

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Considering

$$\Phi(x) = e^{-i\mathcal{P}\cdot\mathcal{X}} f(z),$$

we obtain

$$-f''(z) - [(d-1)A(z) + \phi(z)]' f'(z) + m_5^2 e^{2A(z)} f(z) = M^2 f(z),$$

and with the next change

$$f(z) = \exp\left[\frac{1}{2}(\phi(z) - (d-1)A(z))\right]\psi(z),$$

obtain an Schrödinger like equation

$$[-\partial_z^2 + V(z)]\psi(z) = M^2\psi(z),$$

where potential is

$$V(z) = \frac{1}{4}(-3A'(z) + \phi'(z))^2 - \frac{1}{2}(-3A''(z) + \phi''(z)) + m_5^2 e^{2A(z)}.$$

⋆ Usual SW Model.

- AdS metric: $A(z) = ln(z/R)$.
- Dilaton: $\phi(z) = \kappa^2 z^2$.

In this case

$$V(z) = \frac{1}{z^2}\left[\frac{15}{4} + m_5^2\right] + z^2\kappa^4 + 2\kappa^2.$$

$$\psi(z) = N \exp^{-\frac{1}{2}z^2\kappa^2} z^{(\frac{1}{2}+\sqrt{4+m_5^2})} L_n^l(z^2\kappa^2).$$

$$M^2 = 4\kappa^2\left(n + \frac{1}{2}\sqrt{4+m_5^2} + 1\right).$$

A Family of Dilatons for AdS / QCD Models [2]

[2] A. V and P. Cabrera, In progress.

V_1 are $\widehat{V}_1$ strictly isospectral if [3],

$$\widehat{V}_1(z) = V_1(z) - 2\frac{d^2}{dz^2}\ln[I(z)+\lambda],$$

where

$$I(z) = \int_0^z \psi_1^2(z')dz'.$$

here ψ_1 it is the ground state of V_1.

$$I(z) = 1 - \frac{\Gamma(1+\sqrt{4+m_5^2}, z^2\kappa^2)}{\Gamma(1+\sqrt{4+m_5^2})}.$$

So, in general

$$V_1(z) - 2\frac{d^2}{dz^2}\ln[I(z)+\lambda] = \frac{1}{4}(-3A'(z)+\phi'(z))^2 - \frac{1}{2}(-3A''(z)+\phi''(z)) + m_5^2 e^{2A(z)}.$$

[3] F. Cooper, A. Khare and U. Sukhatme, Phys. Rept. **251**, 267 (1995).

Considering a defined metric, we get an equation for a family of dilatons. For AdS metrics equation for dilatons is

$$V_1(z) - 2\frac{d^2}{dz^2}\ln[I(z)+\lambda] = \frac{15}{4z^2} + \frac{m_5^2 R^2}{z^2} + \frac{3\phi'(z)}{2z} + \frac{1}{4}\phi'^2(z) - \frac{\phi''(z)}{2},.$$

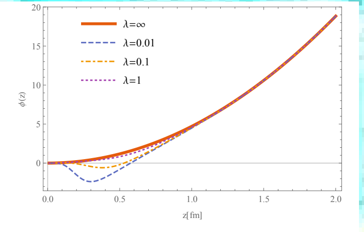


Figure: Some dilatons from family studied for case $m_5 = 0$.

In this case: $\phi(z \to 0) = \kappa_0^2 z^2$ and $\phi(z \to \infty) = \kappa_0^2 z^2$.

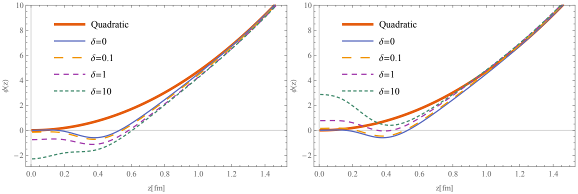
A Family of Dilatons for AdS / QCD Models

To get dilatons where $(\phi(z \to 0) = \kappa_0^2 z^2$ and $\phi(z \to \infty) = \kappa_\infty^2 z^2)$, important in some cases [4], it is possible to consider.

$$\frac{1}{z^2}\left(\frac{15}{4} + m_5^2\right) e^{-\delta z^2} + z^2\kappa^4 + 2\kappa^2 - 2\frac{d^2}{dz^2}\ln[I(z)+\lambda] = \frac{1}{z^2}\left(\frac{15}{4}+m_5^2\right) + \frac{3\phi'(z)}{2z} + \frac{1}{4}\phi'^2(z) - \frac{\phi''(z)}{2},$$

or

$$\frac{1}{z^2}\left(\frac{15}{4} + m_5^2\right) + z^2\kappa^4 + 2\kappa^2 - 2\frac{d^2}{dz^2}\ln[I(z)+\lambda] = \frac{1}{z^2}\left(\frac{15}{4}+m_5^2\right) e^{-\delta z^2} + \frac{3\phi'(z)}{2z} + \frac{1}{4}\phi'^2(z) - \frac{\phi''(z)}{2}.$$



[4] e.g see T. Gherghetta, J. I. Kapusta and T. M. Kelley, Phys. Rev. D **79**, 076003 (2009); K. Chelabi, Z. Fang, M. Huang, D. Li and Y. L. Wu, arXiv:1511.02721 [hep-ph]..

Final Comments and Conclusions

- Techniques to get Isospectral potentials in Schrödinger equations can be interesting to study AdS / QCD models.
- With this transforms it is posible to get families of potentials strictly isospectrals, and from these to get families of dilatons and/or metrics.
- We focus this talk on a family of dilatons to models with AdS metric.
- The family of dilatons that we got, can solve some problems that appear in AdS / QCD models without destroy the good spectrum obtained in models with quadratic dilatons.

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That's all Folks!