QCD running in neutrinoless double beta decay: Short range mechanism

In Press in PRD/arXiv:1511.03945

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6-12 January 2016 **High Energy Physics in the LHC Era**

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FUNDADA EN 1931

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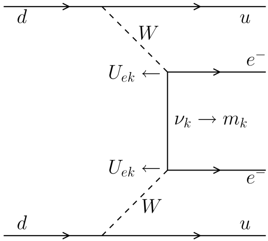
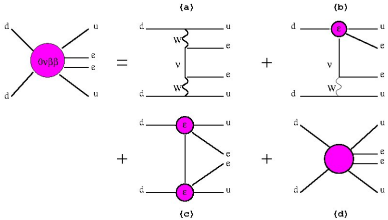
Introduction





Effective Lagrangian and base of operators

Effective Lagrangian

$$\mathcal{L}_{\text{eff}}^{0\nu\beta\beta} = \frac{G_F^2}{2m_p} \sum_i C_i^{XY}(\mu) \cdot \mathcal{O}_i(\mu), \quad (1)$$

with the operator basis:

$$\mathcal{O}_1^{XY} = 8(\bar{u}P_X d)(\bar{u}P_Y d)\, j, \quad (2)$$
$$\mathcal{O}_2^{XX} = 8(\bar{u}\sigma^{\mu\nu}P_X d)(\bar{u}\sigma_{\mu\nu}P_X d)\, j, \quad (3)$$
$$\mathcal{O}_3^{XY} = 8(\bar{u}\gamma^{\mu}P_X d)(\bar{u}\gamma_{\mu}P_Y d)\, j, \quad (4)$$
$$\mathcal{O}_4^{XY} = 8(\bar{u}\gamma^{\mu}P_X d)(\bar{u}\sigma_{\mu\nu}P_Y d)\, j^{\nu}, \quad (5)$$
$$\mathcal{O}_5^{XY} = 8(\bar{u}\gamma^{\mu}P_X d)(\bar{u}P_Y d)\, j_{\mu} \quad (6)$$

$$j = \bar{e}P_X e^c\,, \quad j_\mu = \bar{e}\gamma_\mu P_X e^c\ .$$

Half-life

Applying standard nuclear theory methods, one finds for the half-life:

Half-life

$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = G_1 \left|\sum_{i=1}^{3} C_i(\mu_0)\mathcal{M}_i\right|^2 + G_2 \left|\sum_{i=4}^{5} C_i(\mu_0)\mathcal{M}_i\right|^2 \qquad (7)$$

Here, $G_1 = G_{01}$ and $G_2 = (m_e R)^2 G_{09}/8$ are phase space factors in the convention of [Doi et al., 1985], and $\mathcal{M}_i = \langle A_f|\mathcal{O}_i^{\mathrm{h}}|A_i\rangle$ are the nuclear matrix elements defined in Ref. [Päs et al., 2001].

QCD running motivations

Why we need the running?

Energy scales involved:

- LNV scale at $\backsim$ 1 Tev
- Hadronic scale at $\backsim$ 1 Gev
- Nuclear scale at $P_F \backsim 200$ Mev

$\Rightarrow$ QCD running has to be taken into account.

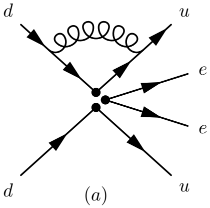


Figure : One-loop QCD corrections to the short range mechanisms of $0\nu\beta\beta$ decay in the effective theory.

$$T^a_{\alpha\beta} T^a_{\gamma\rho} = -\frac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{1}{2}\delta_{\alpha\delta}\delta_{\gamma\rho} \quad (8)$$

Color Mismatch effect.

Renormalization

Bare operator matrix elements.

$$\langle \mathcal{O}_i \rangle^{(0)} = \left[\delta_{ij} + \frac{\alpha_s}{4\pi} b_{ij} \left(\frac{1}{\epsilon} + \ln \left(\frac{\mu^2}{-p^2} \right) \right) \right] \langle \mathcal{O}_j \rangle_{\mathrm{tree}}. \tag{9}$$

[Buras, 1998] :

$$\mathcal{O}_i^{(0)} = Z_{ij} \mathcal{O}_j, \quad C_i^{(0)} = Z_{ij}^c C_j \tag{10}$$

It can be shown $\hat{Z}^{cT} = \hat{Z}^{-1}$

$$Z_{ij} = \delta_{ij} + \frac{\alpha_s}{4\pi} (b_{ij} - 2 C_F \delta_{ij}) \frac{1}{\epsilon} + O(\alpha_s^2). \tag{11}$$

RGE

RGE of WC

$$\frac{d\vec{C}(\mu)}{d\ln\mu} = \hat{\gamma}^T\vec{C}(\mu), \tag{12}$$

$$\hat{\gamma}(\alpha_s) = -2\alpha_s\frac{\partial\hat{Z}_1(\alpha_s)}{\partial\alpha_s}, \tag{13}$$

$$\gamma_{ij}(\alpha_s) = \frac{\alpha_s}{4\pi}\gamma_{ij}^{(0)}, \quad \text{with} \quad \gamma_{ij}^{(0)} = -2(b_{ij} - 2C_F\delta_{ij}), \tag{14}$$

$$\vec{C}(\mu) = \hat{U}(\mu, M_W) \cdot \vec{C}(M_W). \tag{15}$$

In the LO one finds

$$\hat{U}^{(0)}(\mu, M_W) = \hat{V}\,\mathrm{Diag}\left\{ \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\gamma_i^{(0)}/(2\beta_0)} \right\} \hat{V}^{-1}. \tag{16}$$

The LO QCD running coupling constant is as usual

$$\alpha_s(\mu) = \frac{\alpha_s(M_z)}{1 - \beta_0 \frac{\alpha_s(M_z)}{2\pi} \log\left(\frac{Mz}{\mu}\right)} \tag{17}$$

with $\beta_0 = (33 - 2f)/3$, where f is the number of the quark flavors with masses $m_f < \mu$.

Anomalous Dimensions

$$\hat{\gamma}_{31}^{LR,LR(0)} = -2 \begin{pmatrix} -\frac{3}{N} & -6 \\ 0 & 6\,C_F \end{pmatrix} \tag{18}$$

$$\hat{\gamma}_{12}^{LL,RR(0)} = -2 \begin{pmatrix} 6\,C_F - 3 & \frac{1}{2N} + \frac{1}{4} \\ -12 - \frac{24}{N} & -3 - 2\,C_F \end{pmatrix} \tag{19}$$

$$\gamma_{3}^{LL,RR(0)} = -2 \left(\frac{3}{N} - 3 \right) \tag{20}$$

$$\gamma_{5}^{LR,RL(0)} = -3\gamma_{4}^{LR,RL(0)} = -12\, C_F, \tag{21}$$

$$\hat{\gamma}_{45}^{LL,RR(0)} = -2 \begin{pmatrix} 9 - 2\,C_F & 3i - \frac{6i}{N} \\ i + \frac{2i}{N} & 6\,C_F + 1 \end{pmatrix}. \tag{22}$$

$$\begin{aligned}
\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = \; & G_1 \Big| \beta_1^{XX}\left(C_1^{LL}(\Lambda)+C_1^{RR}(\Lambda)\right)+\beta_1^{LR}\left(C_1^{LR}(\Lambda)+C_1^{RL}(\Lambda)\right)+ \quad (23)\\
& +\beta_2^{XX}\left(C_2^{LL}(\Lambda)+C_2^{RR}(\Lambda)\right)+\\
& +\beta_3^{XX}\left(C_3^{LL}(\Lambda)+C_3^{RR}(\Lambda)\right)+\beta_3^{LR}\left(C_3^{LR}(\Lambda)+C_3^{RL}(\Lambda)\right)\Big|^2+\\
+ \; & G_2 \Big| \beta_4^{XX}\left(C_4^{RR}(\Lambda)+C_4^{RR}(\Lambda)\right)+\beta_4^{LR}\left(C_4^{LR}(\Lambda)+C_4^{RL}(\Lambda)\right)+\\
& +\beta_5^{XX}\left(C_5^{RR}(\Lambda)+C_5^{RR}(\Lambda)\right)+\beta_5^{LR}\left(C_5^{LR}(\Lambda)+C_5^{RL}(\Lambda)\right)\Big|^2,
\end{aligned}$$

where

$$\beta_1^{XX} = \mathcal{M}_1 \overbrace{U_{11}^{XX}}^{1} + \mathcal{M}_2 \overbrace{U_{21}^{XX}}^{0}, \qquad \beta_1^{LR} = \mathcal{M}_3^{(+)} U_{31}^{LR} + \mathcal{M}_1 U_{11}^{LR}, \quad (24)$$

$$\beta_2^{XX} = \mathcal{M}_1 U_{12}^{XX} + \mathcal{M}_2 U_{22}^{XX}, \quad (25)$$

$$\beta_3^{XX} = \mathcal{M}_3^{(-)} U_{33}^{XX}, \qquad \beta_3^{LR} = \mathcal{M}_3^{(+)} U_{33}^{LR} \quad (26)$$

$$\beta_4^{XX} = -|\mathcal{M}_4| \; U_{44}^{XX} + |\mathcal{M}_5| \; U_{54}^{XX}, \quad \beta_4^{LR} = |\mathcal{M}_4| \; U_{44}^{LR}, \quad (27)$$

$$\beta_5^{XX} = -|\mathcal{M}_4| \; U_{45}^{XX} + |\mathcal{M}_5| \; U_{55}^{XX}, \quad \beta_5^{LR} = |\mathcal{M}_5| \; U_{55}^{LR}. \quad (28)$$

^{A}X	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3^{(+)}$	$\mathcal{M}_3^{(-)}$	$\lvert\mathcal{M}_4\rvert$	$\lvert\mathcal{M}_5\rvert$
^{76}Ge	9.0	-1.6×10^3	1.3×10^2	2.1×10^2	$\lvert 1.9 \times 10^2 \rvert$	$\lvert 1.9 \times 10^1 \rvert$
^{136}Xe	4.5	-8.5×10^2	6.9×10^1	1.1×10^2	$\lvert 9.6 \times 10^1 \rvert$	$\lvert 9.3 \rvert$

Table : The numerical values of the nuclear matrix elements $\mathcal{M}_i$ taken from Ref. [Deppisch et al., 2012].

$$T_{1/2}^{0\nu\beta\beta}(^{76}Ge) \geq T_{1/2}^{0\nu\beta\beta-exp}(^{76}Ge) = 3.0\ 10^{25} \text{ yrs}, \quad (29)$$

$$T_{1/2}^{0\nu\beta\beta}(^{136}Xe) \geq T_{1/2}^{0\nu\beta\beta-exp}(^{136}Xe) = 3.4\ 10^{25} \text{ yrs}. \quad (30)$$

^{A}X	$\lvert C_1^{XX}(M_W)\rvert$	$\lvert C_1^{XX}(\Lambda_{LNV})\rvert$	$\lvert C_1^{XX(0)}\rvert$	$\lvert C_1^{LR,RL}(M_W)\rvert$	$\lvert C_1^{LR,RL}(\Lambda_{LNV})\rvert$	$\lvert C_1^{LR,RL(0)}\rvert$
^{76}Ge	5.0×10^{-10}	3.8×10^{-10}	2.6×10^{-7}	8.6×10^{-8}	6.2×10^{-8}	2.6×10^{-7}
^{136}Xe	3.4×10^{-10}	2.6×10^{-10}	1.8×10^{-7}	6.0×10^{-8}	4.3×10^{-8}	1.8×10^{-7}

^{A}X	$\lvert C_2^{XX}(M_W)\rvert$	$\lvert C_2^{XX}(\Lambda_{LNV})\rvert$	$\lvert C_2^{XX(0)}\rvert$	–
^{76}Ge	3.5×10^{-9}	5.2×10^{-9}	1.4×10^{-9}	–
^{136}Xe	2.4×10^{-9}	3.5×10^{-9}	9.4×10^{-10}	–

^{A}X	$\lvert C_3^{XX}(M_W)\rvert$	$\lvert C_3^{XX}(\Lambda_{LNV})\rvert$	$\lvert C_3^{XX(0)}\rvert$	$\lvert C_3^{LR,RL}(M_W)\rvert$	$\lvert C_3^{LR,RL}(\Lambda_{LNV})\rvert$	$\lvert C_3^{LR,RL(0)}\rvert$
^{76}Ge	1.5×10^{-8}	1.6×10^{-8}	1.1×10^{-8}	2.0×10^{-8}	2.1×10^{-8}	1.8×10^{-8}
^{136}Xe	9.7×10^{-9}	1.1×10^{-8}	7.4×10^{-9}	1.4×10^{-8}	1.4×10^{-8}	1.2×10^{-8}

^{A}X	$\lvert C_4^{XX}(M_W)\rvert$	$\lvert C_4^{XX}(\Lambda_{LNV})\rvert$	$\lvert C_4^{XX(0)}\rvert$	$\lvert C_4^{LR,RL}(M_W)\rvert$	$\lvert C_4^{LR,RL}(\Lambda_{LNV})\rvert$	$\lvert C_4^{LR,RL(0)}\rvert$
^{76}Ge	5.0×10^{-9}	3.9×10^{-9}	1.2×10^{-8}	1.7×10^{-8}	1.9×10^{-8}	1.2×10^{-8}
^{136}Xe	3.4×10^{-9}	2.7×10^{-9}	7.9×10^{-9}	1.2×10^{-8}	1.3×10^{-8}	7.9×10^{-9}

^{A}X	$\lvert C_5^{XX}(M_W)\rvert$	$\lvert C_5^{XX}(\Lambda_{LNV})\rvert$	$\lvert C_5^{XX(0)}\rvert$	$\lvert C_5^{LR,RL}(M_W)\rvert$	$\lvert C_5^{LR,RL}(\Lambda_{LNV})\rvert$	$\lvert C_5^{LR,RL(0)}\rvert$
^{76}Ge	2.3×10^{-8}	1.4×10^{-8}	1.2×10^{-7}	3.9×10^{-8}	2.8×10^{-8}	1.2×10^{-7}
^{136}Xe	1.6×10^{-8}	9.5×10^{-9}	8.2×10^{-8}	2.8×10^{-8}	2.0×10^{-8}	8.2×10^{-8}

Table : Upper limits on the Wilson coefficients in eq. (1) with $C_i(\Lambda)$ and $C_i(M_W)$ calculated for two different matching scales, $\Lambda = 1$ TeV and M_W. For comparison we also give $C_i^{(0)}$, i.e. limits without QCD running.

Summary

- We have calculated QCD running to the complete set of Lorentz-invariant operators for the short-range (SR) part of the NDBD amplitude.
- We derived 1-loop improved limits on all the Wilson coefficients appearing in the SR contributions.
- Improved nuclear physics calculations can be easily implemented with our results.
- We showed that the QCD corrections are indeed important principally because of the operator mixing

Thanks for your attention

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Backup

An example:

$$\mathcal{O}_1^{XX} = 8(\bar{u}_\alpha P_X d_\alpha)(\bar{u}_\beta P_X d_\beta)\ j$$
$$\tilde{\mathcal{O}}_1^{XX} = 8(\bar{u}_\alpha P_X d_\beta)(\bar{u}_\beta P_X d_\alpha)\ j$$

Schematically:

$$\mathcal{O}_1^{XX} \longrightarrow \text{QCD corrections} \longrightarrow C_1 \mathcal{O}_1^{XX} + \tilde{C}_1 \underbrace{\tilde{\mathcal{O}}_1^{XX}}_{\mathcal{O}_2^{XX}}$$