FAILURES OF HOMOGENEOUS & ISOTROPIC COSMOLOGIES IN EXTENDED QUASI-DILATON MASSIVE GRAVITY (arXiv:1706.01872)

Saurabh Kumar,
Department of Physics,
Case Western Reserve University,
Cleveland, Ohio.

Co-authors: Stefano Anselmi (IAP & Observatoire de Paris),
Diana Lopez-Nacir (CERN),
Glenn Starkman (CWRU).

FROM MASSLESS (GR) TO MASSIVE GRAVITY (dRGT)

- *Einstein's GR is a **massless** spin-2 theory.*
- *Linearized GR + Mass Term (Fierz-Pauli Action):*

$$S = \int d^D x - \frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2).$$

  - *5 degrees of freedom (2 in GR).*
  - ***vDVZ Discontinuity**: Take m → 0, you can't recover GR!*

(van Dam & Veltman Nucl. Phys. B 1970, Zakharov JETP Lett. 1970)

- *Non-linear Kinetic but Linear Potential Term:*

$$S=\frac{1}{2\kappa^2}\int d^Dx\left[(\sqrt{-g}R)-\frac{1}{4}m^2\eta^{\mu\alpha}\eta^{\nu\beta}\left(h_{\mu\nu}h_{\alpha\beta}-h_{\mu\alpha}h_{\nu\beta}\right)\right]$$

  - *leads to* ***BD Ghost*** *(6th dof).* *(Boulware, Deser PRD 1972)*

- ***dRGT theory*** *is ghost free in all orders of interactions:*

$$S=\frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}[R-2\Lambda+2m_g^2\mathcal{L}_{\rm MG}],$$

(de Rham, Gabadadze & Tolley PRL 2011, Hassan & Rosen PRL 2012)

with $\mathcal{L}_{\rm MG}=\mathcal{L}_2+\alpha_3\mathcal{L}_3+\alpha_4\mathcal{L}_4$, where

$$\mathcal{L}_2=\frac{1}{2}([\mathcal{K}]^2-[\mathcal{K}^2]),$$

$$\mathcal{L}_3=\frac{1}{6}([\mathcal{K}]^3-3[\mathcal{K}][\mathcal{K}^2]+2[\mathcal{K}^3]),$$

$$\mathcal{L}_4=\frac{1}{24}([\mathcal{K}]^4-6[\mathcal{K}]^2[\mathcal{K}^2]+3[\mathcal{K}^2]^2+8[\mathcal{K}][\mathcal{K}^3]-6[\mathcal{K}^4]),$$

$$\mathcal{K}^\mu_\nu=\delta^\mu_\nu-(\sqrt{g^{-1}f})^\mu_\nu.$$

FLRW SOLUTIONS FROM dRGT

- *dRGT theory does not admit FLAT FLRW solutions!*
  (Guido D'Amico et.al. PRD 2011)
- ***Alternatives:***
  - *Bi-metric gravity,* *(Hassan and Rosen, JHEP 2012)*
  - *Graviton Mass a field,* *(Huang et.al, PRD 2012)*
  - *Background coupled to a Scalar Field:*
    - *Q-Dilaton Field (Quasi-Dilaton Massive Gravity (QDMG))*
      (Guido D'Amico et.al. PRD 2013, R. Gannouji et.al. PRD 2013)
    - *Extended Quasi-Dilaton Massive Gravity (EQDMG)*
      (Felice & Mukhoyama, Phys. Lett. B, 2014)

QDMG THEORY

$$S = S_{\rm EH} + S_\sigma$$

Non-linear Kinetic Term of Graviton

$$= \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\Big[R - \frac{\omega}{M_{\rm Pl}^2}\partial_\mu\sigma\partial^\mu\sigma$$

Kinetic Term of Quasi-Dilaton

$$+ 2m_g^2(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4)\Big],$$

Metric-Quasi-Dilaton Interaction

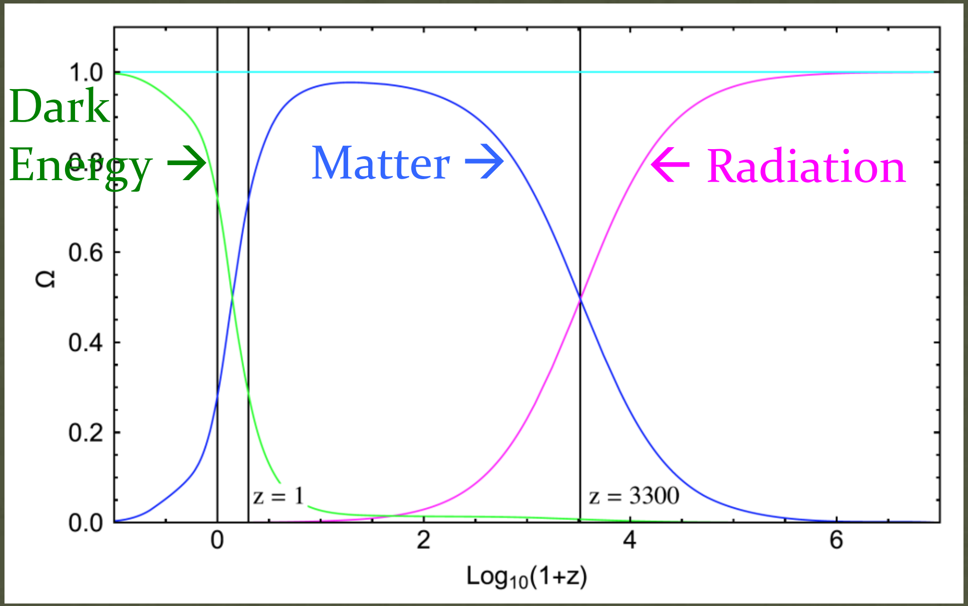
$$\mathcal{L}_2 = \frac{1}{2}([\mathcal{K}]^2 - [\mathcal{K}^2]),$$

$$\mathcal{L}_3 = \frac{1}{6}([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]),$$

$$\mathcal{L}_4 = \frac{1}{24}([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]).$$

$$\mathcal{K}^\mu{}_\nu \equiv \delta^\mu{}_\nu - e^{\sigma/M_{\rm Pl}}\left(\sqrt{g^{-1}f}\right)^\mu{}_\nu \qquad f_{\mu\nu} \equiv \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

- **SUCCESS**
  - *We reproduce observed expansion history.* *(Anselmi et.al. PRD 2015)*



- **FAILURE**
  - ***Pathology:*** *Scalar Perturbations are unstable:*
    - *Wrong sign kinetic term for large wavenumbers.* *(Gumrukcuoglu et. al. PRD 2013, Guido D'Amico et.al. PRD 2013)*

EXTENDED QDMG

- *Pathology can be treated by introducing a new coupling term to the fiducial metric.*

$$S = S_{\text{EH}} + S_\sigma$$
$$= \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} \Big[R - \frac{\omega}{M_{\text{Pl}}^2} \partial_\mu \sigma \partial^\mu \sigma + 2m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \Big],$$

Felice & Mukhoyama, Phys. Lett. B, 2014

$$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b - \frac{\alpha_\sigma}{M_{\text{Pl}}^2 m_g^2} e^{-2\sigma/M_{\text{Pl}}} \partial_\mu \sigma \partial_\nu \sigma.$$

EQDMG: A CLOSER LOOK (arXiv:1706.01872)

- ***BD GHOST***
  - *Is the ghost exorcised by construction?*
- ***FIXED-POINT ANALYSIS***

Copeland et.al. Int.J.Mod.Phys. 2006

  - ***LINEAR STABILITY***
    - *Dynamical variables (H, $\Omega_{m,r,DE}$) attain Fixed Points in future.*
    - *Fixed points are attractors.*
  - ***EVOLUTION***
    - *Numerical investigation of stability.*

I. KEY RESULTS (arXiv:1706.01872)

- *Fixed-point solutions are de Sitter.*
- *Standard stability analysis is inadequate because of divergences in the Jacobian. We propose an improved approach.*
- *Linear stability does not guarantee attainability of fixed-point solutions. There exists an unavoidable singularity in the theory as the universe evolves towards the fixed point.*

II. KEY RESULTS (arXiv:1706.01872)

- *BD ghost vanishes only at the fixed point.*
- *Away from the fixed point, BD ghost can only be avoided after an "awkward" fine tuning of the theory.*

This is supported by independent studies following ours.

(Golovnev et.al. arXiv:1706.07215, Gumrukcuoglu et.al. arXiv:1707.02004, Mukhoyama, arXiv:1309.2146v2)

CONCLUSIONS

- *dRGT theory doesn't allow flat FLRW solutions.*
  - *All homogeneous & isotropic solutions are unstable under perturbations. (Felice et.al., PRL 2012)*
  - *Existing alternatives (Bi-metric gravity, dRGT + Scalar Field) are problematic too. (Lagos & Ferreira, JCAP 2014)*
  - *Recent candidates (minimal theory of massive gravity (Felice et.al. arXiv:1701.01581), new quasi-dilaton theory (Mukhoyama, JCAP 2014)) claim to remedy the above problems but need further study.*
- *Despite a theoretical triumph, Massive Gravity is not doing well in phenomenology. Cosmic acceleration problem is still wide open.*