Improving non-perturbative estimates in B-meson decays

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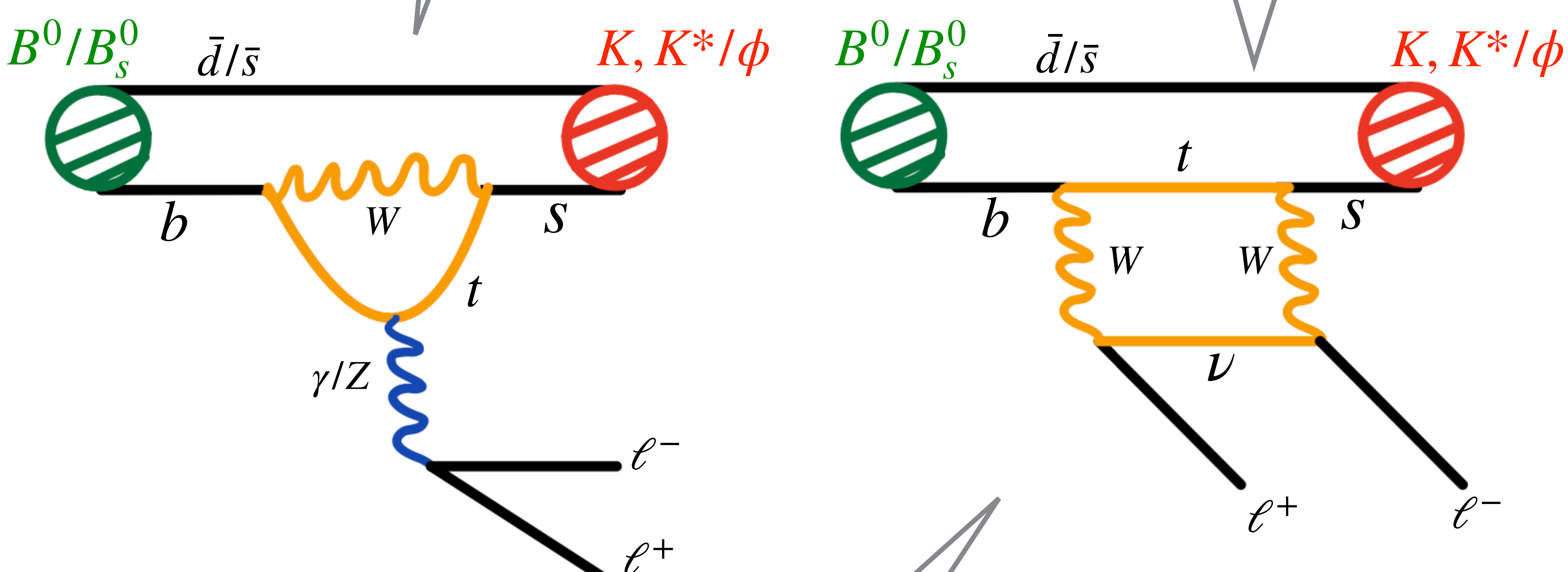
Introduction

small decay rate in SM

sensitive to new particles & interactions

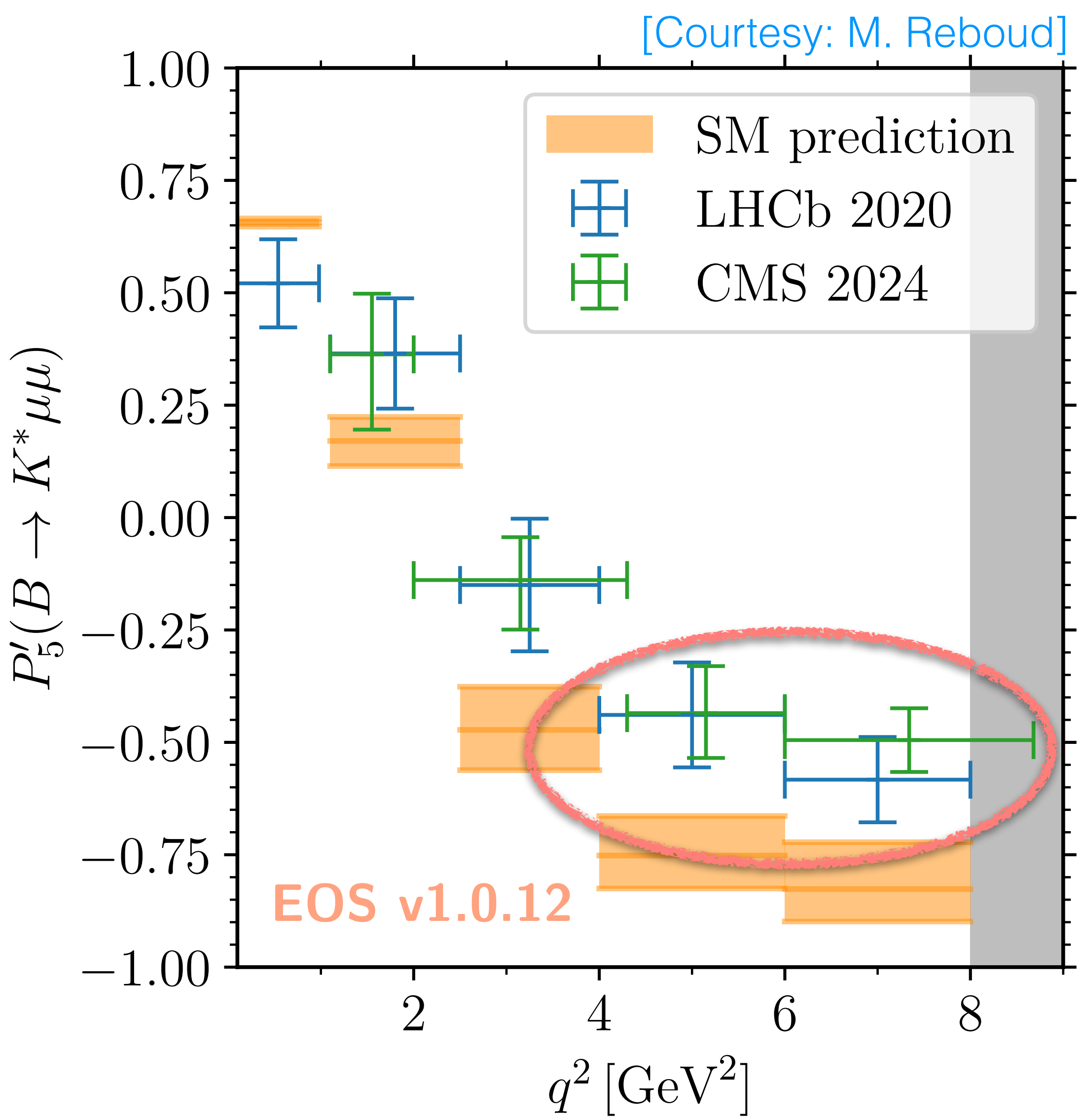


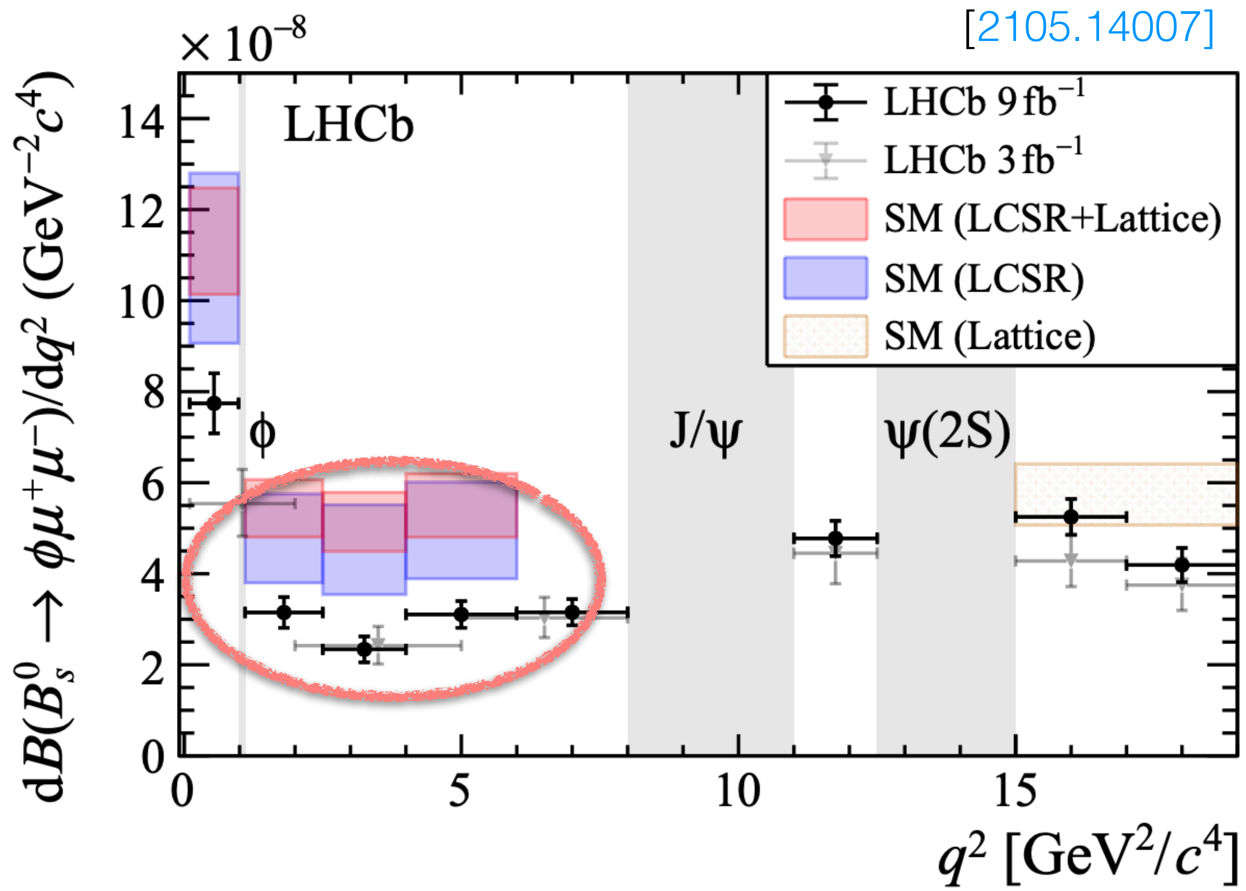
large no. of independent measurements

Interesting FCNC modes

- $B \to K^{(*)}\ell^-\ell^+$
- $B_s \to \phi\ell^-\ell^+$
- $B_s \to$ missing energy

Introduction





Deviations ~3σ observed in `not-so' clean observables

Introduction

- Angular observables $\subset$ short distance + long distance

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Wilson coefficients:
perturbatively calculable

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Form-factors:
non-perturbative estimates
from LCSR, HQET, Lattice …
tremendous effort since past

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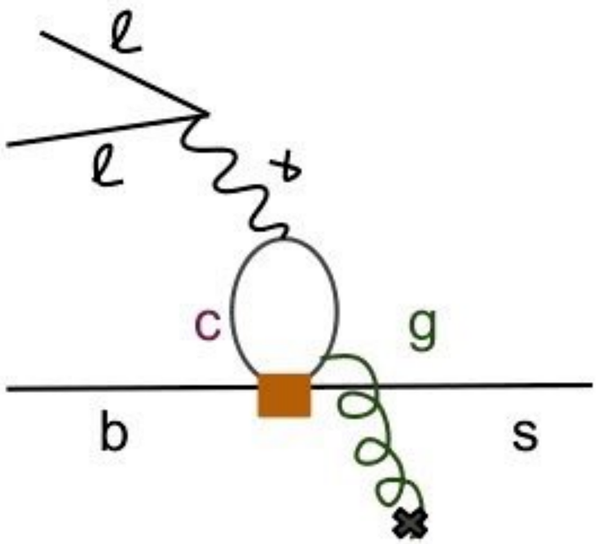
Non-factorizable
contributions:



Limited by unknown
QCD dynamics

- Challenge: Improve precision in estimates

Inverse moment of DA

- Key parameter in Light-Cone Sum Rule estimates of form factor

$$\lambda^{-1}_{B_{(s)}}(\mu) = \int_0^\infty \frac{dk}{k}\, \phi_+^{(s)}(k,\mu)$$

$\lambda_B = 460 \pm 110\,\mathrm{MeV}$ [Braun, Ivanov, Korchemsky '04]

$\lambda_{B_s} = 438 \pm 150\,\mathrm{MeV}$ [Khodjamirian, RM, Mannel '20]

→ Calculated in QCD SR
Large uncertainty

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- Direct extraction of λ_B from $B \to \ell\nu\gamma$ → Lattice might give precise estimate

For λ_{B_s} : $B_s \to \ell\ell\gamma$ → contaminated with nonlocal effects

need to understand *SU(3)* violation effects— difference (if any)

- Look for indirect methods to constrain

Form factors

- Use Lattice estimate of form factors $B_{(s)} \to P @ q^2 = 0$ [HPQCD '22]
  — small uncertainty

  — compare with LCSR predictions

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- QCD Sum rule methods for non perturbative estimates

Based on Operator product expansion [Shifman *et al.* '79]

Perturbatively calculable amplitudes + Quark & gluon condensate: charactarises QCD vacuum or distribution amplitudes in LCSR

Dispersion relation [Khodjamirian *et al.* '05,'07]

Physical hadronic parameters

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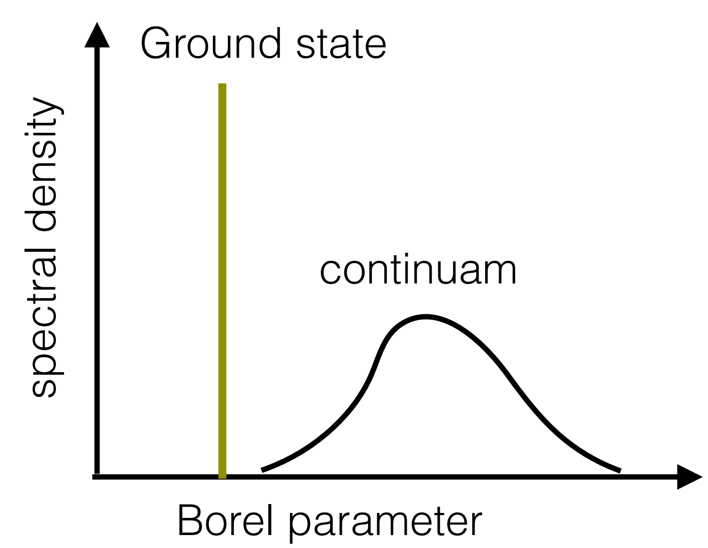
Physical hadronic parameters

- Limitations: hadronic parameter extraction depends on model ansatzs for the spectrum



Form factors $B \to P$

- Correlation function with interpolating & weak currents

$$\mathcal{F}^{\mu\nu}(q,k) = i\int \mathrm{d}^4x\, e^{ik\cdot x}\, \langle 0|\, \mathcal{T}\{J^{\nu}_{int}(x), J^{\mu}_{weak}(0)\}\, |\bar{B}_{q_2}(q+k)\rangle$$

$J^{\nu}_{int}(x)$: $\bar{q}_2(x)\gamma^{\nu}\gamma_5 q_1(x)$

$J^{\mu}_{weak}(0)$: $\bar{q}_1(0)\Gamma^{\mu}_{w} b(0)$

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$J^{\nu}_{int}(x) \to \bar{q}_2(x)\gamma^{\nu}\gamma_5 q_1(x)$ $\qquad$ $J^{\mu}_{weak}(0) \to \bar{q}_1(0)\Gamma^{\mu}_{w}b(0)$

- Dispersion relation for hadronic representation

$$\mathcal{F}^{\mu\nu}_{\text{had}}(q,k) = \frac{\langle 0\,|\bar{q}_2\gamma^{\nu}\gamma_5 q_1|\,P(k)\rangle\,\langle P(k)\,|\bar{q}_1\Gamma^{\mu}_{w}b|\,\bar{B}(q+k)\rangle}{m_P^2 - k^2} + \ldots$$

decay constant for P : $if_P k^{\nu}$

Form factors

$$\langle P(k)|\,\bar{q}_1\gamma^{\mu}b\,|B(p)\rangle = \left[(p+k)^{\mu} - \frac{m_B^2 - m_P^2}{q^2}q^{\mu}\right] f_+ + \frac{m_B^2 - m_P^2}{q^2}q^{\mu}\, f_0$$

$$\langle P(k)|\,\bar{q}_1\sigma^{\mu\nu}\,q_{\nu}b\,|B(p)\rangle = \frac{if_T}{m_B + m_P}\,\left[q^2\,(p+k)^{\mu} - (m_B^2 - m_P^2)\,q^{\mu}\right]\,.$$

Form factors $B \to P$

- OPE part: correlator expanded near the light-cone $k^2 \ll m_{q_1}^2$, $q^2 \ll (m_b + m_{q_1})^2$

$$\mathcal{F}^{\mu\nu}_{\rm OPE}(q,k) = \int d^4x\, e^{ik\cdot x} \int \frac{d^4 l}{(2\pi)^4} e^{-il\cdot x} \left[\gamma^\nu \gamma_5 \frac{\not{l} + m_{q_1}}{m_{q_1}^2 - l^2} \Gamma^\mu_w\right]_{\alpha\beta} \langle 0|\bar{q}_2^\alpha(x) b^\beta(0)|\bar{B}(q+k)\rangle$$

Non-perturbative inputs:

LCDAs: $\phi_\pm$, $g_\pm$

Parametrised in terms of λ_B in HQET

[Braun *et al.* '17]

Form factors $B \to P$

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Parametrised in terms of λ_B in HQET
[Braun *et al.* '17]

- Matching hadronic representation with OPE using

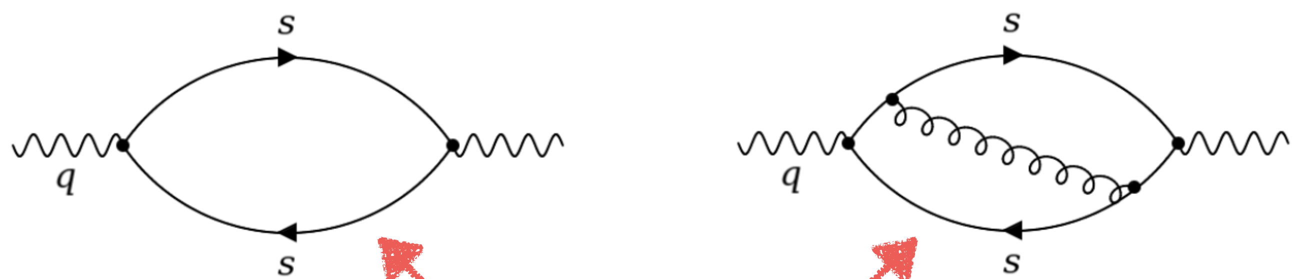
— Quark-hadron duality assumption

— Borel transform to suppress excited & continuum state contributions

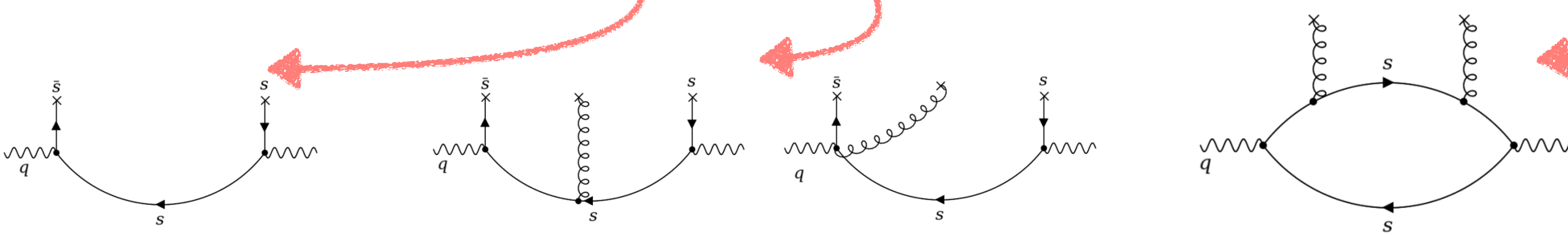
Sum rules for form factors
$$F(q^2) = f(q^2, M^2, s_0, \lambda_B)$$

Decay constant

- Effective threshold s_{th} determined from decay constant sum rule



$$f_{\eta_s}^2 m_{\eta_s}^2 e^{-m_{\eta_s}^2/M^2} = \int_{4m_s^2}^{s_{th}} ds\, e^{-s/M^2} \left[\rho_{\rm pert}^{\rm LO}(s) + \frac{\alpha_s}{\pi}\rho_{\rm pert}^{\rm NLO}(s) + \rho^{\langle GG\rangle}(s)\langle GG\rangle\right]$$
$$- 4m_s e^{-m_s^2/M^2}\left(1 - \frac{m_s^2}{4M^2} - \frac{m_s^2 m_0^2}{16M^4}\right)\langle\bar{s}s\rangle + \left(\frac{5}{24} - \frac{m_s^2}{4M^2}\right) e^{-4m_s^2/M^2}\langle GG\rangle$$



- Solve s_{th} from

$$m_{\eta_s}^2 = \frac{\frac{d}{d[-1/M^2]}\left[F e^{-m_{\eta_s}^2/M^2}\right]}{F e^{-m_{\eta_s}^2/M^2}}$$

Results

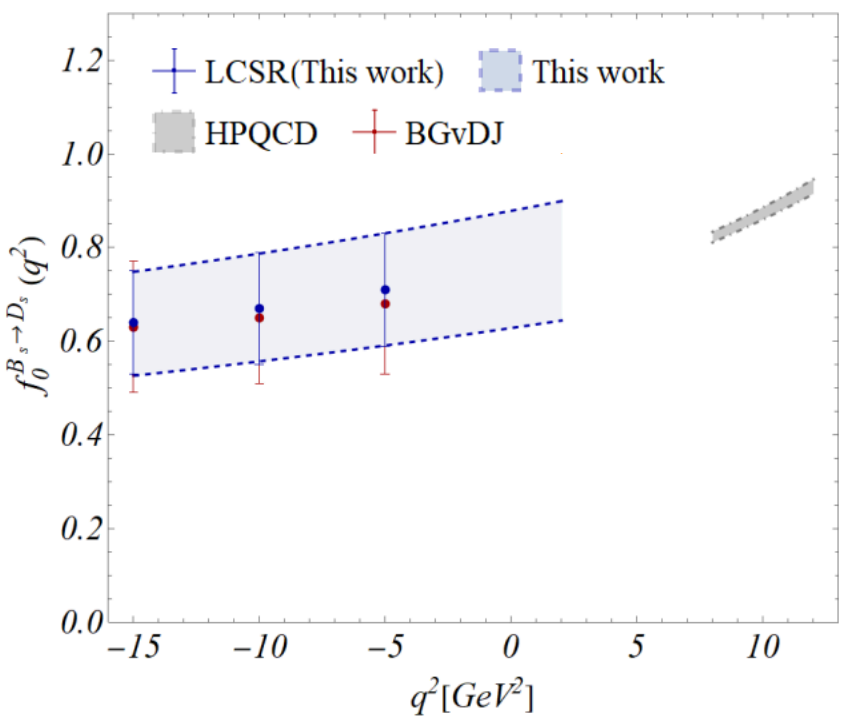
- Fit to Lattice data

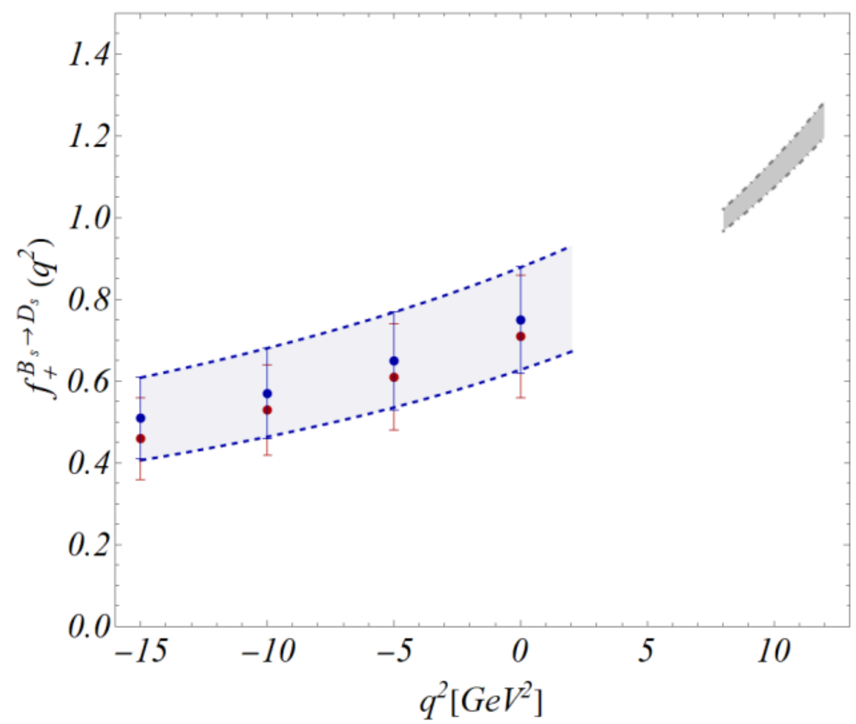
$$\chi^2 = \sum_{i,j} (O_i^{\text{Lattice}} - O_i^{\text{theo}}).\, Cov_{ij}^{-1}.\, (O_j^{\text{Lattice}} - O_j^{\text{theo}}) + \chi^2_{\text{nuis}}$$

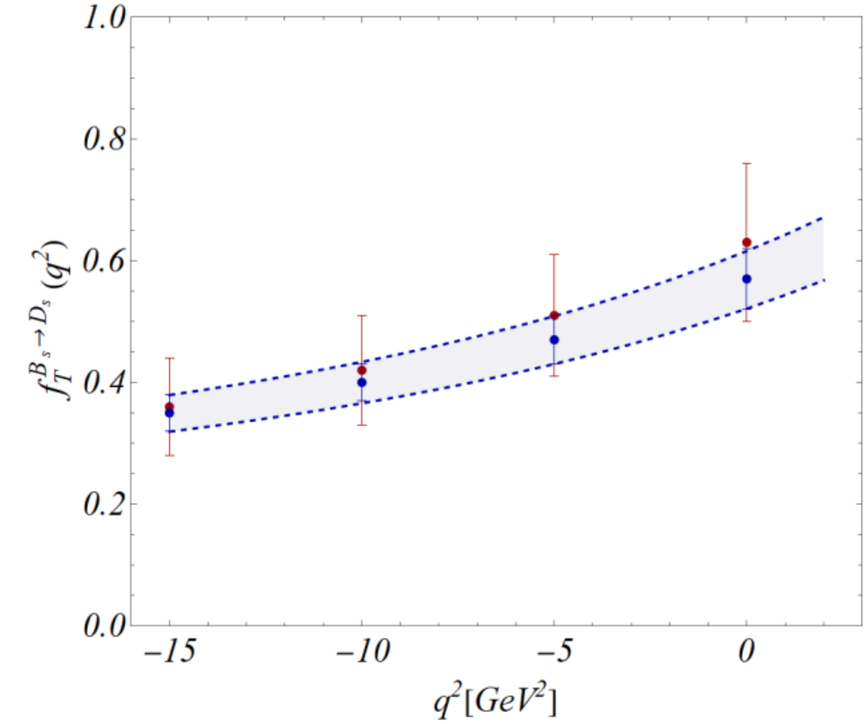
Form factor	Our work	LCSR
$B \to K$	$\lambda_B(1\,\text{GeV}) = 338^{+68}_{-9}\,\text{MeV}$ [RM, Nandi, Ray *PLB '23*]	$383 \pm 153\,\text{MeV}$
$B_s \to \eta_s$	$\lambda_{B_s}(1\,\text{GeV}) = 480^{+92}_{-83}\,\text{MeV}$ [RM, Patil, Ray *JHEP '24*]	$438 \pm 150\,\text{MeV}$ [Khodjamirian, RM, Mannel *JHEP'20*]

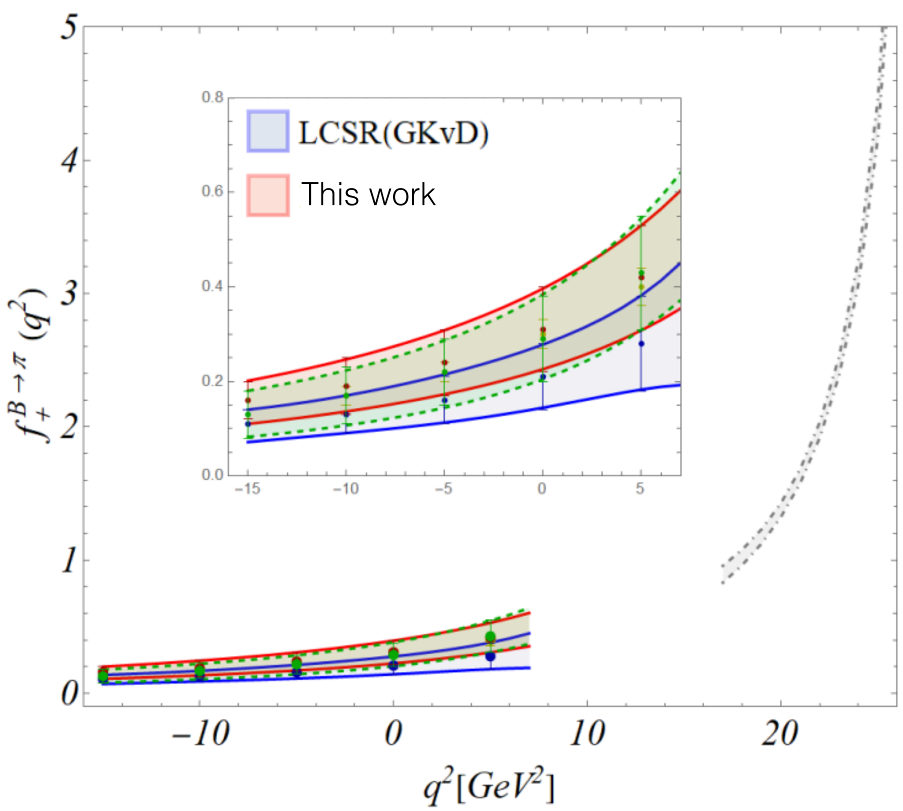
~2 times improvement in uncertainty

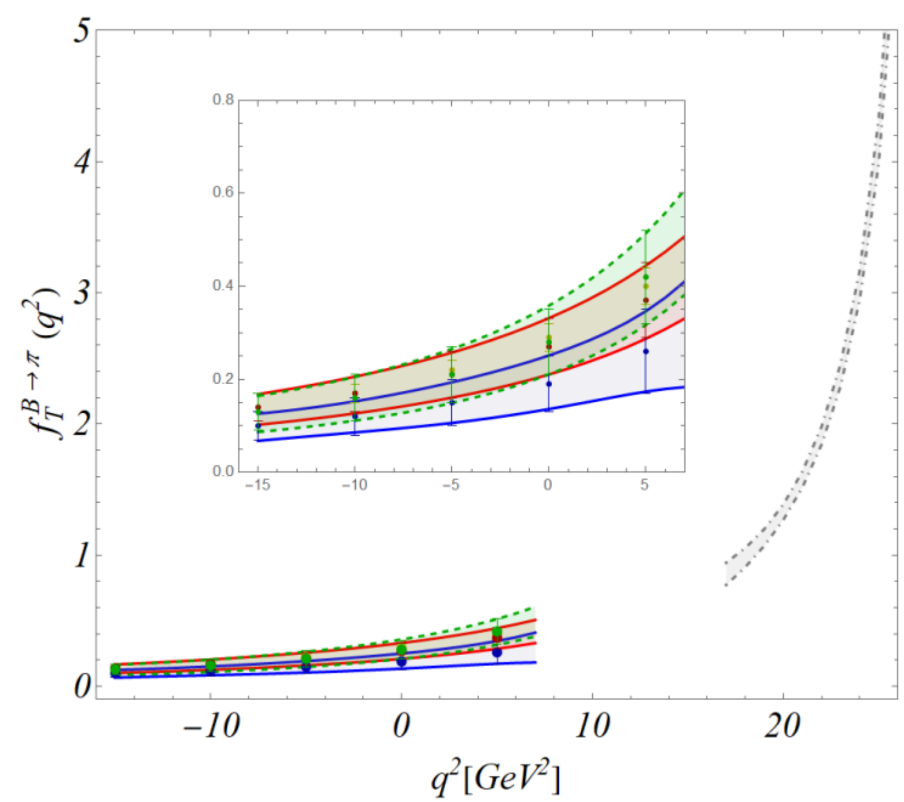
Results


LCSR(This work)
This work
HPQCD
BGvDJ
$f_0^{B_s \to D_s}(q^2)$
$q^2[GeV^2]$

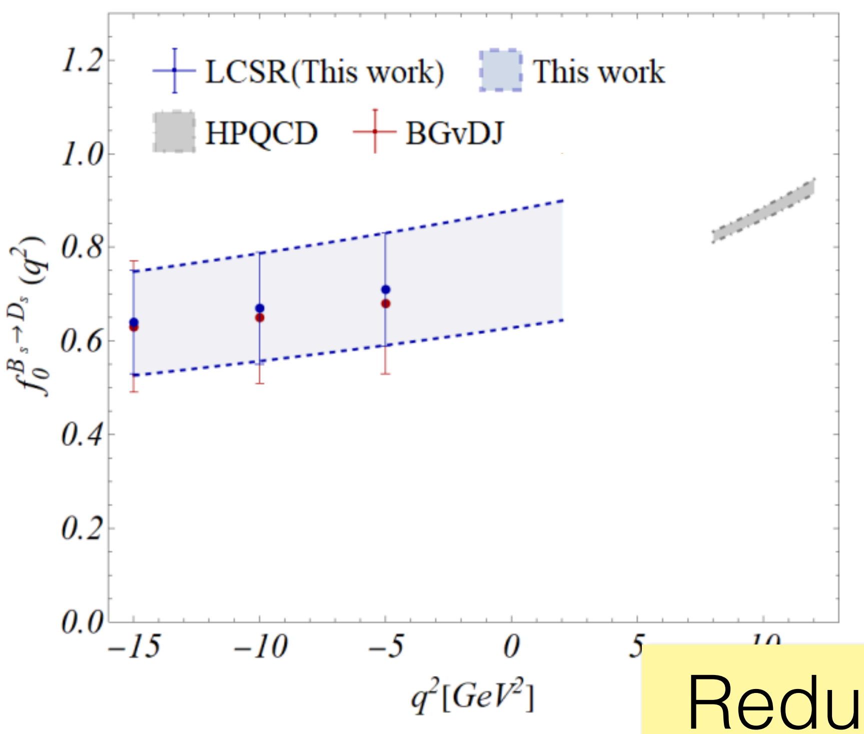

$f_+^{B_s \to D_s}(q^2)$
$q^2[GeV^2]$

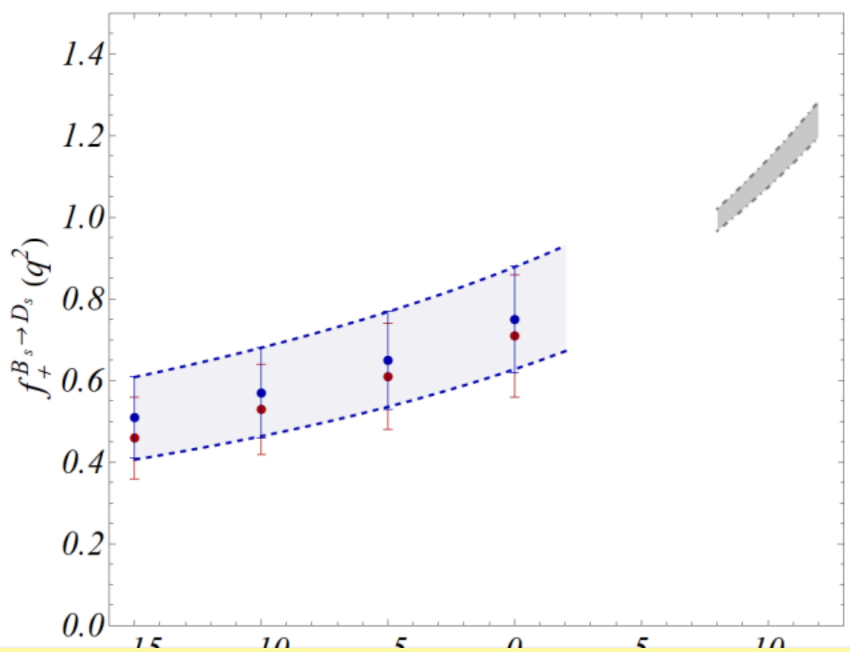

$f_T^{B_s \to D_s}(q^2)$
$q^2[GeV^2]$

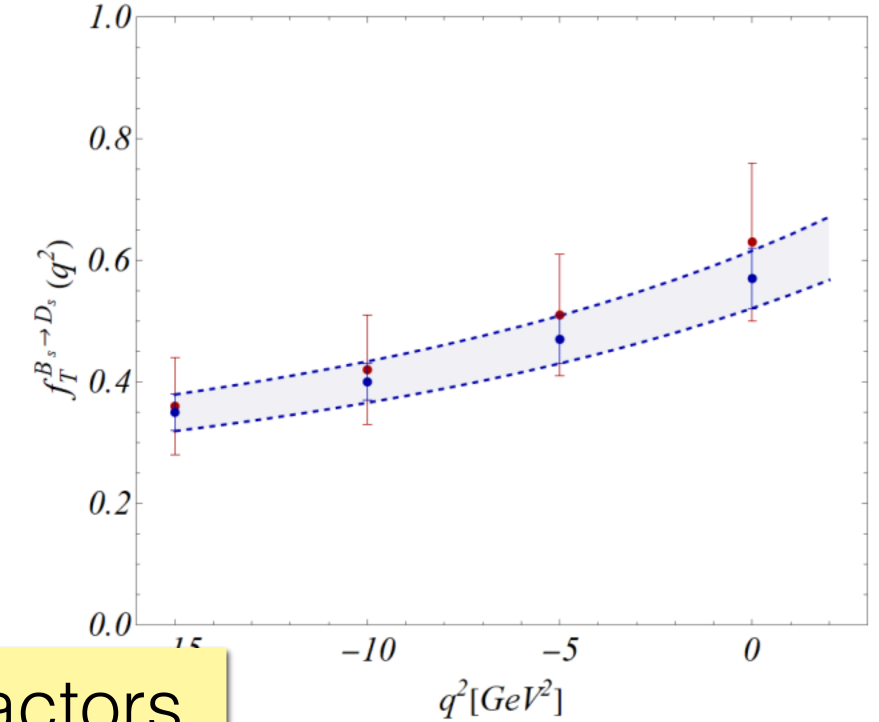

LCSR(GKvD)
This work
$f_+^{B \to \pi}(q^2)$
$q^2[GeV^2]$


$f_T^{B \to \pi}(q^2)$
$q^2[GeV^2]$

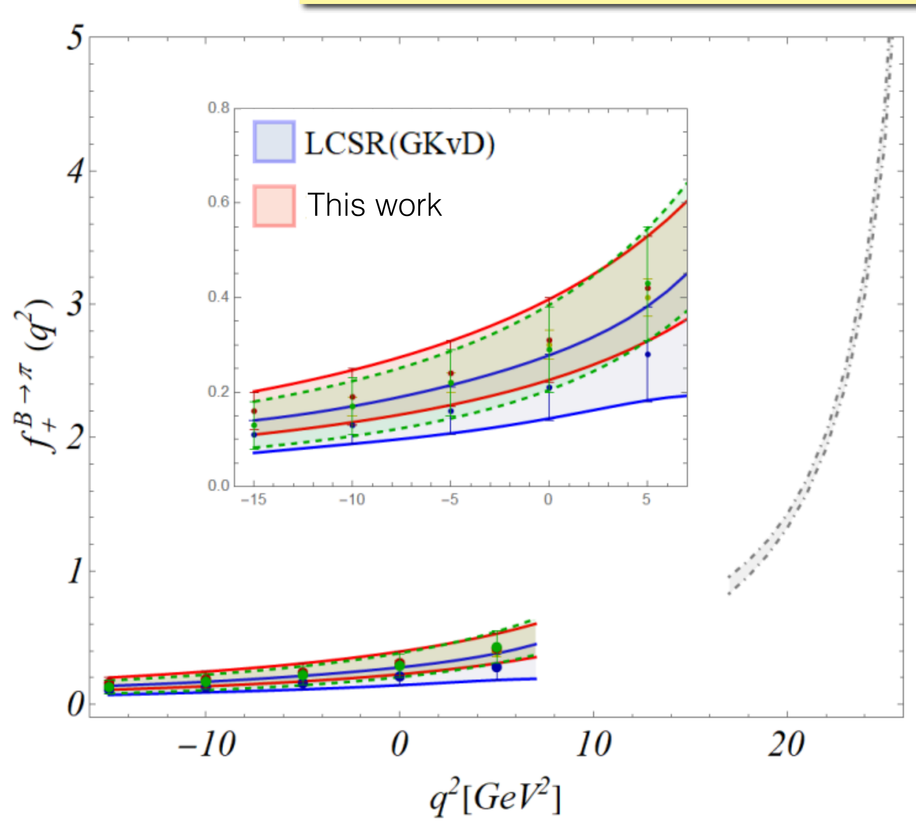
Results

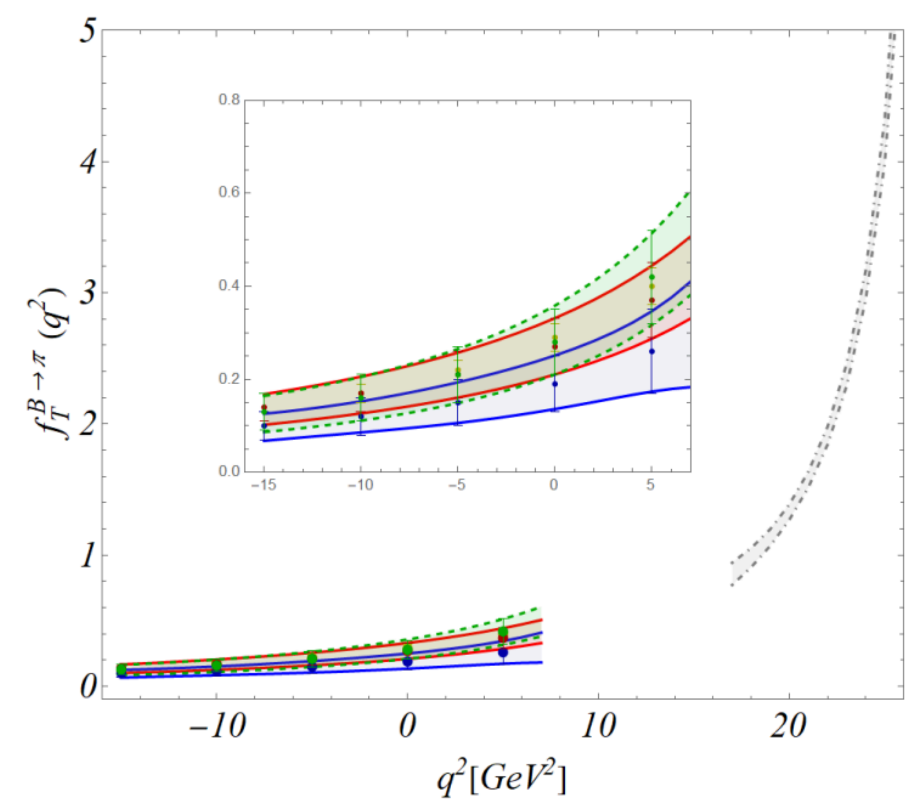






Reduced uncertainty in Form Factors





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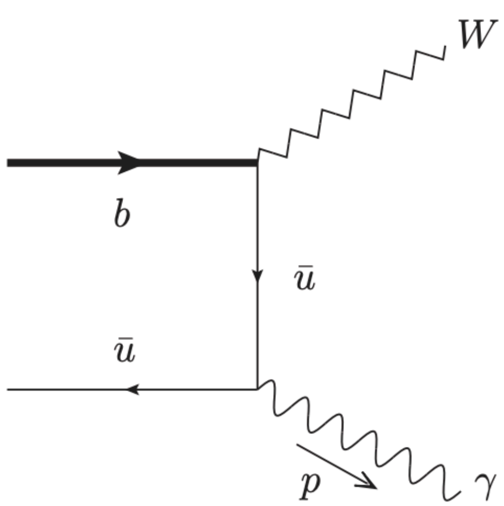
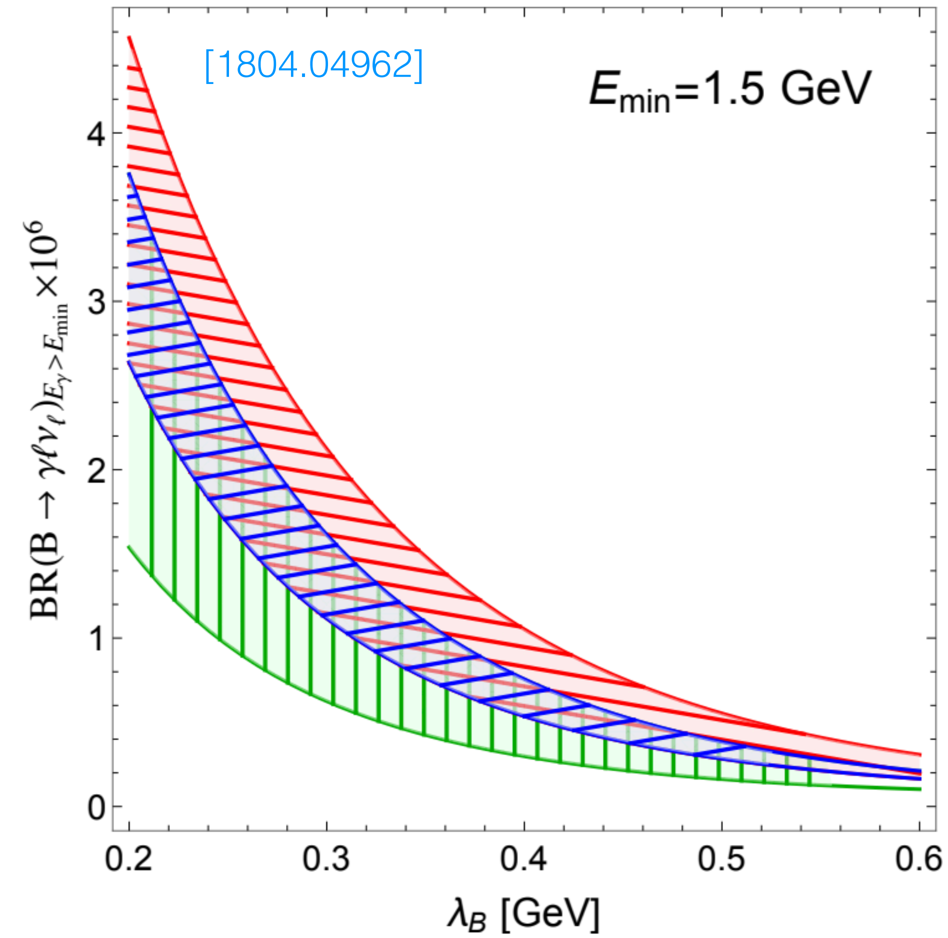


With $1/E_\gamma$ & $1/m_b$

power corrections



LO diagram to $B \to \ell \nu \gamma$

- Constrained prediction $\mathcal{B}(B \to \gamma \ell \nu) = \begin{cases} (0.89 \pm 0.10) \times 10^{-6} & (E_\gamma > 1.5\,\text{GeV}) \\ (0.34 \pm 0.02) \times 10^{-6} & (E_\gamma > 2.0\,\text{GeV}) \end{cases}$

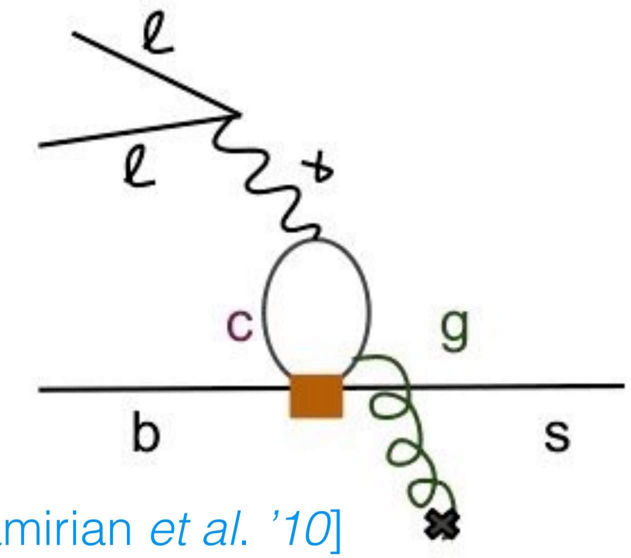
- Implications of updated $\lambda_{B_{(s)}}$ value can be seen in other modes

  — weak decay predictions using QCD factorization

Non-local FFs

$$A(B_s \to \phi \ell^+ \ell^-) = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \Bigg[\Bigg\{ C_9 \langle \phi | \bar{s} \gamma^\mu P_L b | B_s \rangle - \frac{2C_7}{q^2} \langle \phi | \bar{s} i \sigma^{\mu\nu} q_\nu (m_b P_R + m_s P_L) b | B_s \rangle$$

$$\frac{16\pi^2}{q^2} \mathcal{H}^\mu \Bigg\} \bar{\ell} \gamma_\mu \ell + C_{10} \langle \phi | \bar{s} \gamma^\mu P_L b | B_s \rangle \, \bar{\ell} \gamma_\mu \gamma_5 \ell \Bigg],$$



[Khodjamirian *et al.* '10]

$$\mathcal{H}^\mu |_{\text{non-fac}} = 2Q_c \left(C_2 - \frac{C_1}{2N_c} \right) \langle \phi(k) | \widetilde{O}^\mu(q) | B_s(k+q) \rangle$$

— calculated in LCSR

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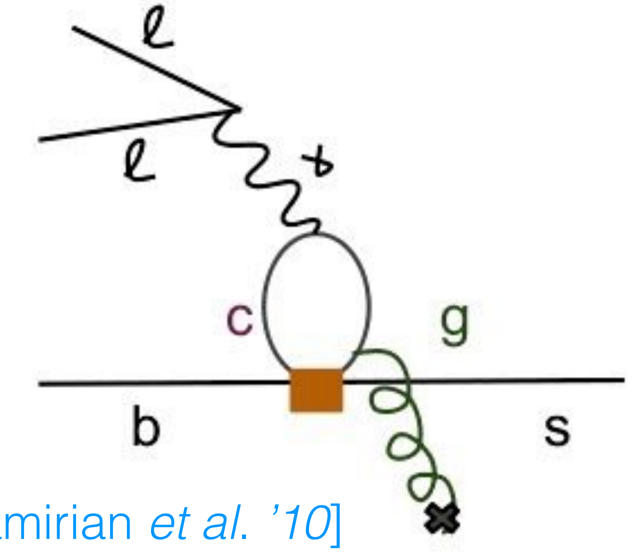

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— calculated in LCSR

Leading contribution from quark-gluon-antiquark non-local operator

→ 3-particle distribution amplitudes

→ Causes shift in C_9: $\Delta C_9^\lambda \propto \dfrac{\mathscr{H}_\lambda}{\mathscr{F}_\lambda}$

Crucial in order to distinguish from new physics

Non-local FFs (Preliminary)

- O(3) difference in $\mathscr{H}_\lambda$ in two previous calculations

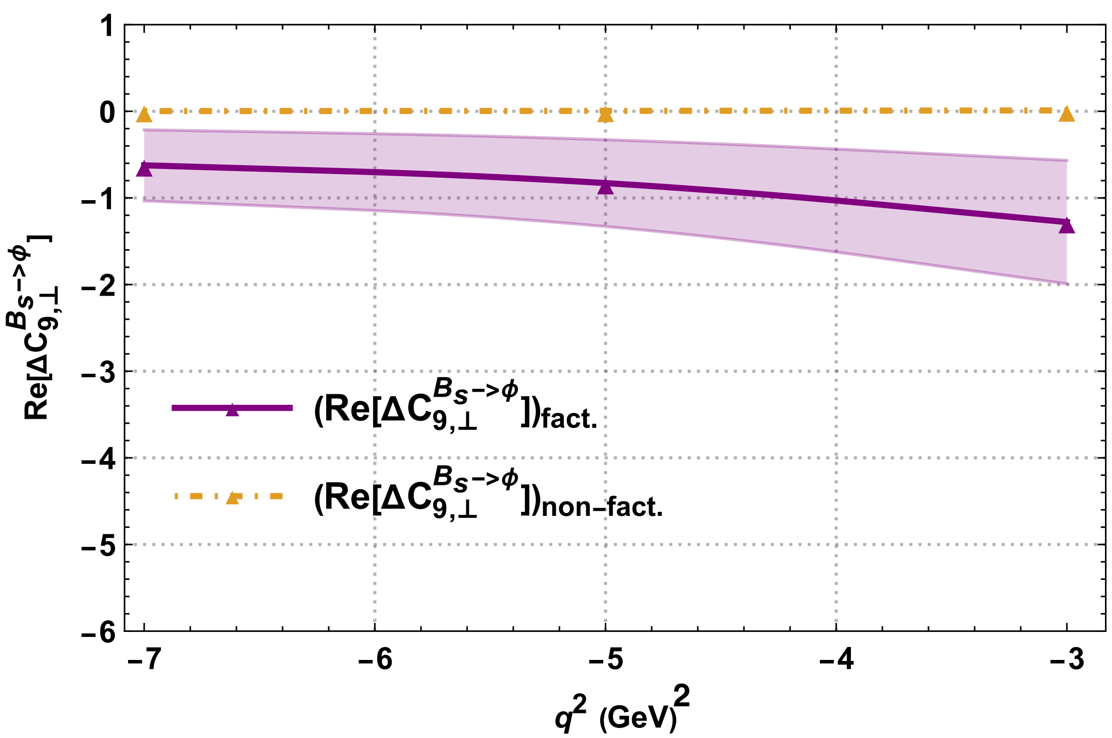
Transition	$\tilde{\mathcal{V}}(q^2 = 1\,\mathrm{GeV}^2)$	[Gubernari *et al.* '21]	[Khodjamirian *et al.* '10]
$B \to K$	$\tilde{\mathcal{A}}$	$(+4.9 \pm 2.8) \cdot 10^{-7}$	$(-1.3^{+1.0}_{-0.7}) \cdot 10^{-4}$
$B \to K^*$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 3.6) \cdot 10^{-7}\,\mathrm{GeV}$	$(-1.5^{+1.5}_{-2.5}) \cdot 10^{-4}\,\mathrm{GeV}$
	$\tilde{\mathcal{V}}_2$	$(+3.3 \pm 2.0) \cdot 10^{-7}\,\mathrm{GeV}$	$(+7.3^{+14}_{-7.9}) \cdot 10^{-5}\,\mathrm{GeV}$
	$\tilde{\mathcal{V}}_3$	$(+1.1 \pm 1.0) \cdot 10^{-6}\,\mathrm{GeV}$	$(+2.4^{+5.6}_{-2.7}) \cdot 10^{-4}\,\mathrm{GeV}$
$B_s \to \phi$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 5.6) \cdot 10^{-7}\,\mathrm{GeV}$	—
	$\tilde{\mathcal{V}}_2$	$(+4.3 \pm 3.1) \cdot 10^{-7}\,\mathrm{GeV}$	—
	$\tilde{\mathcal{V}}_3$	$(+1.7 \pm 2.0) \cdot 10^{-6}\,\mathrm{GeV}$	—

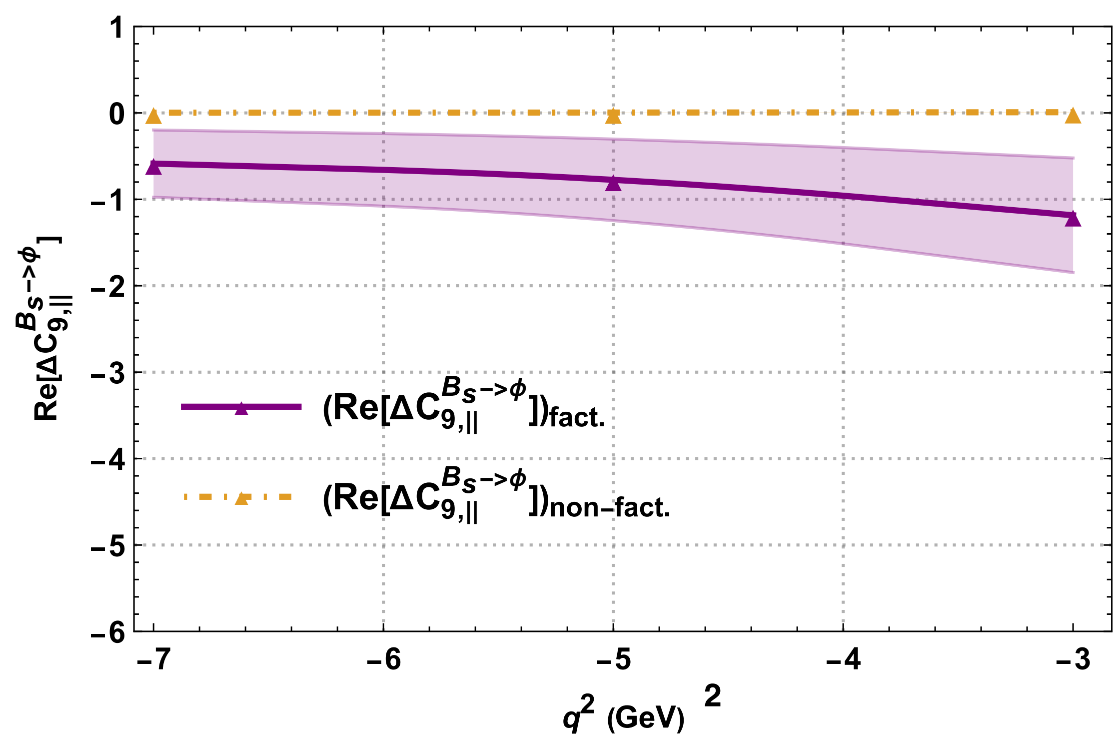
- Improve with complete set of LCDAs [Ongoing work with: Alam Khan, Patil, Ray]

Revisiting the cancellation between different twist contributions

→ 10 times enhancement in Non-local FF estimates

Non-local FFs (Preliminary)



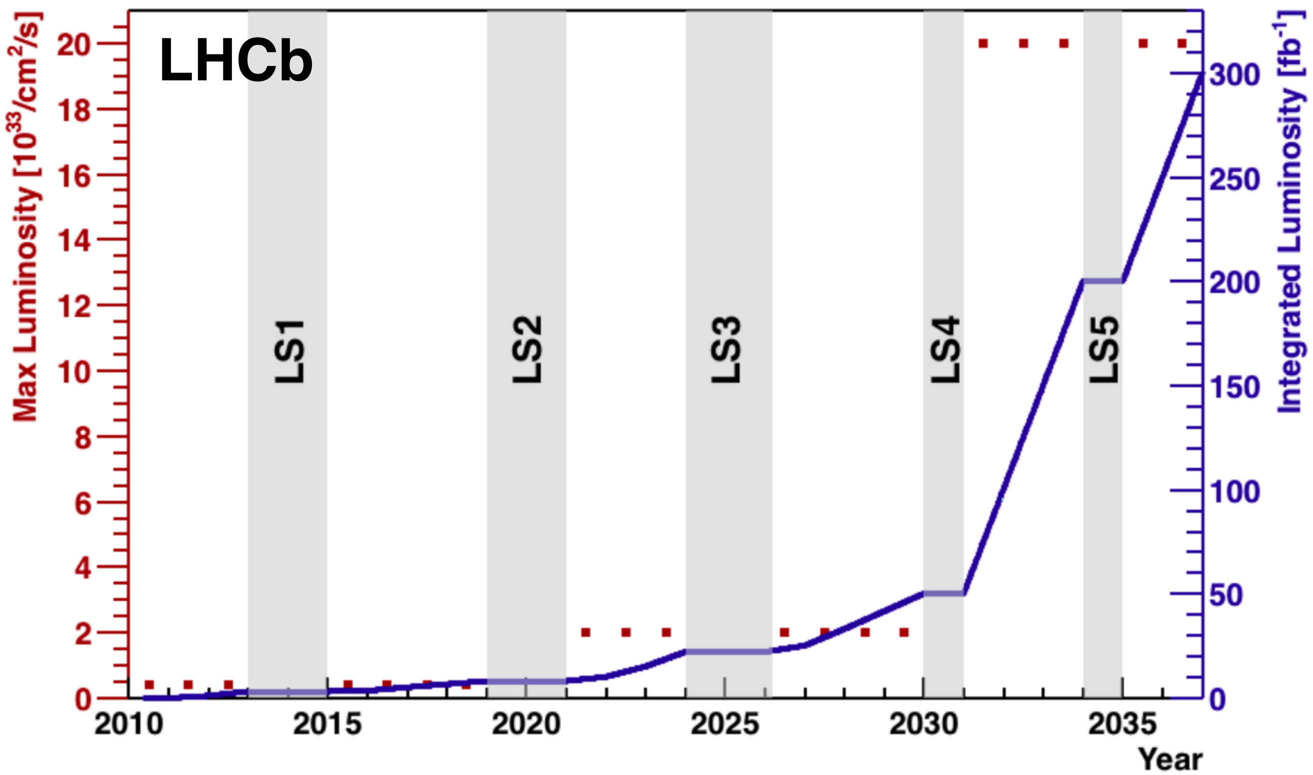


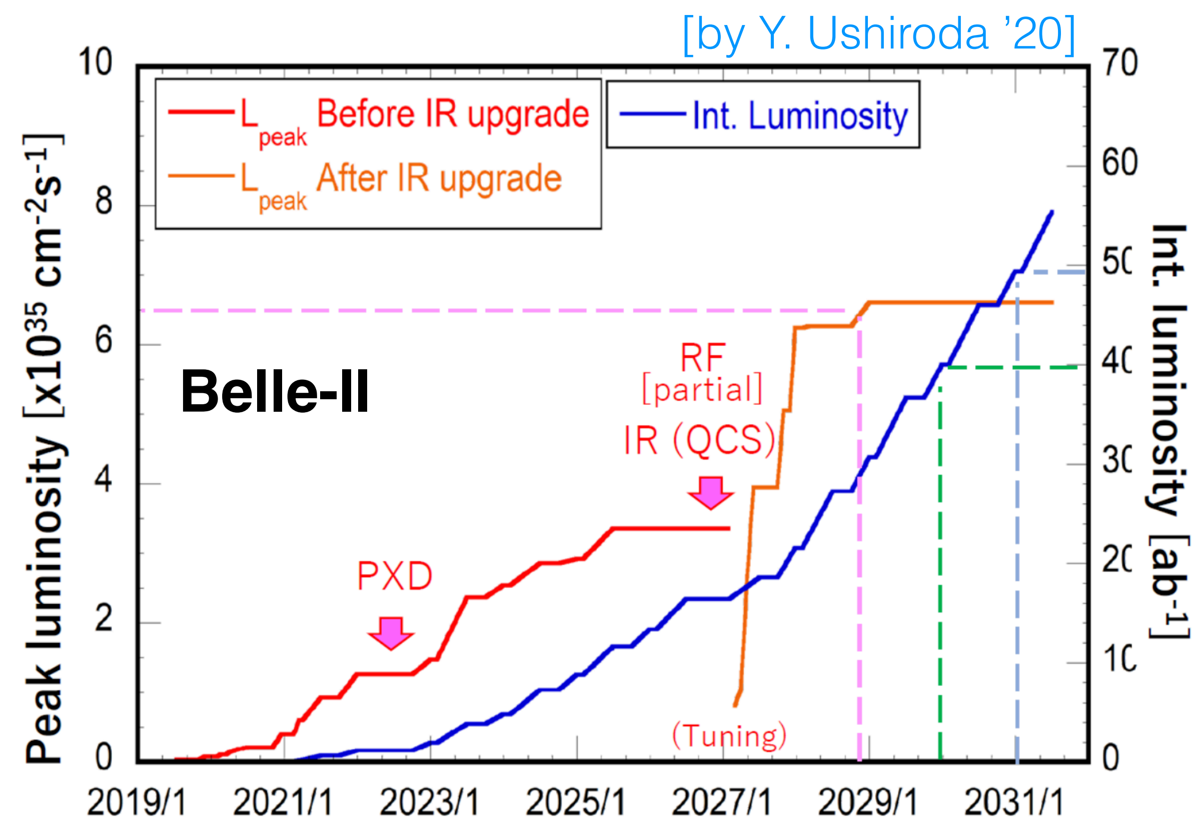
$\Delta C_9^{\lambda} \sim -1$ → Also hinted from global fits to angular observables

- Need to investigate the effect in physical region via dispersion relation

— Model ansatz & use of data on Charmonium states

Outlook





Both LHCb & Belle-II will be running for at least next 10 years:

— validation of discrepancies observed so far…

— plenty of *b*- & *c*-hadron decay channels are to be explored

Thank you for your attention

Back ups

LCDAs

- 2-particle: B to vacuum is HQET

$$\langle 0|\,\bar{s}^{\alpha}(x)h_v^{\beta}(0)\,|\bar{B}_s(v)\rangle = -\frac{if_{B_s}m_{B_s}}{4}\int_0^{\infty}d\omega\Big\{(1+\not{v})\Big[\phi_+(\omega)-g_+(\omega)\partial_\lambda\partial^\lambda + \frac{1}{2}\left(\bar{\phi}(\omega)-\bar{g}(\omega)\partial_\lambda\partial^\lambda\right)\gamma^\rho\partial_\rho\Big]\gamma_5\Big\}^{\beta\alpha} e^{-ir\cdot x}\Big|_{r=\omega v}$$

where

$$\bar{\phi}(\omega) \equiv \int_0^{\omega} d\eta\,(\phi_+(\eta)-\phi_-(\eta))\,,\ \text{and}\quad \bar{g}(\omega)\equiv\int_0^{\omega}d\eta\,(g_+(\eta))-g_-(\eta)))\ .$$

$$\phi_+(\omega) = \frac{\omega}{\omega_0^2}e^{-\omega/\omega_0},\quad \phi_-(\omega) = \frac{1}{\omega_0}e^{-\omega/\omega_0} - \frac{\lambda_E^2-\lambda_H^2}{18\omega_0^5}\left(2\omega_0^2-4\omega\omega_0+\omega^2\right)e^{-\omega/\omega_0}\,,$$

$$g_+(\omega) = -\frac{\lambda_E^2}{6\omega_0^2}\Big\{(\omega-2\omega_0)\mathrm{Ei}\left(-\frac{\omega}{\omega_0}\right)+(\omega+2\omega_0)e^{-\omega/\omega_0}\left(\ln\frac{\omega}{\omega_0}+\gamma_E\right)-2\omega e^{-\omega/\omega_0}\Big\} + \frac{e^{-\omega/\omega_0}}{2\omega_0}\omega^2\Big\{1-\frac{1}{36\omega_0^2}(\lambda_E^2-\lambda_H^2)\Big\},$$

$$g_-(\omega) = \omega\Big\{\frac{3}{4}-\frac{\lambda_E^2-\lambda_H^2}{12\omega_0^2}\left[1-\frac{\omega}{\omega_0}+\frac{1}{3}\left(\frac{\omega}{\omega_0}\right)^2\right]\Big\}e^{-\omega/\omega_0}$$

$\omega_0 = \lambda_{B_s}$.

LCDAs

- 3-particle: *B* to vacuum is HQET

$$\langle 0|\bar{s}^a(x)\delta\left[\omega_2 - in_+\cdot D\right]G_{\sigma\tau}h_v^b(0)\left|\bar{B}(v)\right\rangle\Big|_{x\simeq n_+,x^2\simeq 0}$$

$$= \frac{f_{B_s}M_{B_s}}{4}\int_0^\infty d\omega_1 e^{-i\omega_1 v\cdot x}\Big\{(1+\not{v})\Big[(v_\sigma\gamma_\tau - v_\tau\gamma_\sigma)[\psi_A-\psi_V] - i\sigma_{\sigma\tau}\psi_V$$
$$+ (\partial_\sigma v_\tau - \partial_\tau v_\sigma)\overline{X}_A - (\partial_\sigma\gamma_\tau - \partial_\tau\gamma_\sigma)[\overline{W}+\overline{Y}_A] + i\epsilon_{\sigma\tau\alpha\beta}\partial^\alpha v^\beta\gamma_5\overline{\tilde{X}}_A$$
$$- i\epsilon_{\sigma\tau\alpha\beta}\partial^\alpha\gamma^\beta\gamma_5\overline{\tilde{Y}}_A - (\partial_\sigma v_\tau - \partial_\tau v_\sigma)\not{\partial}\overline{\overline{W}} + (\partial_\sigma\gamma_\tau - \partial_\tau\gamma_\sigma)\not{\partial}\overline{\overline{Z}}\Big]\gamma_5\Big\}^{ba}$$

$$\overline{X}(\omega_1,\omega_2) \equiv \int_0^{\omega_1} d\eta_1 X(\eta_1,\omega_2)$$
$$\overline{\overline{X}}(\omega_1,\omega_2) \equiv \int_0^{\omega_1} d\eta_1 \overline{X}(\eta_1,\omega_2)$$

Combinations of LCDAs