Second Order QCD corrections to SIDIS

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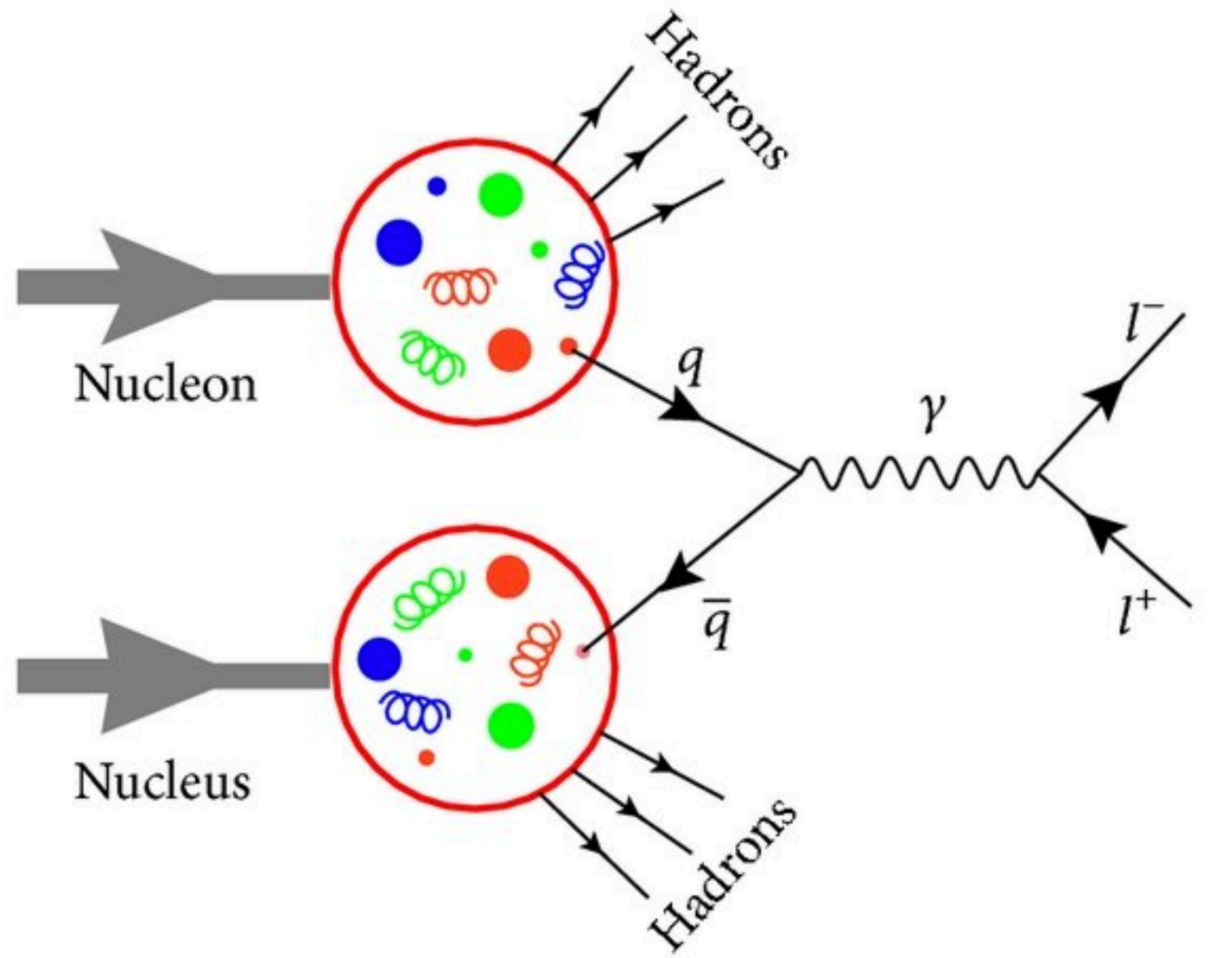
- Introduction to SIDIS
- Hadronic Cross section
- QCD improved Parton Model
- NNLO QCD effects
- Checks on our results
- Conclusion

Drell-Yan Production at the LHC

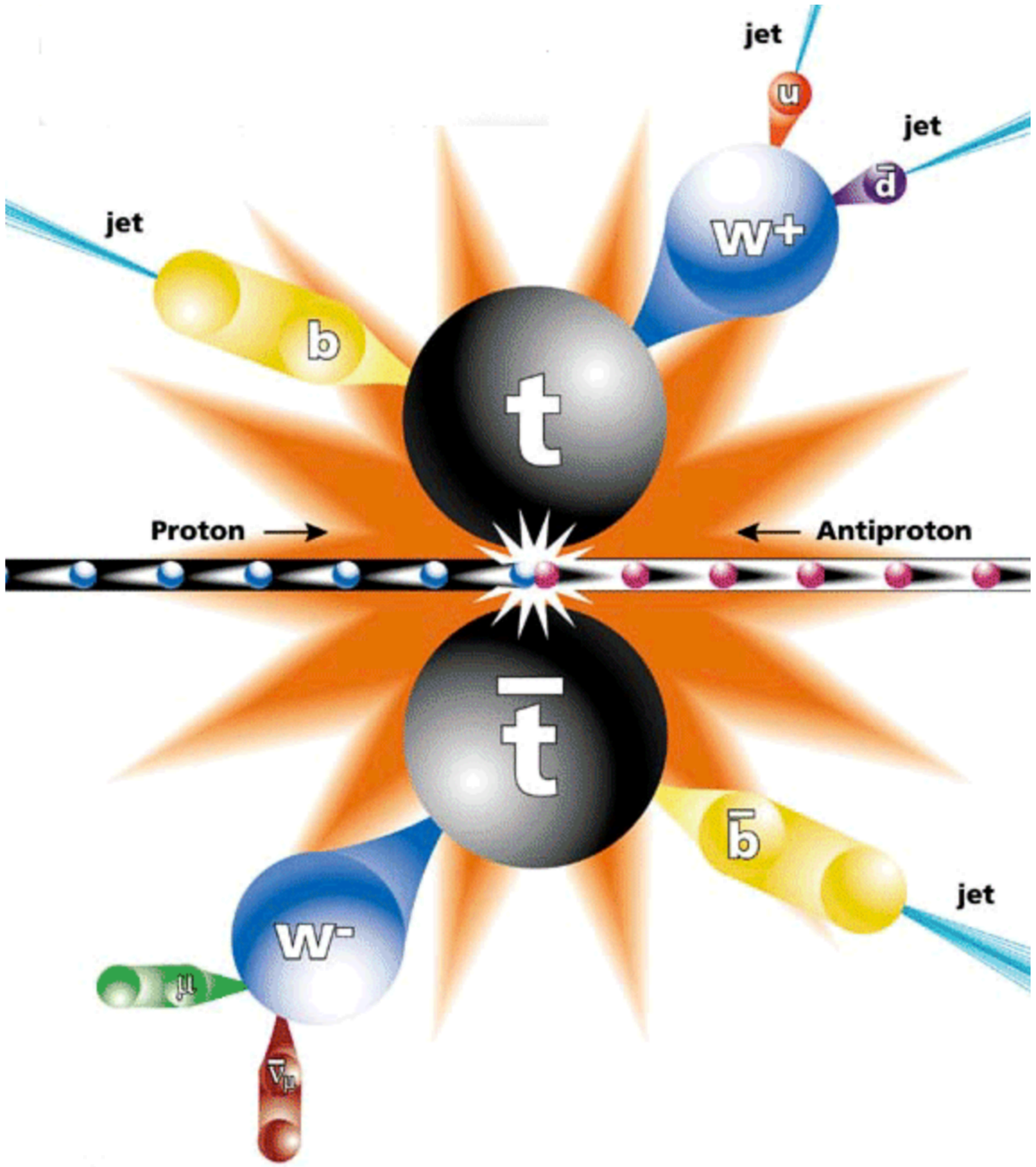
$$\sigma(q^2,\tau) = \sigma_0(\mu_R^2)\int \frac{dz}{z}\Phi_{ab}\left(\frac{\tau}{z},\mu_F^2\right)\Delta_{ab}(q^2,\mu_F^2,z)$$



Parton Distribution Function

$$\Phi_{ab}(\mu_F^2,z) = \int \frac{dy}{y} f_a(y,\mu_F^2) f_b\left(\frac{z}{y},\mu_F^2\right)$$

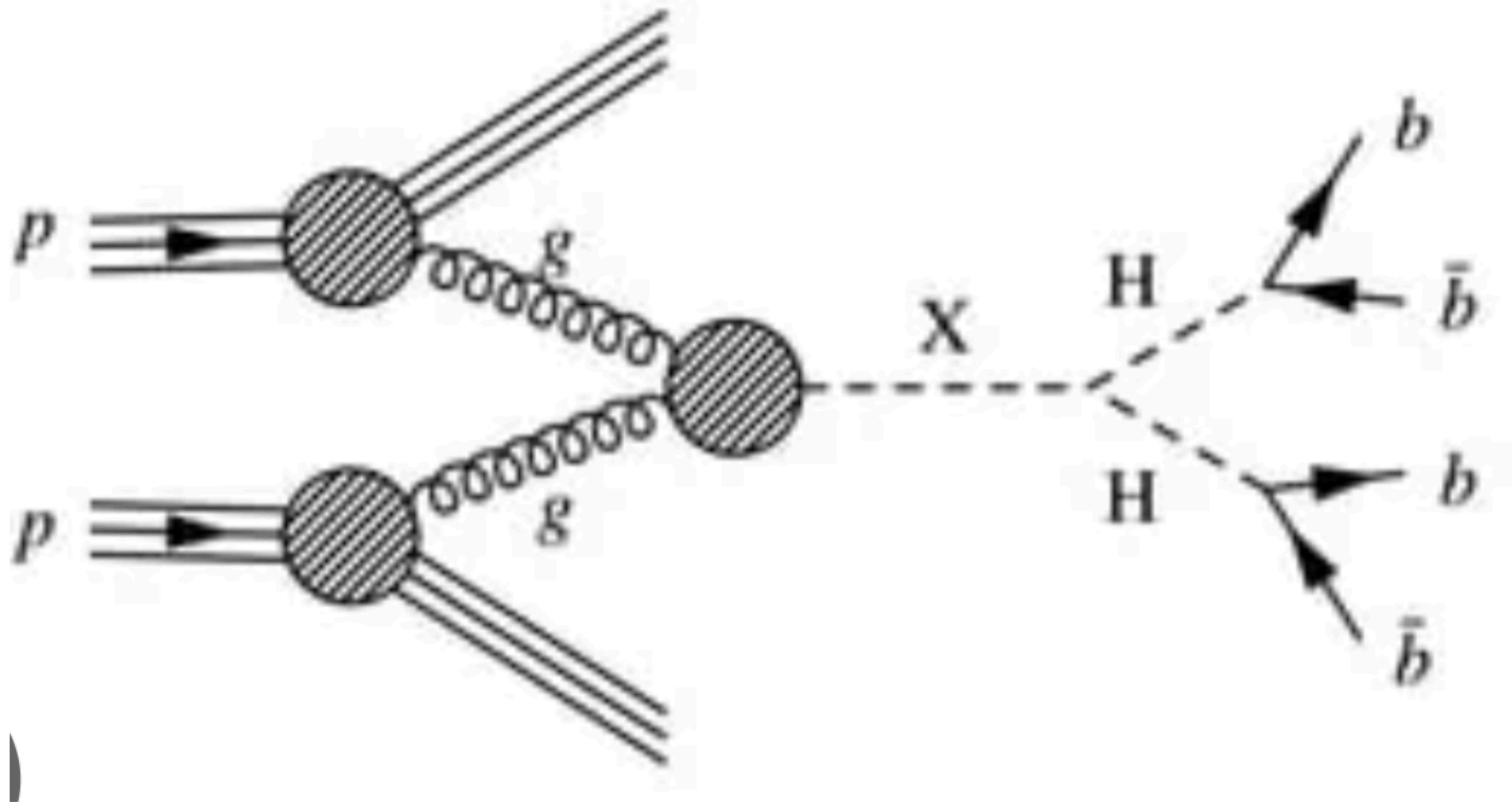
Top pair Production at the LHC



Parton Distribution Function

$$\Phi_{ab}(\mu_F^2, z) = \int \frac{dy}{y} f_a(y, \mu_F^2) f_b\left(\frac{z}{y}, \mu_F^2\right)$$

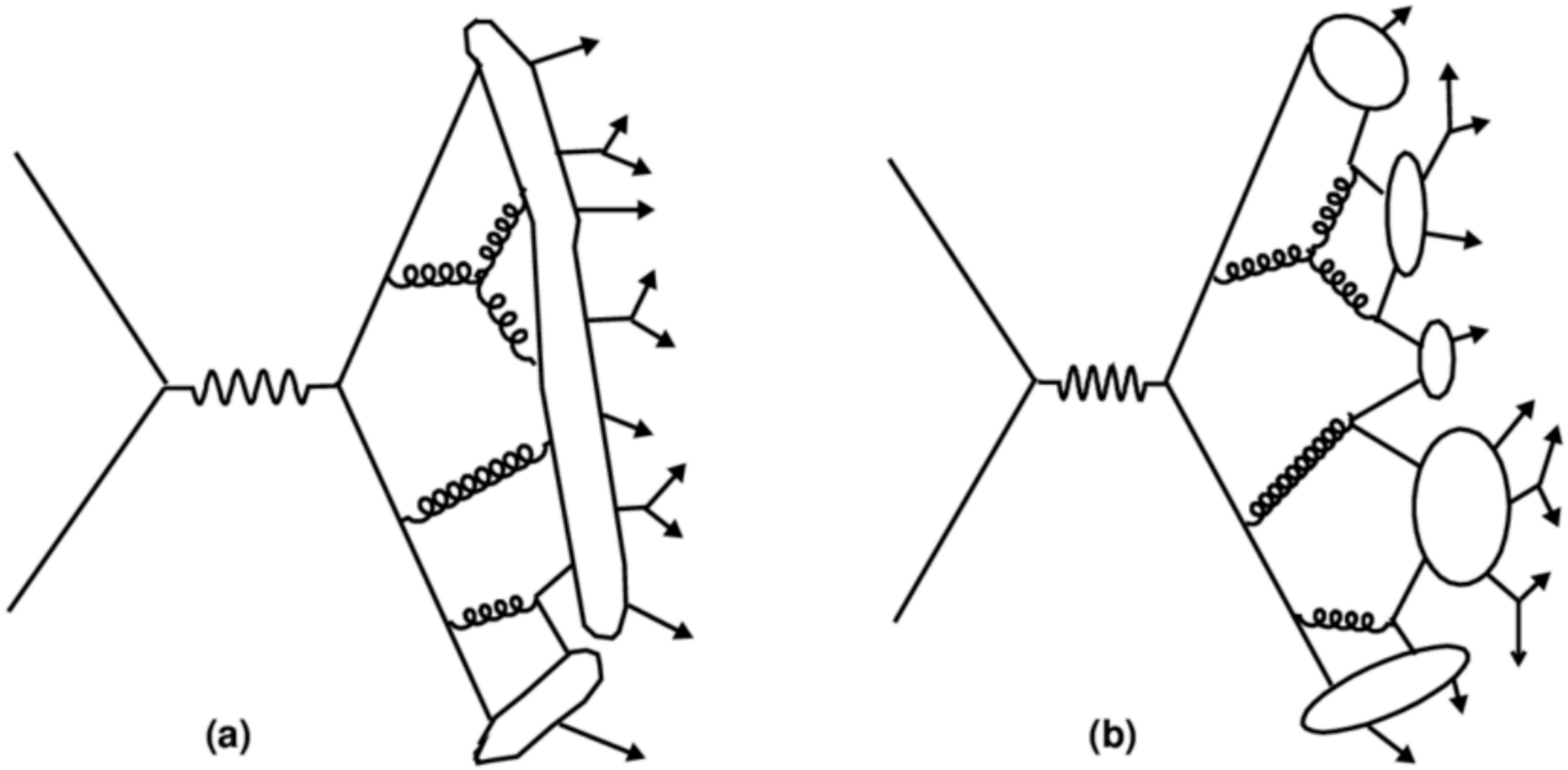
Higgs Production at the LHC



Gluon flux from Parton Distribution Function

$$\Phi_{ab}(\mu_F^2, z) = \int \frac{dy}{y} f_a(y, \mu_F^2) f_b\left(\frac{z}{y}, \mu_F^2\right)$$

Hadronization:



Fragmentation Function:

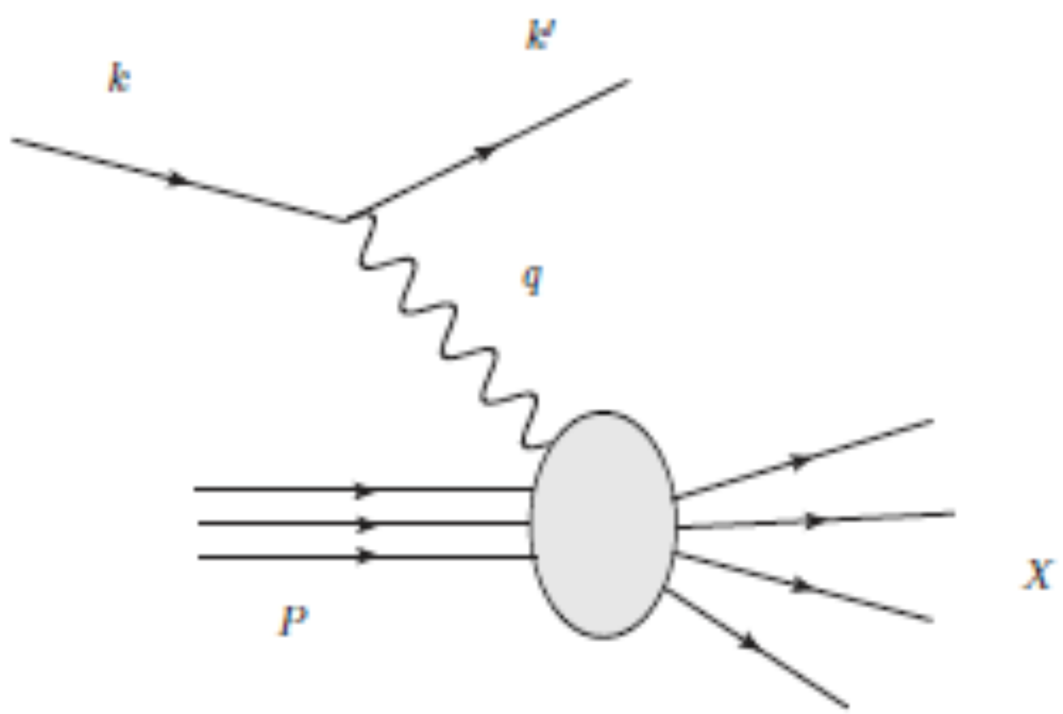
Probability of a Parton converting to Hadron

what is DIS?

Inclusive DIS (Deep Inelastic Scattering),

lepton + hadron -> lepton + X

one sums up all the particles in the final state, except the scattered lepton



Depends on Parton Distribution Function (PDF) of the incoming hadron.

- HERA: deep structure of proton at highest Q^2 and smallest x

PDF extraction

GRV, GJR ...

MRST, MSTW ...

CTEQ, CT# ...

NNPDF

ABM, ABKM

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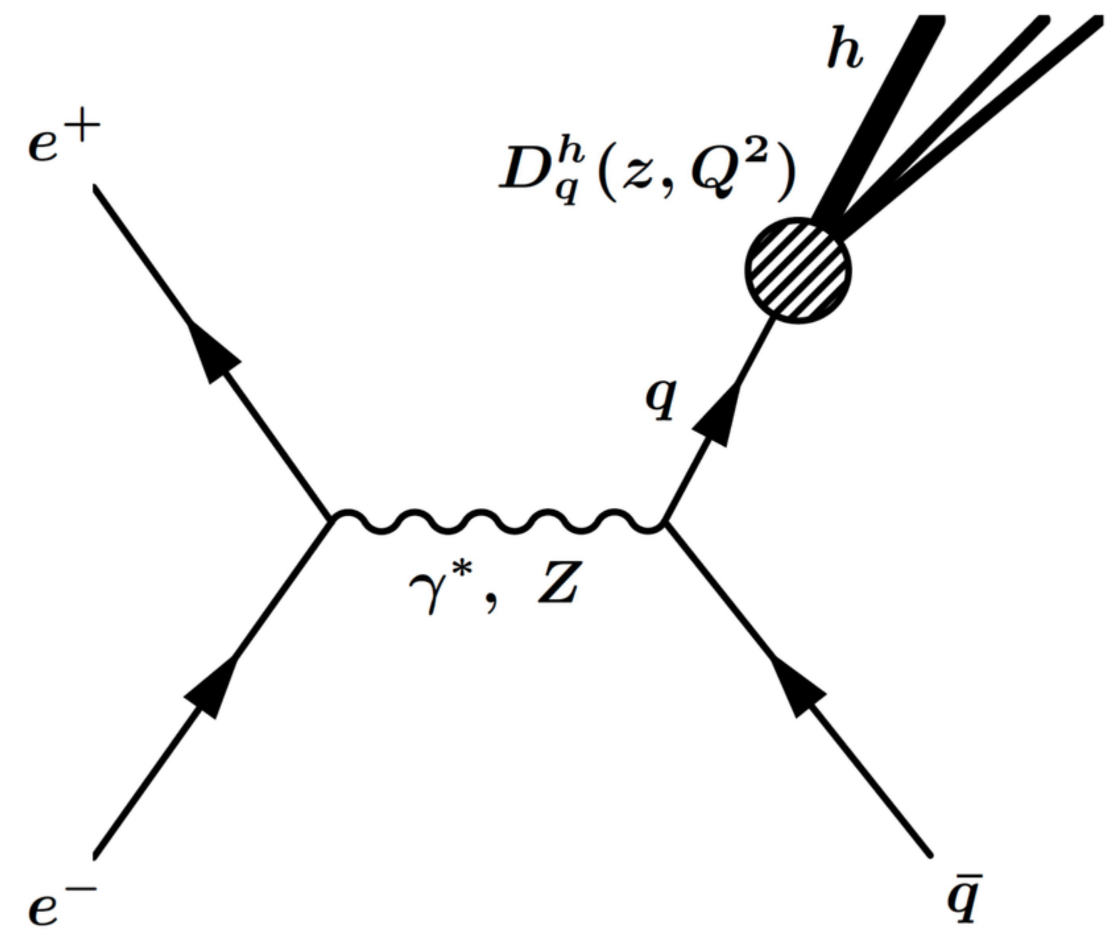
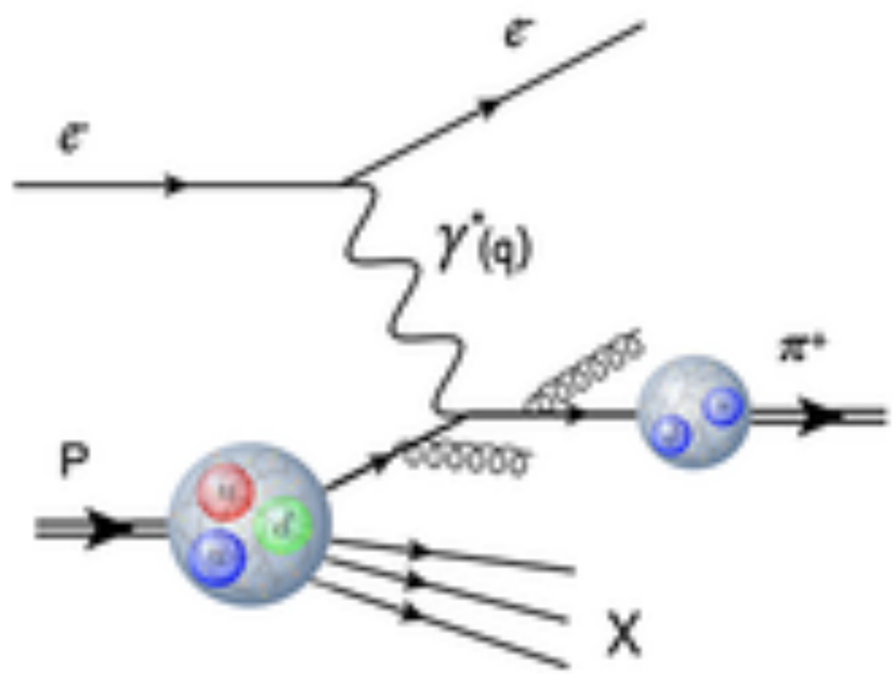
Fragmentation Function:



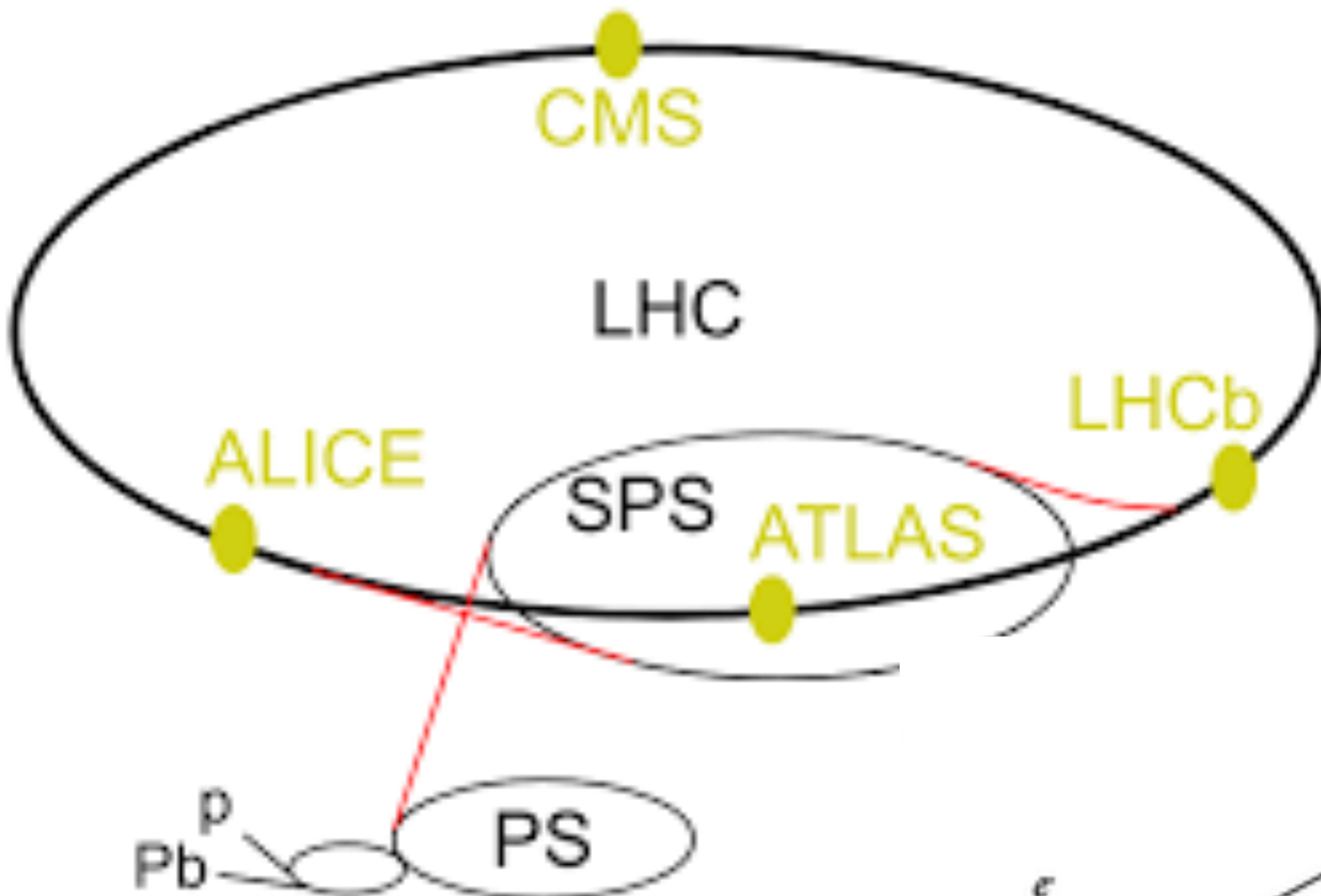
SIDIS?

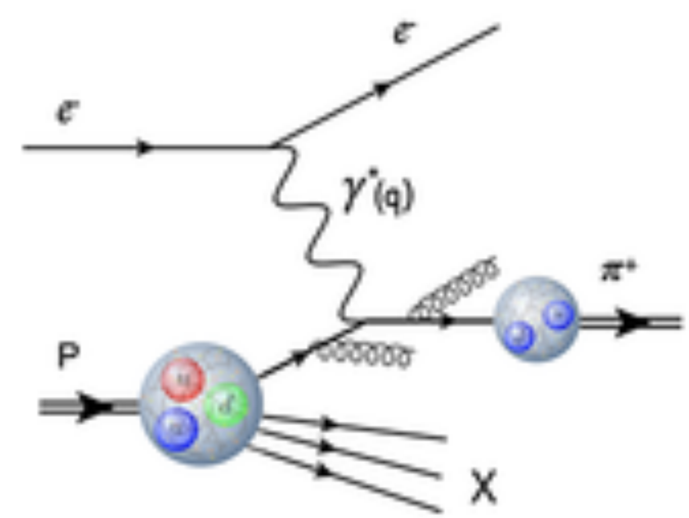
In SIDIS, in addition to the scattered lepton, we tag one of the



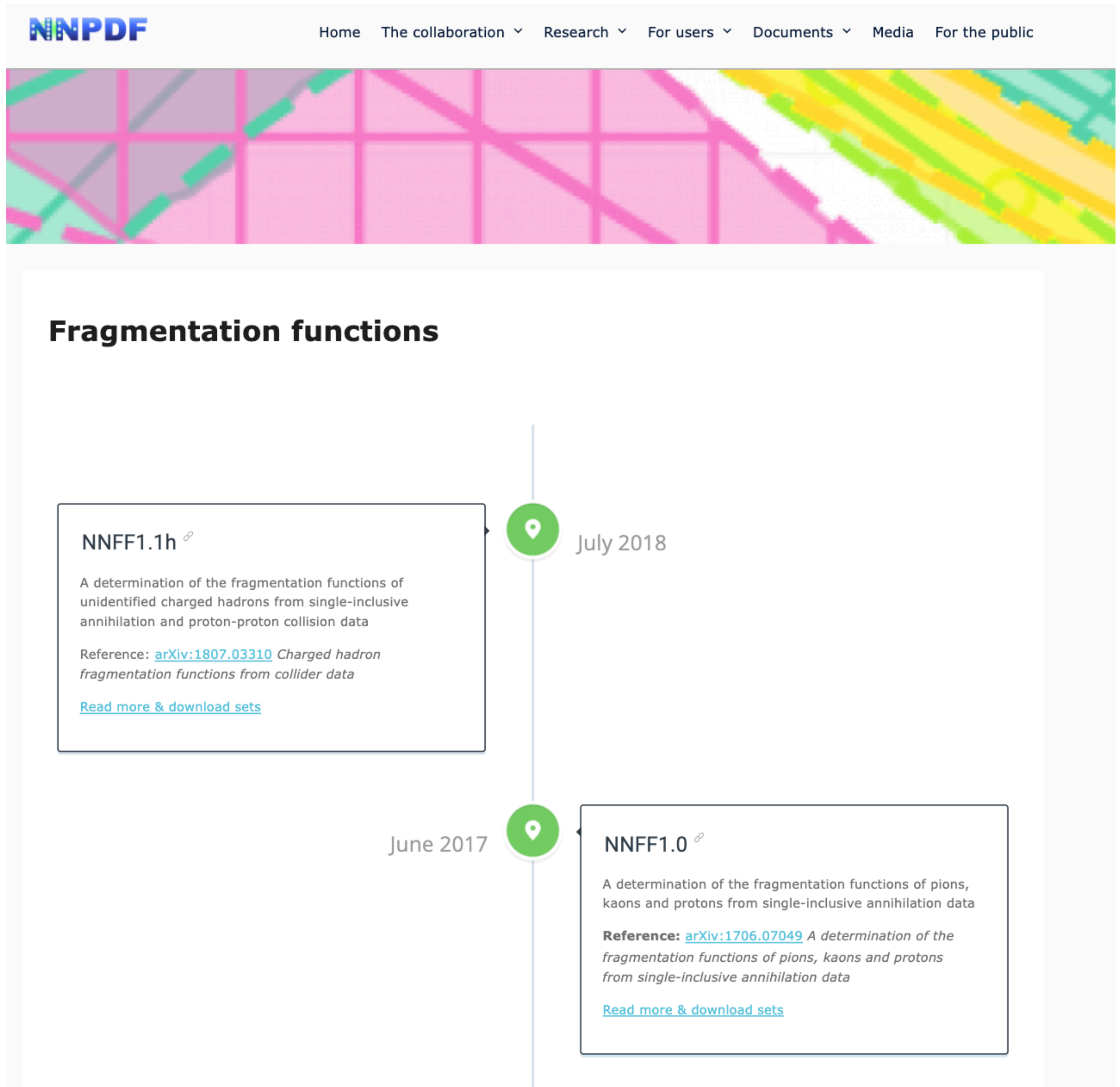
SIDIS depends on Parton Distribution Function (PDF) of the incoming hadron and Parton Fragmentation (FF) of the final state hadron.

COMPASS collaboration for FF

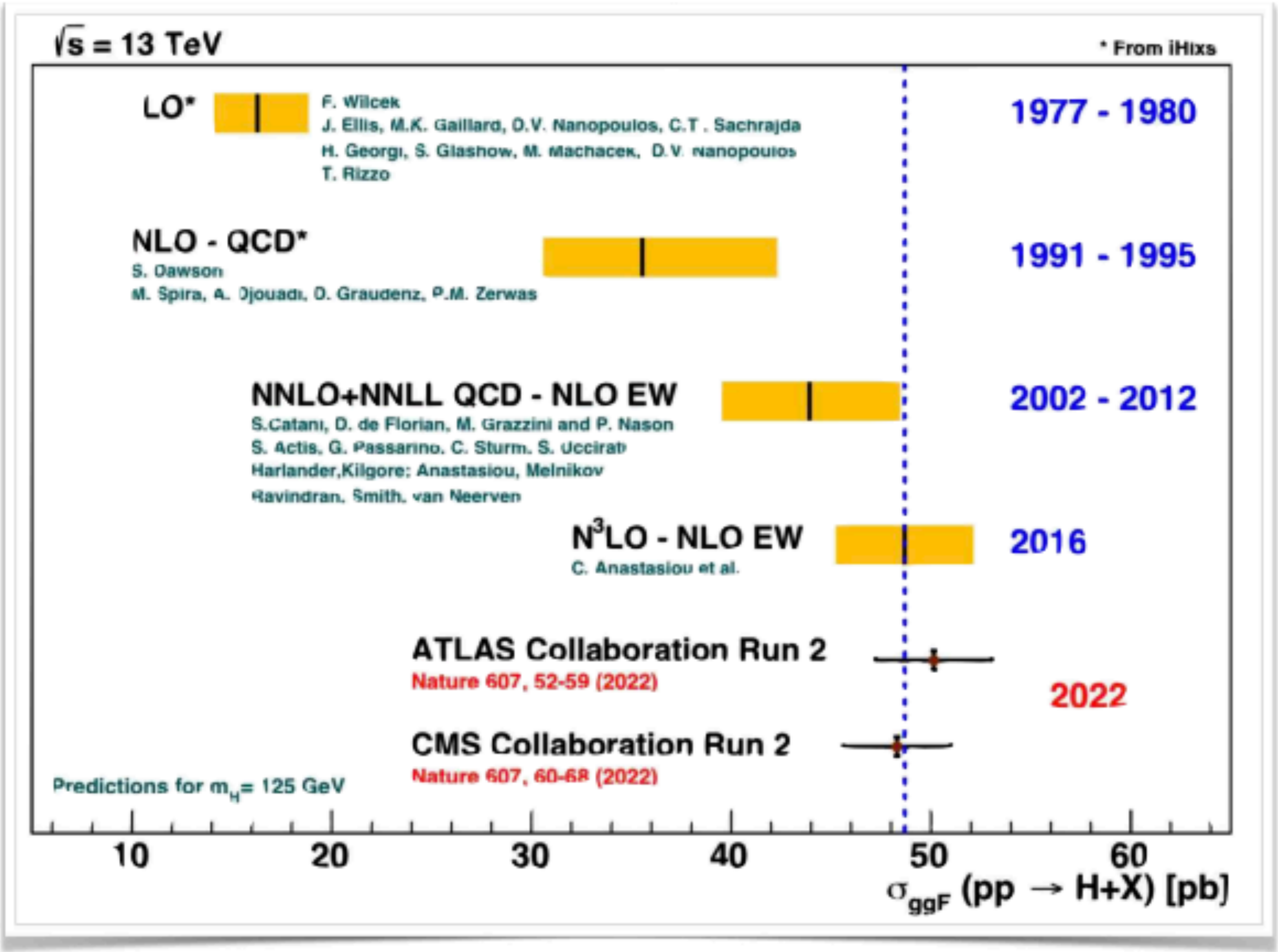




Fragmentation Function:

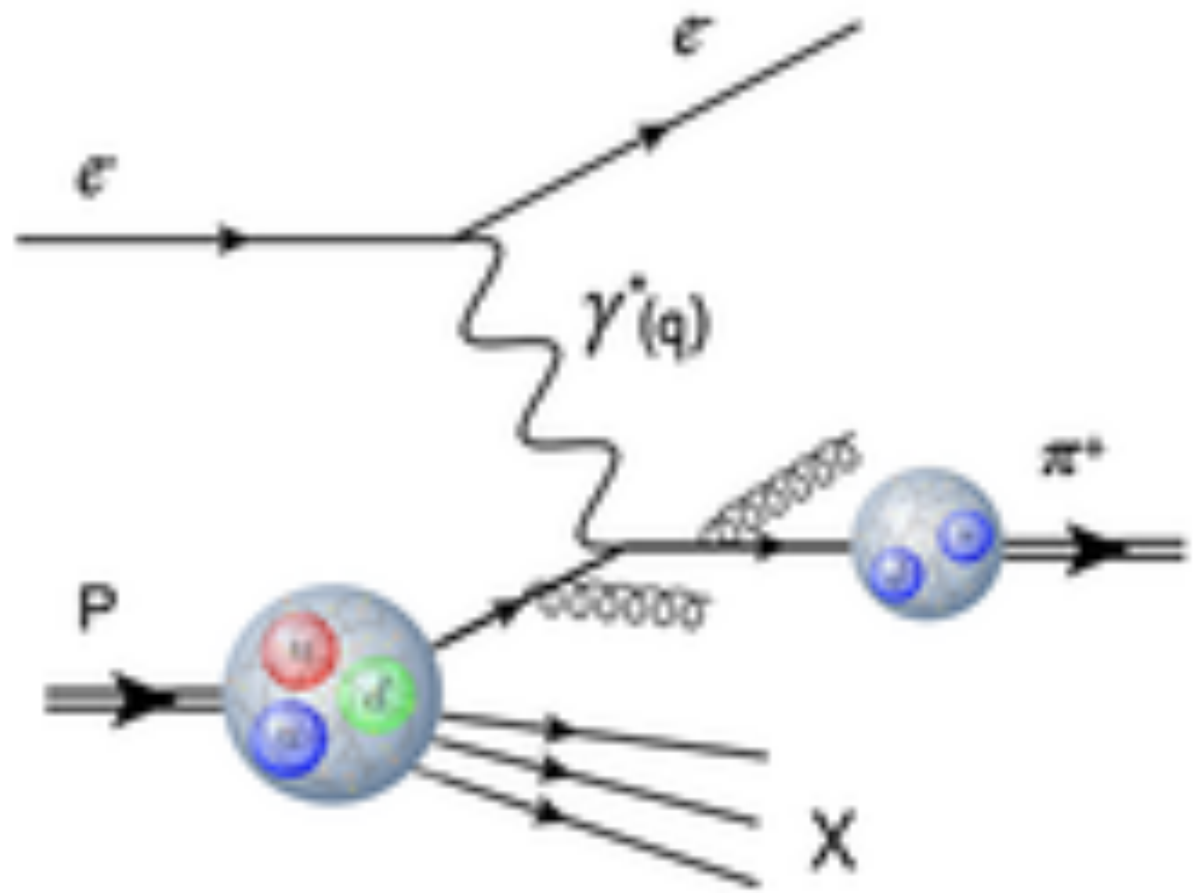


Why "Second order QCD corrections?"



Cross-section for inclusive Higgs production in gluon-gluon fusion

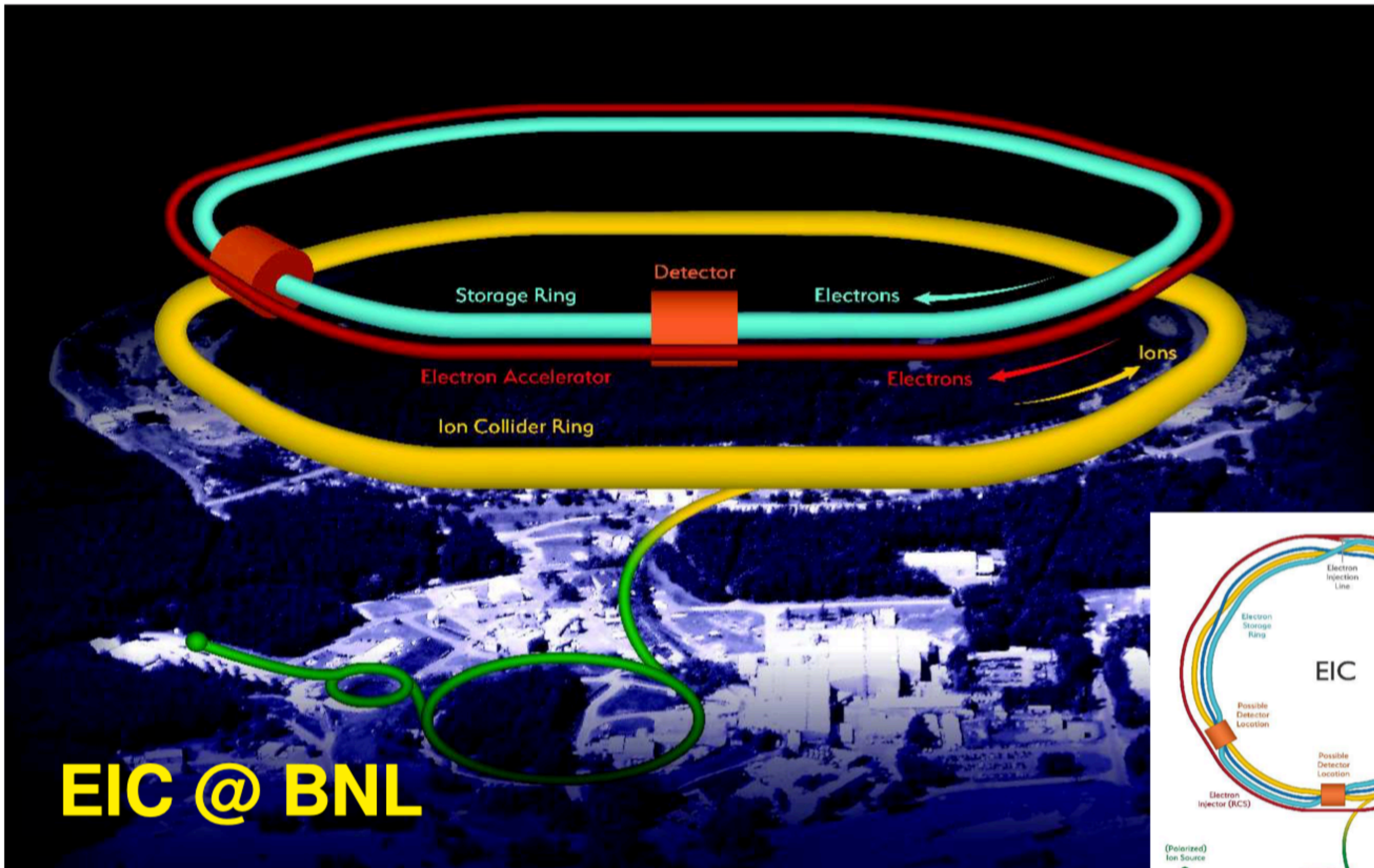
Why "Semi-Inclusive DIS (SIDIS) ?"

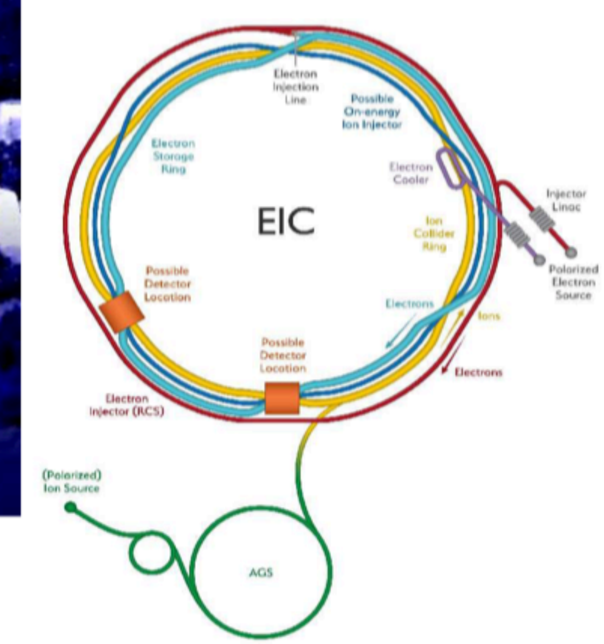


Sensitive to Parton Distribution Function

Fragmentation Function - mechanism for Hadronisation

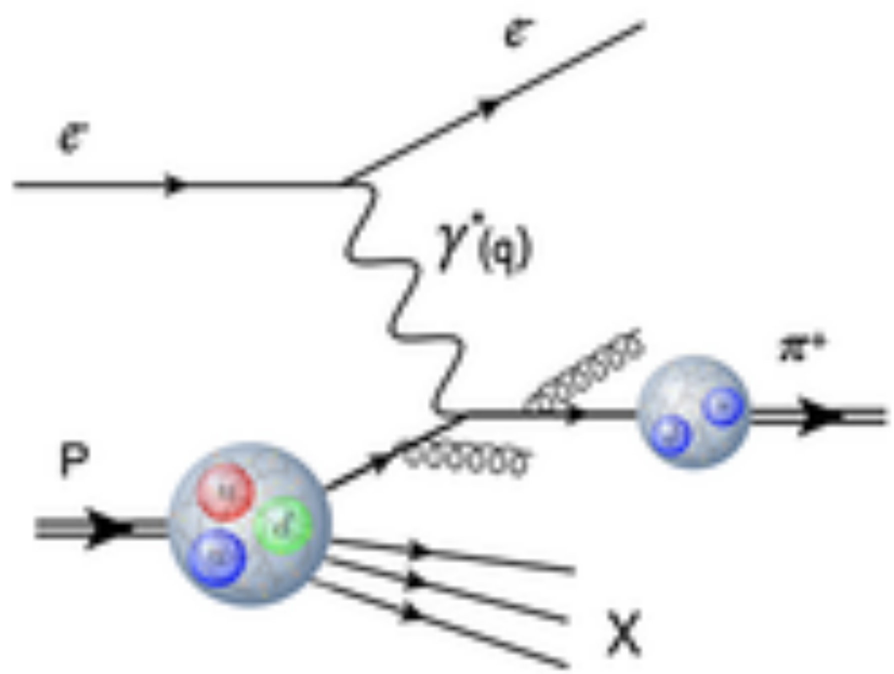
- Electron-Ion Collider
  A machine that will unlock the secrets of the strongest force in Nature





what is SIDIS?

In SIDIS, in addition to the scattered lepton, we tag one of the final-state hadrons.



SIDIS depends on Parton Distribution Function (PDF) of the incoming hadron and Parton Fragmentation (FF) of the final state hadron.

Predictions for SIDIS

Semi Inclusive Deep Inelastic Scattering (SIDIS) helps to study hadron structure both incoming hadron as well as the hadron that fragments in the final state.

Perturbative QCD provides framework to compute SIDIS cross sections order by order in strong coupling constant.

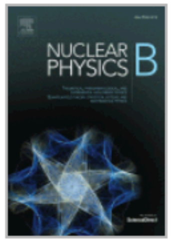
Leading order results are sensitive to theoretical uncertainty

1. Renormalisation scale dependence
2. Factorisation scale dependence from PDFs anfd FFs
3. Choice of PDFs and FFs

Higher order predictions are essential to resolve them

NLO Altarelli et al 1979

Nuclear Physics B
Volume 160, Issue 2, 3 December 1979, Pages 301-329



Processes involving fragmentation functions beyond the leading order in QCD ☆

G. Altarelli, R.K. Ellis, G. Martinelli, So-Young Pi

S+V NNLO

Vogelsang et al 2022

Threshold resummation at N^3LL accuracy and approximate N^3LO corrections to semi-inclusive DIS

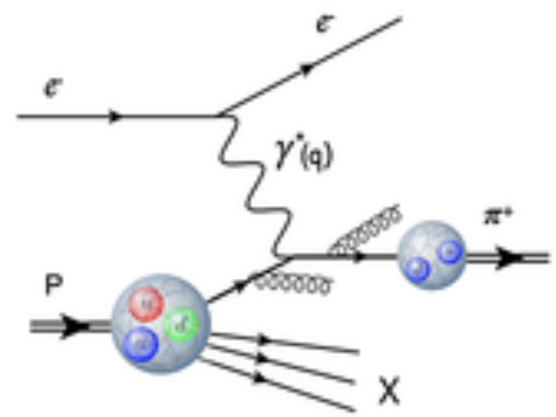
March 16, 2022

Maurizio Abele[a], Daniel de Florian[b], Werner Vogelsang[a]

Approximate NNLO QCD corrections to semi-inclusive DIS

March 16, 2022

Semi-Inclusive DIS - Second order QCD effects



NNLO QCD corrections to polarized semi-inclusive DIS #2

Saurav Goyal, Roman N. Lee, Sven-Olaf Moch, Vaibhav Pathak, Narayan Rana et al. (Apr 15, 2024)

e-Print: 2404.09959 [hep-ph]

Next-to-Next-to-Leading Order QCD Corrections to Semi-Inclusive Deep-Inelastic Scattering #3

Saurav Goyal (IMSc, Chennai and HBNI, Mumbai), Sven-Olaf Moch (Hamburg U., Inst. Theor. Phys. II), Vaibhav Pathak (IMSc, Chennai and HBNI, Mumbai), Narayan Rana (NISER, Jatni), V. Ravindran (IMSc, Chennai and HBNI, Mumbai) (Dec 29, 2023)

Published in: *Phys.Rev.Lett.* 132 (2024) 25, 251902 • e-Print: 2312.17711 [hep-ph]

Scattering Process:

$$e^-(k_l) + H(P) \to e^-(k_l^{'}) + H'(P_H) + X'$$

Factorises as

$$\frac{d^2\sigma_{e-H}}{dE_l' d\Omega dz} = \frac{E_l'}{E_l}\frac{\alpha_e^2}{Q^4} L^{\mu\nu}(k_l, k'_l, q) W_{\mu\nu}(q, P, P_H).$$

Leptonic Tensor

$$L_{\mu\nu} = 2\Big[k_\mu k_\nu' + k_\mu' k_\nu - \frac{Q^2}{2} g_{\mu\nu}\Big]$$

Hadronic Tensor

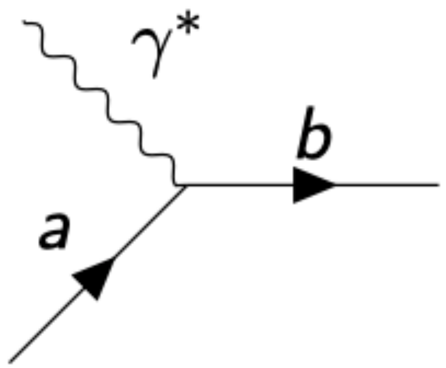
$$W^{\mu\nu} = F_1\Big[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\Big] + F_2\Big[\frac{1}{P.q}(P^\mu - \frac{P.q}{q^2}q^\mu)(P^\nu - \frac{P.q}{q^2}q^\nu)\Big]$$

Paron Model for SIDIS

$$F_I = x^{I-1} \sum_{a,b} \int_x^1 \frac{dx_1}{x_1} f_a(x_1, \mu_F^2) \int_z^1 \frac{dz_1}{z_1} D_b(z_1, \mu_F^2)$$
$$\times \mathcal{F}_{I,ab}\left(\frac{x}{x_1}, \frac{z}{z_1}, Q^2, \mu_F^2\right).$$

- $f_a dx_1$: The probability of finding a parton of type 'a' which carries a momentum fraction x_1 of the parent hadron H.
- $D_b dz_1$: The probability that a parton of type 'b' will fragment into hadron H' which carries a momentum fraction z_1 of the parton.
- $\mathcal{F}_{I,ab}$ are the finite coefficient functions (CFs) that can be computed perturbatively, it is related to partonic cross section.

Partonic Cross sections



$$\hat{\sigma}_{I,ab} = \frac{\mathcal{P}_I^{\mu\nu}}{4\pi} \int \mathrm{dPS}_{X'+b} \,\overline{\Sigma} |M_{ab}|^2_{\mu\nu}\, \delta\Big(\frac{z}{z_1} - \frac{p_a \cdot p_b}{p_a \cdot q}\Big)$$

Fragmentaion

$|M_{ab}|^2$ is the squared amplitude for the process

$$a(p_a) + \gamma^*(q) \to \text{“}b\text{”}(p_b) + X'$$

$\mathcal{P}_I^{\mu\nu}$ are the projectors to project out CFs

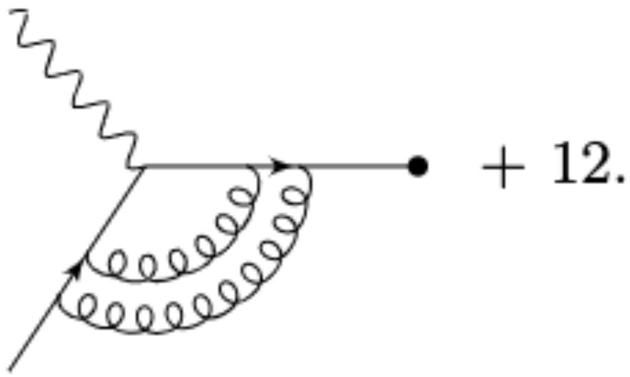
$$\mathcal{P}_1^{\mu\nu} = \frac{1}{(D-2)}\big(T_1^{\mu\nu} + 2x T_2^{\mu\nu}\big)$$

$$\mathcal{P}_2^{\mu\nu} = \frac{2x}{(D-2)x_1}\big(T_1^{\mu\nu} + 2x(D-1) T_2^{\mu\nu}\big).$$

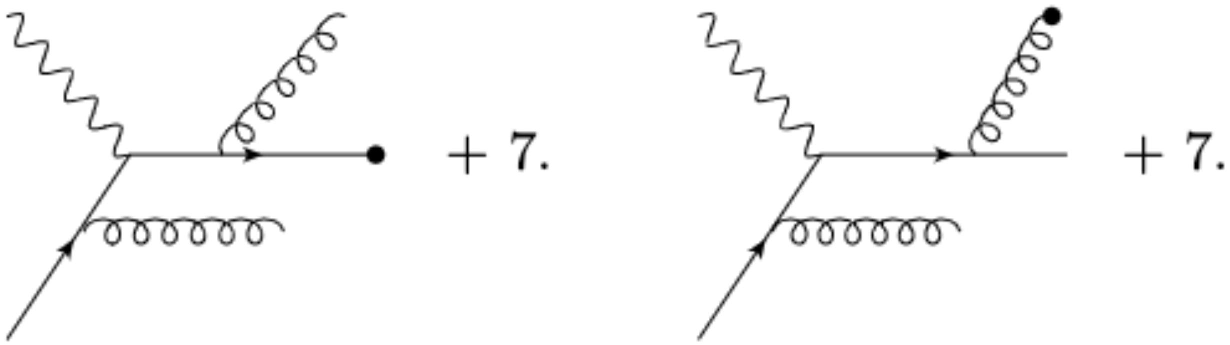
Partonic Subprocesses:

LO		$\gamma^* q \to q$
NLO	1 Loop:(V)	$\gamma^* q \to q$
		$\gamma^* q \to q + g$
		$\gamma^* g \to q + \overline{q}$
	2 Loop:(VV)	$\gamma^* q \to q$
NNLO	1 Loop:(RV)	$\gamma^* q \to q + g$
		$\gamma^* q \to q + g + g$
		$\gamma^* q \to q + q_i + \overline{q}_i$
	1 Loop:(RV)	$\gamma^* g \to q + \overline{q}$
		$\gamma^* g \to q + \overline{q} + g$

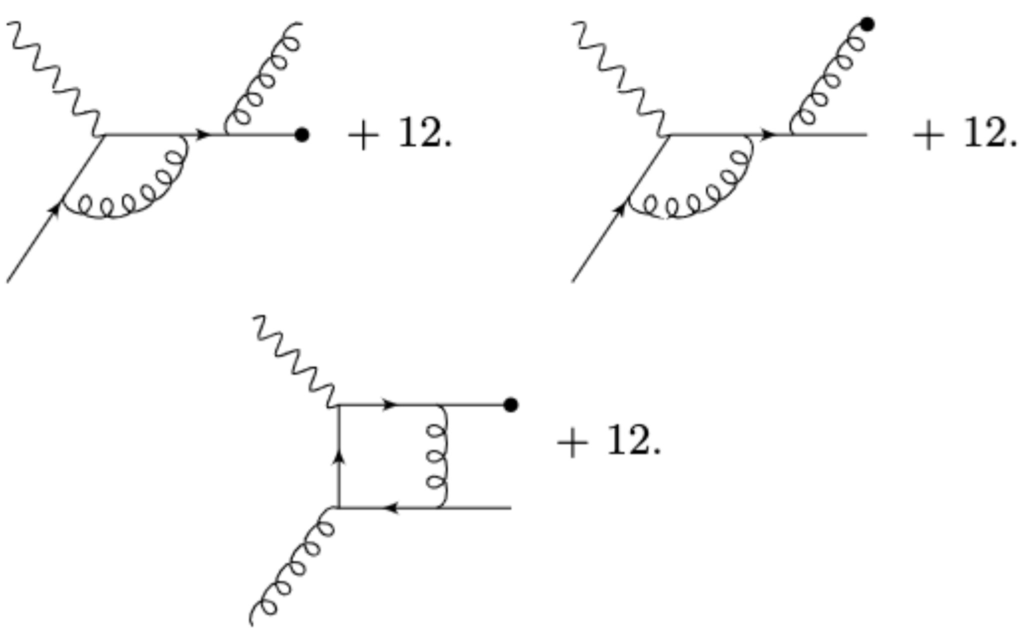
Partonic Subprocesses:



Pure Virtual



Pure Real



Mixed Real-Virtual

Loops

Loop Integrals:

We encounter a large number of loop integrals:

Integration-by-parts identities reduce them to fewer Master Integrals (MIs) .

$$\int d^D l \frac{\partial}{\partial l^\mu} \left[\frac{l^\mu, p^\mu}{D_1^{\nu_1} D_2^{\nu_2} \ldots D_n^{\nu_n}} \right] = 0$$

We choose a convenient set of families

Maping the loop integrals onto these Integral families is done by shifting of momenta ('Reduze') .

We used 'LiteRed' package perform IBP reduction to obtain MIs

Legs

Phase Space Integrals:

3-Body Phase Space, $p_a + q \to$ "p_b" $+ k_1 + k_2$,

$$\int [dPS]_3 =$$

$$\frac{1}{(2\pi)^{2D-3}} \int d^D k_1 \int d^D k_2 \int d^D p_b \delta(k_1^2)\delta(k_2^2)\delta(p_b^2)\delta^D(p_a + q - p_b - k_1 - k_2)\delta(z' - \frac{p_a.p_b}{p_a.q})$$

Reverse Unitarity method :

$$\delta(p^2 - m^2) \to \frac{i}{p^2 - m^2} - \quad c.c.$$

can be almost forget

We get total '21' MIs in phase space calculation.

Next task: Solving the Master Integrals

Master Integrals

Generalization with set of MIs

$$\vec{I} = \Big(I_1, I_2, \cdot, \cdot, \cdot, I_N\Big)$$

$\{I_i(\vec{x})\}$ depend on Scaling variables

$$x_i = f_i\left(\frac{s_{ij}}{Q^2}\right)$$

$$\vec{x} = \Big(x_1, x_2, \cdot, \cdot, \cdot, x_M\Big)$$

Differential equation:

$$d\vec{I} = \sum_{i=1}^{M} \mathbf{A}_i dx_i \; \vec{I}$$

$$\frac{\partial}{\partial x_i} \begin{bmatrix} I_1 \\ \cdot \\ I_N \end{bmatrix} = \begin{bmatrix} \mathbf{A_{11}} & \cdots & \mathbf{A_{1N}} \\ \cdot & \cdots & \cdot \\ \mathbf{A_{N1}} & \cdots & \mathbf{A_{NN}} \end{bmatrix} \begin{bmatrix} I_1 \\ \cdot \\ I_N \end{bmatrix}$$

$$i = 1, 2, \cdot, \cdot, \cdot M$$

Master Integrals

Consider Diff equation:

$$s\frac{\partial}{\partial s}I(s,n) = A(s,n)I(s,n)$$

Expand around n = 4

$$I(s,n) = I^{(0)}(s) + (n-4)I^{(1)}(s) + \mathcal{O}((n-4)^2)$$

$$A(s,n) = A^{(0)}(s) + (n-4)A^{(1)}(s) + \mathcal{O}((n-4)^2)$$

0th order $$s\frac{\partial}{\partial s}I^{(0)}(s) = A^{(0)}(s)I^{(0)}(s)$$

Solution $$I^{(0)}(s) = I^{(0)}(s_0)e^{\int_{s_0}^{s}\frac{d\lambda}{\lambda}A^{(0)}(\lambda)}$$

1st order $$s\frac{\partial}{\partial s}I^{(1)}(s) = A^{(0)}(s)I^{(1)}(s) + A^{(1)}(s)I^{(0)}(s)$$

Canonical/Henn's Basis

Consider Diff equation:

$$d\vec{I}(\vec{x},n) = \sum_i \mathbf{A}_i(\vec{x},n)dx_i \; \vec{I}(\vec{x},n)$$

Choose U Transformation such that

$$U^{-1}\mathbf{A}(\vec{x},n)U - U^{-1}dU = (n-4)\;\overline{\mathbf{A}}(\vec{x})$$

Diff equation contains ‚n' independent A

$$d\vec{\bar{I}}(\vec{x},n) = (n-4)\sum_i \overline{\mathbf{A}}_i(\vec{x})dx_i \; \vec{\bar{I}}(\vec{x},n)$$

Solution $\vec{\bar{I}}(\vec{x},n) = \vec{\bar{I}}(\vec{x}_0,n) \; \mathbf{P}\exp\left((n-4)\int \frac{d\lambda}{\lambda}\overline{\mathbf{A}}(\lambda)\right)$

$\mathbf{P}$ - Path Ordered exponential

Canonical/Henn's Basis

Start with Henn's Diff equation:

$$s\frac{\partial}{\partial s}\overline{I}(s,n) = (n-4)\overline{A}(s)\ \overline{I}(s,n)$$

If $\overline{A}$ contains poses at s_i

$$\overline{A}(s) = \sum_i \frac{\tilde{A}_i(s_i)}{s - s_i}$$

$$\overline{I}(s,n) = \overline{I}^{(0)}(s_0) + (n-4)\sum_i \tilde{A}_i(s_i)\log\left(\frac{s-s_i}{s_0-s_i}\right)$$

$$+(n-4)^2\sum_i \tilde{\tilde{A}}(s_i)\mathcal{L}_i(s_0,s_i) + \cdots$$

Polylogarithms - Uniform transcendental terms

Computation

Dimensionally regulated integrals contain functions that require correct analytic continuation:

$$(z' - x')^{a\epsilon - b}, (1 - z' - x')^{c\epsilon - d}$$

Feynman $+i\varepsilon$ prescription of propagators

$$x' \equiv x' - i\varepsilon \quad \text{and} \quad z' \equiv z' - i\varepsilon.$$

Partial fractioning and theta function to separate different sectors.

$$\left(\frac{z'-x'}{1-x'}\right)^{\varepsilon} = \left|\frac{z'-x'}{1-x'}\right|^{\varepsilon} \Big(\theta(z'-x') + (-1+i\varepsilon)^{\varepsilon}\theta(x'-z')\Big)$$

$$\left(\frac{1-z'}{1-z'-x'}\right)^{\varepsilon} = \left|\frac{1-z'}{1-z'-x'}\right|^{\varepsilon} \Big(\theta(1-z'-x') + (-1-i\varepsilon)^{\varepsilon}\theta(z'+x'-1)\Big)$$

`+` Distributions

Dimensionally regulated integrals contain divergences as x ->1 and z->1

$$\frac{(1-x)^{\epsilon}}{1-x} = \frac{1}{\epsilon}\delta(1-x) + \sum_{i=0}^{\infty} \frac{\epsilon^i}{i!}\left[\frac{\ln^i(1-x)}{1-x}\right]_+$$

$$= \frac{1}{\epsilon}\delta(1-x) + \sum_{i=0}^{\infty} \frac{\epsilon^i}{i!} D_i(x)$$

`+` distributions

$$\int_0^1 dx f(x)[g(x)]_+ = \int_0^1 dx \big(f(x) - f(1)\big) g(x)$$

Double distribution,

$$\int_0^1 dx \int_0^1 dz \frac{f(x,z)}{[1-x]_+[1-z]_+} =$$

$$\int_0^1 dx \int_0^1 dz \frac{f(x,z) - f(1,z) - f(x,1) + f(1,1)}{(1-x)(1-z)}$$

Mass factorization

Soft divergences cancels among virtual and real emission processes,

The collinear divergences related to the a and b partons in the initial state and the final fragmentation state remain.

These divergences can be factored out into Altarelli-Parisi (AP) kernels (mass factorisation) at factorisation scale,

$$\frac{\hat{\sigma}_{I,ab}(\epsilon)}{x'^{I-1}} = \Gamma_{c\leftarrow a}(\mu_F^2,\epsilon) \otimes \mathcal{F}_{I,cd}(\mu_F^2,\epsilon) \tilde{\otimes} \tilde{\Gamma}_{b\leftarrow d}(\mu_F^2,\epsilon),$$

Convolution: $[f \otimes g](x) = \int_x^1 \frac{dt}{t} f(t) g(\frac{x}{t})$

Results for Spin-depependent (independent) at NNLO

NNLO results:

Dominant contribution:

1. Quark Initiated processes and
2. Quarks fragmenting to hadrons

The MIs were computed using two different methods

Initial and final collinear singularities were removed using mass factorization

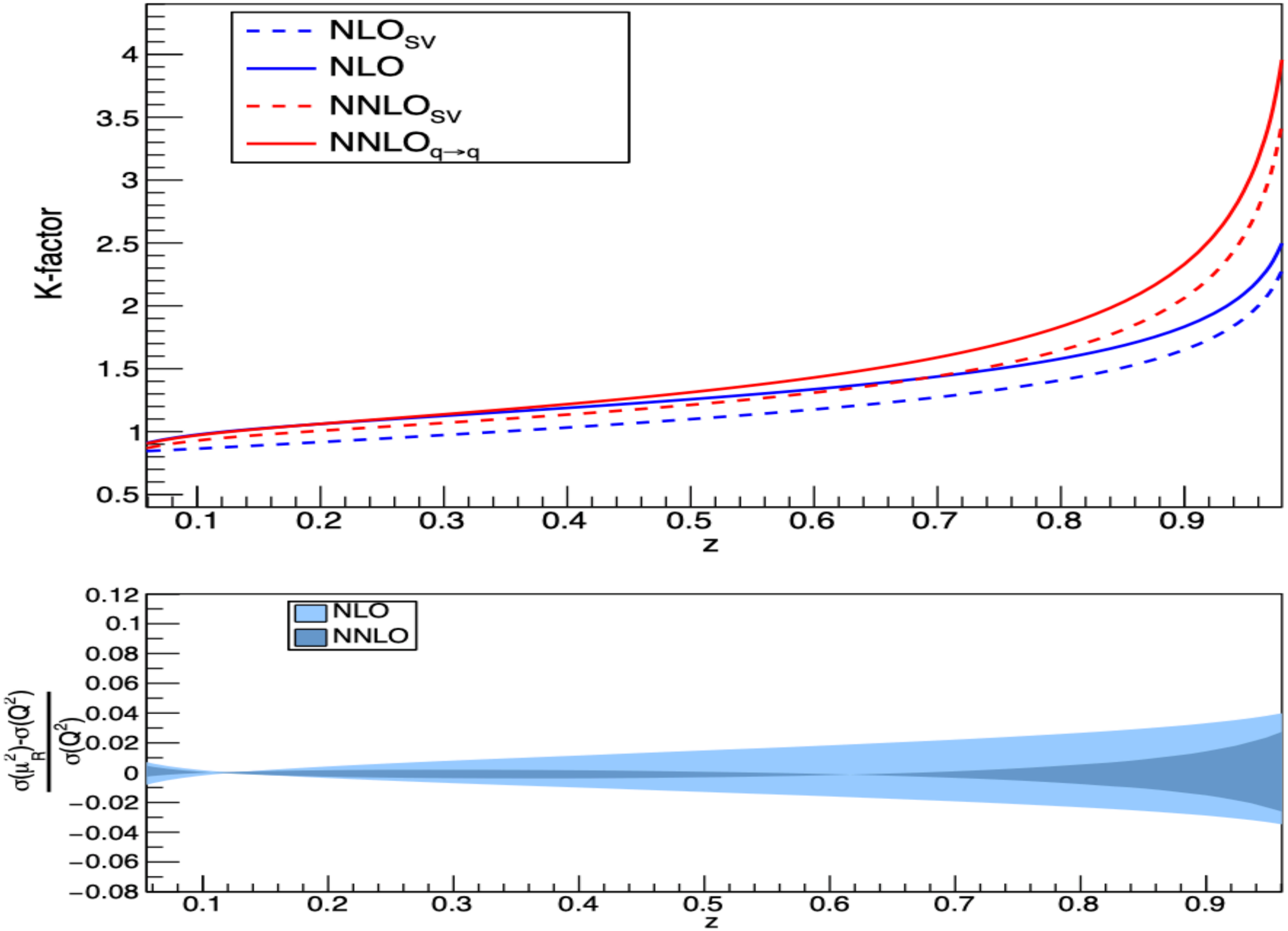
We confirmed the + distributions and Dirac delta contributions, called Soft-Virtual part in the literature

We obtain regular contributions for the first time

Numerical Impact:

Spin-independent

$$d\sigma = \frac{1}{4}\sum_{s,S,s',S_H} d\sigma^{s',S}_{s,S}$$



[5] S. Moch et.al.{arXiv:0404111}

$$xq(x,\mu_F^2) = 0.6x^{-0.3}(1-x)^{3.5}(1+5.0x^{0.8}),$$
$$xg(x,\mu_F^2) = 1.6x^{-0.3}(1-x)^{4.5}(1-0.6x^{0.3}).$$

Numerical Impact:

Spin-dependent

$$d\Delta\sigma = \frac{1}{2}\sum_{s',S_H}\left(d\sigma^{s',S_H}_{s=\frac{1}{2},S=\frac{1}{2}} - d\sigma^{s',S_H}_{s=\frac{1}{2},S=\cdot}\right.$$