Gravitational waves from bubble dynamics: Beyond the Envelope

Masahiro Takimoto (WIS, KEK)

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with Ryusuke Jinno (IBS-CTPU)

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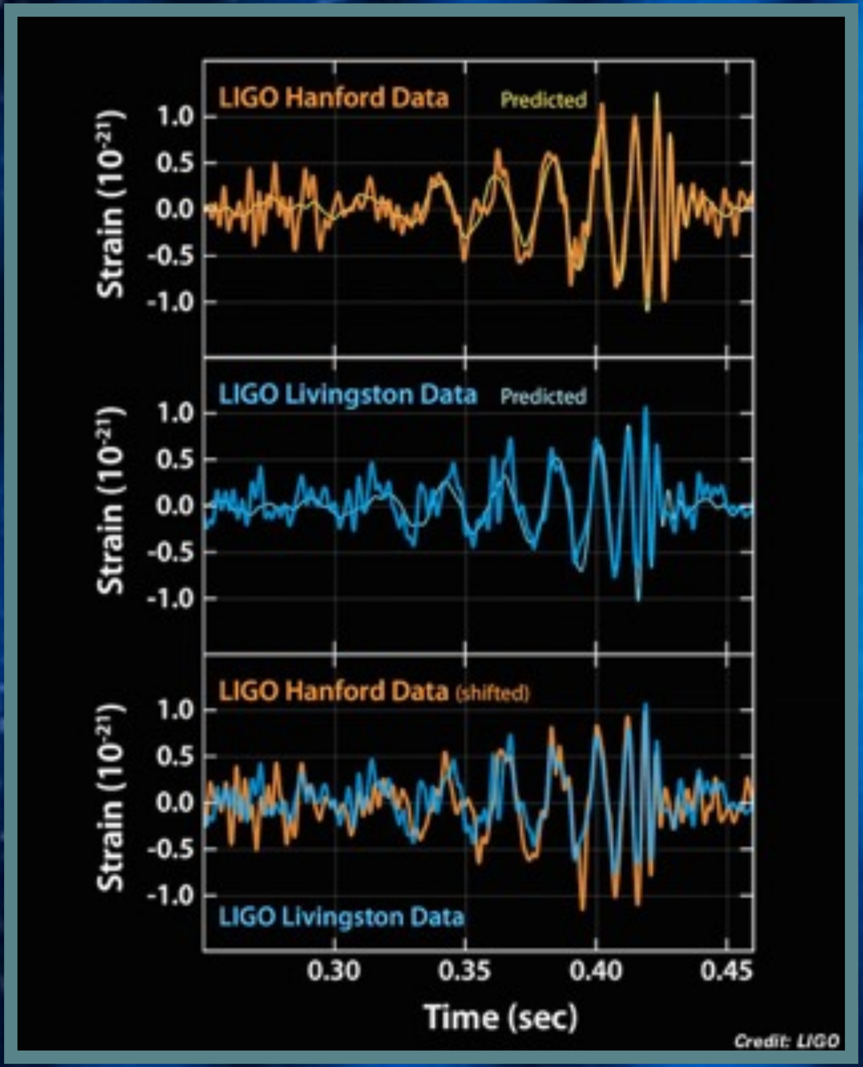
Introduction

ERA OF GRAVITATIONAL WAVES

- First detection of GWs from BH binaries → **GW astronomy** has started

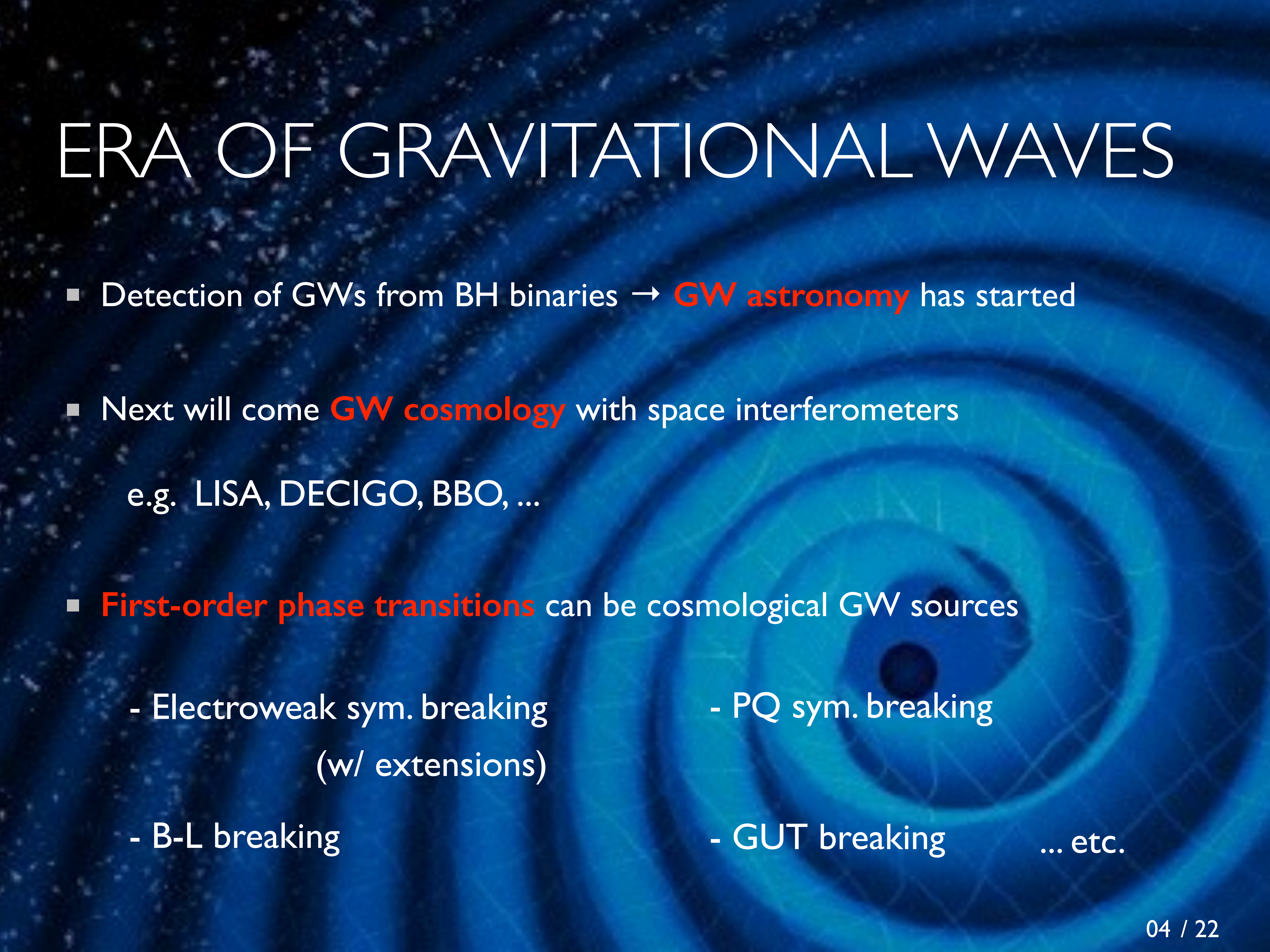


[LIGO]

- Black hole binary $36M_\odot + 29M_\odot \rightarrow 62M_\odot$

- Frequency ~ 35 to 250 Hz

- Significance > 5.1σ



ERA OF GRAVITATIONAL WAVES

- Detection of GWs from BH binaries → GW astronomy has started

- Next will come GW cosmology with space interferometers

  e.g. LISA, DECIGO, BBO, ...

- First-order phase transitions can be cosmological GW sources

  - Electroweak sym. breaking (w/ extensions)
  - PQ sym. breaking
  - B-L breaking
  - GUT breaking

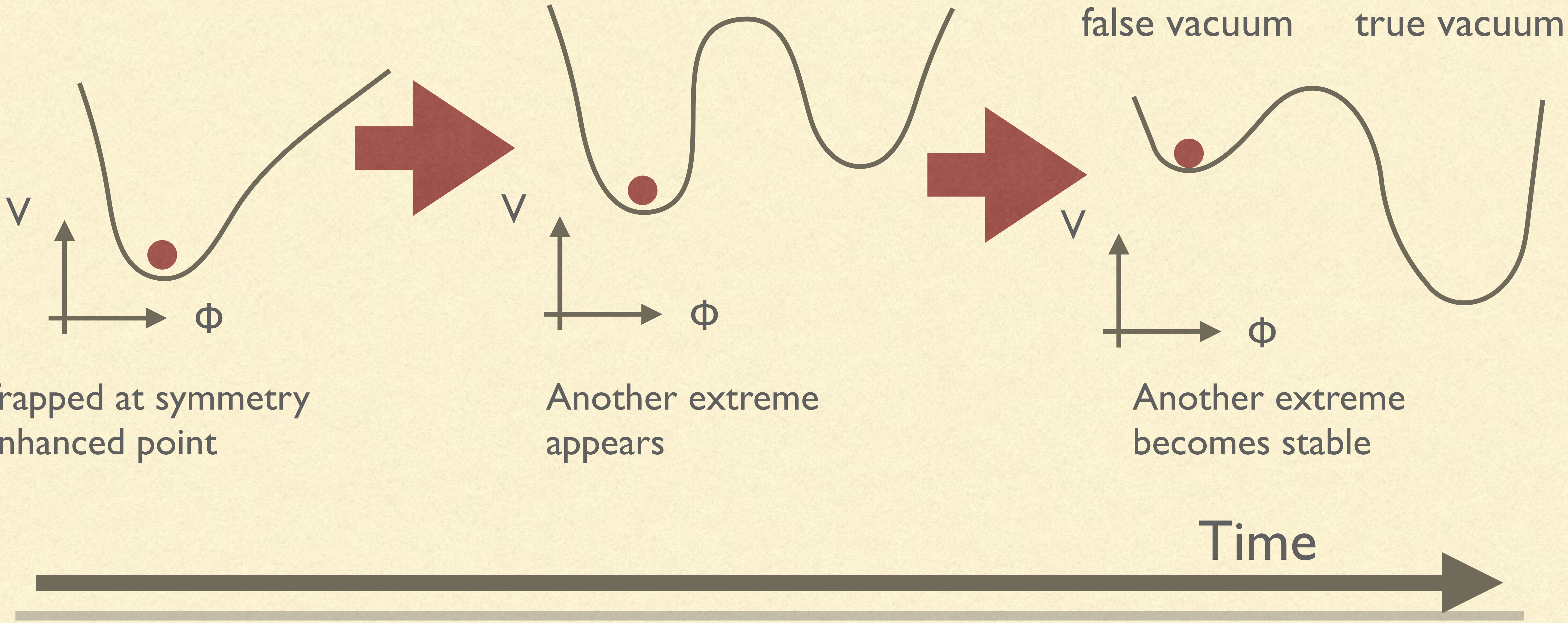
  ... etc.

GRAVITATIONAL WAVES AS A PROBE TO PHASE TRANSITION

- How (first order) phase transition occurs

- High temperature

- Low temperature



Trapped at symmetry enhanced point

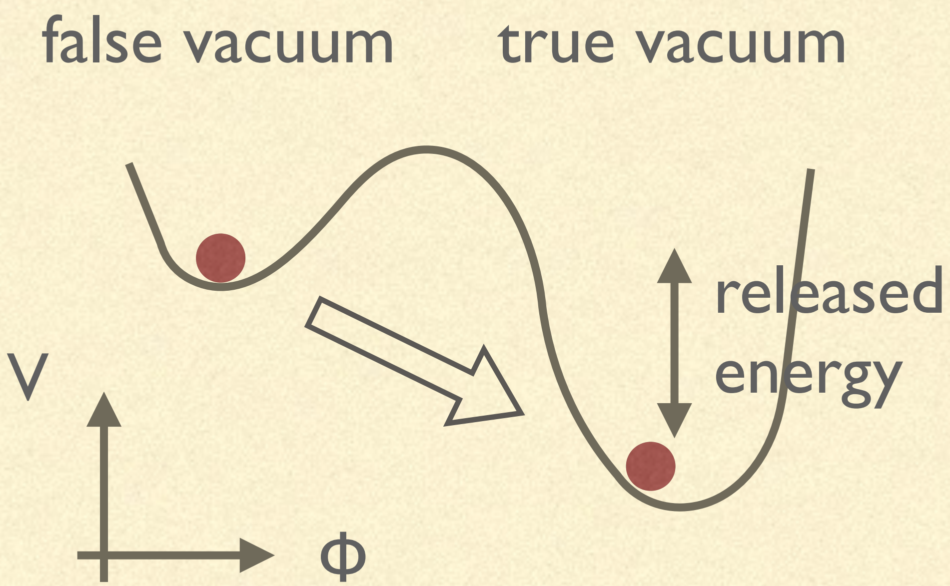
Another extreme appears

Another extreme becomes stable

GWS AS A PROBE TO PHASE TRANSITION

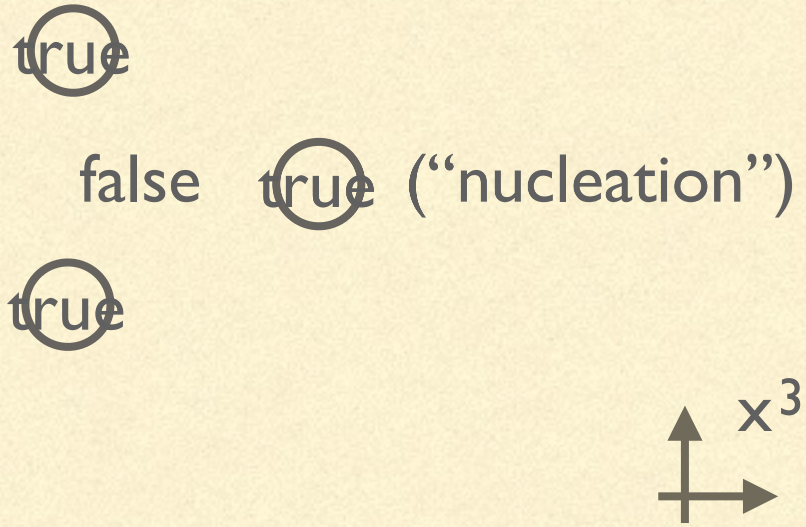
- Thermal first-order phase transition produces GWs in early universe

  - Field space



Quantum tunneling

  - Position space



Bubble formation & GW production

GWS AS A PROBE TO PHASE TRANSITION

- Thermal first-order phase transition produces GWs in early universe

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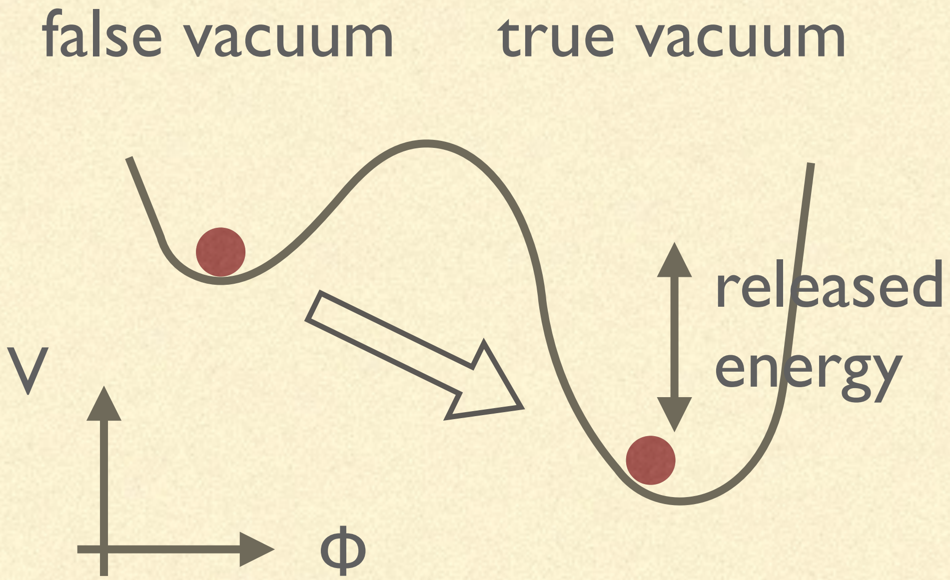
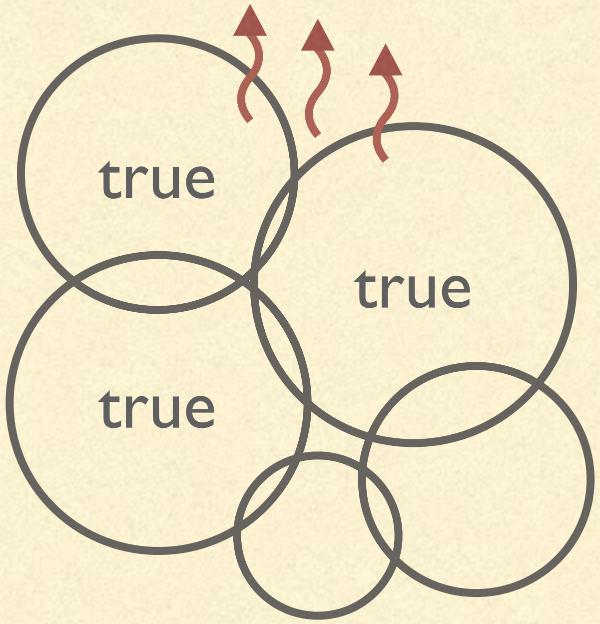


Quantum tunneling

- Position space

GWs $\Box h \sim T$



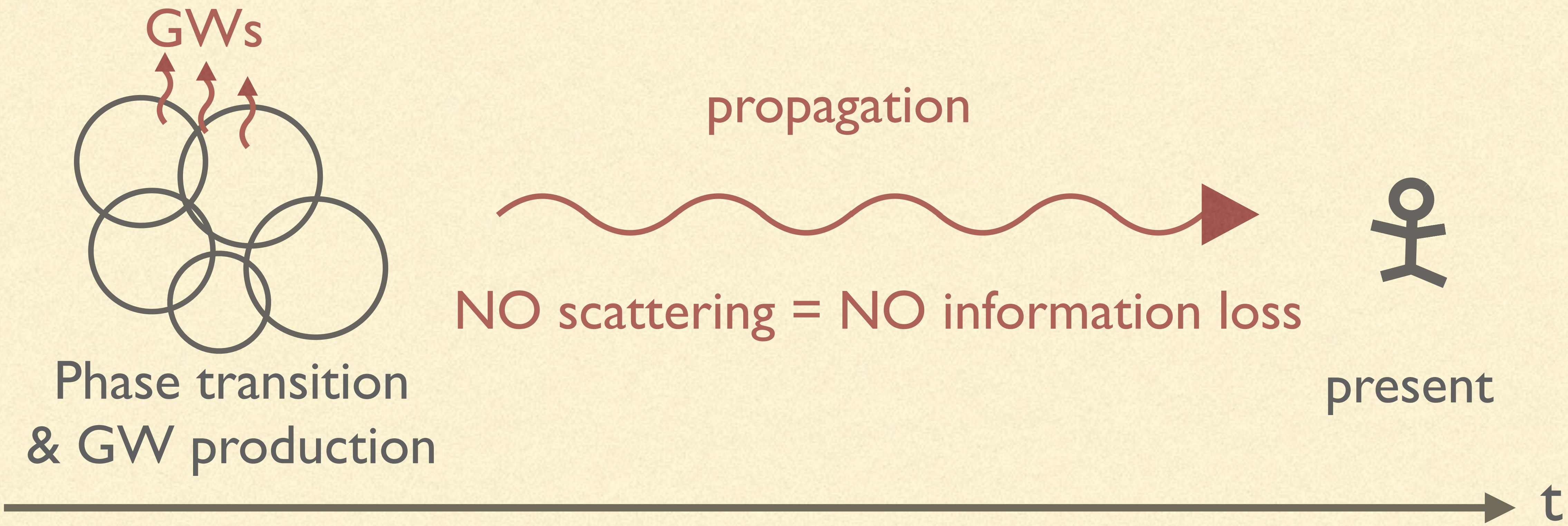
Bubble walls source GWs

Bubble formation & GW production

GRAVITATIONAL WAVES AS A PROBE TO PHASE TRANSITION

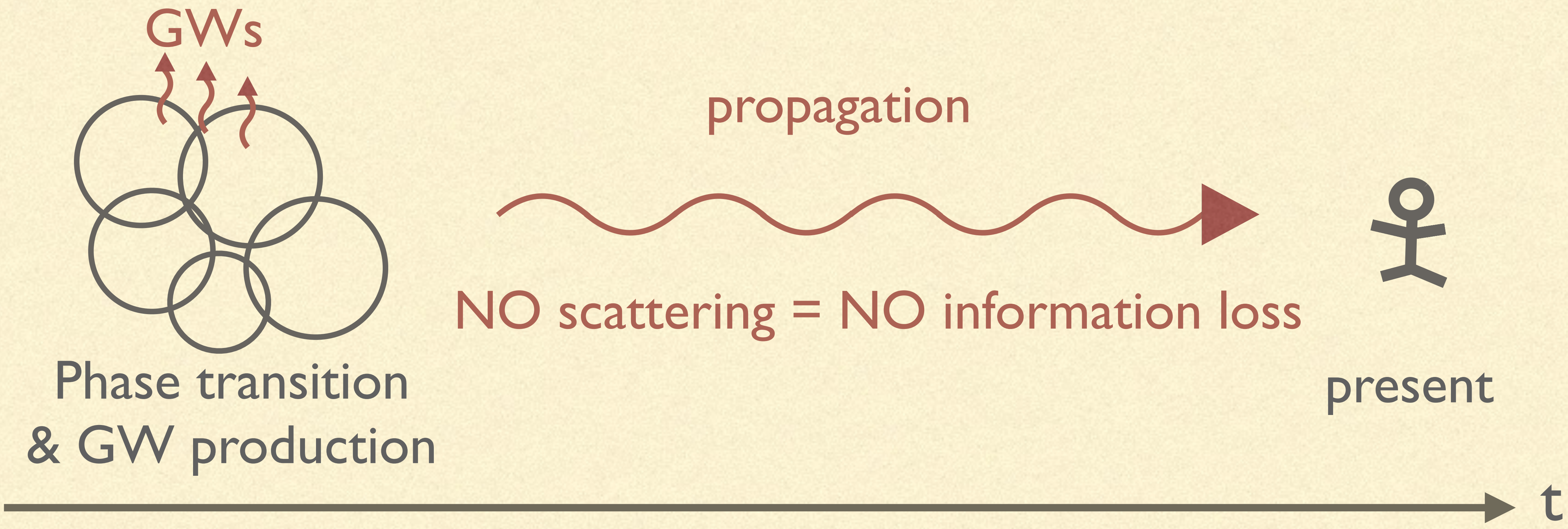
- GWs propagates until the present without losing information

  because of the Planck-suppressed interaction of gravitions



GRAVITATIONAL WAVES AS A PROBE TO PHASE TRANSITION

GWs can be a unique probe
to unknown high-energy particle physics



GWs

propagation

NO scattering = NO information loss

Phase transition
& GW production

present

t

ROUGH SKETCH OF PHASE TRANSITION & GW PRODUCTION

- GWs behave as non-interacting radiation after production

frequency : $f \propto a^{-1}$ (redshift) energy density : $\rho_{\rm GW} \propto a^{-4}$

- Frequency & energy scale correspondence

Present GW frequency $\rightleftarrows$ Energy scale of the Universe @ GW production time

$$f_0 \sim 1 \text{ Hz} \times \left(\frac{\beta}{H}\right)\left(\frac{T}{10^7 \text{ GeV}}\right) \qquad \frac{\beta}{H} \sim \mathcal{O}(10^{1-4})$$

Note: Planed space interferometers are sensitive to (sub) Hz GWs.

→They are sensitive
to new physics around TeV-PeV range!!

TALK PLAN

✓ 0. Introduction

1. Analytic derivation of the GW spectrum

2. Result

3. Conclusion

1. Analytic derivation of GW spectrum

2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- The essence : GW spectrum is determined by

$$\langle T_{ij}(t_x,\mathbf{x})T_{kl}(t_y,\mathbf{y})\rangle_{\mathrm{ens}}$$

$$\begin{tikzcd} \arrow[d, phantom, "\shortparallel" description] \\ \end{tikzcd}$$

Ensemble average

[Caprini et al. '08]

2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- The essence : GW spectrum is determined by $\langle T_{ij}(t_x, \mathbf{x}) T_{kl}(t_y, \mathbf{y}) \rangle_{\mathrm{ens}}$

  - Why ?   Note : indices omitted below

Formal solution of EOM :

$$\Box h \sim T \quad \rightarrow \quad h \sim \int^t dt' \, \mathrm{Green}(t, t') T(t')$$

Energy density of GWs (~ GW spectrum) :

$$\rho_{\mathrm{GW}}(t) \sim \frac{\langle \dot{h}^2 \rangle_{\mathrm{ens}}}{8\pi G} \sim \int^t dt_x \int^t dt_y \cos(k(t_x - t_y)) \boxed{\langle TT \rangle_{\mathrm{ens}}}$$

same as
massless scalar field

substitute
the formal solution

Note : ensemble average
because of the stochasticity
of the bubbles

2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- The essence : GW spectrum is determined by $\langle T_{ij}(t_x,\mathbf{x})T_{kl}(t_y,\mathbf{y})\rangle_{\mathrm{ens}}$

→We have to specify energy momentum tensor of the system.

In thermal phase transition, following 3 ingredients are important.

1. Space-time distribution of the bubbles (nucleation points).

   →Determined by transition rate $\Gamma(t) \sim \Gamma_* e^{-\beta t}$, β : model dependent parameter

2. Energy momentum profile around bubble.

   →We adopt thin wall approximation. Good for low frequencies.

3. Dynamics after bubble collision.

Envelope approximation is often applied.
[Kosowsky, Turner, Watkins, PRD45 ('92)]

Beyond envelope!

[Hindmarsh et al. '14]

*Late time GW enhancement was suggested.

*We did a robust estimate of GW spectrum assuming free propagation!

2. ANALYTIC DERIVATION OF THE GW SPECTRUM

- Estimate of the ensemble average $\langle T(t_x,\mathbf{x})T(t_y,\mathbf{y})\rangle_{\mathrm{ens}}$

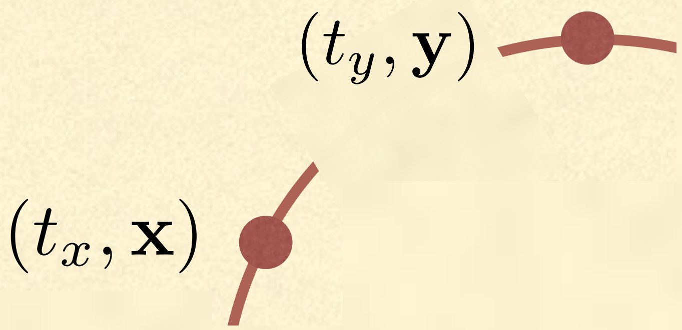
  Note: We have fixed behaviour of energy momentum tensor of the system.

  - Trivial from the definition of ensemble average

(P) Probability part (V) Value part

$$\langle T(t_x,\mathbf{x})T(t_y,\mathbf{y})\rangle_{\mathrm{ens}} = \sum \left(\begin{array}{c}\text{Probability for}\\ T(t_x,\mathbf{x})T(t_y,\mathbf{y}) \neq 0\end{array}\right) \times \left(\begin{array}{c}\text{Value of}\\ T(t_x,\mathbf{x})T(t_y,\mathbf{y})\\ \text{in that case}\end{array}\right)$$

$$\left(\begin{array}{c}\text{Probability for}\\ T(t_x,\mathbf{x})T(t_y,\mathbf{y}) \neq 0\end{array}\right) = \left(\begin{array}{l}\text{Probability that}\\ \text{bubble walls are}\\ \text{passing through}\\ (t_x,\mathbf{x})\&(t_y,\mathbf{y})\end{array}\right)$$



FULL EXPRESSIONS

- Full expression reduces to only ~10-dim. integration [Jinno & Takimoto ‘17]

  A part of the result↓. Calculation is tedious but straightforward.

$(\Omega_{\rm GW} \propto \Delta)$

$$
\begin{aligned}
\Delta^{(d)} = &\int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \\
&\int_0^{\infty} dr \int_{-\infty}^{t_x} dt_{xn} \int_{-\infty}^{t_y} dt_{yn} \int_{t_{xn}}^{t_x} dt_{xi} \int_{t_{yn}}^{t_y} dt_{yi} \int_{-1}^{1} dc_{xn} \int_{-1}^{1} dc_{yn} \int_0^{2\pi} d\phi_{xn,yn} \\
&\frac{k^3}{3} \left[\begin{aligned} &\Theta_{\rm sp}(x_i, y_n)\Theta_{\rm sp}(x_n, y_i) e^{-I(x_i,y_i)} \Gamma(t_{xn})\Gamma(t_{yn}) \\ &\times r^2 \left[j_0(kr)\mathcal{K}_0(n_{xn}, n_{yn}) + \frac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn}, n_{yn}) + \frac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn}, n_{yn}) \right] \\ &\times \partial_{txi} \left[r_B(t_{xi}, t_{xn})^3 D(t_x, t_{xi}) \right] \partial_{tyi} \left[r_B(t_{yi}, t_{yn})^3 D(t_y, t_{yi}) \right] \cos(k t_{x,y}) \end{aligned} \right]
\end{aligned}
$$

TALK PLAN

✓ 0. Introduction

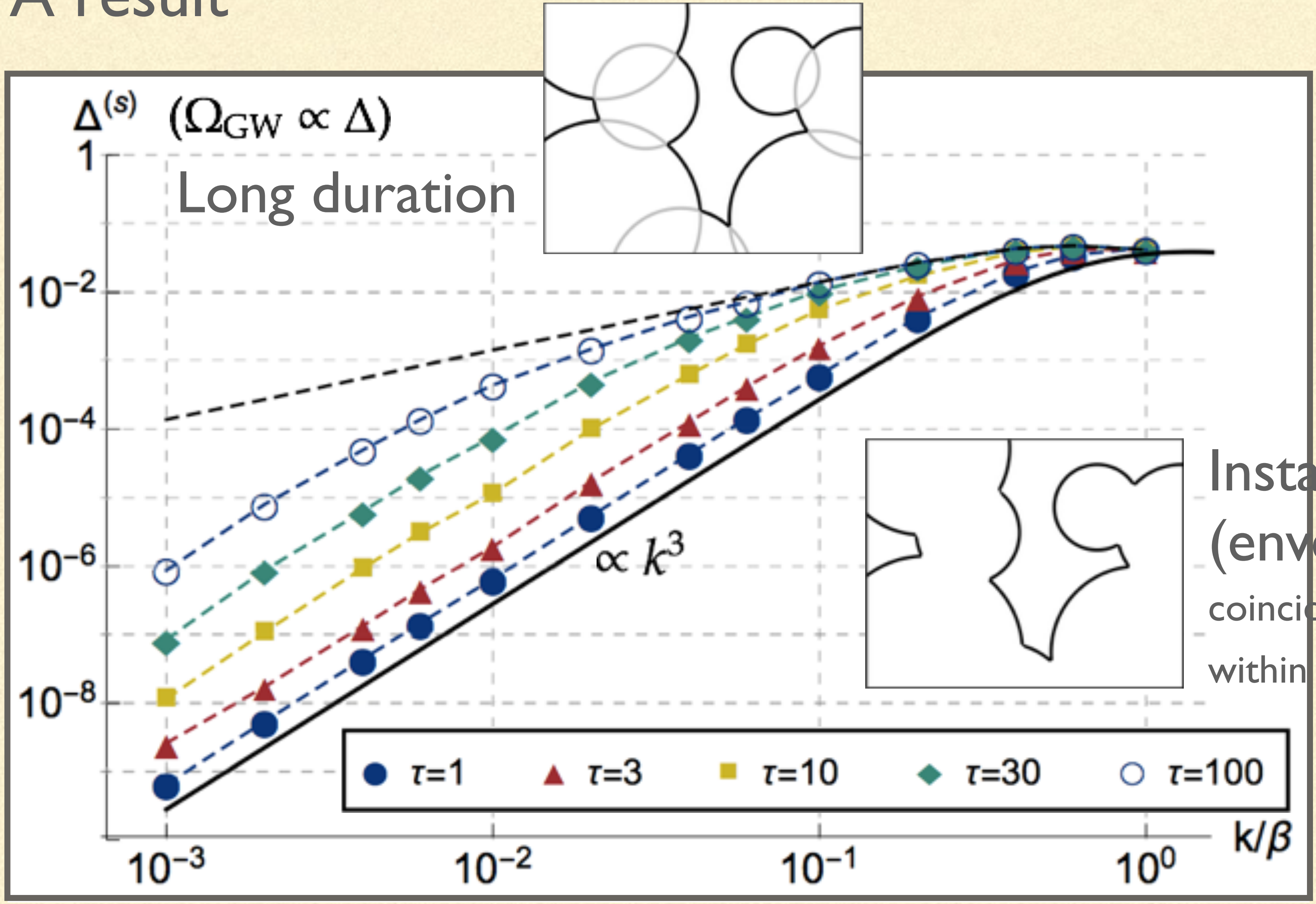
✓ 1. Analytic derivation of the GW spectrum

2. Result

3. Conclusion

NUMERICAL RESULT

- A result



Damping func.

$D \sim e^{-t/\tau}$

after collision

tau: sourcing time after collision.

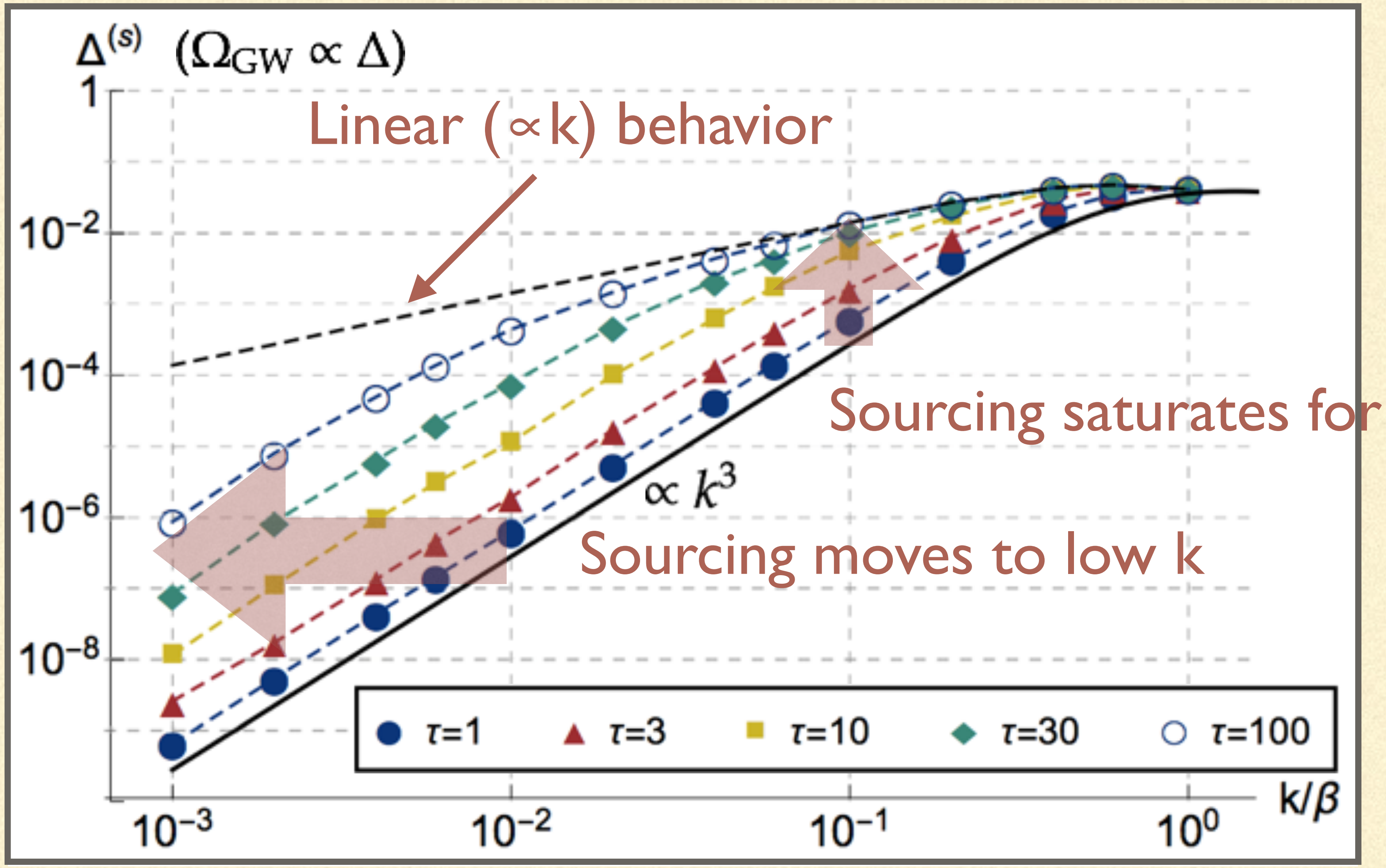
Instant disappearance (envelope) [Jinno&Takimoto '16]

coincide with [Huber&Konstandin '08] within factor 2

[Jinno & Takimoto '17]

NUMERICAL RESULT

- A result



Linear ($\propto$k) behavior

Sourcing saturates for fixed k

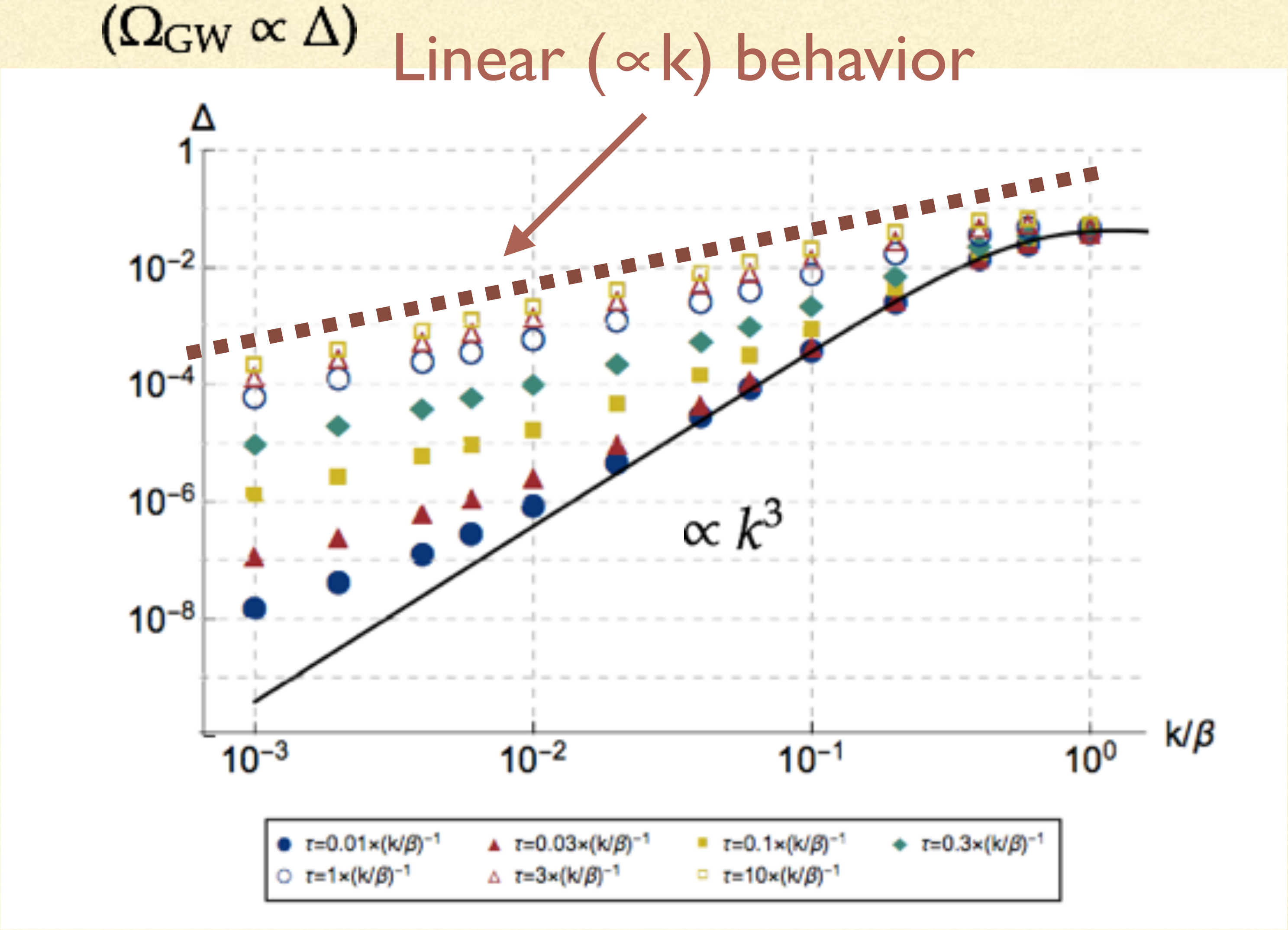
Sourcing moves to low k

[Jinno & Takimoto '17]

NUMERICAL RESULT

- A result

$(\Omega_{\mathrm{GW}} \propto \Delta)$



Linear (∝k) behavior

We confirmed enhancement of GW in low frequencies!!

[Jinno & Takimoto '17]

SUMMARY & FUTURE PROSPECTS

- GW spectrum w/ thin-wall has been derived **ANALYTICALLY**

  - General nucleation rate & wall velocity & damping of wall energy

- We have obtained GW spectrum in lower frequencies.

  - The huge enhancement compared to the envelope approximation

- Various effects can be implemented

  - Cosmic expansion / Nucl. rate dependence / Wall thickness (w/ truncation?)

- Will deepen our understanding on GW sourcing

Back up

GWS AS A PROBE TO PHASE TRANSITION

★Rough range of GW amplitude

~quadrupole factor ~radiation fraction today

$$\Omega_{\mathrm{GM,peak}} \sim \mathcal{O}(10^{-2})\mathcal{O}(10^{-5})(R_* H_*)^2$$

R_*:typical bubble at transition

$$f_{\mathrm{peak}} \sim \frac{1}{H_* R_*}\frac{T_*}{10^8\,\mathrm{GeV}}[\mathrm{Hz}]$$

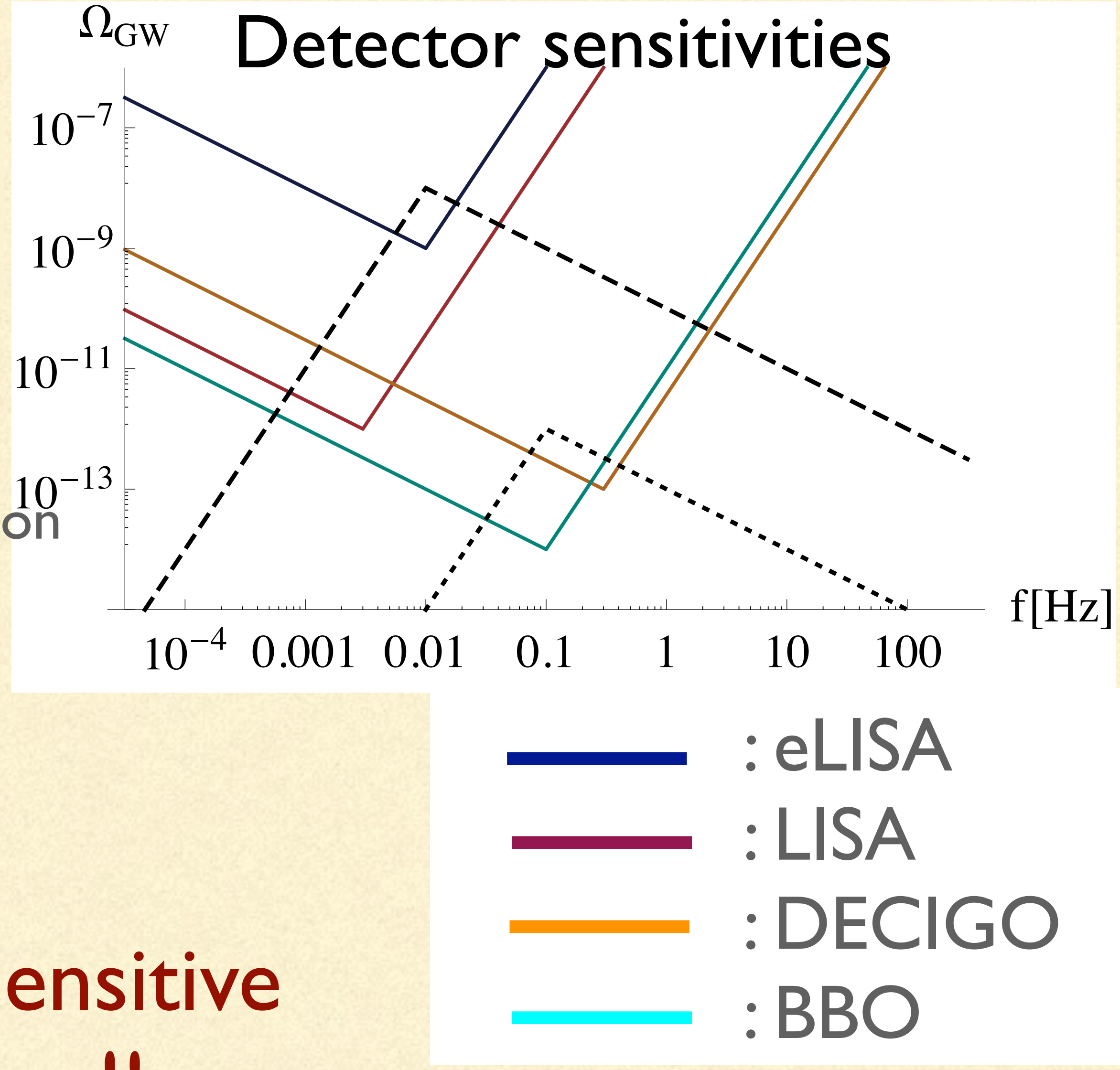
$\Omega_{\mathrm{GW}} = \rho_{\mathrm{GW}}/\rho_{\mathrm{tot}}$

T_* : temp. just after transition

H_* : H just after transition

$$H_* R_* \sim \mathcal{O}(10^{-1} - 10^{-5})$$

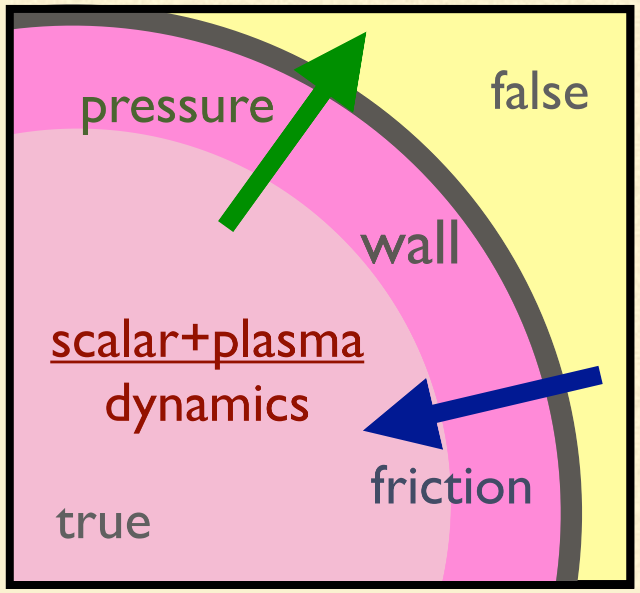
Model dependent parameter



GW from gravitational waves are sensitive to new physics around TeV-PeV range!!

BEHAVIOR OF SINGLE BUBBLE

- Two main players : **scalar field & plasma**



- Walls (where the scalar field value changes) want to expand (“pressure”)

- Walls are pushed back by plasma (“friction”)

- Bubble (wall & surrounding plasma) behavior is determined by

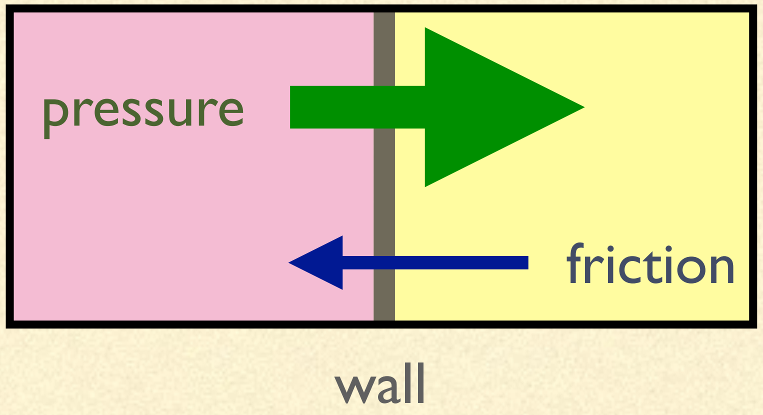
$$\alpha \equiv \frac{\rho_{\rm released}}{\rho_{\rm rad}} \qquad \begin{cases} \alpha \gtrsim \mathcal{O}(0.1) \ : \ \text{Huge energy release} \\ \alpha \lesssim \mathcal{O}(0.1) \ : \ \text{Small energy release} \end{cases}$$

BEHAVIOR OF SINGLE BUBBLE

- Understanding until ~ 2016

[e.g. Bodeker & Moore, JCAP 0905 (2009) 009
Espinosa et al., JCAP 1006 (2010) 028]

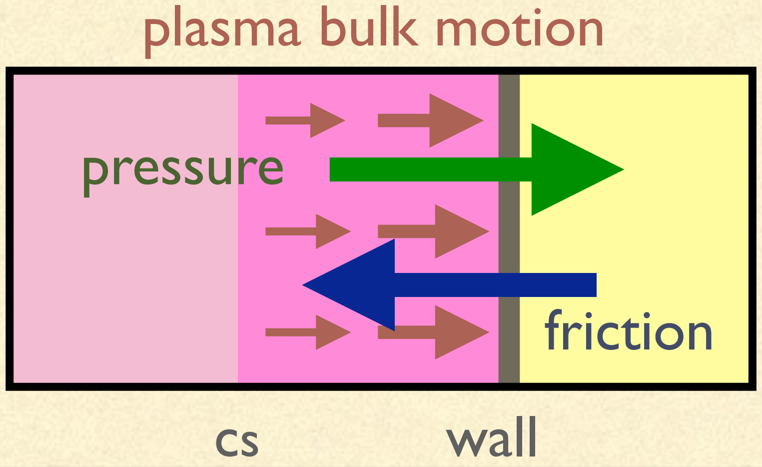
$\alpha \gtrsim \mathcal{O}(0.1)$: Huge energy release → Runaway



- Plasma friction cannot balance with pressure
- Walls approach the speed of light
- Energy accumulates in walls

$\alpha \lesssim \mathcal{O}(0.1)$: Small energy release → Terminal velocity

(to experts :
this is detonation case)



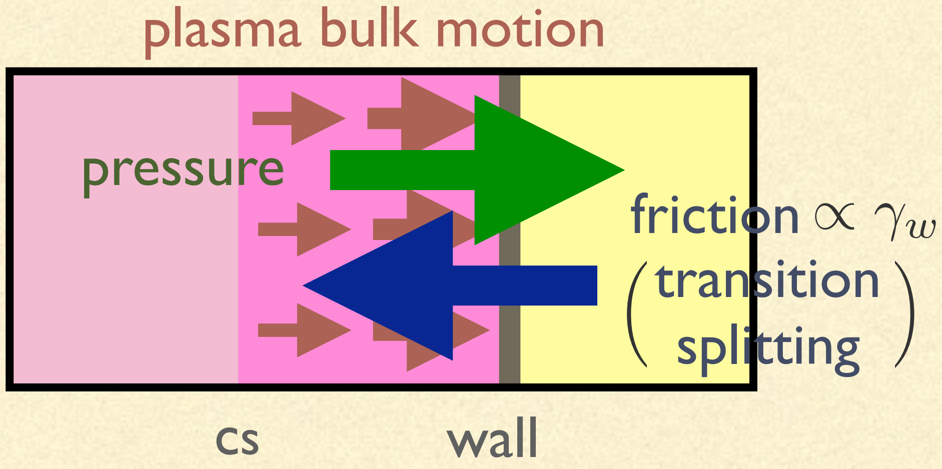
- Plasma friction gets balanced with pressure
- Walls approach terminal velocity
- Energy accumulates in plasma bulk motion

BEHAVIOR OF SINGLE BUBBLE

- Understanding from 2017 ~

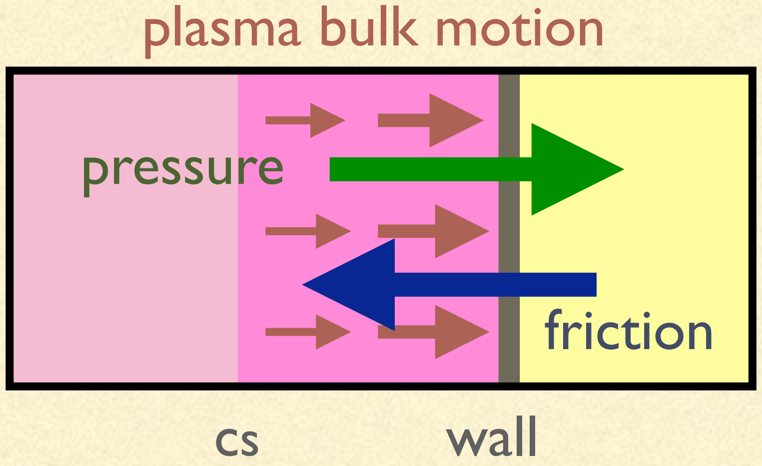
[Bodeker & Moore '17]

$\alpha \gtrsim \mathcal{O}(0.1)$: Huge energy release → ~~Runaway~~ High-terminal velocity



- Plasma friction can balance with pressure
- Walls approach high terminal velocity
- Energy accumulates in plasma bulk motion

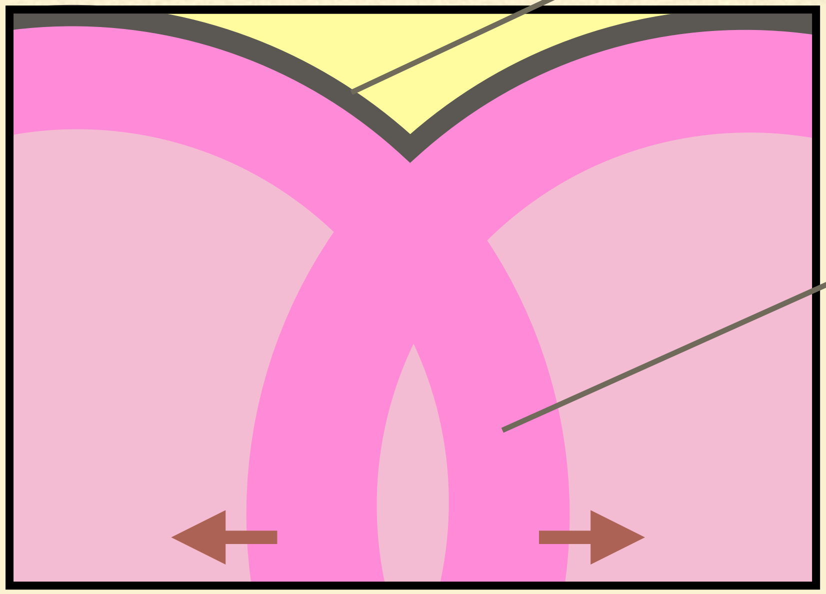
$\alpha \lesssim \mathcal{O}(0.1)$: Small energy release → Low-terminal velocity



- Plasma friction gets balanced with pressure
- Walls approach terminal velocity
- Energy accumulates in plasma bulk motion

BEHAVIOR AFTER COLLISION

- What happens after collisions?



1. Walls (energetically subdominant)

   collide and damp soon

   "bubble collision"

2. Plasma bulk motion continues to propagate

   "sound wave"

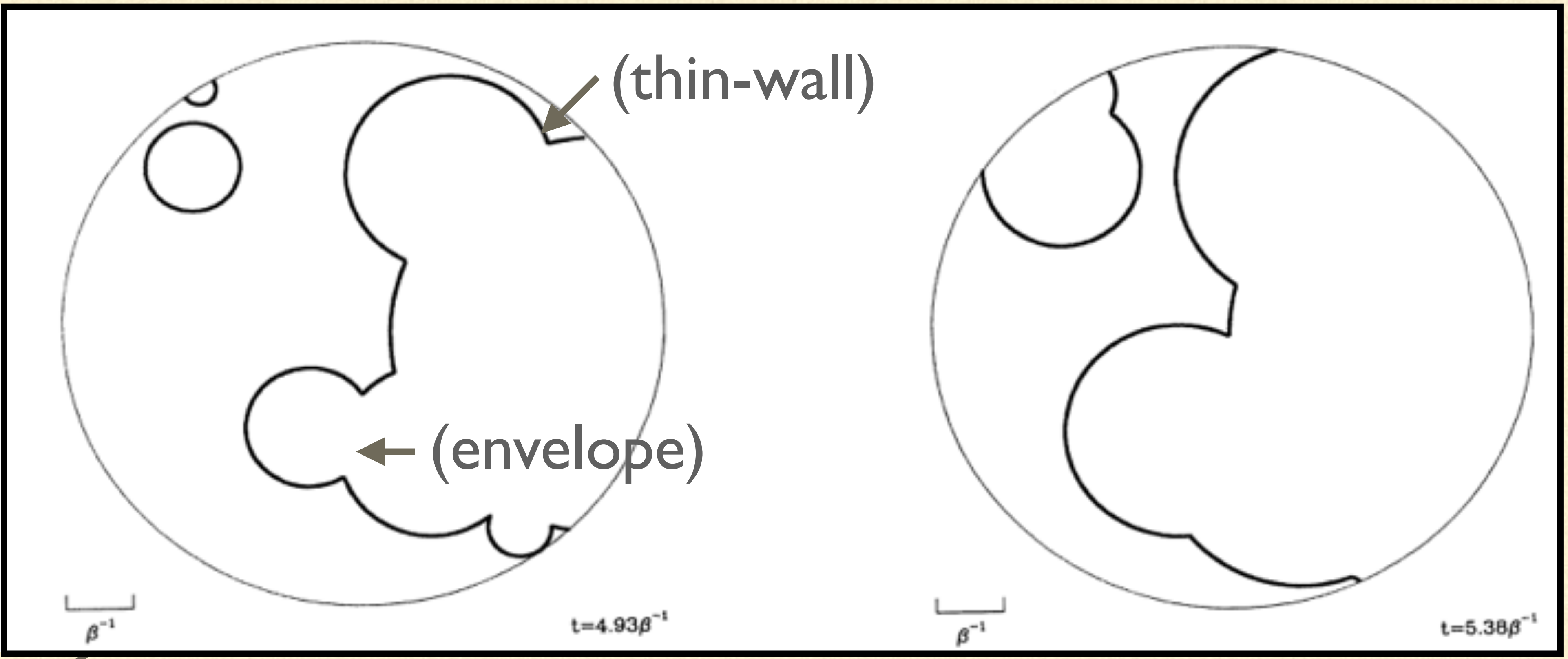
3. At late times,

   sound waves develop into nonlinear regime

   "turbulence"

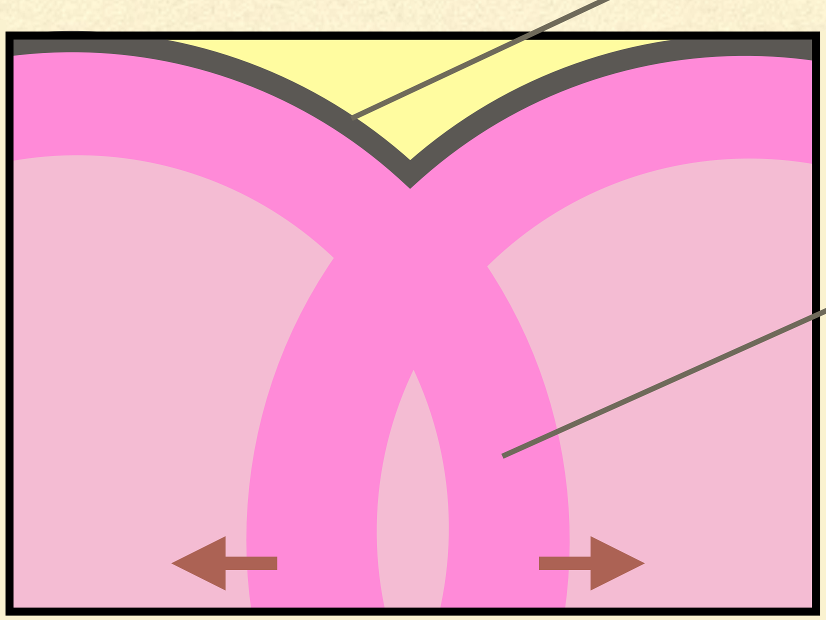
BEHAVIO

- What happens af



[Kosowski et al. '93]

"bubble collision"



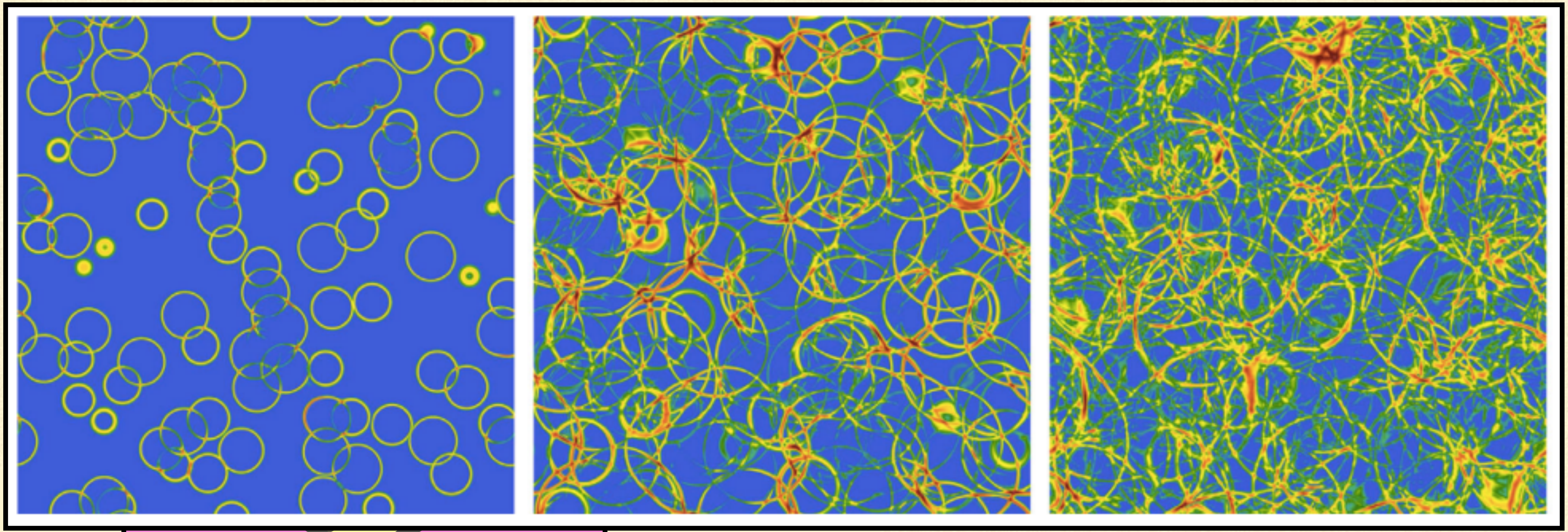
2. Plasma bulk motion continues to propagate

"sound wave"

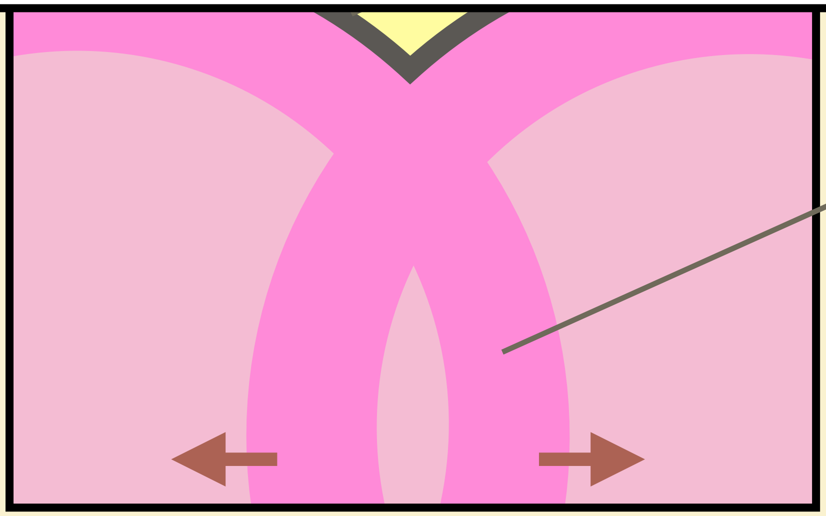
3. At late times,

sound waves develop into nonlinear regime

"turbulence"



[Hindmarsh et al.'15]



2. Plasma bulk motion continues to propagate

“sound wave”

(1) Dynamics is linear for small energy release :

$$\left(\partial_t^2 - c_s^2 \nabla^2\right) u^i = 0 \qquad u^i : \text{fluid velocity field}$$

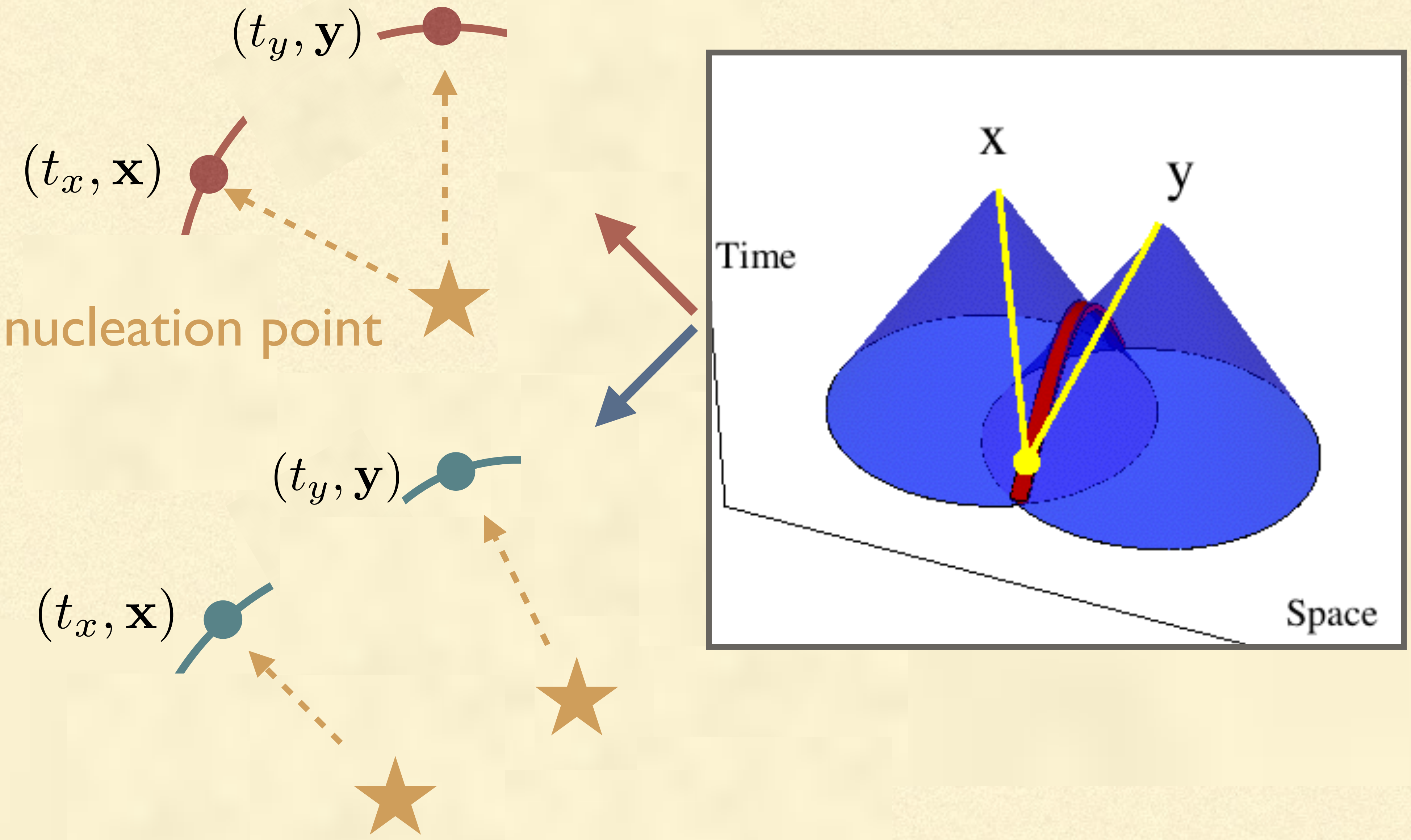
(2) Width remains to be constant after collision

ONLY TWO CASES

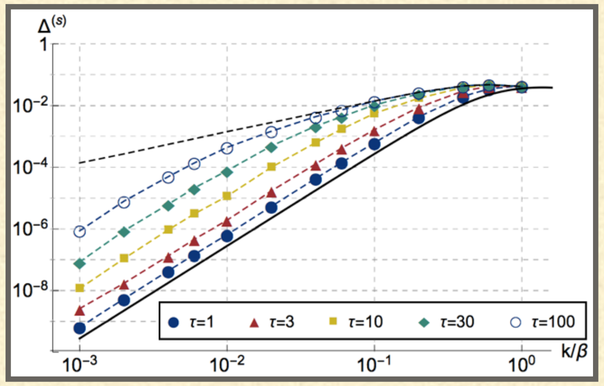
- Following two exhaust $T(x)T(y) \neq 0$ possibilities [Jinno & Takimoto '16]

1. single-bubble

2. double-bubble

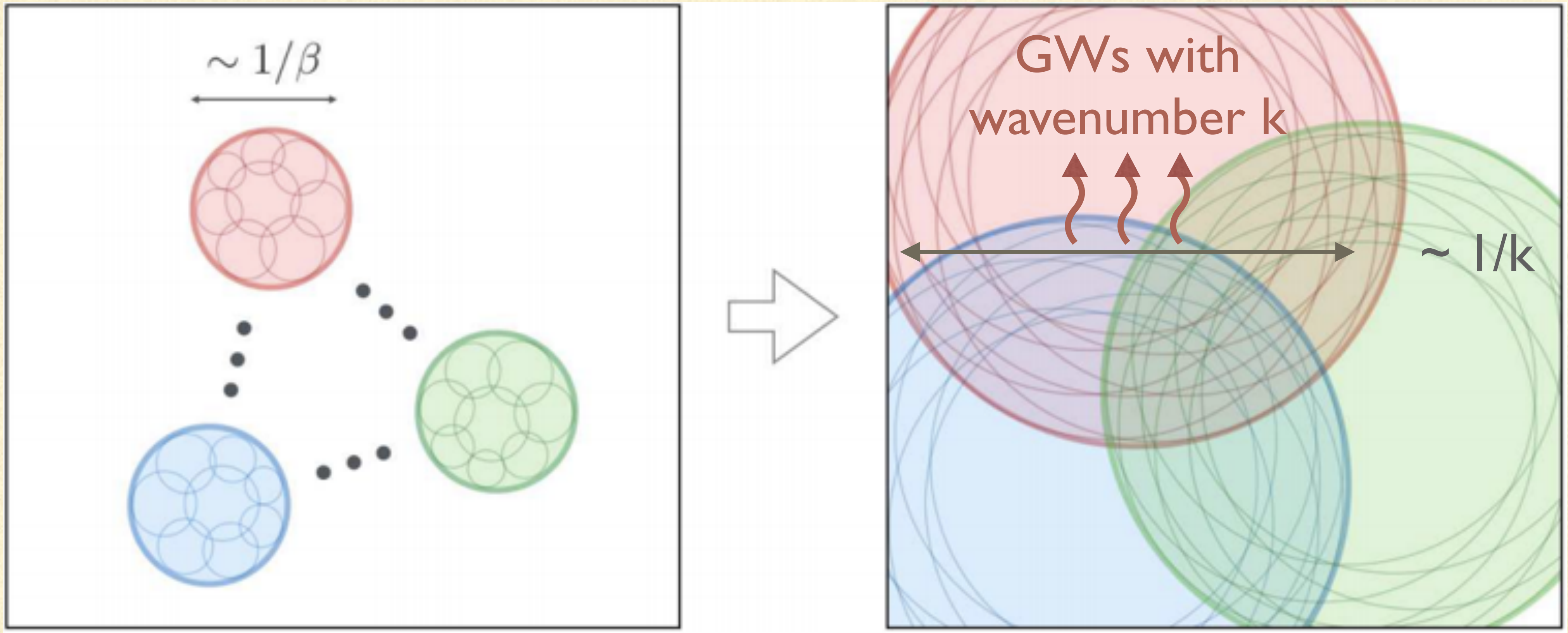


PHYSICAL INTERPRETATION



- Small wavenumbers sourced at late times

  - Wavenumber k sourced when the typical bubble size grows to ~ 1/k



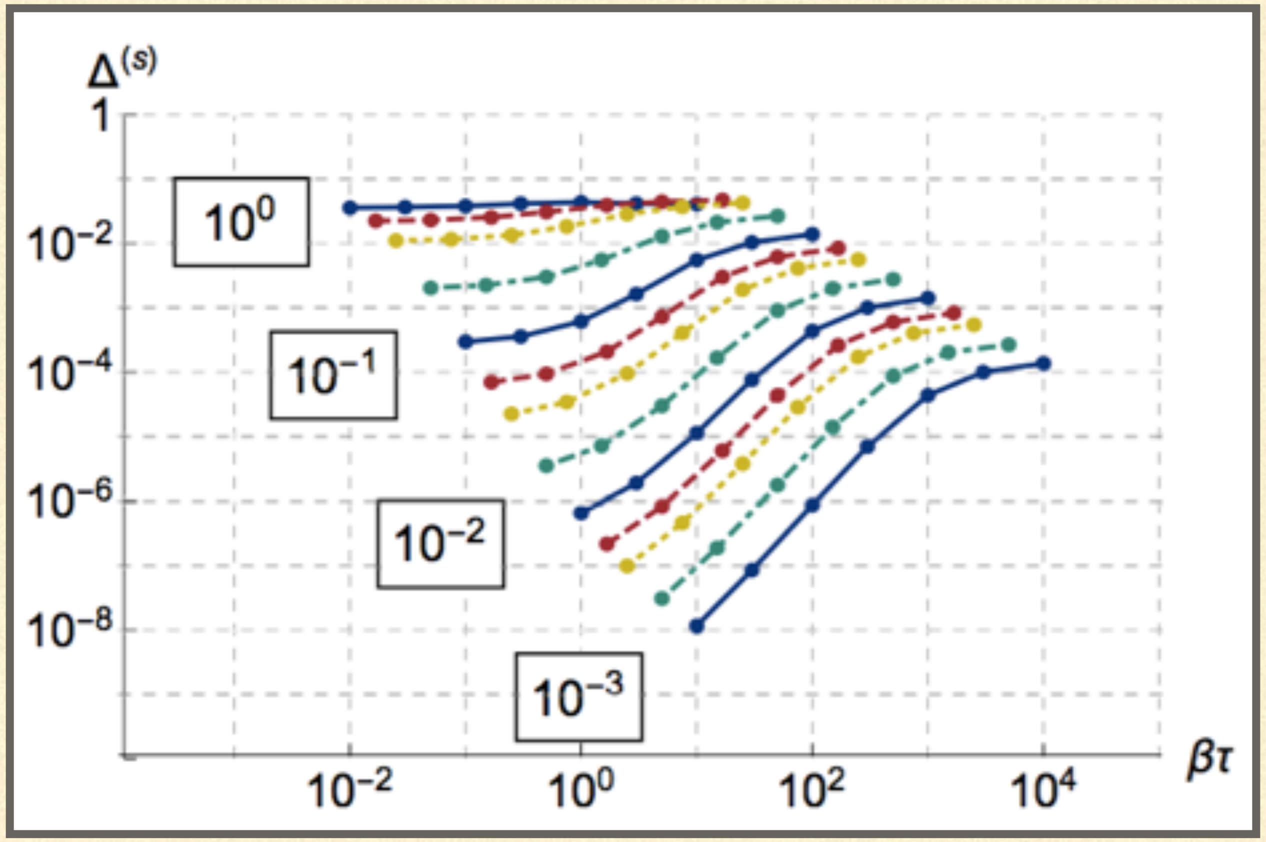
GWs with wavenumber k

~ 1/k

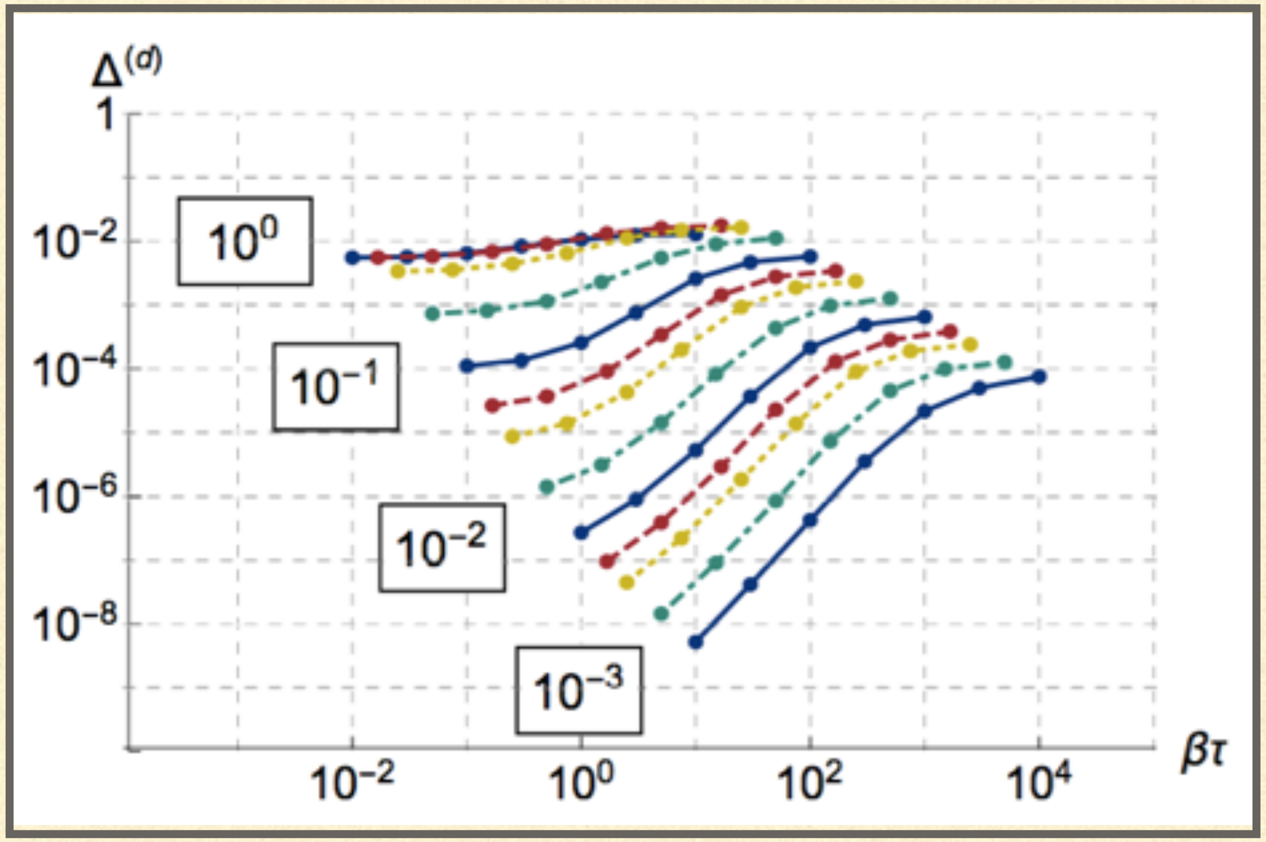
NUMERICAL RESULT

- GW sourcing as a function of time

- Single

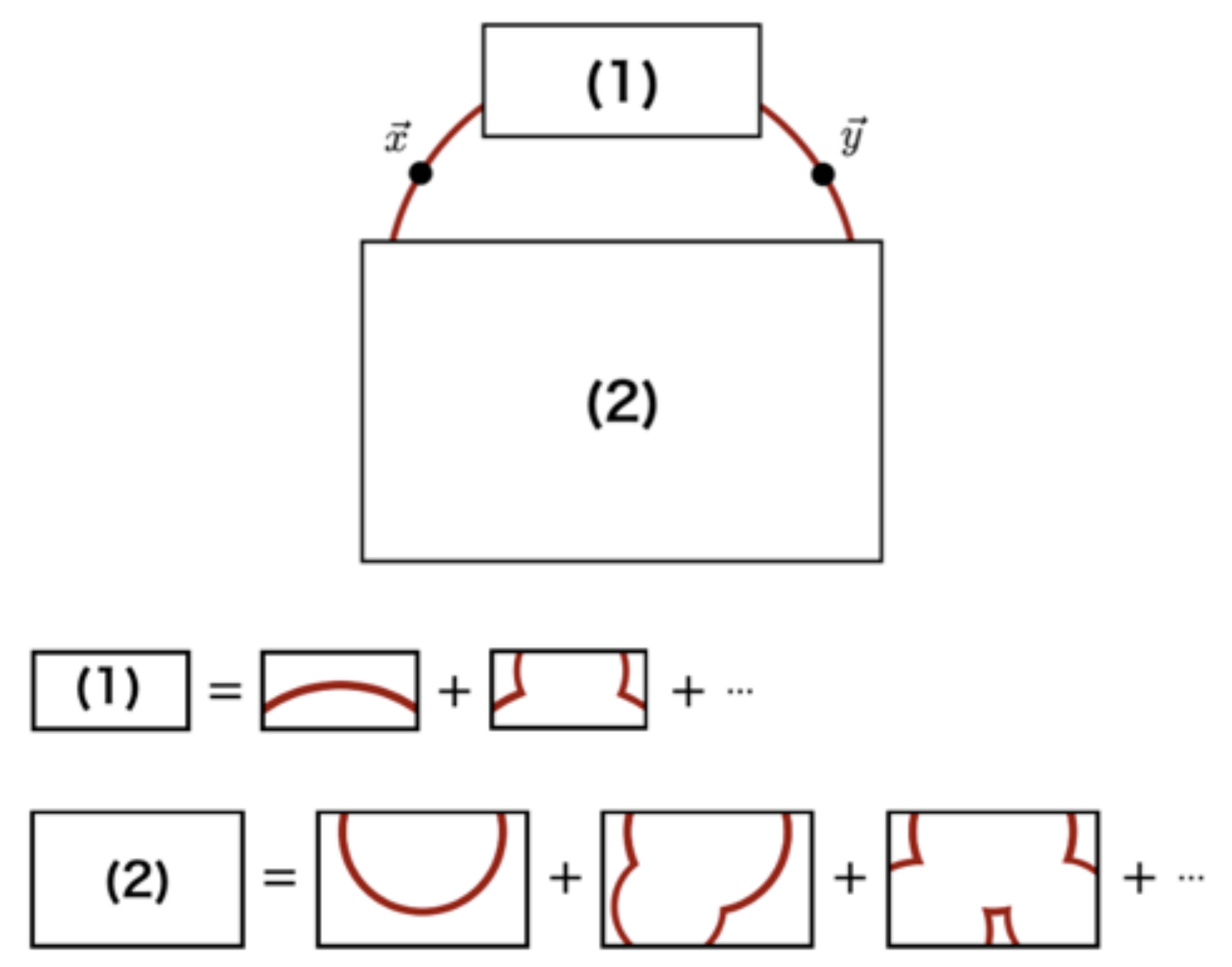


- Double



WHY SINGLE-BUBBLE MATTERS

- Illustration with envelope



- Two bubble-wall fragments must remain uncollided until they reach x and y

- Other parts of the bubble might have collided already

- In this sense, breaking of spherical sym. is automatically taken into account