The matrix model of two-color one-flavor QCD: The ultra-strong coupling regime

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Frontiers in Particle Physics
CHEP, IISc

10 August, 2024

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Based on: arXiv:2406.06055

Outline

- Introduction: Matrix Model
- Why 2-color 1-flavor QCD?
- Overview of the results
- Hamiltonian (H) & its symmetries
- Numerical diagonalization of H in the strong coupling regime
- Results
- Summary & Future Outlook

Introduction: Matrix Model

- Non abelian gauge theories → the governing dynamics of subatomic particles. e.g. QCD explains the strong interaction of quarks and gluons.
- QCD in the strong coupling regime is nonperturbative - hence use of computational methods.
- For computations in the strongly coupled regime: most popular candidate is Lattice QCD - quite successful but computationally intense.
- Gauge Matrix Model:
  1. This approximation captures (some of) the constraints, nonlinearity, and underlying topology!
  2. The existence of axial anomaly has been shown for the $SU(N)$ matrix model.
  3. Provides a simplified computational platform.
  4. The pure $SU(3)$ Yang-Mills matrix model gives a good prediction of light glueball masses.
  5. When coupled to the light quarks, it gives a good numerical prediction of the hadron masses.

Why 2-color 1-flavor QCD?

- Gauge group is $SU(2) \rightarrow$ simplest non-Abelian gauge theory.
- Computationally less challenging.
- Interesting features:
  1. Baryons (diquarks and tetraquarks) are bosonic states.
  2. Presence of additional global symmetry (Pauli-G$\ddot{u}$rsey Symmetry): Fundamental rep of $SU(2)_{col}$ is pseudo-real $\Rightarrow U(1)_B$ is enhanced to $SU(2)_B$.
- This model is extensively studied on the lattice (as there is no sign problem).

Overview of the results

- Quantum phase transitions (QPT) in the sectors $(B, J) = (0, 0), (1, 1), (0, 1)$ due to level crossing.
- QPT are first order: $\langle Q_0 \rangle = \frac{\partial E}{\partial c}$ is discontinuous.
- Interesting division of the total spin between the quark & glue. (Spin Puzzle)
- Signature of the Isgur-Wise symmetry in the heavy quark limit: quark spin is independently conserved.
- Emergence of non-trivial phase structure after adding a Baryon chemical potential $\Rightarrow$ reminiscent of the LOFF phase in 2-col QCD.

$SU(2)$ Matrix Model

- Quantum Mechanical approximation of $SU(2)$ Yang-Mills theory on $\mathbb{R} \times S^3$.
- Building blocks: 3×3 rectangular real matrices M_{ia} and represent our gauge variables.
- Spatial index $i = 1, 2, 3$ & Color index $a = 1, 2, 3 \,(= 2^2 - 1)$.
- Rotations: $M_{ia} \to \mathcal{R}_{ij} M_{ja}, \quad \mathcal{R} \in SO(3)$
  Gauge Transformations: $M_{ia} \to S(h)_{ab} M_{jb}, \quad S(h) \in AdSU(2)$
- The configuration space: $\mathcal{M}_3 / AdSU(2)$,
  $\mathcal{M}_3 =$ space of all 3×3 real matrices.
- Field Strength, $F_{ij} = \left(-\frac{1}{R}\epsilon_{ijk} M_{ka} + f_{abc} M_{ib} M_{jc}\right) T_a, \quad f_{abc} = \epsilon_{abc}; \, T_a \in su(2)$
- The chromoelectric & chromomagnetic fields are

$$E_i^a \equiv F_{0i}^a = \dot{M}_{ia} \,, \quad B_i^a \equiv \frac{1}{2} \epsilon_{ijk} F_{jk}^a = -\frac{1}{R} M_{ia} + \frac{1}{2} \epsilon_{ijk} f_{abc} M_{jb} M_{kc}$$

- The matrix model Lagrangian is

$$L_{YM} \equiv -\frac{R^3}{4g^2} F_{\mu\nu}^a F^{a\mu\nu} = \frac{R^3}{2g^2} \left(E_i^a E_i^a - B_i^a B_i^a\right)$$

Fundamental Fermions in the Matrix Model

- The Dirac fermion Ψ is made up of a left Weyl fermion b and a right Weyl fermion $d^\dagger$:

$$\Psi_{\alpha A} = \begin{pmatrix} b_{\alpha A} \\ -i(\sigma_2)_{\alpha\beta} c^\dagger_{\beta A} \end{pmatrix} \equiv \begin{pmatrix} b_{\alpha A} \\ d^\dagger_{\alpha A} \end{pmatrix} \qquad (\alpha = 1, 2;\ A = 1, 2)$$

- $b(d)$ transforms in the (anti-)fundamental representations of spin and color:

$$b_{\alpha A} \to D^{\frac{1}{2}}(\mathcal{R})_{\alpha\beta} b_{\beta A}; \quad d_{\alpha A} \to \bar{D}^{\frac{1}{2}}(\mathcal{R})_{\alpha\beta} d_{\beta A}$$

$$b_{\alpha A} \to h_{AB} b_{\alpha B}; \quad d_{f\alpha A} \to h^*_{AB} d_{f\alpha B}$$

where $\mathcal{R} \in SO(3)$ and $h \in SU(2)$.

- The Lagrangian coupled with massive quarks is

$$L = L_{YM} + R^3\, \bar{\Psi} \left(i\gamma^\mu \mathcal{D}_\mu - \frac{m}{R} - \frac{\tilde{c}}{R}\gamma^5\gamma^0 \right) \Psi$$

where $\bar{\Psi} = \Psi^\dagger \gamma^0$.

The Hamiltonian

- With $M \to \frac{gM}{R}$, $b \to R^{-\frac{3}{2}} b$, $d \to R^{-\frac{3}{2}} d$, the Hamiltonian works out to be

$$H = \frac{1}{R}\left(H_{YM} + \tilde{c} H_c + g H_{int} + m H_m\right)$$

where

$$H_{YM} = \frac{1}{2}\Pi_{ia}\Pi_{ia} + \frac{1}{2}M_{ia}M_{ia} - \frac{g}{2}\epsilon_{ijk} f_{abc} M_{ia} M_{jb} M_{kc} + \frac{g^2}{4} f_{abc} f_{ade} M_{ib} M_{jc} M_{id} M_{je}$$

$$H_c = (b^{\dagger}_{\alpha A} b_{\alpha A} - d_{\alpha A} d^{\dagger}_{\alpha A})$$

$$H_{int} = M_{ia}(b^{\dagger}_{\alpha A}\sigma^i_{\alpha\beta} T^a_{AB} b_{\beta B} - d_{\alpha A}\sigma^i_{\alpha\beta} T^a_{AB} d^{\dagger}_{\beta B})$$

$$H_m = (b^{\dagger}_{\alpha A} d^{\dagger}_{\alpha A} + d_{\alpha A} b_{\alpha A})$$

- The Gauss's law constraints

$$G_a = f_{abc} M_{ib}\Pi_{ic} + (b^{\dagger}_{\alpha A} T^a_{AB} b_{\alpha B} + d_{\alpha A} T^a_{AB} d^{\dagger}_{\alpha B})$$

- The angular momenta

$$J_i = \epsilon_{ijk} M_{ja}\Pi_{ka} + \frac{1}{2}(b^{\dagger}_{\alpha A}\sigma^i_{\alpha\beta} b_{\beta A} + d_{\alpha A}\sigma^i_{\alpha\beta} d^{\dagger}_{\beta A})$$

Quantization of the Model

- To quantise the system, we impose the canonical (anti)commutation relations

$$[M_{ia}, \Pi_{jb}] = i\delta_{ij}\delta_{ab}$$

$$\{b_{\alpha A}, b^{\dagger}_{\beta B}\} = \delta_{\alpha\beta}\delta_{AB}$$

$$\{d_{\alpha A}, d^{\dagger}_{\beta B}\} = \delta_{\alpha\beta}\delta_{AB}$$

and demand that all physical states be annihilated by the Gauss law:

$$G_a \left|\Psi\right\rangle_{phys} = 0 \quad \text{(colorless states)}$$

The Strong Coupling Regime

- In terms of the rescaled variables and parameters

$$\Pi_{ia} \to g^{\frac{1}{3}} \Pi_{ia}, \;\; M_{ia} \to g^{-\frac{1}{3}} M_{ia}$$

$$c \equiv \tilde{c} g^{-\frac{2}{3}}, \;\; M \equiv m g^{-\frac{2}{3}}, \;\; e_0 \equiv g^{\frac{2}{3}} R^{-1}$$

the Hamiltonian is given by

$$H = e_0 \left[\frac{1}{2} \Pi_{ia} \Pi_{ia} + \frac{1}{2} g^{-\frac{4}{3}} M_{ia} M_{ia} - \frac{1}{2} g^{-\frac{2}{3}} \epsilon_{ijk} f_{abc} M_{ia} M_{jb} M_{kc} \right.$$
$$\left. + \frac{1}{4} f_{abc} f_{ade} M_{ib} M_{jc} M_{id} M_{je} + c\, H_c + M\, H_m + H_{int} \right]$$

- In the double scaling limit: $g \to \infty$, $R \to \infty$, e_0 kept finite $\Rightarrow H$ has a well-defined spectrum.

Symmetries of the 2-color 1-flavor QCD Hamiltonian

- Global Symmetries:
  1. Chiral Symmetry (for $m = 0$) $U(1)_A$

$$\Psi \to e^{i\theta\gamma^5}\Psi \Rightarrow U(1)_A \Rightarrow \text{Generated by } Q_0 = \frac{1}{2}(b^\dagger_{\alpha A} b_{\alpha A} - d_{\alpha A} d^\dagger_{\alpha A})$$
$$\to \text{Anomalously broken to } \mathbb{Z}_2$$
$$\to \text{Explicitly broken to } \mathbb{Z}_2 \text{ when } m \neq 0$$

  2. Vector Symmetry $SU(2)_B$

$$\Psi \to e^{i\theta}\Psi \Rightarrow U(1)_B \to \text{Generated by } B_3 = \frac{1}{2}(b^\dagger_{\alpha A} b_{\alpha A} - d^\dagger_{\alpha A} d_{\alpha A})$$
$$\to \text{Further extends to } SU(2)_B \quad \text{(Pauli-Gürsey Symmetry)}$$
$$\to \text{Generated by } \{B_1, B_2, B_3\}; \quad [B_i, B_j] = i\epsilon_{ijk} B_k,$$
$$[B_i, H] = 0,\ [B_j, Q_0] = 0,\ [B_i, J_j] = 0,\ [B_i, G_a] = 0.$$

- The residual symmetry is

$$SO(3)_{rot} \otimes \mathbb{Z}_2 \otimes SU(2)_B$$

Numerical diagonalization of the Hamiltonian on the Hilbert Space of the Low-lying Energy States ($J = 0, 1$)

- Symmetries of the Hamiltonian $\rightarrow$ the *physical states* can be organised into representations of the Spin ($SO(3)_{rot}$) and Baryon charge ($SU(2)_B$) Groups.
- The Spin-0 and Spin-1 hadrons can be arranged in 5 different sectors. Each sector is labelled by B($SU(2)_B$ charge) and J (Total Spin).

$$(B, J) = (0, 0),\ (0, 1),\ (1, 0),\ (1, 1),\ (2, 0)$$

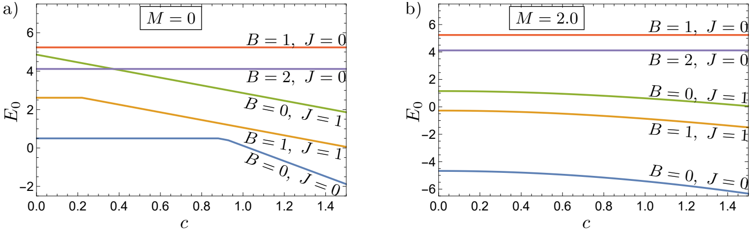
- $B = 0 \Rightarrow$ mesons ($B_3 = 0$).

  $B = 1 \Rightarrow$ mesons ($B_3 = 0$), diquarks ($B_3 = 1$), anti-diquarks ($B_3 = -1$).

  $B = 2 \Rightarrow$ mesons ($B_3 = 0$), diquarks ($B_3 = 1$), anti-diquarks ($B_3 = -1$), teraquarks ($B_3 = 2$), anti-tetraquarks ($B_3 = -2$).
- $SU(2)_B$ symmetry $\Rightarrow$ The states in a given $SU(2)_B$ multiplet (same B, different B_3) are degenerate.

Continued...

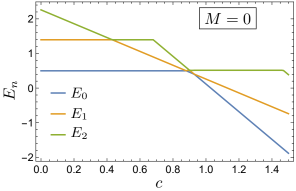
- $\mathcal{H} = \mathcal{H}_{Fermion} \otimes \mathcal{H}_{Boson}$
- $\mathcal{H}_{Boson}$ is infinite dimensional. We truncate it to a given boson number.
- The low-energy spectrum of H is estimated using the Rayleigh-Ritz method.
- Low-lying energy eigenvalues as a function of c from each sector.

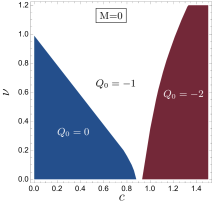


- The ground state is unique and belongs to the the $B = 0, J = 0$ sector.

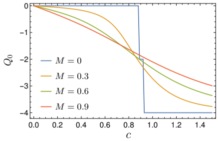
Results: Quantum Phase Transition

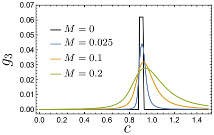
- Level crossing in the $(B, J) = (0, 0)$ is rather special $\Rightarrow$ Triple crossing.
- Plot of $\nu \, (= g^{-2/3})$ vs c shows three distinct phases. For $g \to \infty$ or $\nu \to 0$ two transition lines merge at the triple point.

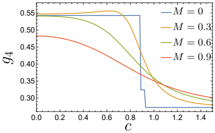




- Critical point $(c, M) \sim (0.928, 0) \Rightarrow Q_0$ is discontinuous; third and forth Binder cumulants (g_3 and g_4) show singular behaviour.

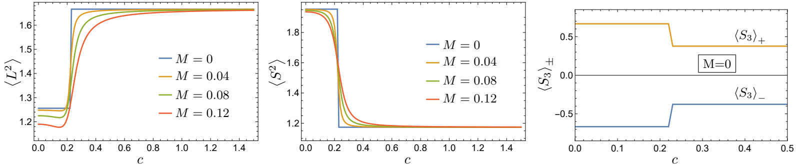






A possible solution to the Spin Puzzle?

- Quarks carry $(4-24\%)$ of proton spin: Proton Spin Puzzle (EMS 1988)
- For $(B,J)=(1,1)$ QPT occurs at $(c,M)\approx(0.22,0)\equiv(c_1^*,M)$.
- Glue (L) and Quark (S) spin contribution in the ground state:

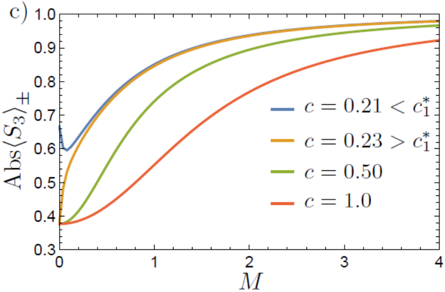


- When $c<c_1^*$ quark spin contributes significantly and it is opposite for $c>c_1^*$.
- Distribution of spin is further clarified by $\langle S_3\rangle_\pm$:

$$\text{At } M=0: \langle S_3\rangle_\pm = \begin{cases} \pm 0.67 & c<c_1^* \\ \pm 0.33 & c>c_1^* \end{cases}$$

Continued...

- Isgur-Wise symmetry: quark spin is independently conserved.

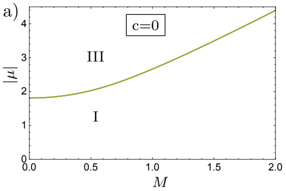


At the heavy quark limit ($M \gg 1$), $\langle S_3 \rangle_\pm \approx 1$, irrespective of c.

Addition of the Baryon Chemical Potential

- By adding the Baryon chemical potential $g^{-2/3}\mu B_3$, we break the $SU(2)_B \to U(1)_B$ explicitly.
- $$E(\mu) = E(\mu = 0) + \mu B_3$$

  The degeneracy between mesons, diquarks and tetraquarks is lifted in a given sector.
- New phases emerge:



Phase-I: *Spin-0 meson*, Phase-II: *Spin-1 diquark*, Phase-III: *Spin-0 tetraquark*

- When in phase-II, the ground state is a spin-1 di-quark $\Rightarrow SO(3)_{rot}$ is spontaneously broken $\Rightarrow$ reminiscent of LOFF phase in 2-col QCD.

Summary & Future Outlook

- $SU(2)$ gauge theory coupled to a fundamental Dirac fermion.
- Enhanced global symmetry (Pauli-G*ü*rsey): $U(1)_B \to SU(2)_B$
- QPTs (when we tune c) in different (B,J) sectors – Level crossings in the gs.
- QPTs are 1st order: $\langle Q_0 \rangle = \frac{\partial E_0}{\partial c}$, is discontinuous.
- We studied the distribution of Spin among the quark and the glue & found signatures of the Isgur-Wise symmetry.
- Addition of Baryon chemical potential:
  - $SU(2)_B \xrightarrow{\text{Explicitly broken to}} U(1)_B$
  - With sufficiently large μ spin-1 (anti-)diquark can become the gs $\Rightarrow SO(3)_{rot}$ is spontaneously broken $\Rightarrow$ reminiscent of the LOFF phases in 2-col QCD.

Ongoing Work:

- SU(2) gauge theory plus one (or more) adjoint Weyl fermion $\Rightarrow \mathcal{N} = 1$ SUSY.
- 3-color 3-flavor Matrix Model $\Rightarrow$ Light-quark QCD

Thank You