Zero-Range Effective Field Theory for Resonant Wino Dark Matter

EVAN JOHNSON - *OHIO STATE UNIVERSITY*

IN COLLABORATION WITH ERIC BRAATEN AND HONG ZHANG

Talk Outline

- Wino dark matter and annihilation
  - Nonperturbative electroweak interactions
- Bound-state effects
  - **Motivation:** Need to understand bound-state contributions to provide **tightest constraints** on dark matter models
- Zero-Range Effective Field Theory: a three part story
  - I. Framework [arXiv: 1706.02253]
  - II. Coulomb Resummation [arXiv 1708.xxxxx]
  - III. Annihilation effects
- Example bound-state formation calculation
- Conclusion and outlook

Wino WIMP dark matter

Motivation: ‘wimp miracle’

- A TeV scale weakly interacting particle naturally produces correct DM relic density

Fundamental theory is either:

- SM with one additional electroweak triplet: $\tilde{w} = \begin{pmatrix}\tilde{w}^+ & \tilde{w}^0 & \tilde{w}^-\end{pmatrix}$
- MSSM where the Lightest Supersymmetric Particle is a wino-like neutralino
- Either case: Refer to the DM particle as a ‘wino’

Wino masses:

- Neutral wino mass $M\sim$ few TeV, charged winos $M+\delta$
  - Radiative corrections give $\delta = 170$ MeV, insensitive to M

(Pierce et al. NPB 1997)

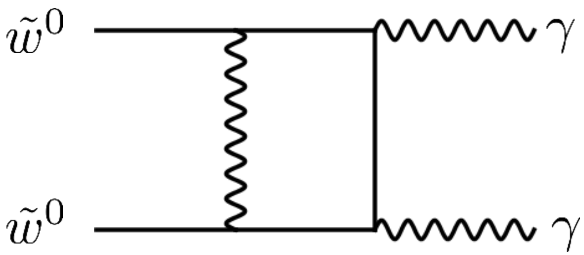
Wino interactions and nonperturbative effects

A pair of neutral winos can annihilate into a pair of electroweak gauge bosons

$$\left.\begin{aligned}\tilde{w}^0\tilde{w}^0 &\to Z^0Z^0 \\ &\to W^+W^-\end{aligned}\right\}\text{Continuous } \gamma\text{-ray and positron signals}$$

$$\left.\begin{aligned}\tilde{w}^0\tilde{w}^0 &\to \gamma\gamma \\ &\to \gamma Z^0\end{aligned}\right\}\text{Monochromatic } \gamma\text{-ray signals}$$

Leading-order (LO) annihilation cross-section for a pair of photons:



$$(v\sigma_{\mathrm{ann}})_{\mathrm{LO}} \sim \alpha^2\alpha_2^2/m_W^2$$

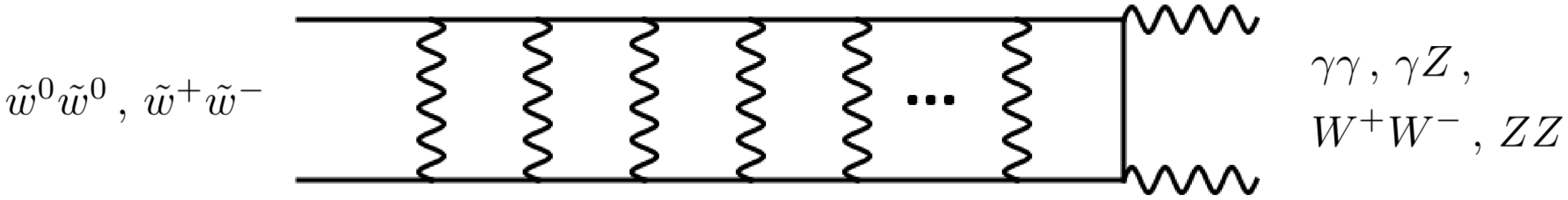
(Hisano et al. PRD 2005)

$(v\sigma_{\mathrm{ann}})_{\mathrm{LO}}$ **exceeds unitarity bound** $4\pi/vM^2$ for sufficiently large M

Higher order diagrams must be included to calculate the annihilation rate

Wino interactions and nonperturbative effects

Higher order diagrams for direct pair annihilation involve exchanges of EW gauge bosons:



Ladder diagrams must be summed to all orders to compute $v\sigma_{\mathrm{ann}}$

- Each 'rung' of the ladder gives a factor of $\alpha_2 M/m_W$ **(Hisano et al. PRD 2005)**
- For large enough M, $\alpha_2 M/m_W \sim 1$
- The annihilation cross sections receive enhancements: the "Sommerfeld enhancements"

Difficult to calculate in the fundamental field theory (summing up diagrams to all orders)

- Instead, calculate by solving a coupled-channel Schrödinger equation

Solving the Schrödinger equation for scattering

Numerically solve a coupled-channel Schrödinger equation:

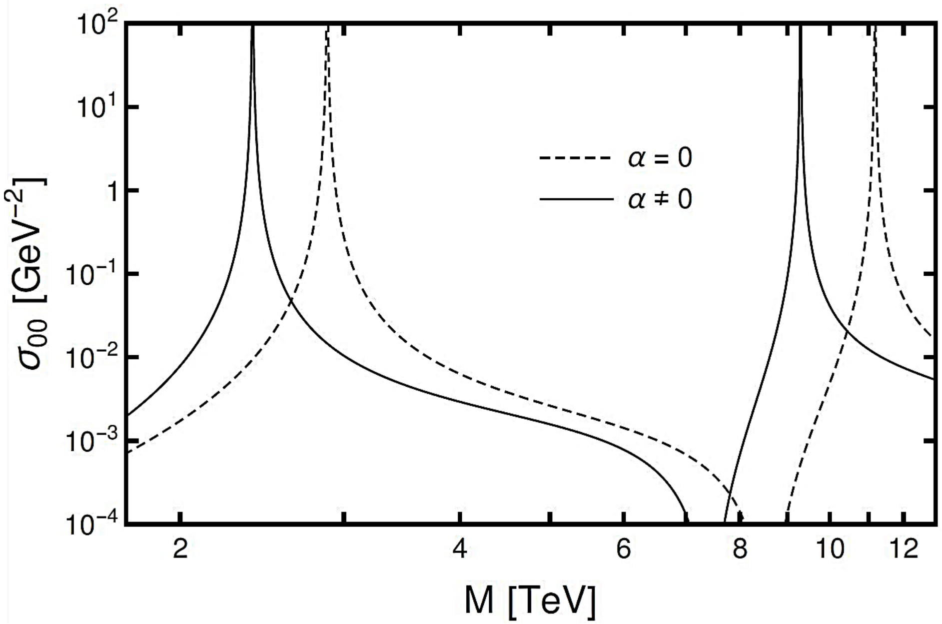
- Wino pairs $\tilde{w}^0\tilde{w}^0$ and $\tilde{w}^+\tilde{w}^-$ interact through the matrix potential

$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\,\alpha_2 \frac{e^{-m_W r}}{r} \\ -\sqrt{2}\,\alpha_2 \frac{e^{-m_W r}}{r} & -\frac{\alpha}{r} - \alpha_2 c_W^2 \frac{e^{-m_Z r}}{r} \end{pmatrix}$$

Channel	Threshold Energy
$\tilde{w}^0\tilde{w}^0$	0
$\tilde{w}^+\tilde{w}^-$	2δ

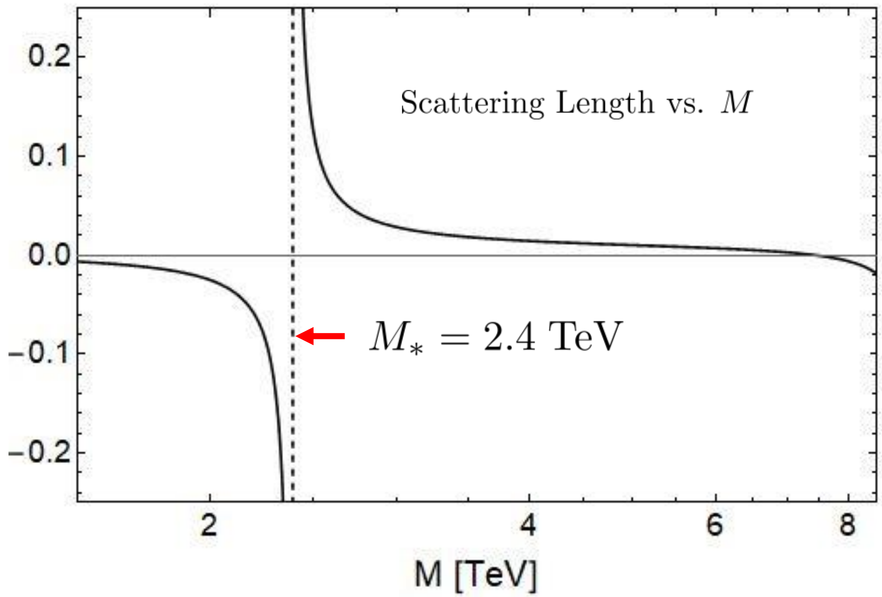
W , γ , Z exchange

- Sequence of critical masses where a resonance exists at the $\tilde{w}^0\tilde{w}^0$ scattering threshold
  - Cross section is resonantly enhanced: **resonant** Sommerfeld enhancement
  - First critical mass at 2.4 TeV with Coulomb potential included
  - Shifts to 2.9 TeV without the Coulomb potential



Resonant Wino DM and bound states

Resonant enhancement of the cross section can be understood in terms of a scattering length



For neutral wino **S-wave** scattering:

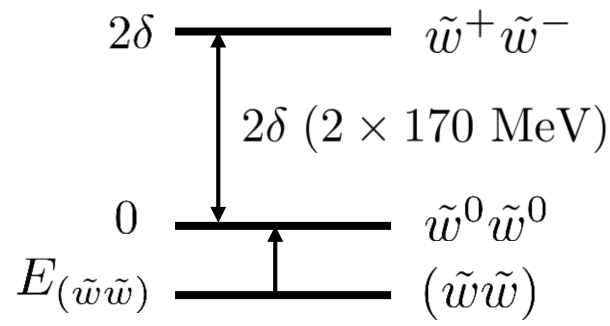
$$\tilde{w}^0\tilde{w}^0 \to \tilde{w}^0\tilde{w}^0$$

- Scattering length a_0 diverges at the critical mass M_*
- Cross section diverges as a_0^2 at small energies and saturates the unitarity bound

When $M > M_*$, the resonance is a real S-wave bound state, denoted $(\tilde{w}\tilde{w})$

- Binding momentum determined by inverse scattering length near the resonance
- Vanishes at the critical mass as $1/a_0 \to 0$

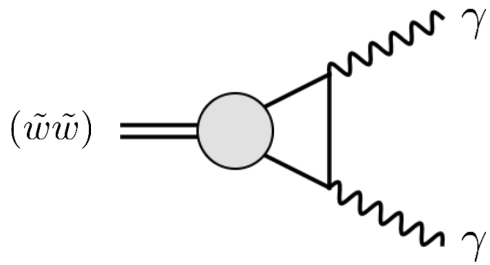
Relative energies:



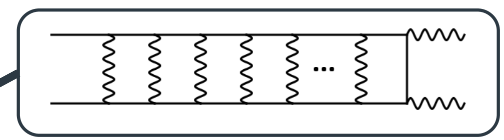
Bound state annihilation

Bound-state production can be followed by annihilation of the bound state into electroweak gauge bosons:

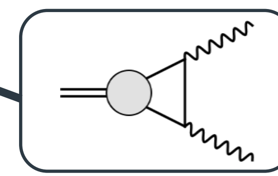
$$(\tilde{w}\tilde{w}) \;\rightarrow\; \gamma\gamma$$



Direct annihilation rates and bound state annihilation rates **add together:**



$$\sigma v[\tilde{w}^0\tilde{w}^0 \rightarrow \gamma\gamma]_{\rm total} = \sigma v[\tilde{w}^0\tilde{w}^0 \rightarrow \gamma\gamma]_{\rm direct} + \sigma v[\tilde{w}^0\tilde{w}^0 \rightarrow (\tilde{w}\tilde{w}) \rightarrow \gamma\gamma]$$



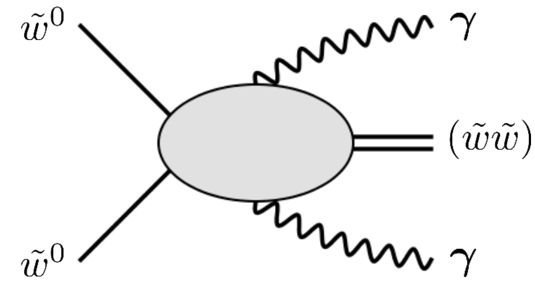
- Theoretical predictions for indirect detection signals are **enhanced** by additional production mechanisms
- May allow **tighter constraints** to be placed on models

Example bound-state formation mechanisms

S-wave bound states can form in neutral wino scattering:

- Through a double radiative transition:

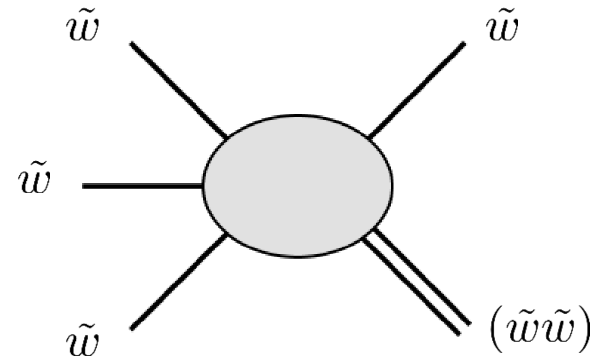
$$\tilde{w}^0\tilde{w}^0 \rightarrow (\tilde{w}\tilde{w}) + \gamma\gamma$$



(See Baumgart et al. 1610.07617, and P. Fitzpatrick talk (8/8) for single photon emission, p-wave case)

- Through three body recombination:

$$\tilde{w}\tilde{w}\tilde{w} \rightarrow (\tilde{w}\tilde{w}) + \tilde{w}$$



Calculations of the rates can be simplified by using a new tool:
Zero-Range Effective Field Theory

Zero-Range Effective Field Theory (ZREFT)

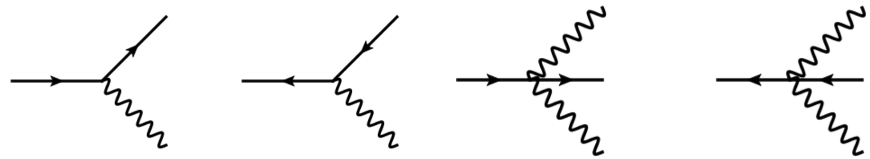
Lagrangian:

$$\mathcal{L} = \tilde{w}^{0\dagger}\left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M}\right)\tilde{w}^0 + \sum_{\pm} \tilde{w}^{\pm\dagger}\left(iD_0 + \frac{\boldsymbol{D}^2}{2M} - \delta\right)\tilde{w}^{\pm} + \mathcal{L}_{\text{EM}} + \mathcal{L}_{\text{zero-range}}$$

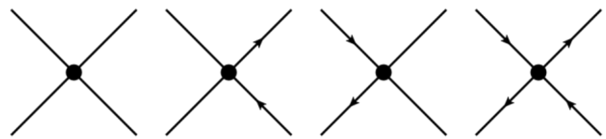
- Photon interactions arise from covariant derivatives for charged winos:

$$D_0\tilde{w}^{\pm} = (\partial_0 \pm ieA_0)\tilde{w}^{\pm}$$
$$\boldsymbol{D}\tilde{w}^{\pm} = (\boldsymbol{\nabla} \mp ie\boldsymbol{A})\tilde{w}^{\pm}$$

- Single and double photon vertices:

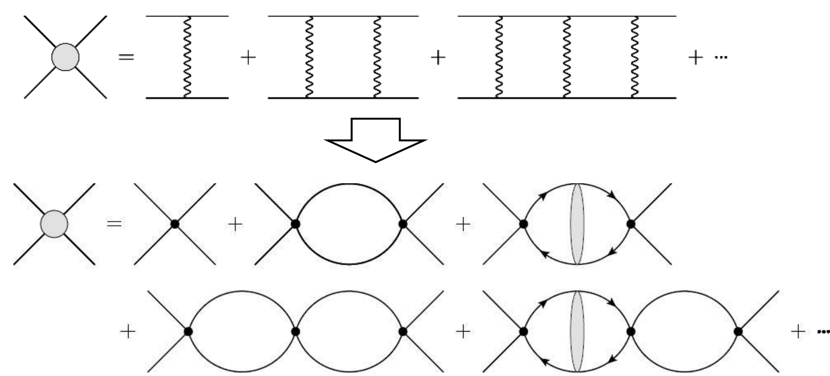


- Zero-range contact interactions for pairs of winos:



(Arrow on line: charged wino)

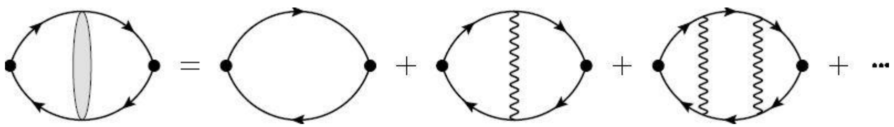
- Non-perturbative electroweak interactions reproduced by summing bubble diagrams to all orders:
- Must resum over any number of photons exchanged between intermediate charged winos



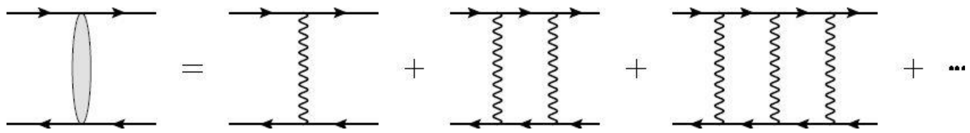
Coulomb resummation at a glance

When charged winos have relative momentum less than their inverse Bohr radius, αM, photon exchange diagrams are not suppressed by powers of α.

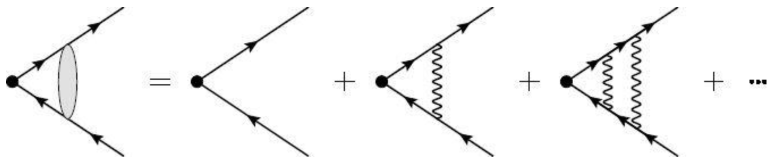
Intermediate charged winos can exchange any number of photons:



For charged-wino elastic scattering, there are additional ladder diagrams which must be summed to all orders:



Finally, incoming and outgoing charged winos can also exchange photons:



See upcoming paper II. Coulomb Resummation for much more details

The renormalization and power counting of the Zero-Range EFT is governed by its RG fixed points

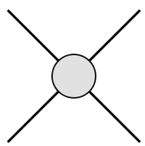
(Lensky and Birse, EPJ 2011)

- Three fixed points for a two-channel theory correspond to the number of fine-tuned parameters:
- 0, 1, or 2 resonances at the scattering threshold requiring 0, 1, or 2 fine-tunings

If only the wino mass M is tuned to its critical value M_*, expect a **single resonance** at the neutral-wino threshold

- Single resonance channel becomes a linear combination of $\tilde{w}^0 \, \tilde{w}^0$ and $\tilde{w}^+ \, \tilde{w}^-$ with mixing angle ϕ
- No scattering in the orthogonal channel

Analytic elastic scattering amplitude at leading order (LO):



$$= \boldsymbol{\mathcal{T}}(E) = \frac{8\pi \cos^2\phi / M}{-\gamma + K(E)\sin^2\phi - \boldsymbol{i}\sqrt{ME}\cos^2\phi}$$

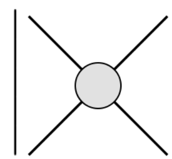
$K(E)$: Function of energy from Coulomb resummation

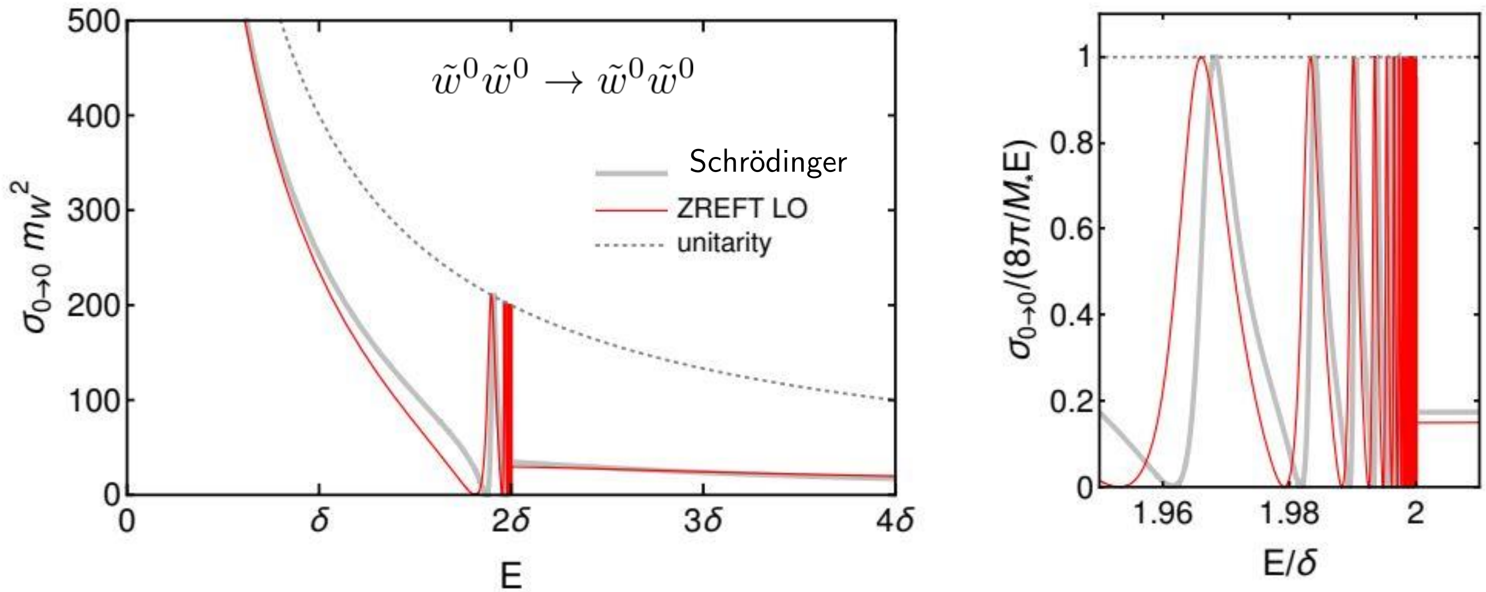
- Fine-tuning of physical quantity M becomes a fine-tuning of scattering parameter γ
- Saturate unitarity bound: Denominator of $\boldsymbol{\mathcal{T}}(E)$ must vanish at $E = 0$ when $M = M_*$
- Fixes $\gamma = K(0)\sin^2\phi$
- The ZREFT at LO has **one free parameter**: ϕ
- Matching analytic results to numerical results from solving the Schrödinger equation gives $\phi = 41°$

Predictions at Leading Order

Schrödinger equation solved numerically with potential $V(r) = \begin{pmatrix} 0 & -\sqrt{2}\,\alpha_2 \frac{e^{-m_W r}}{r} \\ -\sqrt{2}\,\alpha_2 \frac{e^{-m_W r}}{r} & -\frac{\alpha}{r} - \alpha_2 c_W^2 \frac{e^{-m_Z r}}{r} \end{pmatrix}$

ZREFT result obtained analytically at LO: $\sigma_{\rm el}(E) = \frac{M^2}{8\pi} \left| \;\cdot\; \right|^2 = \frac{M^2}{8\pi} |\boldsymbol{\mathcal{T}}(E)|^2$





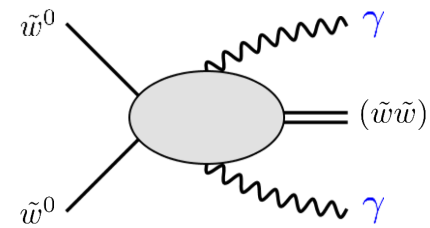
Quantitative and qualitative behavior reproduced by LO result with a **single parameter** $\phi = 41^\circ$

- Unitarity bound saturated as $E \to 0$
- Resonances at bound states in Coulomb potential below charged-wino threshold reproduced

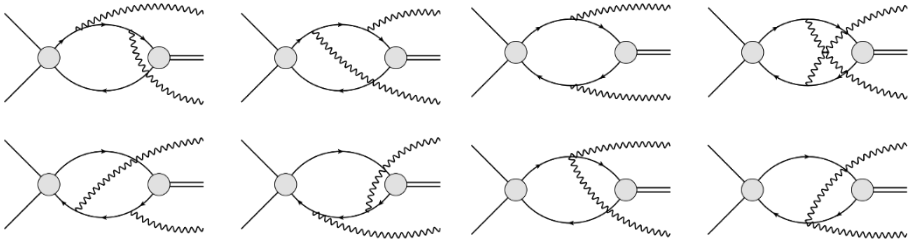
Example calculation in ZREFT: Production of wino-pair bound state via radiative transition

To form an **S-wave** bound state, the wino pair loses energy by radiating photons

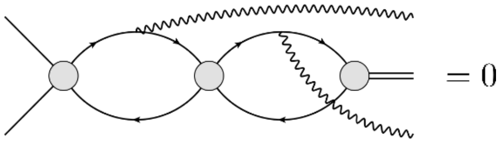
$\tilde{w}^0\tilde{w}^0 \rightarrow (\tilde{w}\tilde{w}) +$ soft photons



At leading order in α, there are eight contributing diagrams:



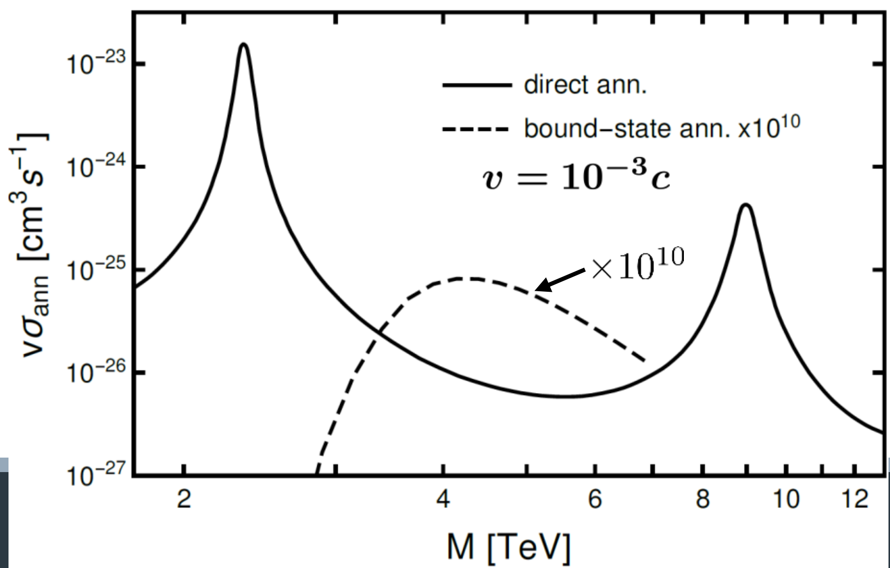
Eight more two-bubble diagrams with one photon attached to each bubble vanish by parity:



Analytic result from leading order ZREFT at small energies is relatively simple:

$$v\sigma_{\text{ann}} \approx \frac{\tan^4\phi\,\alpha^2\,M^2\,\hbar^3}{53760\,a_0\,\delta^5\,c^2}\,(E/Mc^2)^6$$

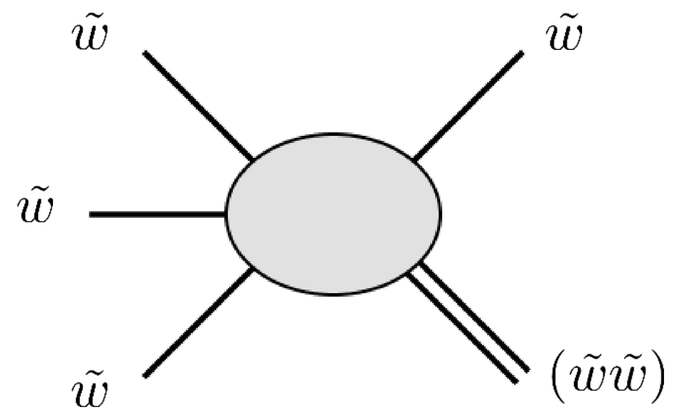
Highly suppressed compared to direct annihilation → Not relevant for indirect detection



Three body recombination

Annihilation rate of bound states formed through three body recombination (a number changing process) can be calculated in ZREFT

Qualitative properties known from cold atom physics: (Braaten, Hammer Phys. Rept. 2006)



$$\frac{\mathrm{d}n}{\mathrm{d}t} \sim K\, n^3 \quad , \quad K \sim \frac{1}{M^5}\langle v^{-4}\rangle$$

- n : wino number density
- K : rate constant

Scales as $\boldsymbol{v^{-4}}$ vs. $\boldsymbol{v^{-2}}$ for direct pair annihilation

- Could become more important at **small wimp velocity** such as in dwarf galaxies

Proportional to $\boldsymbol{n^3}$ vs $\boldsymbol{n^2}$ for direct pair annihilation

- Could become more important at **higher wimp density**, such as at centers of dark matter halos

Conclusion and outlook

Zero-Range EFT describes low energy wino **scattering very well**

- **Single parameter** at LO reproduces the results from conventional method of solving the Schrödinger equation
- Can be systematically improved with two more parameters at NLO

Annihilation of dark matter bound states provides an additional mechanism to produce a dark matter annihilation signal

- Can be difficult to calculate using the conventional methods
- ZREFT is a new tool that can be used to calculate bound-state formation rates
- ZREFT provides a framework to explore which bound state processes are of phenomenological interest to provide the strongest theoretical constraints on dark matter annihilation

In the future:

- Compute the three-body recombination rate in ZREFT
- Develop ZREFTs for other forms of dark matter, such as a higgsino-like wimp candidate or a general MSSM neutralino

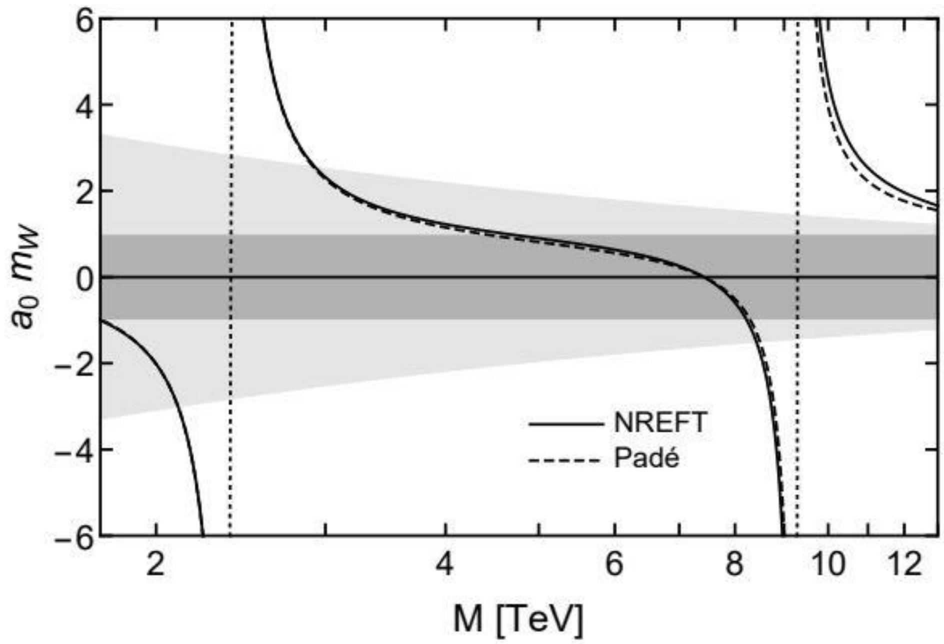
Thank you!

Backup slides

Range of validity of ZREFT

ZREFT is valid when the s-wave scattering length is large compared to the smaller of two scales:

- $1/m_W$, the range of the weak interactions (dark grey band)
- $1/\sqrt{2M\delta}$, the range associated with transitions between a pair of neutral and charged winos (light grey band)

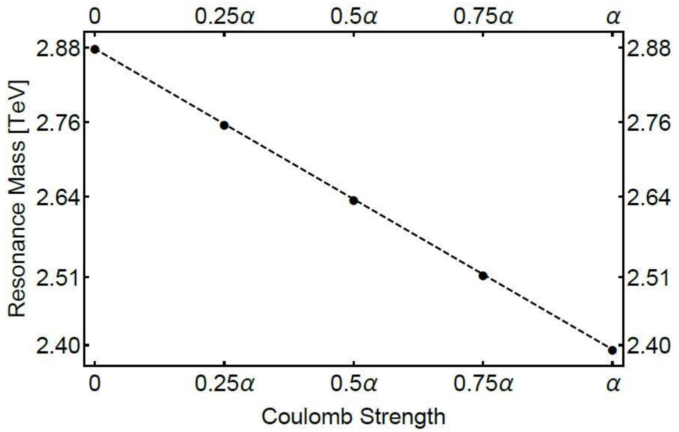


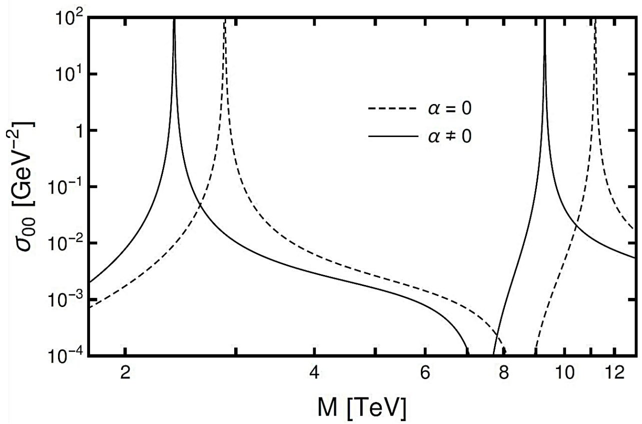
Why ignore the Coulomb potential?

$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\,\alpha_2 \frac{e^{-m_W r}}{r} \\ -\sqrt{2}\,\alpha_2 \frac{e^{-m_W r}}{r} & -\frac{\alpha}{r} - \alpha_2 c_W^2 \frac{e^{-m_Z r}}{r} \end{pmatrix}$$

Channel	Threshold Energy
$\tilde{w}^0\tilde{w}^0$	0
$\tilde{w}^+\tilde{w}^-$	2δ

Nonperturbative scattering dynamics create a resonance around 2.5 TeV. Location of resonance mass is linear in the strength of the Coulomb potential.





With Coulomb potential: $M_* = 2.4$ TeV

Without Coulomb potential: $M_* = 2.9$ TeV