On the applicability of Eddington's inversion methods to direct dark matter searches

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in collaboration with

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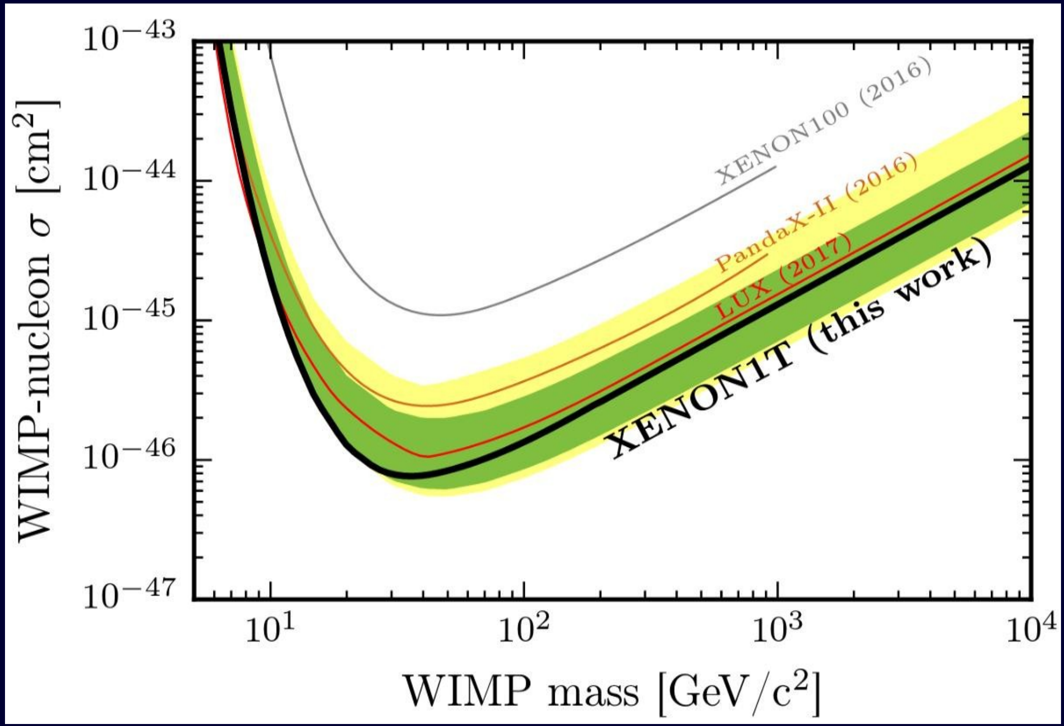
The dark matter velocity distribution

Direct searches

Local velocity distribution needed to compute the differential event rate :

$$\frac{\mathrm{d}R}{\mathrm{d}E_{\mathrm{r}}} \propto \rho_{\odot} \int_{v_{\mathrm{min}}<|\vec{v}|<v_{\mathrm{esc}}} \mathrm{d}^3\vec{v}\, \frac{f(\vec{v})}{|\vec{v}|}$$

Basic ingredient to derive constraints on the WIMP-nucleon cross-section



[Aprile+ 17]

Indirect searches

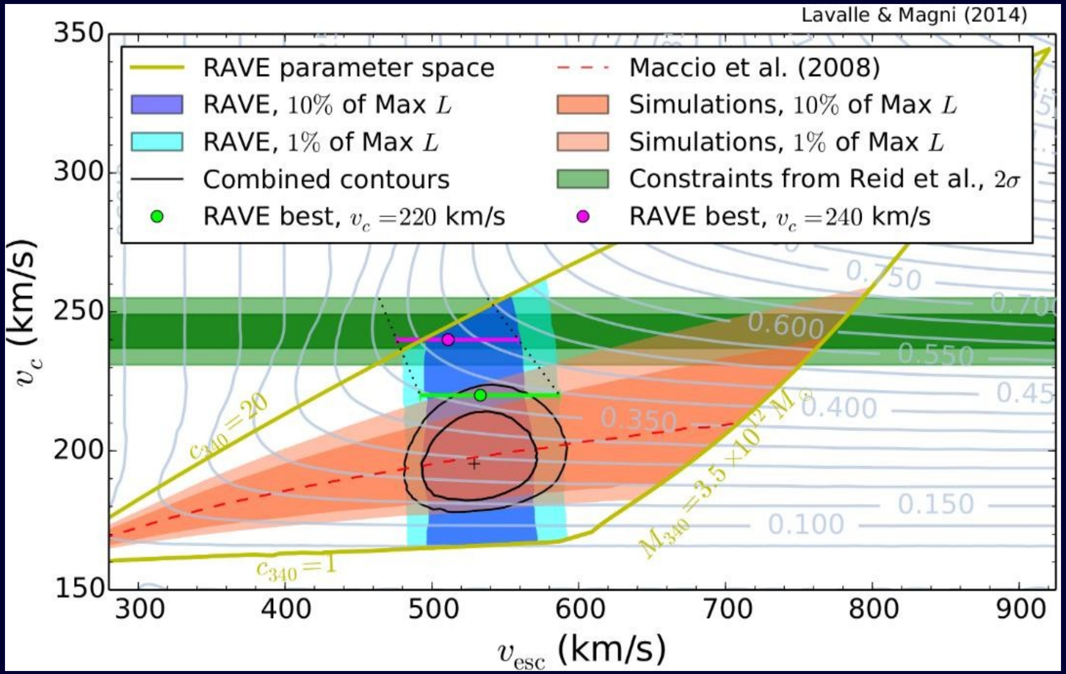
p-wave annihilation (*e.g.* K. Boddy's talk yesterday)
capture and annihilation in the Sun/Earth

Importance of dynamical constraints

The Milky Way is a *dynamically constrained* system
[see *e.g.* Catena & Ullio 10, McMillan 17]

- Impact on astrophysical uncertainties ?

- Correlation between dynamical quantities ?



[Lavalle & Magni 14]

Case of the Standard Halo Model :

$$f(\vec{v}) \propto \begin{cases} e^{-v^2/v_c^2} - e^{-v_{\rm esc}^2/v_c^2} & \text{if } |\vec{v}| < v_{\rm esc} \\ 0 & \text{if } |\vec{v}| \geq v_{\rm esc} \end{cases}$$

- isothermal sphere not realistic

- dynamical correlations ignored

The Eddington formalism (1)

A method to derive the DM phase-space distribution function (DF) starting from a Milky Way mass model

Maximal symmetry is assumed [Eddington 1916, Binney & Tremaine 87]
- spherical system
- isotropic velocity distribution

The DF can then be written as $f(\vec{r},\vec{v}) = f(\mathcal{E}(r,v))$

where

$$\begin{aligned} \Psi(r) &= \phi(R_{\max}) - \phi(r) \\ \mathcal{E}(r,v) &= \Psi(r) - \frac{v^2}{2} \end{aligned}$$

relative potential >0 (can include baryons)

relative energy >0

The relation between the density profile and the DF $\rho(r) = \int \mathrm{d}^3\vec{v}\, f(\mathcal{E})$

can be written has an Abel equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}\Psi} = \sqrt{8}\pi \int_0^{\Psi(r)} \frac{f(\mathcal{E})}{\sqrt{\Psi - \mathcal{E}}} \mathrm{d}\mathcal{E}$$

The Eddington formalism (2)

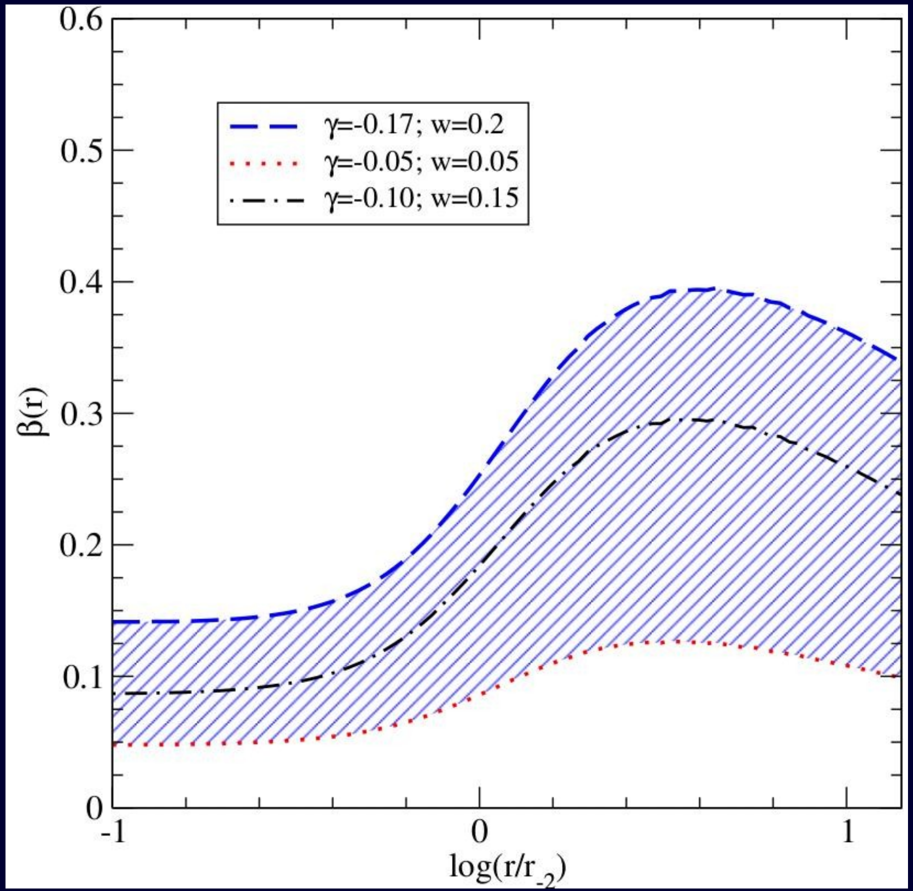
Dark matter DF : $$f(\mathcal{E}) = \frac{1}{\sqrt{8}\pi^2}\left[\frac{1}{\sqrt{\mathcal{E}}}\left(\frac{\mathrm{d}\rho}{\mathrm{d}\Psi}\right)_{\Psi=0} + \int_0^{\mathcal{E}} \frac{\mathrm{d}^2\rho}{\mathrm{d}\Psi^2}\frac{\mathrm{d}\Psi}{\sqrt{\mathcal{E}-\Psi}}\right]$$

Can be generalized to system with non-zero anisotropy parameter :

$$\beta(r) = 1 - \frac{\sigma_{\mathrm{t}}^2}{2\sigma_{\mathrm{r}}^2}$$

- Constant anisotropy [Cuddeford 91]
- Radially-increasing anisotropy [Osipkov 79, Merritt 85]

The previous DFs can be combinated to reproduce the anisotropy found in cosmological simulations → [Bozorgnia+ 13]



[For state-of-the-art DF computation, see Binney+ 15, Piffl+ 15, Posti+ 15]

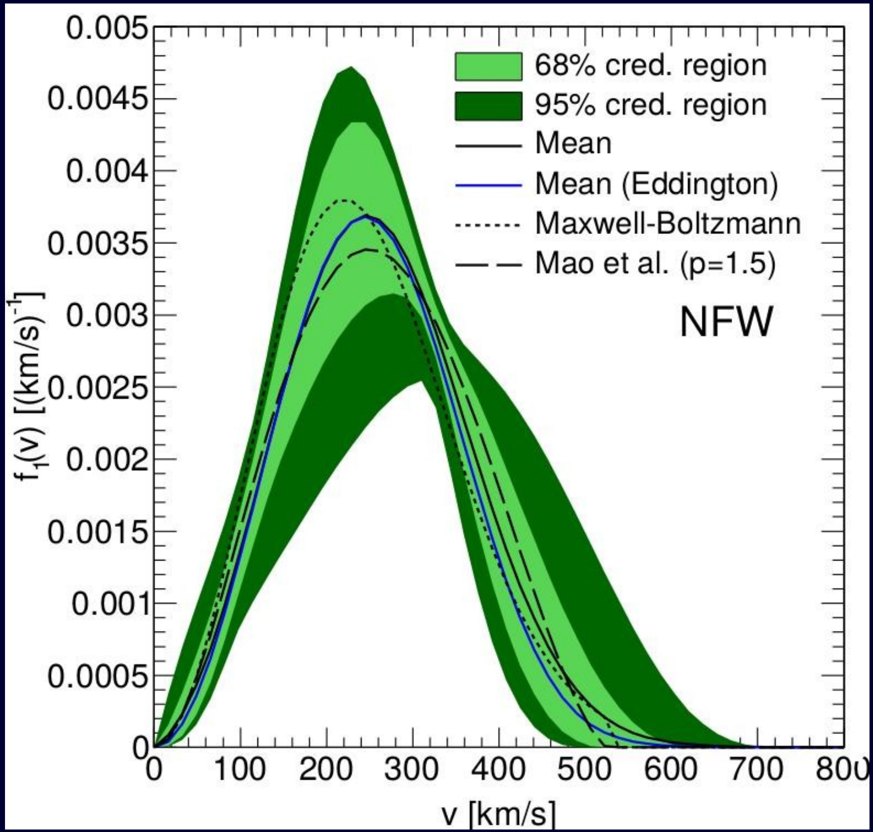
Application to direct searches

Extensive use of Eddington's formalism in direct detection studies
[Ullio & Kamionkowski 01, Vergados & Owen 03, An & Evans 06, Wojtak+ 08, Bozorgnia+ 13, Fornasa & Green 14]

BUT

Eddington's formalism suffers from theoretical limitations :

- System of finite extension
- Non-physical solutions



[Fornasa & Green 14]

Finite extension

Finite extension R_{max} due to an other potential well nearby
(e.g. the Andromeda galaxy in the case of the Milky Way : R_{max} = 500 kpc)

Impacts the escape speed :

$$\begin{aligned} v_{\mathrm{esc}}(r) &= \sqrt{2\Psi(r)} \\ &= \sqrt{2(\phi(R_{\mathrm{max}}) - \phi(r))} \end{aligned}$$

Impacts the DF :

$$f(\mathcal{E}) = \frac{1}{\sqrt{8}\pi^2}\left[\frac{1}{\sqrt{\mathcal{E}}}\left(\frac{\mathrm{d}\rho}{\mathrm{d}\Psi}\right)_{\Psi=0} + \int_0^{\mathcal{E}} \frac{\mathrm{d}^2\rho}{\mathrm{d}\Psi^2}\frac{\mathrm{d}\Psi}{\sqrt{\mathcal{E}-\Psi}}\right]$$

Divergence at $\mathcal{E}$=0

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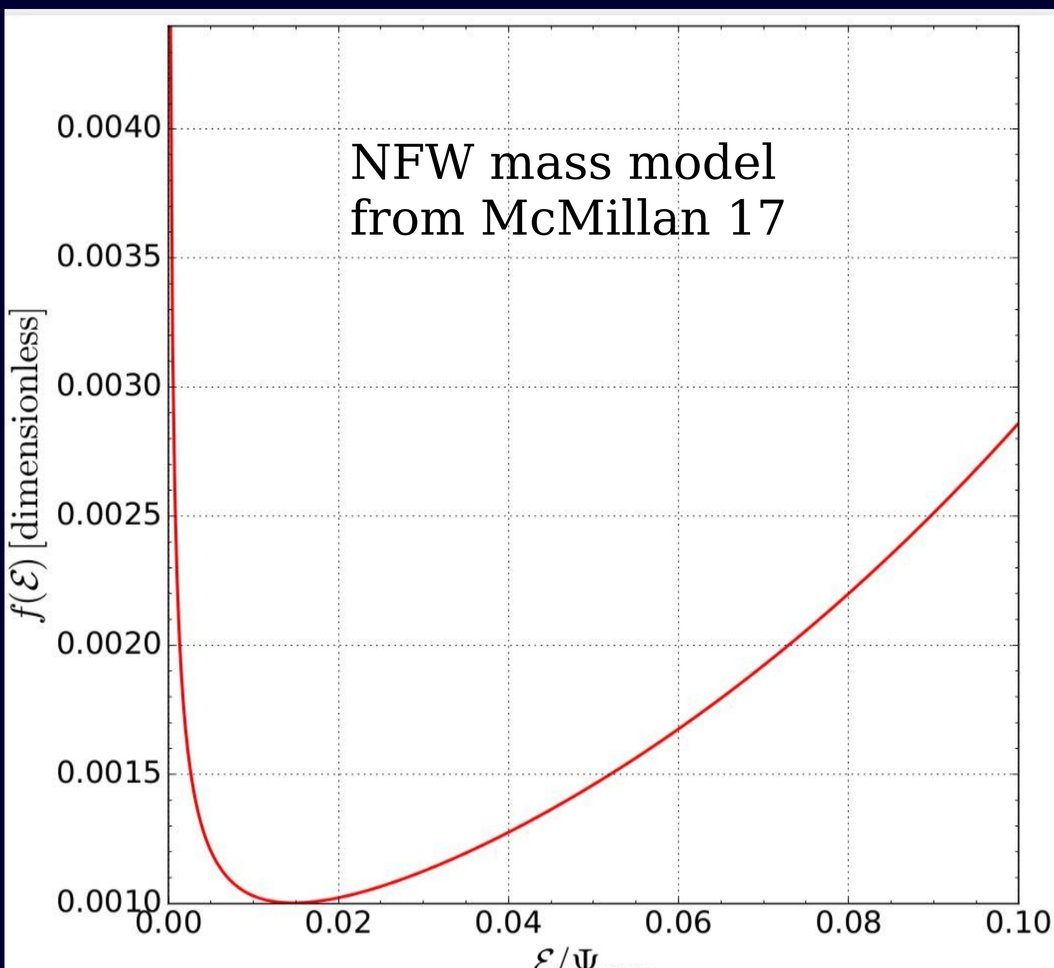
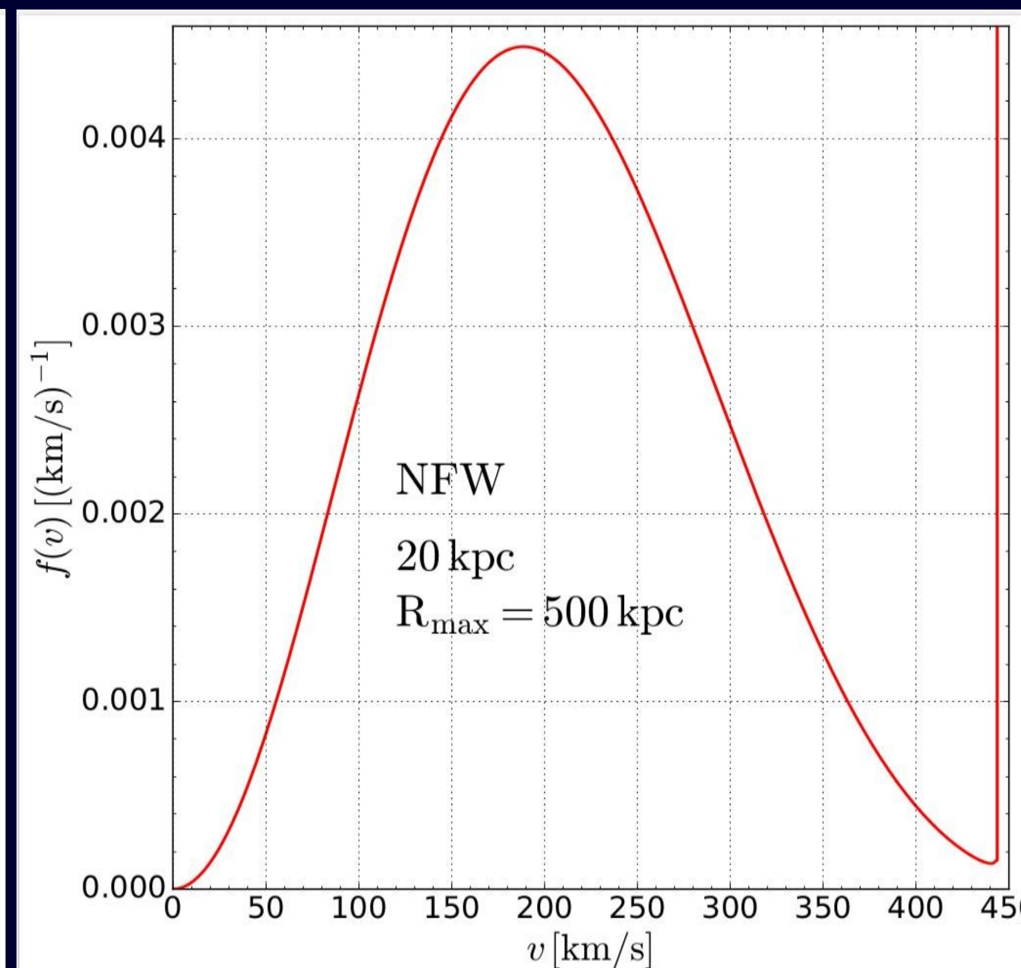
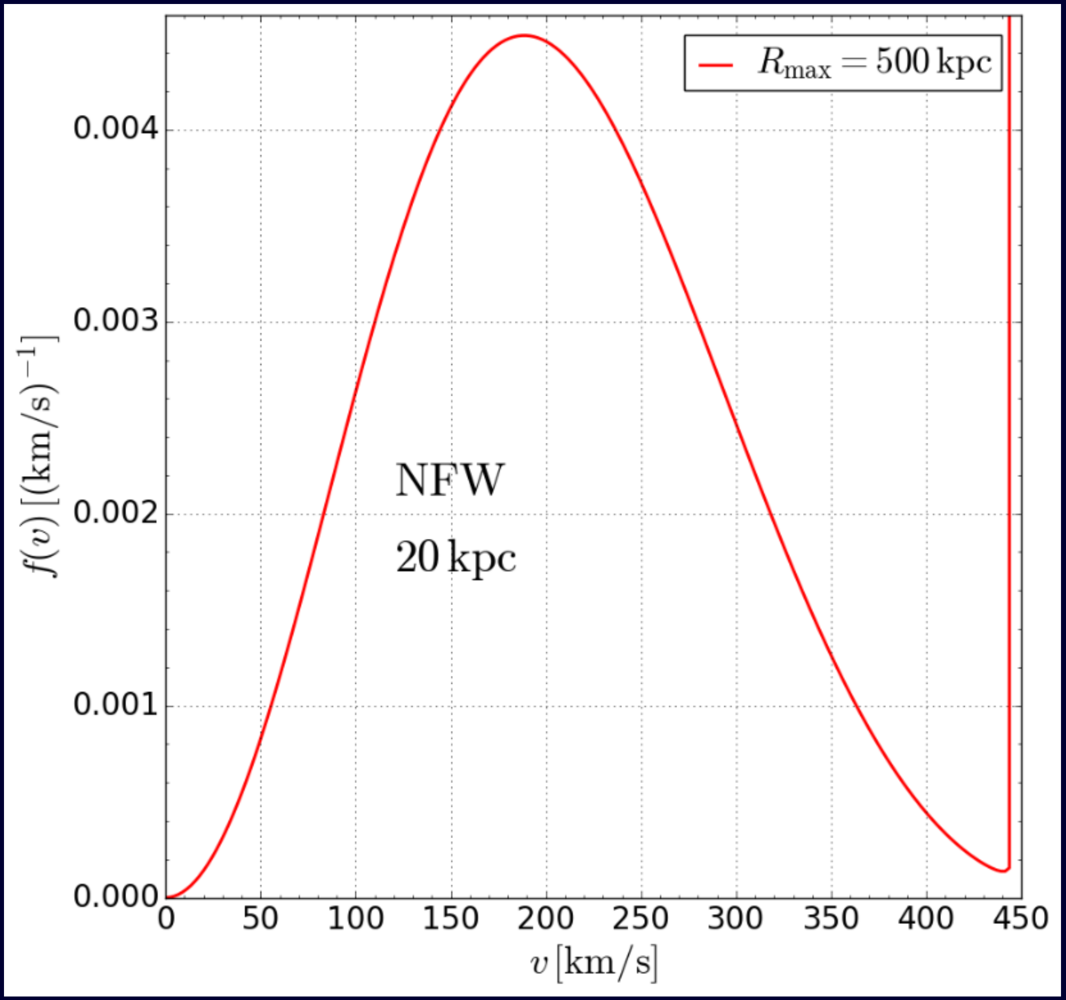




Treating the divergence

Two ways of dealing with the divergence :



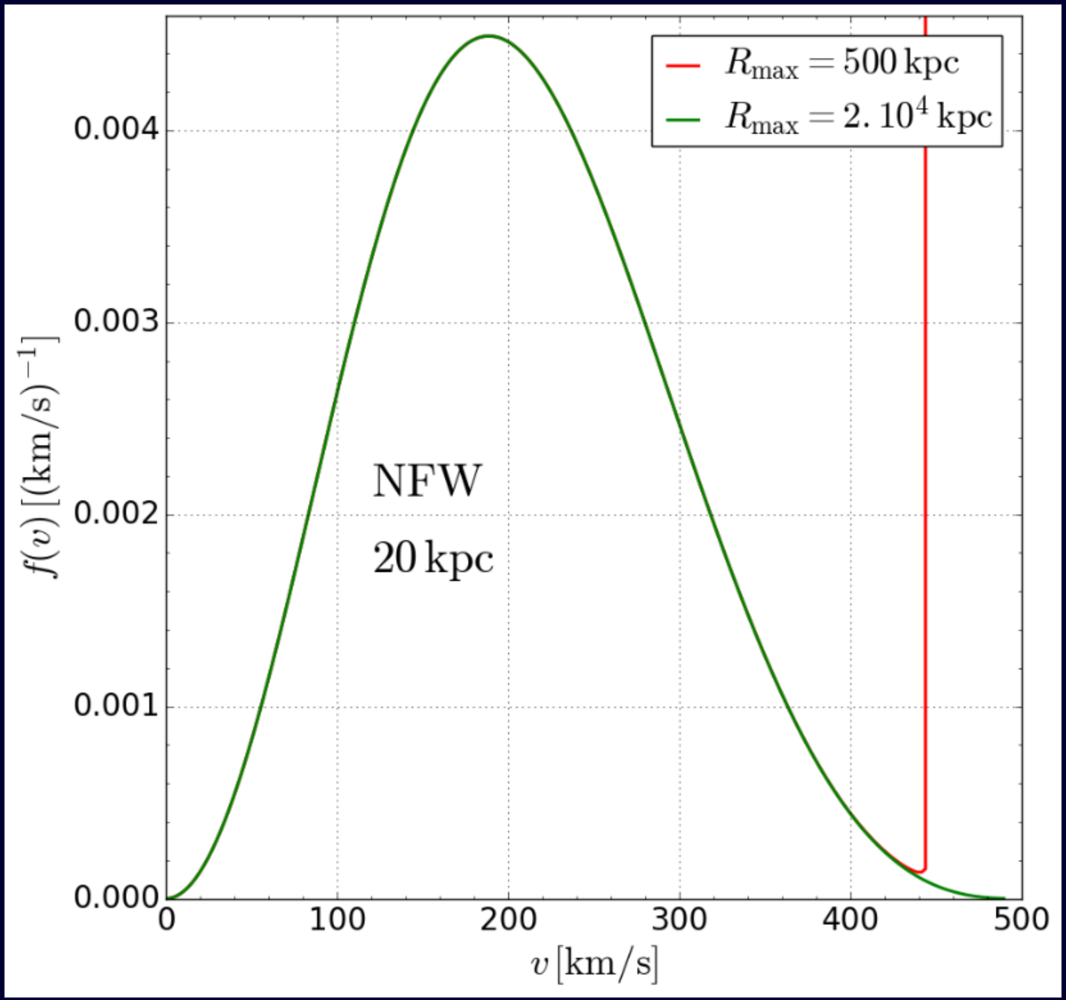
Treating the divergence

Two ways of dealing with the divergence :

- $R_{max} \longrightarrow \infty$

➡ increases the escape speed, which is a physical quantity

8% difference at 8 kpc
25% at 100 kpc



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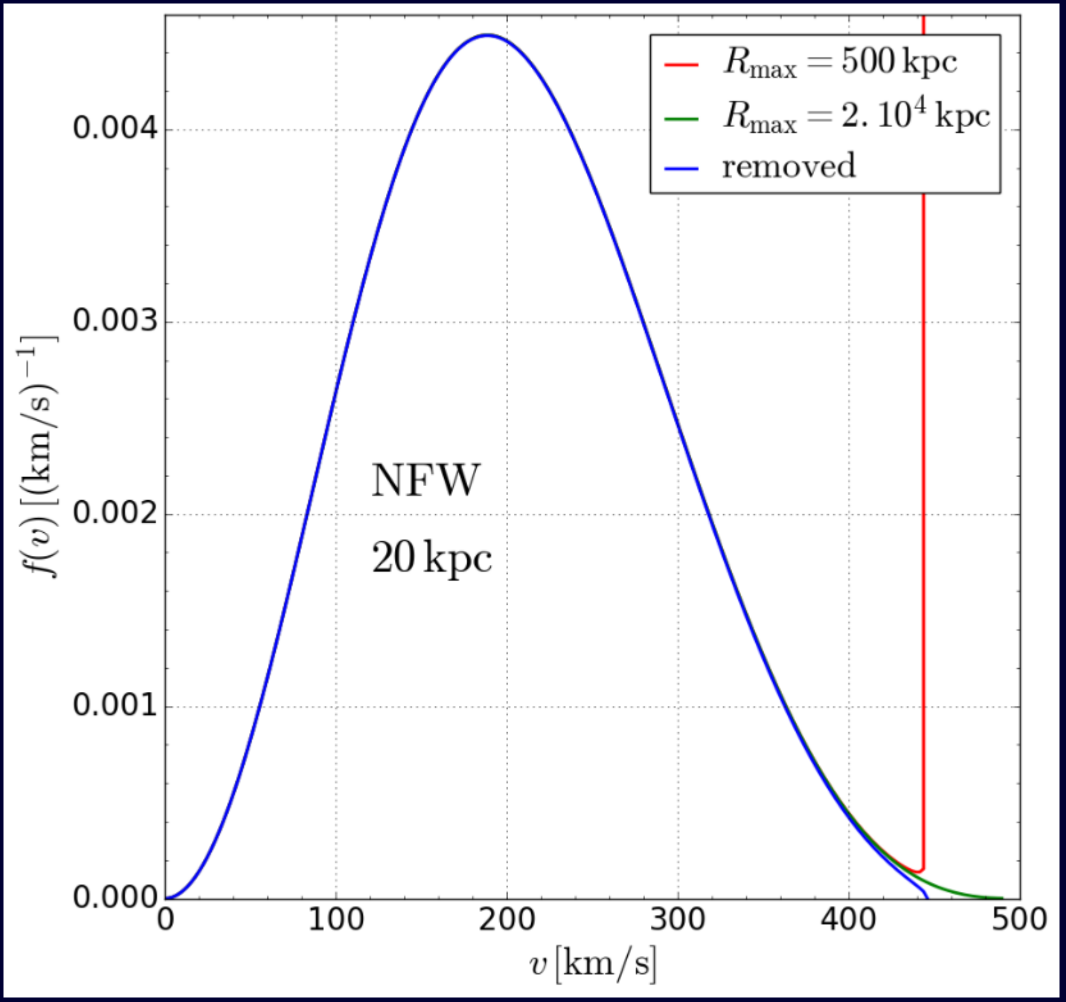
- Removing the divergence by hand

➡ Escape speed unchanged

➡ loss of self-consistency : the density profile is not fully reconstructed from the DF



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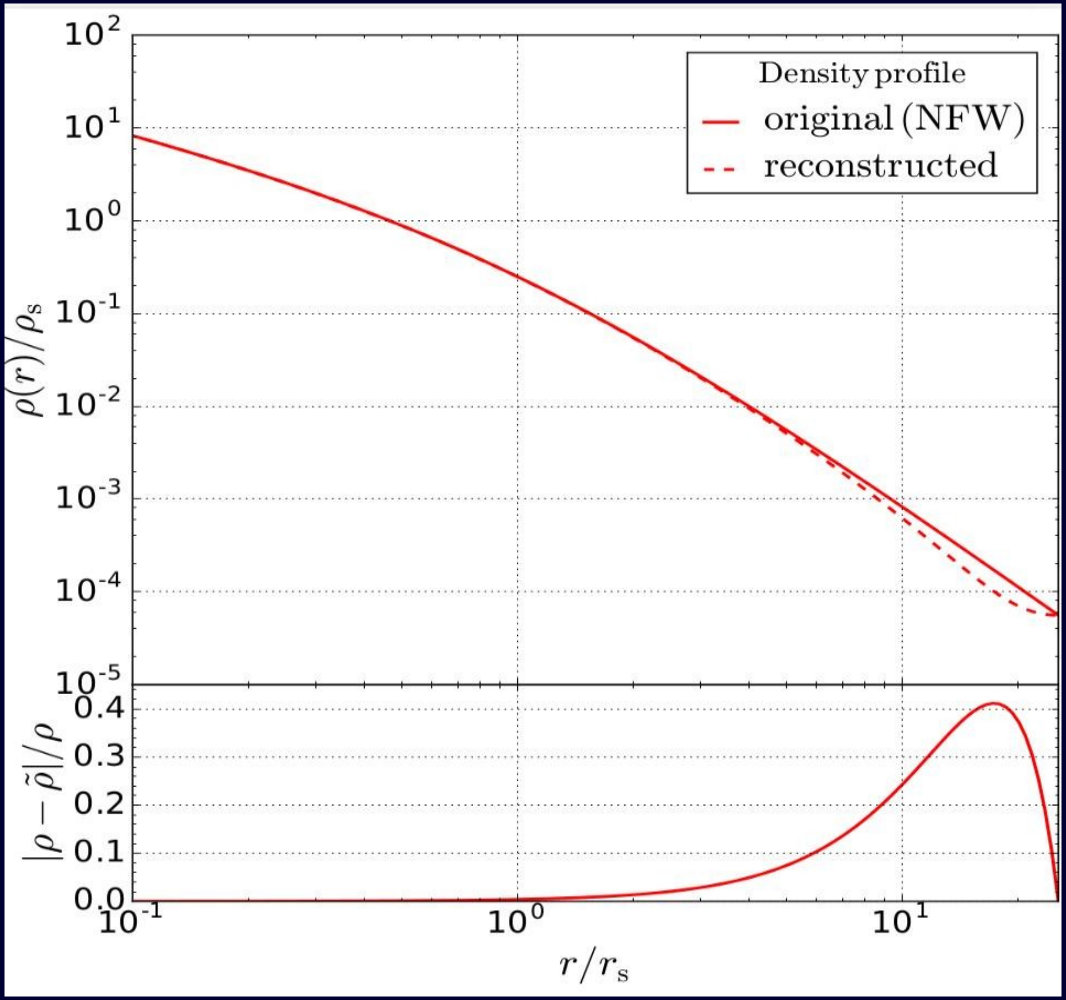


Positive distribution function

Trivial requirement for a good DF : $f(\vec{r}, \vec{v}) \geq 0$

No guarantee that Eddington’s method leads to a positive solution !

- Positivity condition derived for multi-component systems [Ciotti 96]

- Problems can arise for DM-only systems :

$$\frac{\mathrm{d}\rho}{\mathrm{d}\Psi} = \sqrt{8}\pi \int_0^{\Psi(r)} \frac{f(\mathcal{E})}{\sqrt{\Psi - \mathcal{E}}} \mathrm{d}\mathcal{E}$$

→ If the derivative cancels, the DF is negative

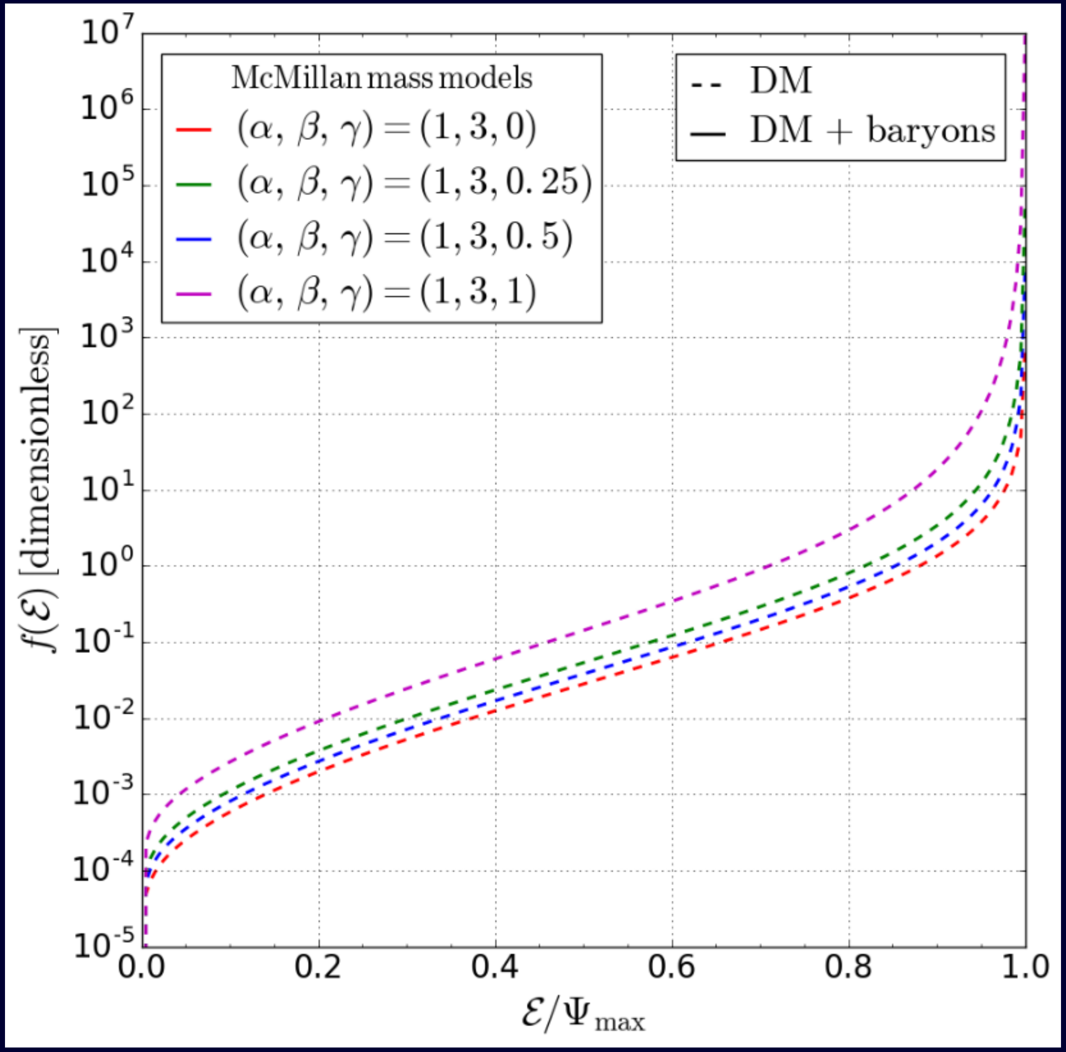
Case of a cored profile : $\rho(r) = \rho_\mathrm{s} \left[1 + \left(\frac{r}{r_\mathrm{s}}\right)^\alpha\right]^{-\beta/\alpha}$ needs $\alpha \leq 2$

- In general, the DF goes negative at large $\mathcal{E}$:
  central region of the halo (<< 8 kpc)

- Baryons always make things worse

Is positivity enough ?

NO ! Non-physical features can arise even if the DF is positive



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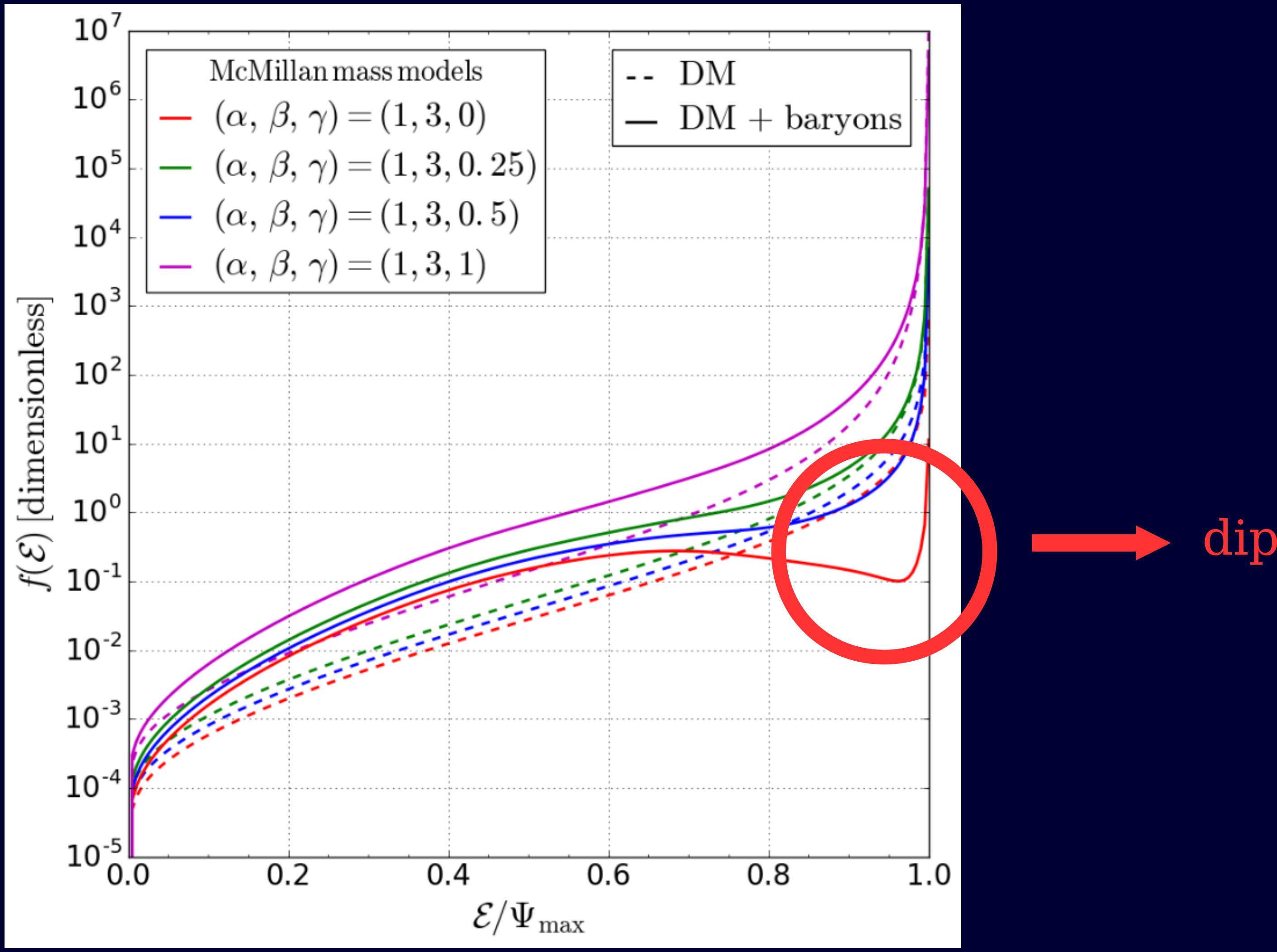

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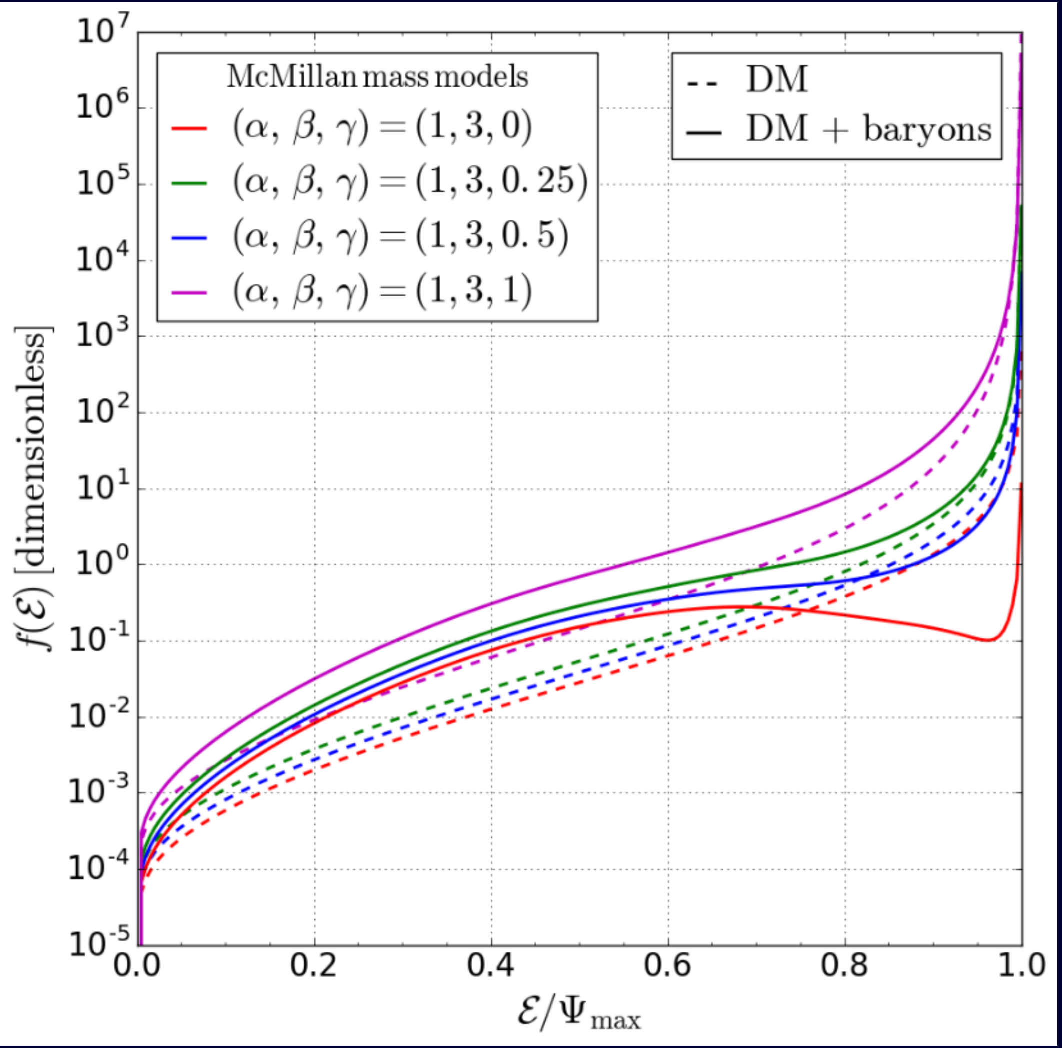


$$\frac{\mathrm{d}\rho}{\mathrm{d}\Psi} = 2\sqrt{8}\pi \int_0^{\Psi} \sqrt{\Psi - \mathcal{E}}\, \frac{\mathrm{d}f}{\mathrm{d}\mathcal{E}}\, \mathrm{d}\mathcal{E}$$

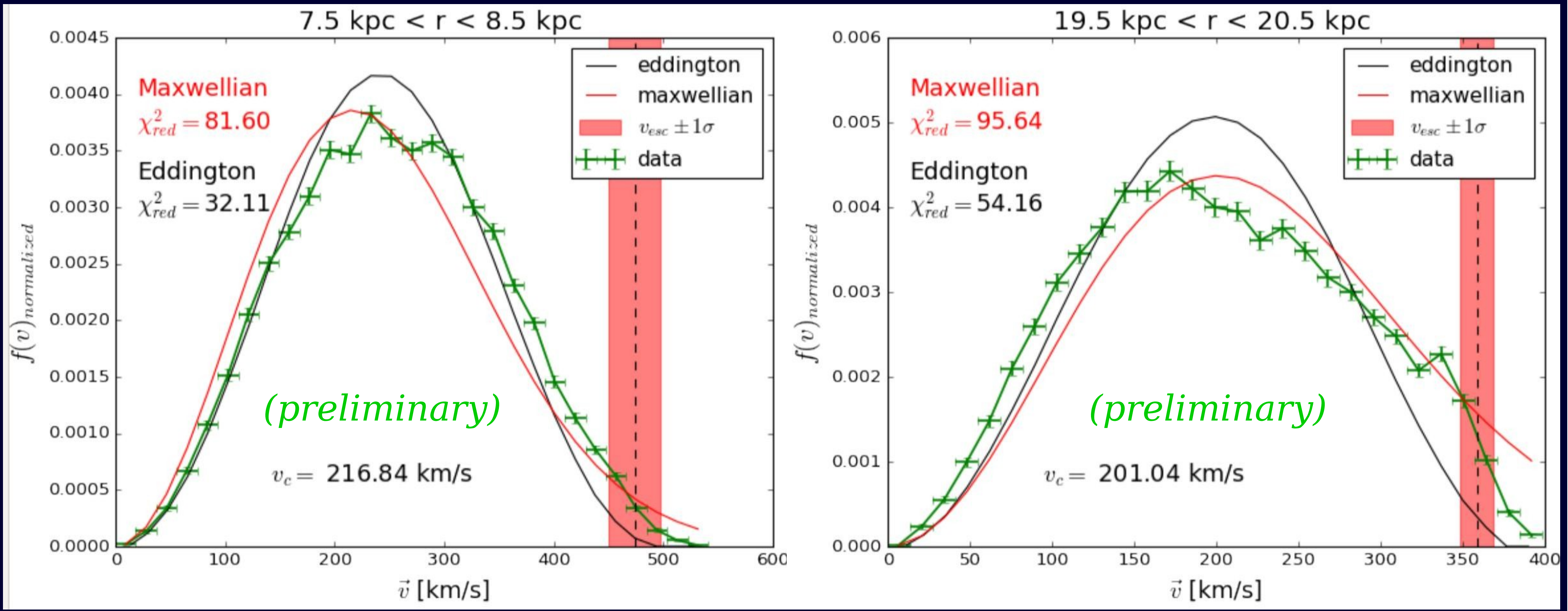
Criterion for physical solution :

$$\frac{\mathrm{d}f}{\mathrm{d}\mathcal{E}} \iff \frac{\mathrm{d}^2\rho}{\mathrm{d}\Psi^2} > 0$$

- DM-only systems pass the test (except the negative DF cases)
- Cored DM + baryons almost always fails the test

Comparison to simulation

Milky-Way-like halo from hydrodynamical simulation including baryons (see Mollitor+ 15 for simulation details)



8 kpc 20 kpc

Summary

- The Eddington formalism can be used to compute the DM phase-space from a dynamically constrained mass model
- Issues in the outer regions of DM halos (divergence) and in the central regions (negativity, dip), especially for cored profiles and multi-component systems
- Still, Eddington's method captures remarkably well the DM dynamics as observed in hydrodynamical simulations

Shameful self-promotion :

Impact of Galactic subhalos on indirect dark matter searches

with cosmic-ray antiprotons

on Friday, 17h15