Correlation between spatiotemporal patterns and local instability

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Patterns emerge in spatially extended systems at all scales, from astronomical to micrometer. The appearance of galaxies in a spiral, elliptical, or irregular form [1], the patterns formed by a starving colony of slime molds [2], wind-swept sand patterns in rivers and deserts [3], patterns in zebra and snow leopards [4, 5] are a few examples. The hexagonal pattern in Rayleigh-Benard experiment [6] and spiral wave in Belousov-Zhabotinsky chemical reaction [7] are such examples in laboratory experiments. These patterns can be simple or complex, static and dynamic i.e., patterns vary both in space and time, called spatiotemporal patterns. In general, the dynamics of the spatiotemporal patterns attributed to the local instabilities that emerge around their equilibrium point or fixed points when driven out of equilibrium also depends on the nature of the driving field [1]. Then, a natural question arises: do the local instabilities keep all the information about the nature of the spatio-temporal patterns? In other words, can seemingly equivalent linear instability give rise to various spatiotemporal patterns?

To address the question, we consider the spatially extended Ananthakrishna's (AK) model, a dynamical model developed to understand the plastic instability of crystalline materials system [8]. The AK model captures all the generic spatiotemporal patterns that appear in a dynamic experiment as a function of the control parameter [9]. The model has a broad instability spectrum associated with the control parameter. We observed that seemingly identical local linear instability with distinctly different control parameters gives rise to distinct spatiotemporal patterns. To understand the origin of the changing nature of the spatio-temporal pattern as a function of the control parameter, we evaluate the Lyapunov spectrum that calculates the time scales associated with each degree of freedom. Our study reveals that the nature of spatiotemporal patterns depends on the separation of participating time scales. While a large difference in the average value of participating time scales gives rise to static and randomized spatiotemporal patterns, uniformly distributed competing time scales give rise to continuous patterns. This information is not apparent in the local instability and is therefore not adequate to explain various dynamic spatiotemporal patterns in extended systems.

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