Dark Matter Phenomenology at Higher Orders

Ambresh Shivaji

IIISER Mohali, Punjab, INDIA

Frontiers in Particle Physics 2024

CHEP, IISc Bangalore, August 09-11, 2024

Simplified DM models

The study of simplified models with a DM candidate X and a mediator Y connecting DM and the SM is a phenomenologically driven approach.

They fill the gap between EFT approach and complete DM models like MSSM.

Several models with involving *colored* mediators have been studied: 1308.0592, 1308.2679, 1403.4634, 2307.10367

Gluphilic DM model: 1506.01408

$$\mathcal{L}^{\mathrm{DM}} = \partial_\mu \chi^* \partial^\mu \chi - m_\chi^2 |\chi|^2 + (D_\mu \phi)^\dagger D^\mu \phi - m_\phi^2 |\phi|^2 + \lambda_d \chi^* \chi \phi^\dagger \phi \tag{1}$$

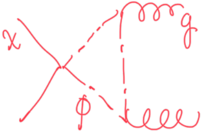
For phenomenological viability, the colored mediator ϕ can interact with quarks.

EFT description in the large m_ϕ limit: 1008.1783

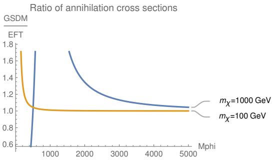
$$\mathcal{L}^{\mathrm{EFT}} = \frac{\alpha_s \lambda_d}{96\pi M_\phi^2} |\chi|^2 G^{\mu\nu a} G^a_{\mu\nu} \tag{2}$$

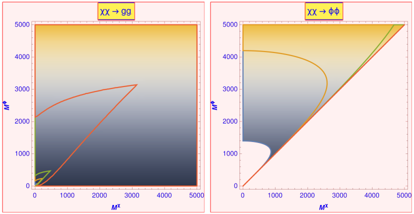
Leading order phenomenology

Annihilation cross section and relic density



$$\sigma v_\chi(gg) = \frac{\lambda_d^2 T_r^2 \alpha_s^2}{64\pi^3 m_\chi^2} |(1 + 2m_\phi^2 C_0)|; \; \sigma v_\chi(\phi\phi) = \frac{\lambda_d^2 T_r}{64\pi m_\chi^2} \sqrt{1 - \frac{m_\phi^2}{m_\chi^2}} \tag{3}$$



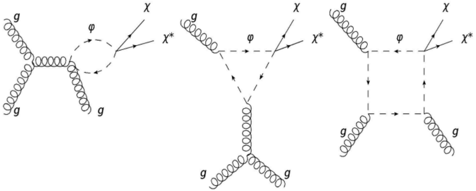


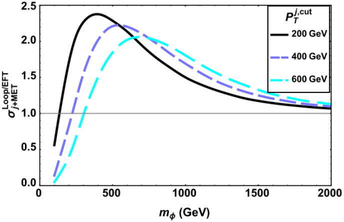
1506.01408

Leading order phenomenology

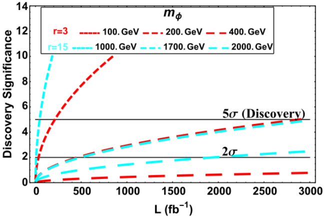
Monojet signatures: studied both in EFT and simplified model

$$gg \to \chi\chi g, \quad q\bar{q} \to \chi\chi g, \quad qg \to \chi\chi q \tag{4}$$





(b) $\sqrt{s}$ =13 TeV



(b) $\sqrt{s}$ = 100 TeV

Large scale uncertainty

1605.04756

Phenomenology at Higher orders

Since the DM couples to colored SM particles via ϕ, the QCD corrections are relevant.

We need both virtual and real contributions. Virtual contributions appear at two-loop.

The calculation of two-loop amplitudes can be organized in terms of form-factors.

Annihilation cross section

$$\mathcal{M}(\chi\chi \to gg) = F\,(p_1.p_2 g^{\mu\nu} - p_1^\nu p_2^\mu)\,\epsilon_\mu(p_1)\epsilon_\mu(p_2)$$

Monojet signatures

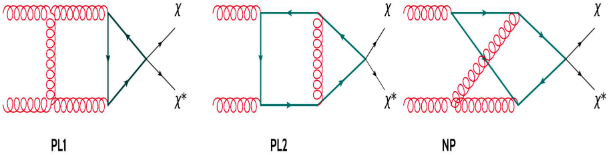
$$\mathcal{M}(gg \to \chi\chi g) = (F_1\, p_1^\nu g^{\rho\mu} + F_2\, p_2^\rho g^{\mu\nu} + F_3\, p_3^\mu g^{\nu\rho} + F_4\, p_1^\nu p_2^\rho p_3^\mu)\,\epsilon_\mu(p_1)\epsilon_\nu(p_2)\epsilon_\rho(p_3)$$

The form factors have expansion in α_s. Projector technique to relate form factors and Feynman diagrams.

Annihilation cross section: Two-loop form factor for gg channel

This channel is very much like the Higgs boson production in SM with single dimensionless parameter, three integral families, and 18 master integrals.

$$\tau = \frac{4m_\phi^2}{s}, x = -\frac{\sqrt{1-\tau}-1}{\sqrt{1-\tau}+1} + i\epsilon \tag{5}$$



PL1	PL2	NP
$\{k_1, 0\}$	$\{k_1, m_\phi\}$	$\{k_1, m_\phi\}$
$\{k_1+p_1, 0\}$	$\{k_1+p_2, m_\phi\}$	$\{k_1-k_2-p_1, 0\}$
$\{k_1+p_1+p_2, 0\}$	$\{k_1+p_1+p_2, m_\phi\}$	$\{k_1+p_1+p_2, m_\phi\}$
$\{k_2+p_1+p_2, m_\phi\}$	$\{k_2+p_1+p_2, m_\phi\}$	$\{k_2+p_1+p_2, m_\phi\}$
$\{k_2+p_1, m_\phi\}$	$\{k_2+p_2, m_\phi\}$	$\{k_2+p_1, m_\phi\}$
$\{k_2, m_\phi\}$	$\{k_2, m_\phi\}$	$\{k_1+p_1, m_\phi\}$
$\{k_1-k_2, m_\phi\}$	$\{k_1-k_2, 0\}$	$\{k_1-k_2, 0\}$

Annihilation cross section: Two-loop form factors for *gg* channel

...after UV renormalization and infrared subtraction,

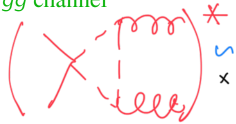
F_{gg}^{2L} [redacted]

$$
\begin{aligned}
&= \frac{1}{2880N(x-1)^4(x+1)}\Big(3(-1+x)\left(3375-6810x+15(454-225x)x^3+192\pi^4(x+x^3)\right)\\
&+20\pi^2(3+(-2+x)x(-11+28x+6x^2))+138240(1-x)\left(3(x+x^3)-2N^2(x+2x^3)\right)\mathrm{HPL}(\{-4\},x)\\
&-23040N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{-2\},x)^2\\
&+11520(1-x)\left(N^2x(23+31x^2)-27(x+x^3)\right)\mathrm{HPL}(\{4\},x)\\
&-184320N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{-3,1\},x)\\
&-92160N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{2,-2\},x)\\
&-184320N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{3,-1\},x)\\
&+46080N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{-2,-1\},x)\log(x)+92160N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{-2,1\},x)\log(x)\\
&+92160N^2(-1+x)^2x(1+x)\mathrm{HPL}(\{2,-1\},x)\log(x)\\
&-1440x\mathrm{HPL}(\{3\},x)\left(-((1+x)(-32+88N^2(-1+x)^2+59x-32x^2)\right)\\
&-5760(1-x)x\mathrm{HPL}(\{-3\},x)\left(-((-27+20N^2)(-1+x^2)\right)\\
&-11520N^2(-1+x)^2(1+x)\log(1-x)^2((-1+x)^2-x\log(x)^2)+\\
&240(1-x)\mathrm{HPL}(\{-2\},x)\left(-((-1+x)(-9(-1-9x+9x^2+x^3)+8N^2(-6+(-15+2\pi^2)x\right.\\
&-12x(-1+x^2)(-27+20N^2+48N^2\log(1-x))\log(x)+48x(N^2(-5+x^2)+2(1+x^2))\log(x)^2)\\
&-60\mathrm{HPL}(\{2\},x)\left(3((-1+x)(1+x)^2(3+x(-16+3x))\right.\\
&+32N^2(1-x)x(4\pi^2(-1+x^2)-3\log(x)(20(-1+x^2)+(3+11x^2)\log(x))))\\
&-16N^2(1-x)\left(630(-1+x)^3(1+x)-2\pi^4x(-19+55x^2)-30\pi^2(-1+x)^2(2+x(7+2x))\right.\\
&-15\log(x)\left(-2(24+x(57+\pi^2(6-14x^2)+3x(-18+x(-13+4x))))\log(x)\right.\\
&+x(21-19x^2)\log(x)^3+12(-1+x)(9+x(4-\pi^2(1+x)+x(-16+3x))\\
&-360x(7(-1+x^2)+2(-7+15x^2)\log(x))\zeta(3))\\
&+180(1-x)(1+x)\log(1-x)\left(\log(x)((1+x)(3+x(-16+3x))-40(-1+x)x\log(x)\right)\\
&+32N^2(1-x)\left(6(-1+x)^2+\log(x)\left(4+2(-2+\pi^2)x-x\log(x)(11+4\log(x))\right)+36x\zeta(3)\right)+\\
&30\left((36+x(930+64\pi^2(-1+x)(1+x^2)+3x(-435+x(-139+3x(71+x)))))\right)\log(x)^2\\
&+2x(1+x)(86+x(-215+134x))\log(x)^3+8(-1+x)x(1+x^2)\log(x)^4\\
&-48x(1+x)(49+x(-103+49x))\zeta(3)\\
&+2\log(x)\left(3+36(-1+x)^3(1+x(10+x))\log(1+x)\right.\\
&+x(336-2\pi^2(1+x)(32+x(-69+32x))-3x(337+x(-337+x(112+x)))\\
&+768(-1+x)(1+x^2)\zeta(3))))\Big)
\end{aligned}
$$

Result in terms of 11 unique harmonic polylogs of weight upto 4 in single variable.

Annihilation cross section: one-loop form factor for gg channel

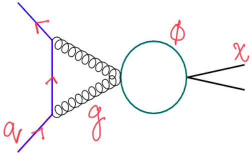
We also need one-loop form factor upto $\mathcal{O}(\epsilon^2)$

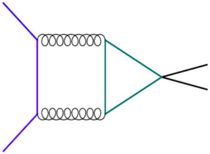


$$
\begin{aligned}
F^{1L}_{gg} &= -1 + \tfrac{2xH(0,0,x)}{(x-1)^2} \\
&- \tfrac{\epsilon}{3(x-1)^2}\bigg(3\bigg(-2xH(0,0,x)\left(\gamma_e + \log\left(m_\phi^2\right) - 3\right) + 4xH(0,-1,0,x) \\
&- 2xH(0,0,0,x) + \gamma_e + x^2\log\left(m_\phi^2\right) - 2x\log\left(m_\phi^2\right) + \log\left(m_\phi^2\right) + \gamma_e x^2 \\
&- 3x^2 - 2\gamma_e x + 6x\zeta(3) + 6x - 3\bigg) + \left(3x^2 + \pi^2 x - 3\right)H(0,x)\bigg) \\
&+ \epsilon^2\bigg(\tfrac{((x+1)}{x-1}H(0,x) - x + 1)\left(\gamma_e + \log\left(m_\phi^2\right) - 2\right) \\
&- \tfrac{x}{36(x-1)^2}\bigg(72\gamma_e\log\left(m_\phi^2\right)H(0,0,x) - 12H(0,x)\left(\pi^2\gamma_e + \pi^2\log\left(m_\phi^2\right) - 12\zeta(3) - \pi^2\right) \\
&+ 36\log^2\left(m_\phi^2\right)H(0,0,x) - 216\log\left(m_\phi^2\right)H(0,0,x) - 144\log\left(m_\phi^2\right)H(0,-1,0,x) \\
&+ 72\log\left(m_\phi^2\right)H(0,0,0,x) + 36\gamma_e^2 H(0,0,x) - 216\gamma_e H(0,0,x) - 144\gamma_e H(0,-1,0,x) \\
&+ 72\gamma_e H(0,0,0,x) - 24\pi^2 H(0,-1,x) + 18\pi^2 H(0,0,x) + 360H(0,0,x) + 144H(0,-1,0,x) \\
&- 72H(0,0,0,x) - 288H(0,-1,-1,0,x) + 144H(0,-1,0,0,x) + 144H(0,0,-1,0,x) - 72H(0,0,0,0,x) \\
&- 216\zeta(3)\log\left(m_\phi^2\right) - 216\gamma_e\zeta(3) + 216\zeta(3) + \pi^4\bigg) \\
&+ \tfrac{(x+1)}{1-x}H(0,x) - 2(x+1)H(-1,0,x) + (x+1)H(0,0,x) - 2x - \tfrac{1}{6}\pi^2(x+1) + 2 \\
&+ \tfrac{1}{12}\left(6\gamma_e^2 + \pi^2\right) + \gamma_e\left(\log\left(m_\phi^2\right) - 2\right) + \tfrac{1}{2}\log^2\left(m_\phi^2\right) - 2\log\left(m_\phi^2\right) + 2\bigg) \\
&+ O\left(\epsilon^3\right)
\end{aligned}
$$

Annihilation cross section: Two-loop form factors for qq channel

DM annihilation to qq channel appears first time at two-loop. Relevant for massive quarks only.





PL1
$\{k_1, m_b\}$
$\{k_2, m_\phi\}$
$\{k_1 + p_1, 0\}$
$\{k_1 - k_2 + p_1, m_\phi\}$
$\{k_1 - p_2, 0\}$
$\{k_2 - p_1 - p_2, m_\phi\}$
$\{k_2 + p_1, 0\}$

Parametrization of integrals in terms of two dimensionless parameters. Only one integral family is needed. Total 20 master integrals.

$$\frac{s}{m_\phi^2} = -\frac{(1-w^2)^2}{w^2}; \quad \frac{s}{m_b^2} = -\frac{(1-z^2)^2}{z^2}; \tag{6}$$

Being LO, the contribution at two-loop is finite, *a strong check on the calculation*

Annihilation cross section: Two-loop form factors for qq channel

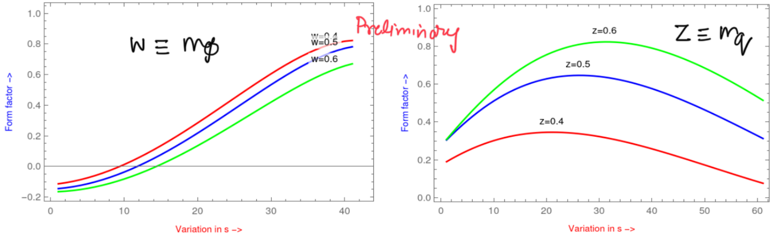
$$\mathcal{F}^{2L}_{qq} = \Big(\frac{wzG[-1,w]}{3(-1+w^2)(-1+z^2)} + \frac{wzG[1,w]}{3(-1+w^2)(-1+z^2)} + \frac{\pi^2 wz(z^2+w^2(-2+(2+w^2)z^2-2z^4))G[-1,0,z]}{18(-1+w^2)^3(1+z^2)^3}$$

$$- \frac{4w(1+w^2)z^3G[0,-1,w]}{3(-1+w^2)^2(-1+z^2)(1+z^2)^2} - \frac{2w^3z(1-6z^2+z^4+w^2(3-2z^2+3z^4))G[0,0,w]}{3(-1+w^2)^3(-1+z^2)(1+z^2)^2}$$

$$- \frac{4w(1+w^2)z^3G[0,1,w]}{3(-1+w^2)^2(-1+z^2)(1+z^2)^2} + G[z^{-1},w]\Bigg(-\frac{1}{18}\frac{\pi^2 w(w-z)z(w+z)(-1+wz)(1+wz)}{(-1+w^2)^3(-1+z^2)(1+z^2)^2}$$

$$- \frac{2w(w-z)z(w+z)(-1+wz)(1+wz)G[0,-1,z]}{3(-1+w^2)^3(-1+z^2)(1+z^2)^2} + \frac{2w(w-z)z(w+z)(-1+wz)(1+wz)G[0,0,z]}{3(-1+w^2)^3(-1+z^2)(1+z^2)^2}$$

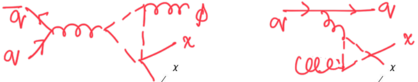
$$- \frac{2w(w-z)z(w+z)(-1+wz)(1+wz)G[0,1,z]}{3(-1+w^2)^3(-1+z^2)(1+z^2)^2}\Bigg) + \ldots$$

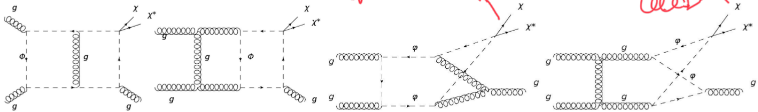


Result in terms of 328 unique generalized polylogs of weight upto 4 in two variables

Monojet signature: Two-loop form factors

4 form factors for $gg \to \chi\chi g$ channel, 2 for qq and qg channels. gg channel is the most challenging.





PL1	PL2	NP
$\{k_1, 0\}$	$\{k_1, m_\phi\}$	$\{k_1, m_\phi\}$
$\{k_1 - p_1, 0\}$	$\{k_1 - p_1, m_\phi\}$	$\{k_1 + p_1, m_\phi\}$
$\{k_1 - p_1 - p_2, 0\}$	$\{k_1 - p_1 - p_2, m_\phi\}$	$\{k_1 - p_2 - p_3, m_\phi\}$
$\{k_1 - p_1 - p_2 - p_3, 0\}$	$\{k_1 - p_1 - p_2 - p_3, m_\phi\}$	$\{k_2, m_\phi\}$
$\{k_2, m_\phi\}$	$\{k_2, m_\phi\}$	$\{k_2 + p_1, m_\phi\}$
$\{k_2 - p_1, m_\phi\}$	$\{k_2 - p_1, m_\phi\}$	$\{k_2 - p_3, m_\phi\}$
$\{k_2 - p_1 - p_2, m_\phi\}$	$\{k_2 - p_1 - p_2, m_\phi\}$	$\{k_1 - k_2, 0\}$
$\{k_2 - p_1 - p_2 - p_3, m_\phi\}$	$\{k_2 - p_1 - p_2 - p_3, m_\phi\}$	$\{k_1 - k_2 - p_2, 0\}$
$\{k_1 - k_2, m_\phi\}$	$\{k_1 - k_2, 0\}$	$\{k_1 - k_2 - p_2 - p_3, 0\}$

A multi-scale problem to solve (s, t, m_χ, m_ϕ). Total 120 master integrals.

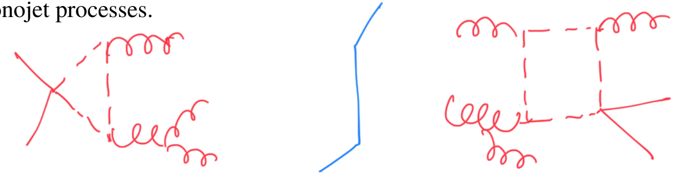
Not all master integral publicly available (?), a bottleneck at the moment.

for computing F_i's at $O(\alpha_s)$.

Conclusion and Outlook

Two-loop form factors are required for higher order phenomenology in gluphilic DM model. Form factors for $\chi\chi \to gg, qq$ are now available, those for $gg \to \chi\chi g$ are being calculated.

Calculation of real corrections for the estimation of annihilation cross section, and monojet processes.



Phase space integration in dimensional regularization, ensuring the cancellation of IR singularities, and subtraction of collinear singularities is non-trivial.

A quantitative comparison between EFT approach and simplified model approach beyond leading order.