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# A global view of the inflationary landscape in future experiments



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**10 August 2024**  
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# Cosmic inflation

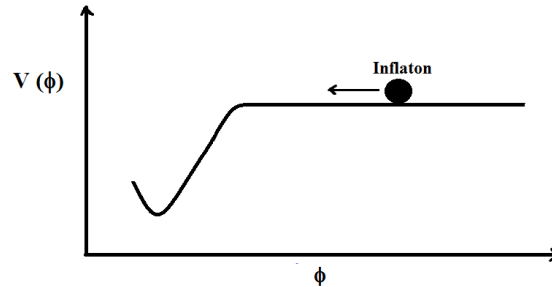
## Inflation in a nutshell

Flatness, horizon, absence of exotic relics problems

## Dynamics

$$\varphi(x^\mu) = \bar{\varphi}(t) + \delta\varphi(x^\mu)$$

$$\ddot{\bar{\varphi}} + 3H\dot{\bar{\varphi}} + V_{E,\bar{\varphi}} = 0$$

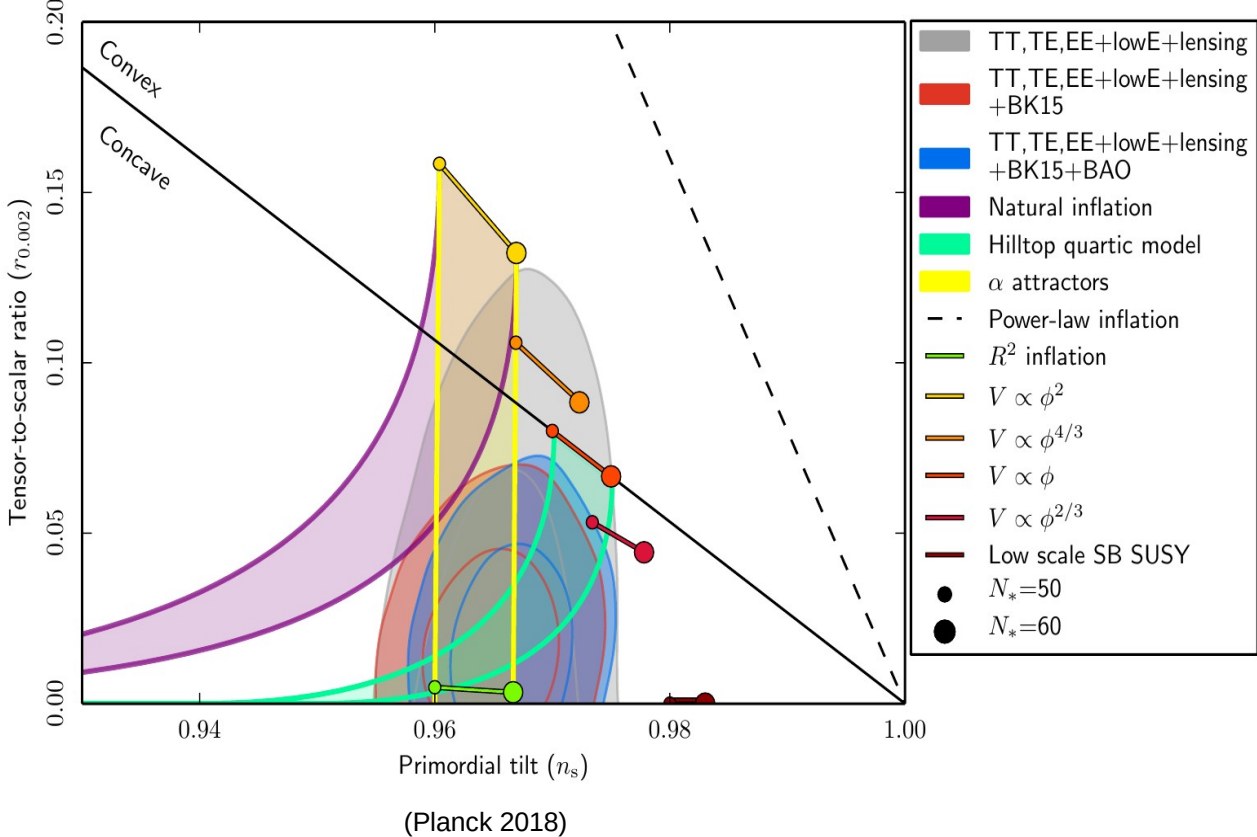


## Perturbation

$$\delta\varphi \rightarrow Q \rightarrow \mathcal{R} \rightarrow \mathcal{P}_{\mathcal{R}} :$$

- CMB temperature and polarisation fluctuations
- Neutral hydrogen density to 21cm brightness fluctuation

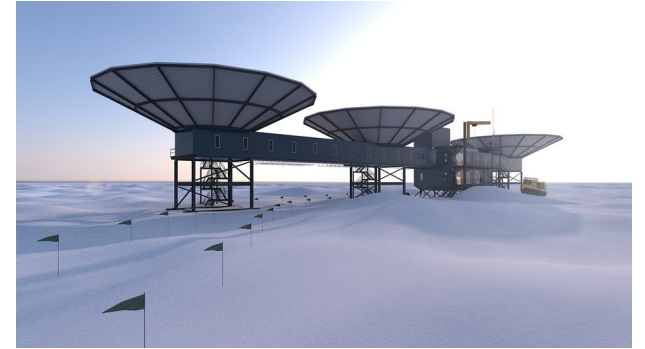
# The Zoo



# CMB

## CMB-S4

12 telescopes: 500,000 cryogenically-cooled superconducting detectors in South Pole and in the Chilean Atacama desert



(CMB-S4 South pole preliminary; Credit: CMB-S4)

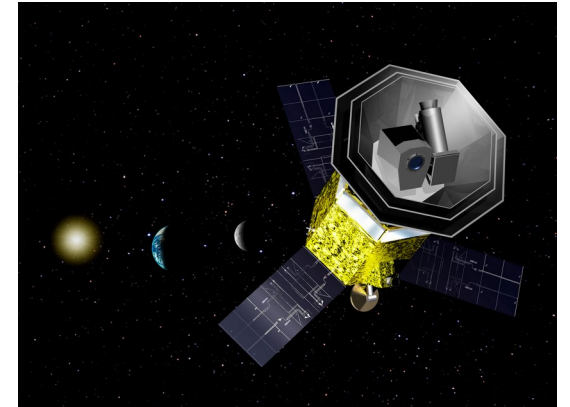
## LiteBIRD

Second Lagrange point, 1.5 million kilometers from Earth

### Current and future of CMB experiments

- Planck
- CMB-S4:  $(\ell \in [30, 3000])$
- LiteBIRD:  $(\ell \in [2, 1350])$
- LiteBIRD low- $\ell$  + CMB-S4 high- $\ell$ :  $(\ell \in [2, 50]) + [50, 3000]$

(Brinckmann et al. JCAP'19)



(LiteBIRD concept art: Credit ISAS/JAXA)

# 21cm intensity mapping by SKA

## Brightness temperature $T_b$

(TM, T. Plehn, L. Röver, and B. M. Schäfer, B. Schosser, SciPost Phys. 15, 047 (2023))

Resonant interaction of CMB photons & the hyperfine transition

$$\nu_0 = 1420.405752 \text{ MHz}, \lambda_0 = 21.11 \text{ cm}$$

## Differential brightness temperature

$$\Delta T_b = \tau \frac{T_b(z) - T_\gamma(z)}{1+z}, \text{ optical depth: } \tau \ll 1$$

## Power Spectrum

$$\overline{\Delta T_b} \simeq 189 \left[ \frac{H_0 (1+z)^2}{H(z)} \right] \Omega_{\text{HI}}(z) \text{ h mK} \quad (\text{Sprenger et al. JCAP'19})$$

$$P_{21}(k, \mu, z) = f_{\text{AP}}(z) \times f_{\text{res}}(k, \mu, z) \times f_{\text{RSD}}(\hat{k}, \hat{\mu}, z) \times b_{21}^2(z) \times P_\delta(\hat{k}, z)$$

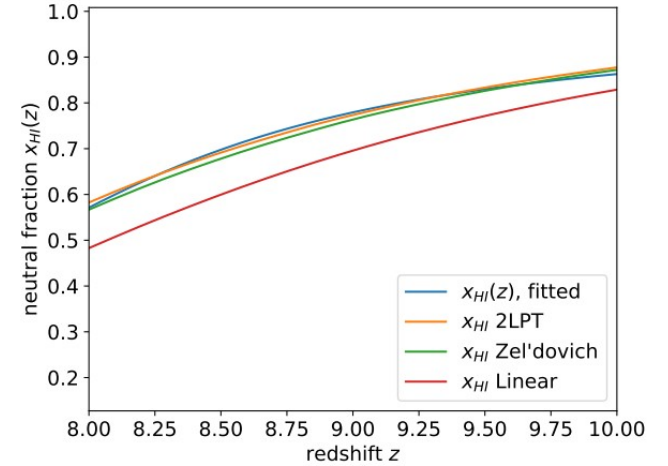
$$b_{21} = \overline{\Delta T_b}(z) b_{\text{HI}}(z)$$

## Neutral hydrogen fraction

$$\Omega_{\text{HI}}(z) = \frac{\rho_{\text{HI}}}{\rho_c} = \Omega_b(1 - Y_P) \left( \frac{H_0}{H(z)} \right)^2 (1 + z)^3 x_{\text{HI}}(z)$$
$$x_{\text{HI}}(z) = \frac{1}{2} \left[ 1 + \frac{2}{\pi} \tan^{-1} (\delta_1(z - \delta_2)) \right]$$

Fit values:  $\delta_1 = 0.9755$ ,  $\delta_2 = 7.7664$

three nuisance parameters:  $\{b_{\text{HI}}, \delta_1, \delta_2\}$



21cmFAST

# SKA1-Low

$$z \in [8, 10] \text{ and } 0.01 \text{ Mpc}^{-1} < k < 0.2 \text{ Mpc}^{-1}$$

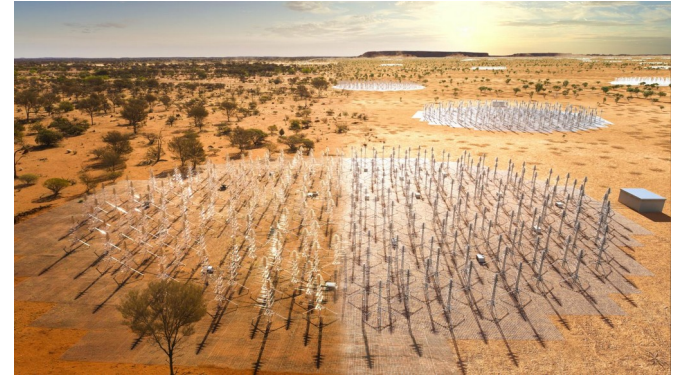
The comoving radial distance line of sight to

$z = 8$ : 8943.21 (Mpc)

$z = 10$ : 9440.25 (Mpc)

The difference translates to 37.09 Mpc/h for 20 bins

Avg. ionized patches few Mpc (Mellema et al. 1210.0197)



(SKA-Low prototype antennas: Credit SKA)

## SKA1-Low specifications

$t_{\text{obs}} = 10000$  hrs,  $\nu_0 = 1420.405752$  MHz,  $\lambda_0 = 21.11$  cm, band [50, 350] MHz, array of 224 stations, size of the station  $D = 40$  each with 256 antennas with area  $3.2 \text{ m}^2$ .

Compared to LOFAR 25% better resolution, eight times the sensitivity, and will be able to survey the sky 135 times faster.

# Hubble slow-roll parameters

(TM, T. Plehn, L. Röver, and B. M. Schäfer, B. Schosser, SciPost Phys. 15, 047 (2023);  
TM, T. Plehn, L. Röver, and B. M. Schäfer, SciPostPhys. Core '22)

- Inflationary dynamics: parametrized by expansion of Hubble parameter

## The parameters

$$H(\varphi) = \sum_{n=0}^N \frac{1}{n!} \frac{d^n H}{d\varphi^n} \Big|_{\varphi_*} (\varphi - \varphi_*)^n$$

$$\epsilon_H = \frac{M_{\text{pl}}^2}{4\pi} \left( \frac{H'}{H} \right)^2,$$

$$\eta_H = \frac{M_{\text{pl}}^2}{4\pi} \left( \frac{H''}{H} \right), \dots$$

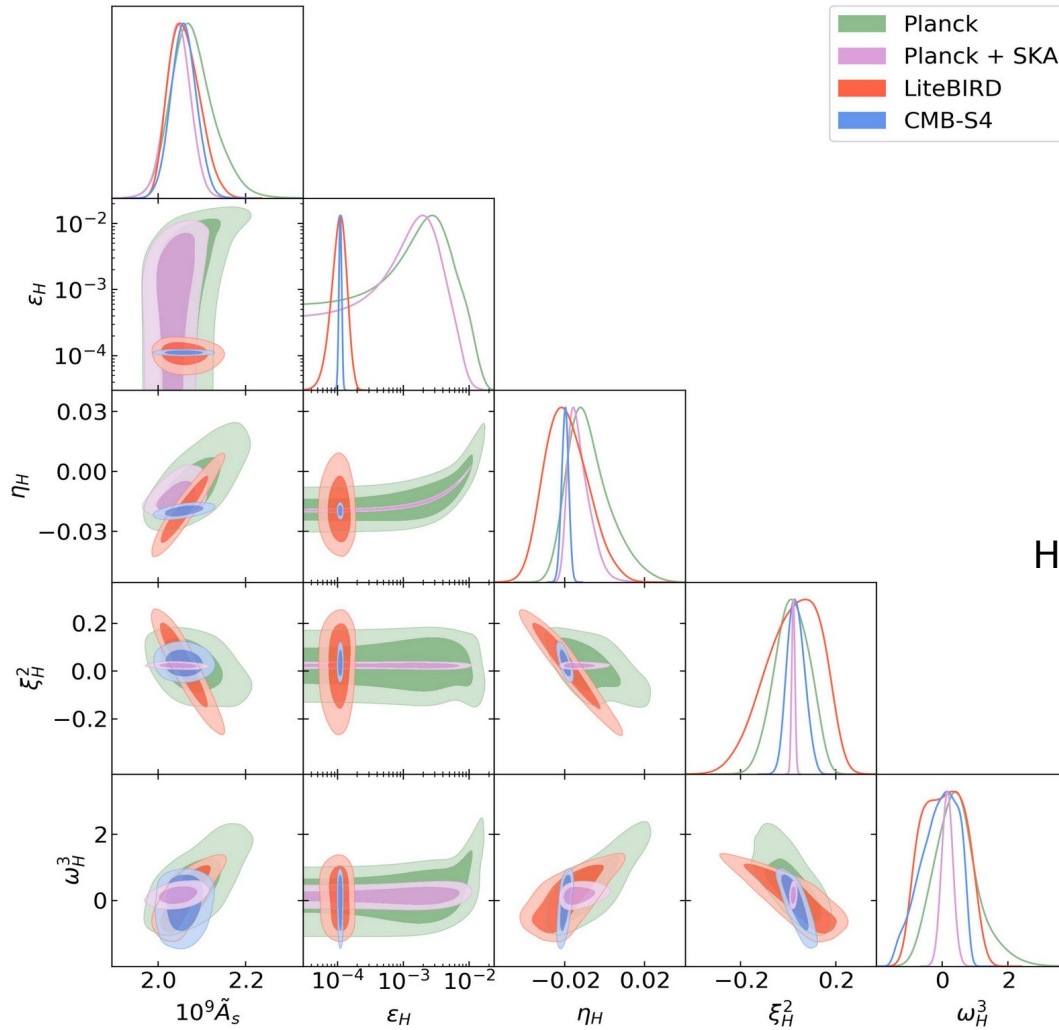
(Lesgourgues et al. JCAP'08, Planck'18)

## Parameters of

$$\{\tilde{A}_s, \epsilon_H, \eta_H, \xi_H^2, \omega_H^3\}$$



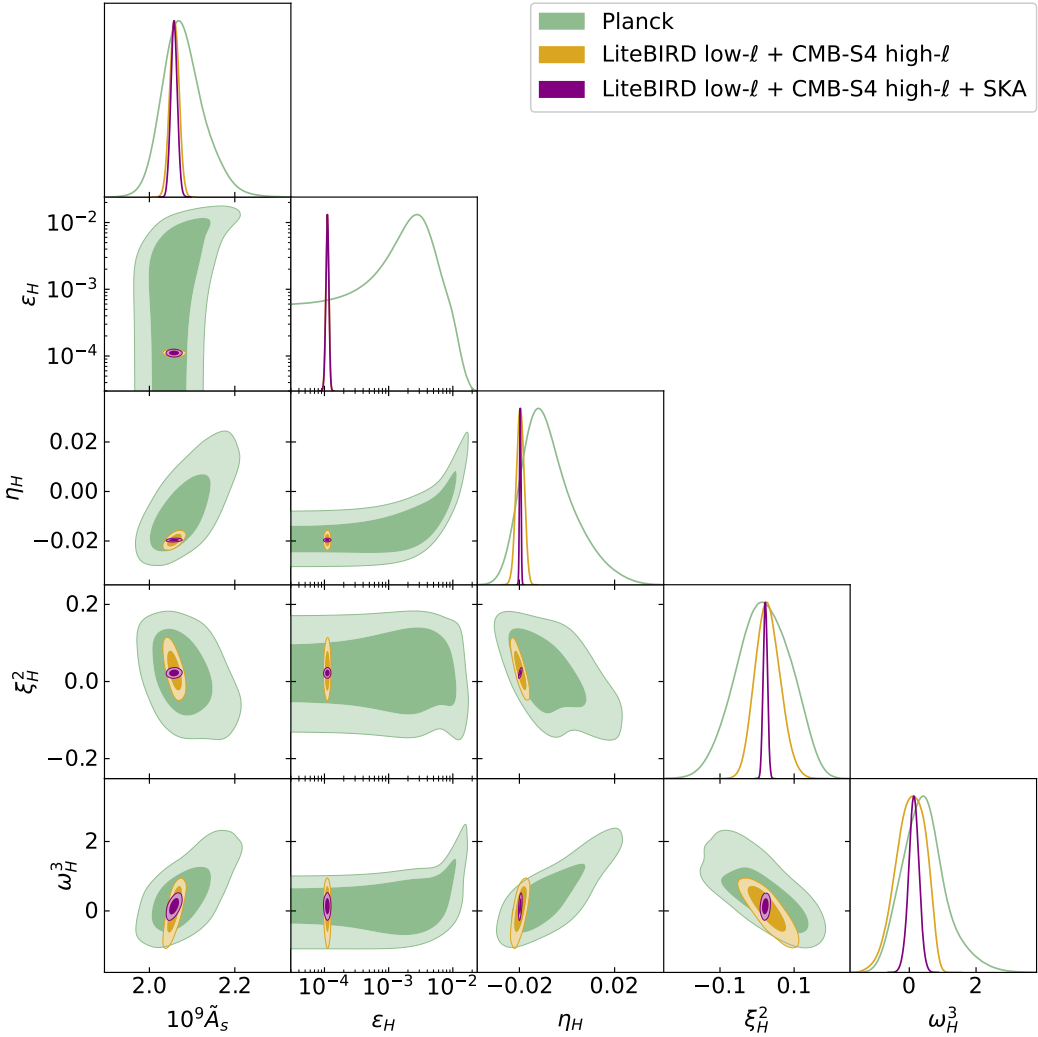
# CMB



Class+MontePython

HALOFIT: For nonlinear corrections

# With SKA



# Modified $\Lambda$ CDM

power spectrum:  $\mathcal{P}_{\mathcal{R}}(k) = \mathcal{A}_s \left( \frac{k}{k_*} \right)^{n_s - 1 + \frac{\alpha_s}{2!} \ln\left(\frac{k}{k_*}\right) + \frac{\beta_s}{3!} \ln\left(\frac{k}{k_*}\right)^2}$

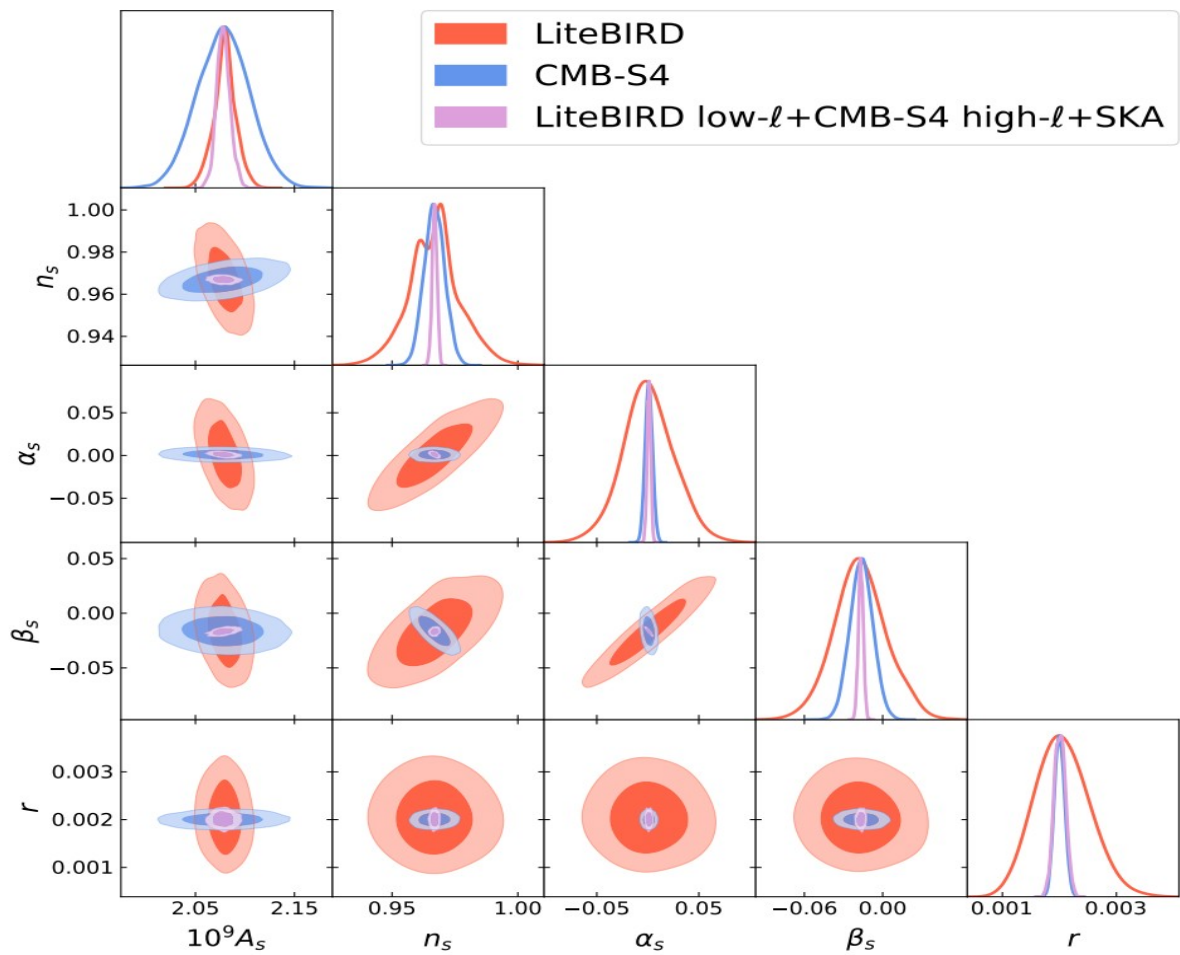
spectral index:  $n_s = 1 + d\mathcal{P}_{\mathcal{R}}(k)/d \ln k$

Runnings of  $n_s$ :  $\alpha_s = dn_s/d \ln k$  and  $\beta_s = d^2n_s/d \ln k^2$

tensor-to-scalar ratio:  $r = \frac{\mathcal{A}_t}{\mathcal{A}_s}$

## Baseline parameters

$$\{\omega_b, \omega_{\text{cdm}}, h, \tau_{\text{reio}}, n_s, \mathcal{A}_s, \alpha_s, \beta_s, r\}$$



# Starobinsky inflation

## Action in different frames

$$S_J = \frac{1}{2} \int d^4x \sqrt{-g_J} f(R),$$

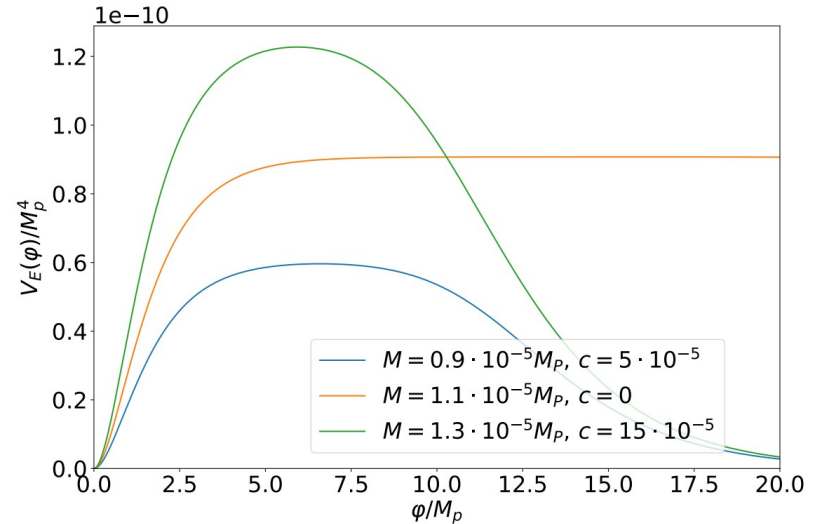
$$f(R) = M_P^2 \left( R + \frac{1}{6M^2} R^2 + \frac{c}{36M^4} R^3 \right)$$

## Potential

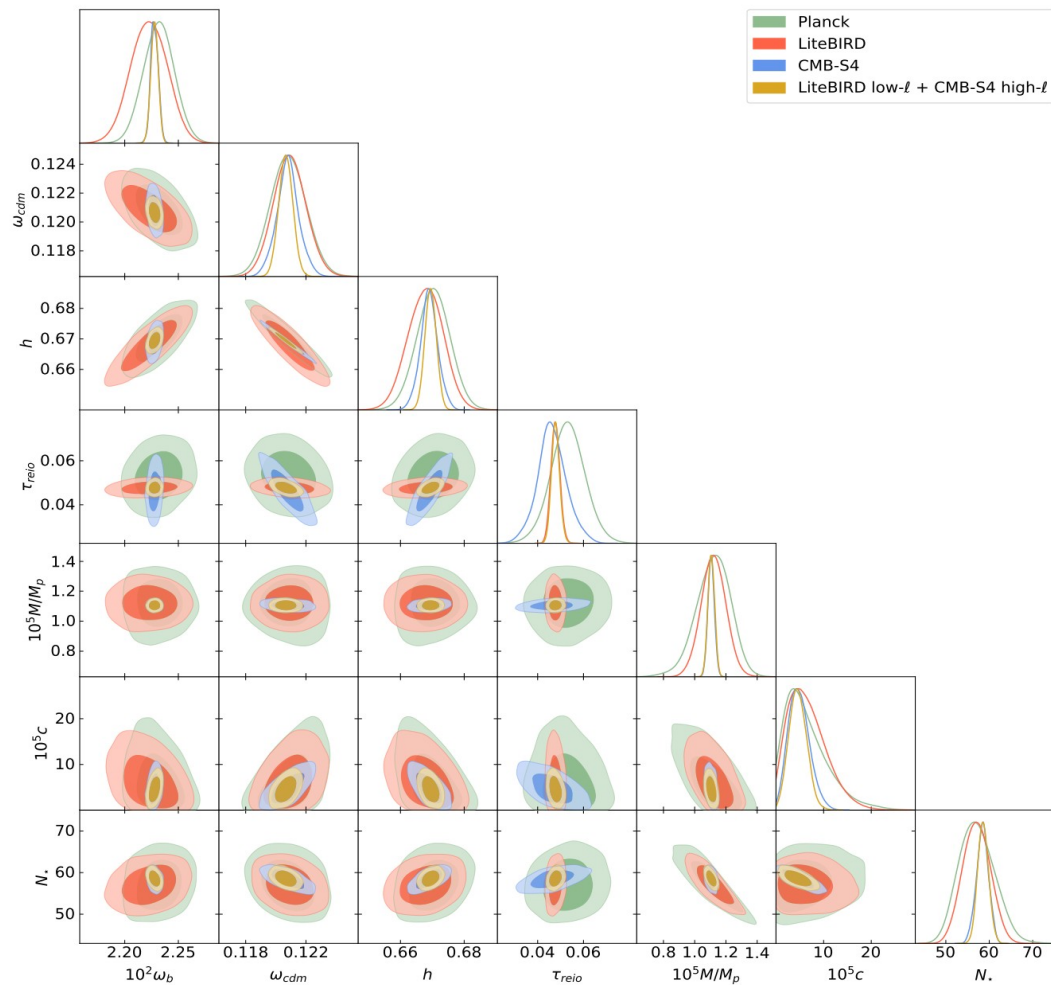
$$V_E(\varphi) = \frac{M_P^2 \left( \frac{cs(\varphi)^3}{M^2} + 3s(\varphi)^2 \right)}{36M^2 \left( 1 + \frac{s(\varphi)}{3M^2} + \frac{cs(\varphi)^2}{12M^4} \right)^2}$$

## Baseline parameters

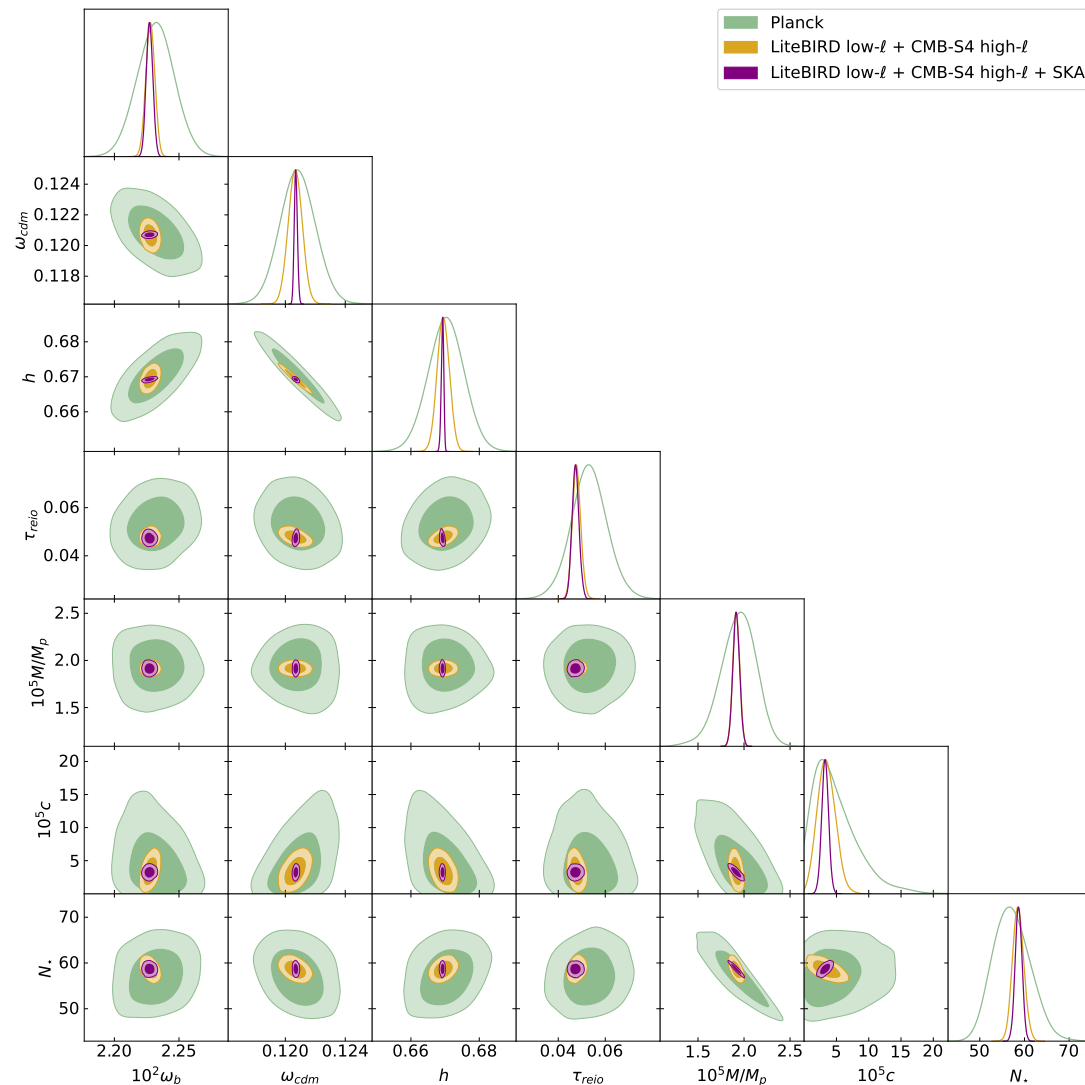
$$\{\omega_b, \omega_{\text{cdm}}, h, \tau_{\text{reio}}, M, c, N_*\}$$



# CMB



# With SKA



Data	Parameters	Best-fit	Mean $\pm\sigma$	95% lower	95% upper
LiteBIRD low- $\ell$	100 $\omega_b$	2.228	2.227 $^{+0.003}_{-0.003}$	2.222	2.232
	$\omega_{\text{cdm}}$	0.1206	0.1207 $^{+0.0001}_{-0.0001}$	0.1205	0.1209
+	$h$	0.6694	0.6692 $^{+0.0004}_{-0.0003}$	0.6685	0.670
CMB-S4 high- $\ell$	$\tau_{\text{reio}}$	0.04792	0.04734 $^{+0.0014}_{-0.0016}$	0.04445	0.05033
+	$10^5 M/M_P$	1.100	1.106 $^{+0.023}_{-0.023}$	1.064	1.148
SKA	$10^5 c$	4.350	4.325 $^{+0.692}_{-0.690}$	2.891	5.734
	$N_*$	58.95	58.68 $^{+0.77}_{-0.75}$	57.20	60.18



# Few words on uncertainties at SKA

## **Biasing**

Assumed linear biasing with a nuisance parameter  $b_{HI}$ . Correct?

## **Neutral hydrogen fraction**

Better modeling for the neutral hydrogen fraction instead of fitting function  
High redshift galaxies observed by JWST

## **Foreground removal**

Foreground: much higher than the actual signal  
Assumed sufficiently smooth to be removed

# Outlook

- Many models of inflation: Need narrow down or discover
- Future CMB experiments will provide sensitive probes
- 21cm intensity mapping by SKA: Complementary probes with CMB

Thank you!!

# Additional Slides

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## Flatness problem

The curvature:  $\Omega - 1 \propto \frac{1}{H^2 a^2}$ ,  $\Omega = \frac{\rho}{\rho_c}$ ,  $\rho_c = \frac{3H^2}{8\pi G}$

$\Omega - 1 = 0 \implies$  flat Universe

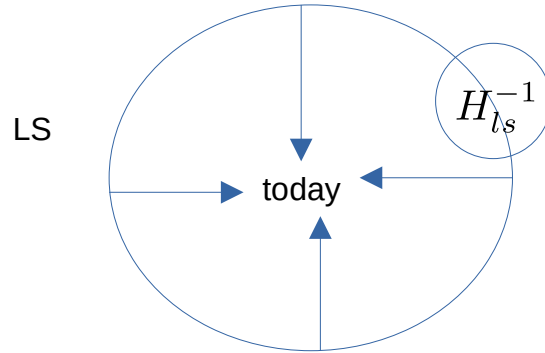
Planck epoch:  $\frac{|\Omega-1|_P}{|\Omega-1|_0} \approx \frac{a_{MP}^2}{a_0^2} \approx \frac{T_0^2}{T_{MP}^2} \approx \mathcal{O}(10^{-64})$ ;

BBN:  $\frac{|\Omega-1|_{BBN}}{|\Omega-1|_0} \approx \frac{a_{BBN}^2}{a_0^2} \approx \frac{T_0^2}{T_{BBN}^2} \approx \mathcal{O}(10^{-16})$ ;

$|\Omega - 1|_0 \sim 10^{-3} \implies$  small curvature at earliest epochs

## Horizon problem

From epoch of last-scattering, photons free-stream and reach us untouched



$$\frac{(H_0^{-1})_{ls}^{-3}}{H_{ls}^{-3}} \sim 10^6 \text{ causally disconnected Hubble volumes}$$

CMB photons:  $> 1^\circ$  or  $\ell < 200$  no causal contact

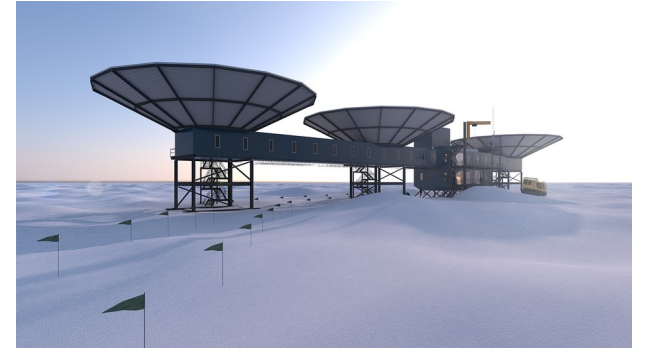
## Exotic relic problem

Magnetic monopoles, ....

# CMB

## Current and future of CMB experiments

- Planck
- CMB-S4: ( $\ell \in [30, 3000]$ )
- LiteBIRD: ( $\ell \in [2, 1350]$ )
- LiteBIRD low- $\ell$  + CMB-S4 high- $\ell$ : ( $\ell \in [2, 50] + [50, 3000]$ )



(CMB-S4 South pole preliminary)

## CMB-S4

12 telescopes: 500,000 cryogenically-cooled superconducting detectors in South Pole and in the Chilean Atacama desert

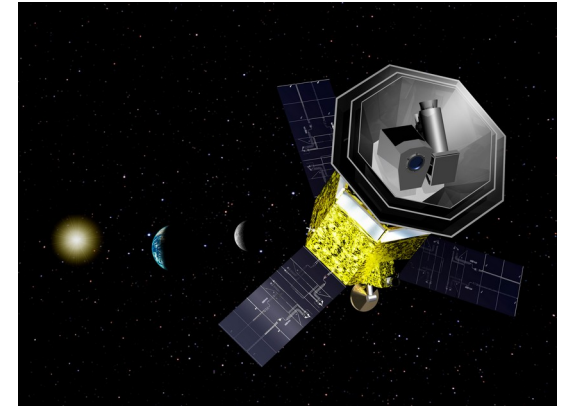
CMB-S4:  $f_{\text{sky}} = 0.4$ , 150 GHz channel, FWHM = 3 arcmin,  $\Delta T = 1.0 \mu\text{K arcmin}$  and  $\Delta P = 1.41 \mu\text{K arcmin}$ .

## LiteBIRD

Second Lagrange point, 1.5 million kilometers from Earth

LiteBIRD: Sky fraction:  $f_{\text{sky}} = 0.7$ , Channel: 140 GHz with full-width-half-max or FWHM = 31 arcmin,  $\Delta T = 4.1 \mu\text{K arcmin}$ , and  $\Delta P = 5.8 \mu\text{K arcmin}$

(Brinckmann et al. JCAP'19)



(LiteBIRD concept art: ISAS/JAXA)

## Quantum fluctuations to CMB anisotropies

- Primordial fluctuation imprinted in CMB temperature and polarization anisotropies
- Scalar (density) perturbations create **E**-modes but no **B**-modes
- Vector (vorticity) perturbations not excited during inflation
- Tensor (gravitational waves) perturbations create both **E**- and **B**-modes

E.g., the power spectrum of temperature and *B*-mode anisotropies:

$$C_{\ell}^{TT} \propto \int k^2 dk \mathcal{P}_{\mathcal{R}} \Delta_{T\ell}(k) \Delta_{T\ell}(k),$$

$$C_{\ell}^{BB} \propto \int k^2 dk \mathcal{P}_{\mathcal{T}} \Delta_{B\ell}(k) \Delta_{B\ell}(k),$$

# Power spectrum

$$P_{21}(k, \mu, z) = f_{\text{AP}}(z) \times f_{\text{res}}(k, \mu, z) \times f_{\text{RSD}}(\hat{k}, \hat{\mu}, z) \times (b_{\text{HI}} \overline{\Delta T_b})^2 \times P_{\delta}(\hat{k}, z)$$

$$P_{21}^{\text{obs}}(k, \mu, z) = P_{21}(k, \mu, z) + P_N(z)$$

## SKA1-Low

$$z \in [8, 10] \text{ and } 0.01 \text{ Mpc}^{-1} < k < 0.2 \text{ Mpc}^{-1}$$

The comoving radial distance line of sight:  $z = 8$ : 8943.21 (Mpc);  $z = 10$ : 9440.25 (Mpc).  
The difference translates to 37.09 Mpc/h.

Avg. ionized patches few Mpc (Mellema et al. 1210.0197)

$t_{\text{obs}} = 10000$  hrs,  $\nu_0 = 1420.405752$  MHz,  $\lambda_0 = 21.11$  cm, band [50, 350] MHz, array of 224 stations, size of the station  $D = 40$  m, maximum baseline  $D_{\text{base}} = 1$  km, 64000 channels,  $f_{\text{sky}} = 0.58$ , field of view of  $\Omega = (1.2\lambda/D)^2$ , an area  $A = N_{\text{dish}}\pi(D/2)^2$  per station, and the covering fraction  $f_{\text{cover}} = N_{\text{dish}}(D/D_{\text{base}})^2$ .

(SKA Red book)



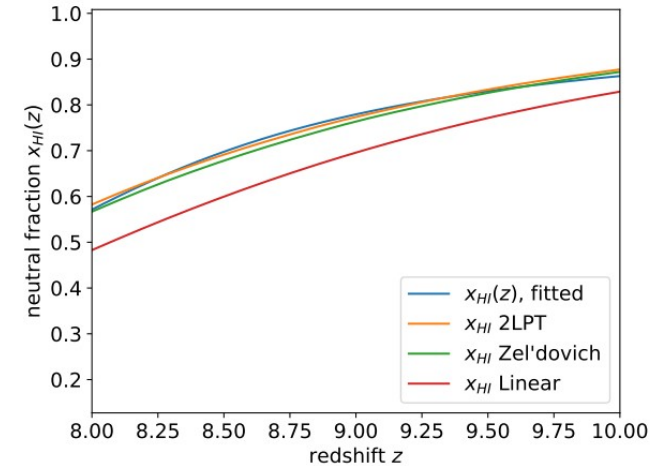
## Neutral hydrogen fraction

$$\Omega_{\text{HI}}(z) = \frac{\rho_{\text{HI}}}{\rho_c} = \Omega_b(1 - Y_P) \left( \frac{H_0}{H(z)} \right)^2 (1 + z)^3 x_{\text{HI}}(z)$$

$$x_{\text{HI}}(z) = \frac{1}{2} \left[ 1 + \frac{2}{\pi} \tan^{-1} (\delta_1(z - \delta_2)) \right]$$

### Fitting

- 21cmFAST
- 1<sup>st</sup> order perturbative approx
- 2<sup>nd</sup> order 2LPT of velocity field



Fit values:  $\delta_1 = 0.9755$ ,  $\delta_2 = 7.7664$

# SKA specifications

$$P_{21}(k, \mu, z) = f_{\text{AP}}(z) \times f_{\text{res}}(k, \mu, z) \times f_{\text{RSD}}(\hat{k}, \hat{\mu}, z) \times b_{21}^2(z) \times P_{\delta}(\hat{k}, z)$$

$$P_{21}^{\text{obs}}(k, \mu, z) = P_{21}(k, \mu, z) + P_N(z)$$

$$P_N(z) = \frac{4\pi T_{\text{sys}}^2 f_{\text{sky}} \lambda^2 y D_A^2}{A \Omega f_{\text{cover}} t_{\text{obs}}}$$

$$T_{\text{sys}} = T_{\text{sky}} + T_{\text{rx}}$$

$$\text{with } T_{\text{sky}} = 25 \text{ K} \left( \frac{408 \text{ MHz}}{\nu} \right)^{2.75} \quad \text{and} \quad T_{\text{rx}} = 0.1 T_{\text{sky}} + 40 \text{ K} \quad (\text{SKA Red book})$$

$t_{\text{obs}} = 1000$  hrs,  $\nu_0 = 1420.405752$  MHz,  $\lambda_0 = 21.11$  cm, band [50, 350] MHz, array of 224 stations, size of the station  $D = 40$  m, maximum baseline  $D_{\text{base}} = 1$  km, 64000 channels,  $f_{\text{sky}} = 0.58$ , field of view of  $\Omega = (1.2\lambda/D)^2$ , an area  $A = N_{\text{dish}} \pi (D/2)^2$  per station, and the covering fraction  $f_{\text{cover}} = N_{\text{dish}} (D/D_{\text{base}})^2$ .

# Noise SKA

$$P_N(z) = \frac{4\pi T_{\text{sys}}^2 f_{\text{sky}} \lambda^2 y D_A^2}{A \Omega f_{\text{cover}} t_{\text{obs}}}$$

$$T_{\text{sys}} = T_{\text{sky}} + T_{\text{rx}} \text{ with } T_{\text{sky}} = 25 \text{ K} \left( \frac{408 \text{ MHz}}{\nu} \right)^{2.75} \text{ and } T_{\text{rx}} = 0.1 T_{\text{sky}} + 40 \text{ K}$$

(SKA Red book)

## SKA1-Low specification

$$z \in [8, 10] \text{ and } 0.01 \text{ Mpc}^{-1} < k < 0.2 \text{ Mpc}^{-1}$$

$t_{\text{obs}} = 10000$  hrs,  $\nu_0 = 1420.405752$  MHz,  $\lambda_0 = 21.11$  cm, band [50, 350] MHz, array of 224 stations, size of the station  $D = 40$  m, maximum baseline  $D_{\text{base}} = 1$  km, 64000 channels,  $f_{\text{sky}} = 0.58$ , field of view of  $\Omega = (1.2\lambda/D)^2$ , an area  $A = N_{\text{dish}}\pi(D/2)^2$  per station, and the covering fraction  $f_{\text{cover}} = N_{\text{dish}}(D/D_{\text{base}})^2$ .

# LiteBIRD and CMB-S4

For LiteBIRD the angular scales are  $\ell = 2 \dots 1350$ , the sky fraction is  $f_{\text{sky}} = 0.7$ , while the channel is taken as 140 GHz with full-width-half-max or FWHM = 31 arcmin,  $\Delta T = 4.1 \mu\text{K arcmin}$ , and  $\Delta P = 5.8 \mu\text{K arcmin}$  (as per arXiv:1808.05955). The CMB-S4 specifications are  $\ell = 30 \dots 3000$ ,  $f_{\text{sky}} = 0.4$ , 150 GHz channel, FWHM = 3 arcmin,  $\Delta T = 1.0 \mu\text{K arcmin}$  and  $\Delta P = 1.41 \mu\text{K arcmin}$ . We need to ensure that the two experiments cover mutually exclusive  $\ell$  ranges, so just as in arXiv:1808.05955 we combine low- $\ell$  from LiteBIRD data and high- $\ell$  CMB-S4 data, separated at  $\ell \leq 50$ . Noise is estimated through minimum variance estimator for both experiments. We use the HALOFIT model for the nonlinear corrections throughout this paper.

(Brinckmann et al. JCAP'19)

$$\Omega_b = 0.0495. \quad Y_p = 0.24672$$

$$k = |\vec{k}| \quad \text{and} \quad \mu = \frac{\vec{k} \cdot \vec{r}}{kr}$$

$$\hat{k}^2 = \left[ \frac{\hat{H}^2}{H} \mu^2 + \frac{D_A}{\hat{D}_A} (1 - \mu^2) \right] k^2,$$

$$\hat{\mu}^2 = \frac{\hat{H}^2}{H} \mu^2 \left[ \frac{\hat{H}^2}{H} \mu^2 + \frac{D_A}{\hat{D}_A} (1 - \mu^2) \right]^{-1}$$

$$f_{\text{AP}}(z) = \frac{D_A^2 \hat{H}}{\hat{D}_A^2 H}$$

$$f_{\text{res}}(k, \mu, z) = \exp \left[ -k^2 \left( \mu^2 (\sigma_{\parallel}^2 - \sigma_{\perp}^2) + \sigma_{\perp}^2 \right) \right].$$

$$\sigma_{\parallel} = \frac{c}{H} (1+z)^2 \frac{\sigma_v}{v_0} \quad \text{and} \quad \sigma_{\perp} = (1+z) D_A \sigma_{\theta},$$

$$\sigma_{\theta} = \frac{1}{\sqrt{8 \ln 2}} \frac{\lambda_0}{D_{\text{base}}} (1+z) z \quad \text{and} \quad \sigma_v = \frac{\delta_v}{\sqrt{8 \ln 2}}.$$

$$f_{\text{RSD}}(\hat{k}, \hat{\mu}, z) = \left( 1 + \beta(\hat{k}, z) \hat{\mu}^2 \right)^2 e^{-\hat{k}^2 \hat{\mu}^2 \sigma_{\text{NL}}^2}, \quad \text{with} \quad \beta(\hat{k}, z) = -\frac{1+z}{2b_{21}(z)} \frac{d \log P_{\delta}(\hat{k}, z)}{dz}$$

$$\chi^2 = \sum_{\text{bins } n} \int_{k_{\min}}^{k_{\max}} k^2 dk \int_{-1}^1 d\mu \frac{V_r(\bar{z}_n)}{2(2\pi)^2} \left[ \frac{(\Delta P_{21}(k, \mu, \bar{z}_n))^2}{(P_{21}(k, \mu, \bar{z}_n) + P_N)^2 + \sigma_{\text{th}}^2(k, \mu, \bar{z}_n)} \right]$$

$$\sigma_{\text{th}}(k, \mu, z) = \left[ \frac{V_r(z)}{(2\pi)^2} k^2 \Delta k \frac{\Delta z}{\Delta \bar{z}} \right]^{1/2} \alpha(k, \mu, z) P_{21}(k, \mu, z).$$

The correlation length  $\Delta k$  is assumed to be  $0.05 h/\text{Mpc}$  as a conservative choice, matching the BAO scale. We also choose  $\Delta z = 1$ , which is slightly lower than the whole redshift range probed by the experiment  $z_{\text{max}} - z_{\text{min}} = 2$ .

### Non-linear Effects:

The prediction of the matter power spectrum, the bias, and RSD  
 0.33% error at  $k = 0.01 h/\text{Mpc}$ , a 1% error at  $k = 0.3 h/\text{Mpc}$ , and  
 a 3% error at  $k = 10 h/\text{Mpc}$

$$\alpha(k, z) = \begin{cases} a_1 \exp\left(c_1 \log_{10} \frac{k}{k_1(z)}\right), & \text{for } \frac{k}{k_1(z)} < 0.3, \\ a_2 \exp\left(c_2 \log_{10} \frac{k}{k_1(z)}\right), & \text{for } \frac{k}{k_1(z)} > 0.3, \end{cases}$$

$$k_1(z) = 1 \frac{h}{\text{Mpc}} (1+z)^{\frac{2}{2+n_s}},$$

with  $a_1 = 1.4806\%$ ,  $a_2 = 2.2047\%$ ,  $c_1 = 0.75056$ , and  $c_2 = 1.5120$

# Power spectrum

(TM, T. Plehn, L. Röver, and B. M. Schäfer, B. Schosser, SciPost Phys. 15, 047 (2023))

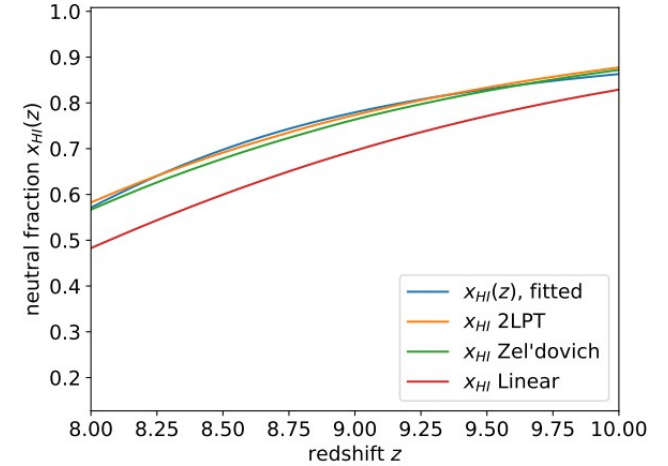
$$P_{21}(k, \mu, z) \sim (b_{HI} \overline{\Delta T_b})^2 \times P_\delta(\hat{k}, z)$$
$$P_{21}^{\text{obs}}(k, \mu, z) = P_{21}(k, \mu, z) + P_N(z)$$

## Neutral hydrogen fraction

$$\Omega_{\text{HI}}(z) = \frac{\rho_{\text{HI}}}{\rho_c} = \Omega_b(1 - Y_P) \left( \frac{H_0}{H(z)} \right)^2 (1 + z)^3 x_{\text{HI}}(z)$$
$$x_{\text{HI}}(z) = \frac{1}{2} \left[ 1 + \frac{2}{\pi} \tan^{-1}(\delta_1(z - \delta_2)) \right]$$

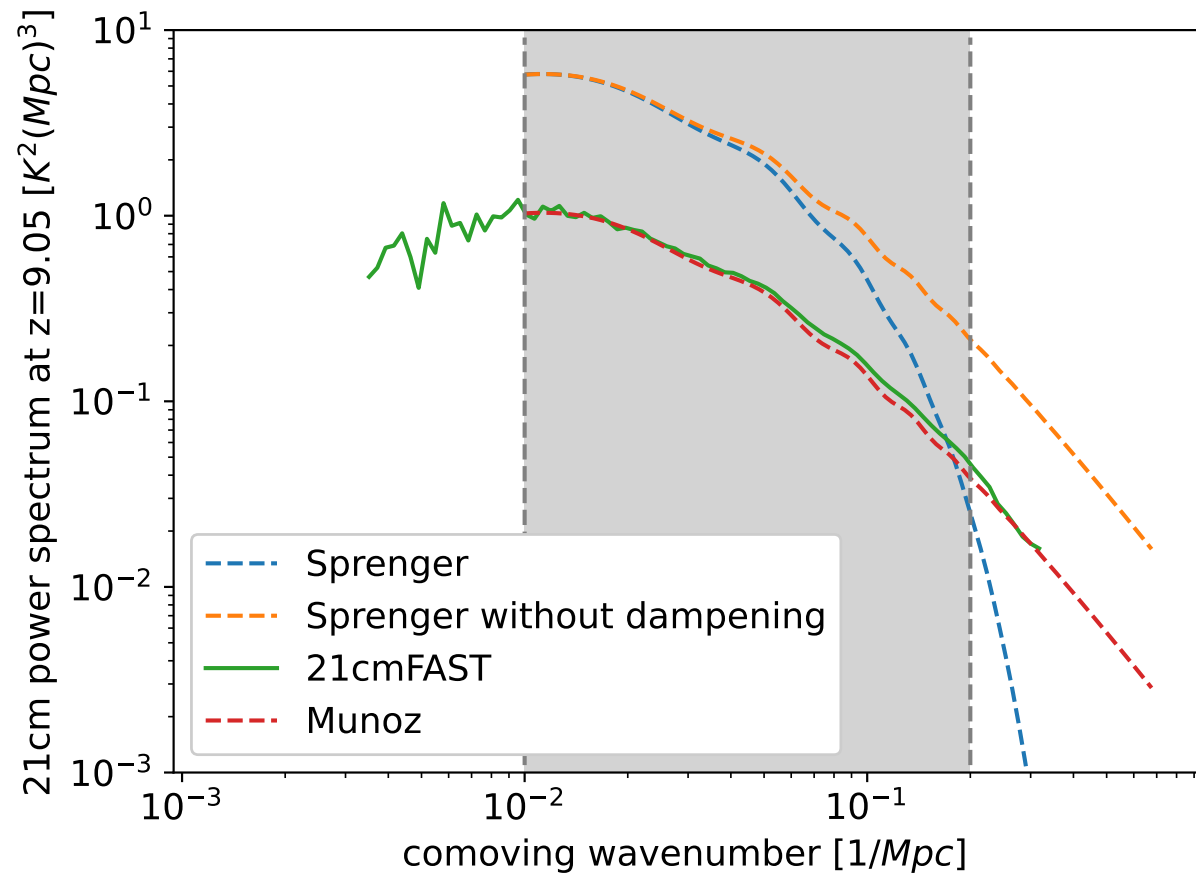
Fit values:  $\delta_1 = 0.9755$ ,  $\delta_2 = 7.7664$

three nuisance parameters:  $\{b_{HI}, \delta_1, \delta_2\}$

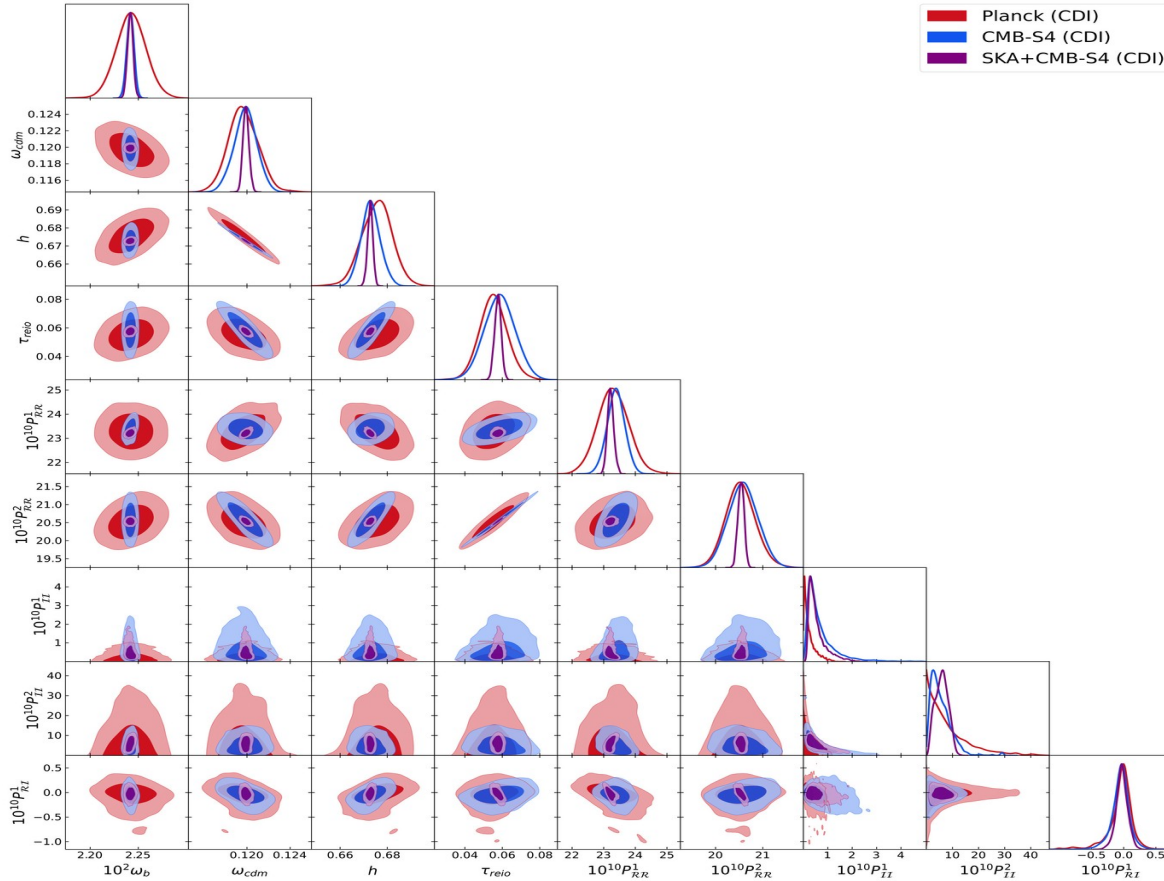


21cmFAST





# Generally correlated isocurvature mode



- Models with one adiabatic and one isocurvature mode
- Parameterized by power spectra of the 2x2 correlation matrix at a certain pivot scale  $P_{RR}, P_{II}, P_{RI}$

CDI = CDM isocurvature

