Axions in Gravity with Torsion

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Outline

Introduction

Einstein (1915): General Relativity

- Torsion-free condition: $\mathcal{T}_{\mu}{}^{\lambda}{}_{\nu} = \Gamma_{\mu}{}^{\lambda}{}_{\nu} - \Gamma_{\nu}{}^{\lambda}{}_{\mu} \equiv 2\Gamma_{[\mu}{}^{\lambda}{}_{\nu]} = 0$.
- *Local* Lorentz Symmetry and diffeomorphisms.
- One gravitational field: the metric.
- Second order equations.

Cartan (1922): First order formalism

- The torsion-free condition is relaxed: $\mathcal{T}_{\mu}{}^{\lambda}{}_{\nu} \neq 0$.
- Equivalent to GR when torsion vanishes.
- Two gravitational fields: the vielbein and the Lorentz connection.
- First order equations.

First order formalism

Gravitational fields

- The vielbein $\mathbf{e}^a = e^a_\mu \, dx^\mu$, defined through $g_{\mu\nu} = \eta_{ab} \, e^a_\mu \, e^b_\nu$.
- The Lorentz connection $\boldsymbol{\omega}^{ab} = \omega_\mu{}^{ab} \, dx^\mu$, which defines the covariant derivative, $\boldsymbol{D}$, with respect to the local Lorentz group.

- Cartan's structure equations

$$\boldsymbol{D}\mathbf{e}^a = \mathbf{d}\mathbf{e}^a + \boldsymbol{\omega}^a{}_c \wedge \mathbf{e}^c = \boldsymbol{\mathcal{T}}^a = \frac{1}{2} \mathcal{T}_\mu{}^a{}_\nu \, dx^\mu \wedge dx^\nu , \tag{1}$$

$$\mathbf{d}\boldsymbol{\omega}^{ab} + \boldsymbol{\omega}^a{}_c \wedge \boldsymbol{\omega}^{cb} = \boldsymbol{\mathcal{R}}^{ab} = \frac{1}{2} \mathcal{R}^{ab}{}_{\mu\nu} \, dx^\mu \wedge dx^\nu . \tag{2}$$

- Bianchi identities

$$\boldsymbol{D}\boldsymbol{\mathcal{T}}^a = \boldsymbol{\mathcal{R}}^a{}_b \wedge \mathbf{e}^b \qquad \text{and} \qquad \boldsymbol{D}\boldsymbol{\mathcal{R}}^{ab} = 0 \tag{3}$$

- Decomposition of the Lorentz connection $\boldsymbol{\omega}^{ab} = \mathring{\boldsymbol{\omega}}^{ab}(e) + \boldsymbol{\mathcal{K}}^{ab}$, where

$$\mathbf{d}\mathbf{e}^a + \mathring{\boldsymbol{\omega}}^a{}_b \wedge \mathbf{e}^b \equiv \mathring{\boldsymbol{D}}\mathbf{e}^a = 0 \quad \text{and} \quad \boldsymbol{\mathcal{T}}^a = \boldsymbol{\mathcal{K}}^a{}_b \wedge \mathbf{e}^b . \tag{4}$$

Einstein–Cartan theory coupled with fermions

- D-dimensional action[1]

$$S = \frac{1}{2\kappa_*^2}\int \boldsymbol{\mathcal{R}}_{ab}\wedge\star\left(\mathbf{e}^a\wedge\mathbf{e}^b\right) - \frac{1}{2}\int\left(\bar{\psi}\boldsymbol{\gamma}\wedge\star\boldsymbol{D}\psi - \boldsymbol{D}\bar{\psi}\wedge\star\boldsymbol{\gamma}\psi\right) \tag{5}$$

where $\kappa_*^2 \sim \frac{1}{M_*^{2+n}}$, $\boldsymbol{\gamma} = \gamma_a\mathbf{e}^a$, $\bar{\psi} = -\imath\psi^\dagger\gamma^0$ and the covariant derivative[2]

$$\boldsymbol{D}\psi = \mathbf{d}\psi + \frac{1}{4}\boldsymbol{\omega}^{ab}\gamma_{ab}\psi. \tag{6}$$

- Decomposing the Lorentz connection $\boldsymbol{\omega}^{ab} = \mathring{\boldsymbol{\omega}}^{ab}(e) + \boldsymbol{\mathcal{K}}^{ab}$, we obtain an equivalent action (up-to-a boundary term)

$$\begin{aligned} S = {} & \frac{1}{2\kappa_*^2}\int \mathring{\boldsymbol{\mathcal{R}}}_{ab}\wedge\star\left(\mathbf{e}^a\wedge\mathbf{e}^b\right) - \frac{1}{2}\int\left(\bar{\psi}\boldsymbol{\gamma}\wedge\star\mathring{\boldsymbol{D}}\psi - \mathring{\boldsymbol{D}}\bar{\psi}\wedge\star\boldsymbol{\gamma}\psi\right) \\ & + \frac{1}{2\kappa_*^2}\int \boldsymbol{\mathcal{K}}_{am}\wedge\boldsymbol{\mathcal{K}}^m{}_b\wedge\star\left(\mathbf{e}^a\wedge\mathbf{e}^b\right) - \frac{1}{8}\int\boldsymbol{\mathcal{K}}^{ab}\wedge\star\bar{\psi}\left\{\boldsymbol{\gamma},\gamma_{ab}\right\}\psi \end{aligned} \tag{7}$$

[1] $D = 4 + n$ and the sum over the fermionic flavour is assumed.
[2] $\gamma_{a_1\ldots a_n} \equiv \gamma_{[a_1}\cdots\gamma_{a_n]}$.

Einstein–Cartan theory coupled with fermions

Equations of motion within the Cartan's formalism

$$\delta \mathbf{e}^a \quad : \quad \mathcal{R}_{ab} - \frac{1}{2}\eta_{ab}\mathcal{R} = \kappa_*^2 \, \tau_{ab} \tag{8}$$

$$\delta \boldsymbol{\omega}^{ab} \quad : \quad \mathcal{T}_a{}^b{}_c - 2\mathcal{T}_{[a}\delta^b_{c]} = -\frac{\kappa_*^2}{2}\, \bar{\psi}\gamma_a{}^b{}_c\psi \tag{9}$$

$$\delta \bar{\psi} \quad : \quad \gamma^a \mathring{D}_a\psi + \frac{1}{4}\mathcal{K}_{abc}\gamma^{abc}\psi = 0 \tag{10}$$

Solving the algebraic Eq. (9) we obtain

$$\mathcal{K}_{abc} = -\frac{\kappa_*^2}{4}\, \bar{\psi}\gamma_{abc}\psi, \tag{11}$$

for the contorsion tensor. Replacing it back into the initial action, leads to the D-dimensional effective theory

$$S_{\text{eff}} = \mathring{S}_{\text{gr}} + \mathring{S}_\psi + \frac{\kappa_*^2}{32}\int d^Dx\, e\, \bar{\psi}\gamma_{abc}\psi\, \bar{\psi}\gamma^{abc}\psi. \tag{12}$$

Strong CP problem and the Peccei–Quinn axions

- The CKM matrix and the θ-vacuum of QCD induce

$$\mathscr{L}_{\text{QCD}} \supset -\bar{\theta}\,\frac{\alpha_s}{2\pi}\,\text{Tr}\left(\boldsymbol{G} \wedge \boldsymbol{G}\right)\,, \tag{13}$$

where $\bar{\theta} = \theta + \arg\det M$.

Strong CP problem

Limits on the neutron's electric dipole moment $\rightarrow \bar{\theta} \leq 10^{-10}$.

Peccei and Quinn solution (1977)

- Extra $U(1)_A$ symmetry, spontaneously broken at $\sim \Lambda_{EW}$.
- Axion coupled to Pontryagin density, i.e. $\sim \phi(x)\,\text{Tr}\left[\boldsymbol{G} \wedge \boldsymbol{G}\right]$.
- Promote $\bar{\theta} \rightarrow \bar{\theta}(x) \sim \bar{\theta} + \phi(x)/f_\phi$ with $\left\langle \bar{\theta}(x) \right\rangle = 0 \rightarrow \langle \phi \rangle = -f_\phi \bar{\theta}$.
- Perturbations around $\langle \phi \rangle$ gives a CP-even $a(x)\,\text{Tr}\left[\boldsymbol{G} \wedge \boldsymbol{G}\right]$.

Axions in gravity with torsion

- Motivation: QFT in background geometry
- $SU(N) \times U(1)$ gauge invariant action coupled with fermions

$$S = \frac{1}{2\kappa_*^2}\int \boldsymbol{\mathcal{R}}_{ab} \wedge \star\left(\mathbf{e}^a \wedge \mathbf{e}^b\right) - \frac{1}{2}\int \left(\bar{\psi}\boldsymbol{\gamma} \wedge \star\boldsymbol{\mathcal{D}}\psi - \boldsymbol{\mathcal{D}}\bar{\psi} \wedge \star\boldsymbol{\gamma}\psi\right) - \frac{1}{2}\int \boldsymbol{F} \wedge \star\boldsymbol{F} - \int \mathrm{Tr}\left[\boldsymbol{G} \wedge \star\boldsymbol{G}\right] - \bar{\theta}\,\frac{\alpha_s}{2\pi}\int \mathrm{Tr}\left[\boldsymbol{G} \wedge \boldsymbol{G}\right] \tag{14}$$

- **Duncan et.al:** *Nucl.Phys.B387,215 (1992)*
  - Impose the classical conservation $\mathbf{d}\star\boldsymbol{\mathcal{S}} = 0$, where $\star\boldsymbol{\mathcal{S}} = \mathbf{e}^a \wedge \boldsymbol{\mathcal{T}}_a$, at quantum level through

$$\mathcal{Z} = \int \prod_{\varphi} \mathcal{D}\varphi\,\mathcal{D}\boldsymbol{\mathcal{S}}\, e^{\imath S[\varphi,\boldsymbol{\mathcal{S}}]} \int \mathcal{D}\phi\, e^{\imath \int \phi\, \mathbf{d}\star\boldsymbol{\mathcal{S}}}. \tag{15}$$

Axions in gravity with torsion

- **Mielke and Sánchez Romero:** *Phys.Rev.D73,043521 (2006)*
  - Argue the appearance of $\mathbf{d}\boldsymbol{\mathcal{S}} \wedge \mathbf{d}\boldsymbol{\mathcal{S}}$ in the axial anomaly.
  - Modified axial-current by the addition of Chern-Simons-type terms

$$\star\hat{\boldsymbol{J}}_5 = \star\boldsymbol{J}_5 + \frac{\alpha_{\text{em}}\,\bar{Q}^2}{\pi}\,\boldsymbol{C}_{FF} + \frac{\alpha_s N_q}{2\pi}\,\boldsymbol{C}_{GG} + \frac{N_f}{8\pi^2}\left(\boldsymbol{C}_{RR} + \boldsymbol{\mathcal{S}} \wedge \mathbf{d}\boldsymbol{\mathcal{S}}\right)\,.$$

  - The conservation of the modified axial-current occurs when $\boldsymbol{\mathcal{S}} \sim \mathbf{d}\phi$, where ϕ is a pseudoscalar potential.

- **Mercuri:** *Phys.Rev.Lett.103,081302 (2009)*
  - Divergent Nieh–Yan term in the $U(1)_A$ rotated fermionic measure.[3]
  - Add to the action (14) the topological Nieh–Yan density, i.e.

$$S \to S + \beta \int \left(\boldsymbol{\mathcal{T}}^a \wedge \boldsymbol{\mathcal{T}}_a - \boldsymbol{\mathcal{R}}_{ab} \wedge \mathbf{e}^a \wedge \mathbf{e}^b\right) = S + \beta \int \mathbf{d}\left(\mathbf{e}^a \wedge \boldsymbol{\mathcal{T}}_a\right).$$

  - Promote the BI parameter to be a field, i.e. $\beta \to \beta(x)$ and absorb the divergence by means of renormalized $\beta(x)$.

[3] Chandía & Zanelli *Phys.Rev.D55,7580 (1997).*

Axions in gravity with torsion

- Integrating out the torsion in either of these approaches leads to[4]

$$S_{\text{eff}} = S_0 + S_\theta - \frac{1}{2f_\Phi^2}\int \boldsymbol{J}_5 \wedge \star \boldsymbol{J}_5 - \frac{1}{2}\int \mathbf{d}\Phi \wedge \star \mathbf{d}\Phi + \frac{1}{f_\Phi}\int \Phi\, \mathbf{d} \star \boldsymbol{J}_5 \,.$$

- Either approach gives a vanishing Nieh–Yan density.
- Replacing the axial anomaly

$$\mathbf{d} \star \boldsymbol{J}_5 = -\frac{\alpha_{\text{em}}\,\bar{Q}^2}{\pi}\boldsymbol{F} \wedge \boldsymbol{F} - \frac{\alpha_s N_q}{2\pi}\,\text{Tr}\,[\boldsymbol{G} \wedge \boldsymbol{G}] - \frac{N_f}{8\pi^2}\mathring{\boldsymbol{\mathcal{R}}}^{ab} \wedge \mathring{\boldsymbol{\mathcal{R}}}_{ab}\,, \qquad (16)$$

gives the explicit form of the effective theory[5]

$$S_{\text{eff}} = S_0 - \frac{1}{2f_\Phi^2}\int \boldsymbol{J}_5 \wedge \star \boldsymbol{J}_5 - \frac{\alpha_{\text{em}}\,\bar{Q}^2}{\pi f_\Phi}\int \Phi\, \boldsymbol{F} \wedge \boldsymbol{F} - \frac{1}{2}\int \mathbf{d}\Phi \wedge \star \mathbf{d}\Phi$$
$$- \frac{1}{8\pi^2}\int \left(\Theta + \frac{N_f}{f_\Phi}\,\Phi\right)\mathring{\boldsymbol{\mathcal{R}}}^{ab} \wedge \mathring{\boldsymbol{\mathcal{R}}}_{ab} - \frac{\alpha_s}{2\pi}\int \left(\bar{\theta} + \frac{N_q}{f_\Phi}\,\Phi\right)\text{Tr}\,[\boldsymbol{G} \wedge \boldsymbol{G}]\,.$$

[4] We have defined $S_0 = \mathring{S}_{\text{gr}} + \mathring{S}_\psi + S_{\text{gk}}$, $f_\Phi = \kappa^{-1}\sqrt{8/3}$ and $\Phi = 4/3 f_\Phi^{-1}\vartheta$, where $\vartheta = \beta, \phi$.

[5] N_f: number of fermionic flavors, N_q: number of quarks and $\bar{Q}^2 = \sum_f Q_f^2$.

Axions in gravity with torsion

Phenomenology

	$M_{Pl} \sim 10^{18}$ [GeV]	$M_* \sim 10^4$ [GeV]	$M_* \sim 10^2$ [GeV]
f_Φ [GeV]	10^{18}	10^4	10^2
m_a [keV]	10^{-15}	10^{-1}	10
$\Gamma_{a\to\gamma\gamma}$ [keV]	10^{-101}	10^{-32}	10^{-19}

- Torsion-descended axions as the dominant dark matter content[6]

$$r \leq 1.6 \times 10^{-9}. \tag{17}$$

[6] M. Lattanzi & S. Mercuri *Phys.Rev.D81,125015.*

Conclusions

- Einstein-Cartan + fermions $\rightarrow$ four-fermion interaction.
- Suitable modifications to such a theory solves the strong CP problem.
- Rather different motivations for the torsion-descended axions leads to the same effective theory.
- The axionic phenomenology is characterized only by the gravitational scale.
- Torsion-descended axions might be dark matter candidates.

Thank you!