Capture and Decay of Electroweak WIMPonium

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based on JCAP 1702 (2017) 005 with

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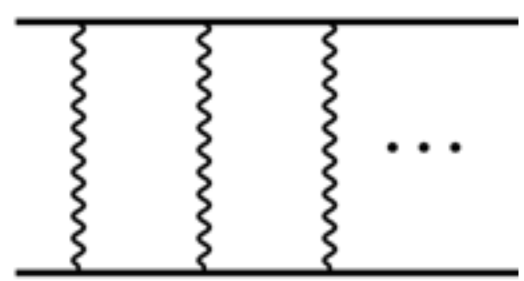
WIMPs

- WIMPs exchange ladder of electroweak gauge bosons

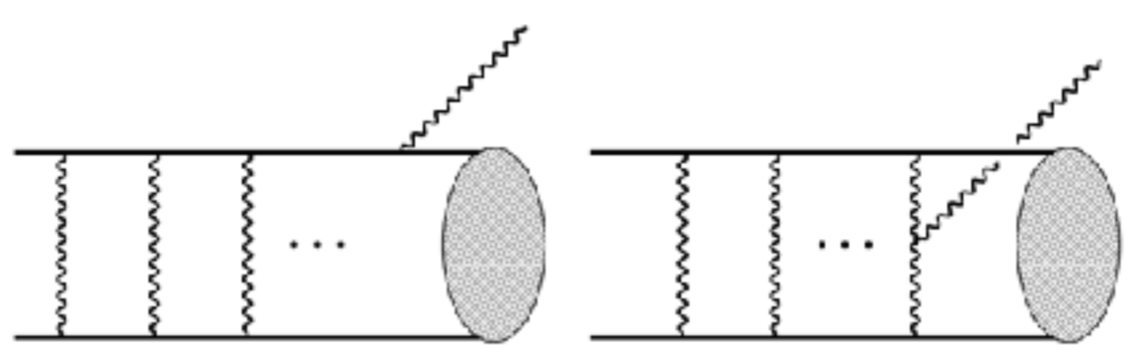


- in NR limit (v ~ 10^(-3)) gives rise to non-local, instantaneous potential

- leads to Sommerfeld enhancement in DM annihilations:

$$\sigma v = \Gamma \left|\psi\left(0\right)\right|^2$$

- WIMP spectrum possesses bound states when WIMP mass sufficiently large relative to mass of electroweak gauge bosons —> WIMPonium



- alternative annihilation channel for DM…significant effect on radiative signals?

wino

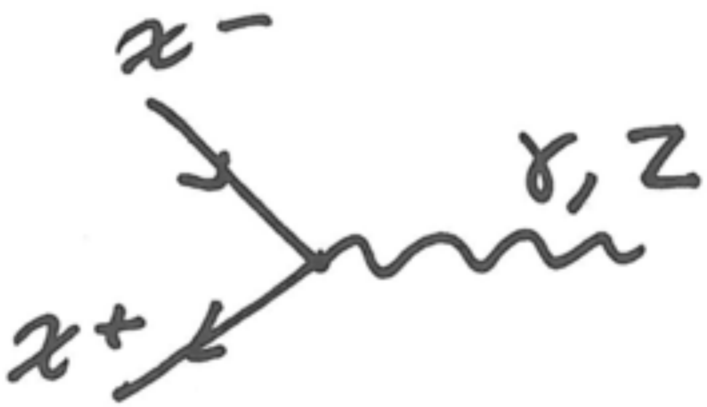
- SU(2)$_L$ triplet Majorana fermion χ_a , zero hypercharge, mass M_χ

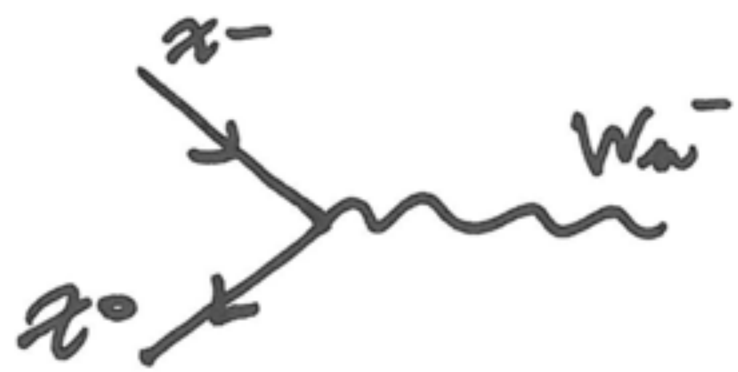
$$\mathcal{L} = i\chi^{a\dagger}\left(\bar{\sigma}^\mu \partial_\mu \delta^{ac} + ig\bar{\sigma}^\mu W^b_\mu T^b_{ac}\right)\chi^c - \frac{1}{2}M_\chi\left(\chi^a\chi^a + h.c.\right)$$

- in mass eigenbasis: $\{\chi^1, \chi^2, \chi^3\} \rightarrow \{\chi^0, \chi^\pm\}$

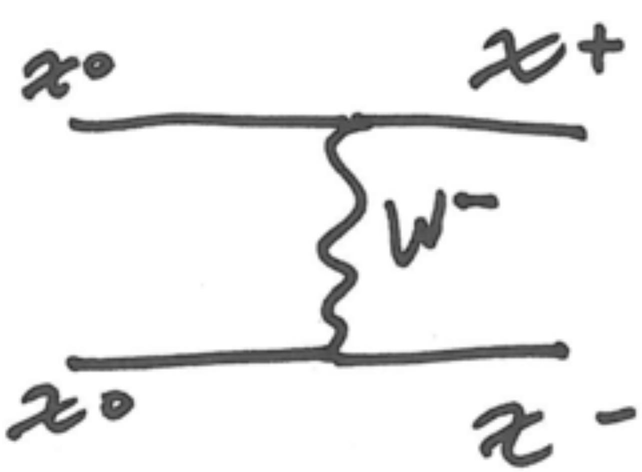
- mass splitting: $\delta M \equiv M_{\chi^\pm} - M_{\chi^0} = 165 MeV$

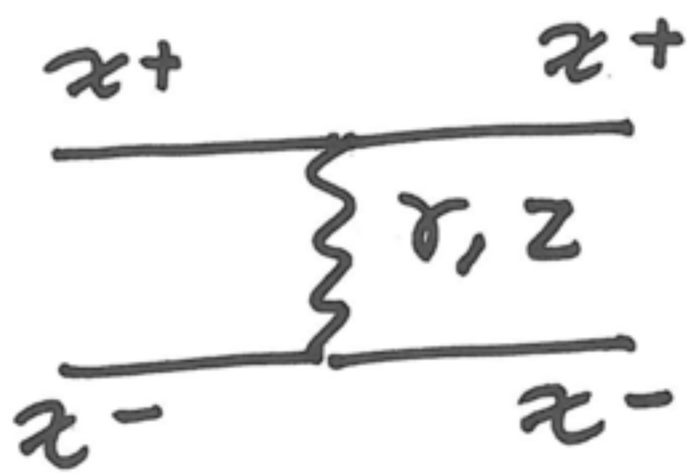
- interactions with electroweak gauge bosons:





- pair states, starting with a pair of neutral winos:





Schrödinger eqtn.

- wino pair states $\Psi = \begin{pmatrix} \psi_N \, (\equiv \chi^0 \chi^0) \\ \psi_C \, (\equiv \chi^+ \chi^-) \end{pmatrix}$

- in NR limit evolve in the Schrödinger eqtn:

$$i\partial_t \Psi = H^0 \, \Psi = \left[-\frac{\nabla_X^2}{4M_\chi} - \frac{\nabla_r^2}{M_\chi} + V(r) \right] \Psi$$

- under the potential:

$$V_{L+S\,\mathrm{even}}(r) = \begin{pmatrix} 0 & -\sqrt{2}\alpha_W \frac{e^{-m_W r}}{r} \\ -\sqrt{2}\alpha_W \frac{e^{-m_W r}}{r} & 2\delta M - \frac{\alpha}{r} - \alpha_W c_W^2 \frac{e^{-m_Z r}}{r} \end{pmatrix} \quad V_{L+S\,\mathrm{odd}}(r) = \begin{pmatrix} 0 & 0 \\ 0 & 2\delta M - \frac{\alpha}{r} - \alpha_W c_W^2 \frac{e^{-m_Z r}}{r} \end{pmatrix}$$

- initial (neutral) positive-energy scattering state Ψ_i $\quad E_{\mathrm{CM}} = \frac{M_\chi v_{\mathrm{rel}}^2}{4}$ $\quad V_{L+S\,\mathrm{even}}$

$$\int d^3\mathbf{r} \Psi_{i,\mathbf{p}}^\dagger (\mathbf{r}) \, \Psi_{i,\mathbf{p}'} (\mathbf{r}) = \delta^3 \left(\mathbf{p} - \mathbf{p}^{'} \right)$$

$$H_{L+S\,\mathrm{even}}^0 \; \Psi_i = \frac{M_\chi v_{\mathrm{rel}}^2}{4} \Psi_i$$

…radiative transition $\Delta L = \pm 1$ to

- purely chargino negative-energy bound state Ψ_f $\quad E_n \lesssim \mathcal{O}\left(\alpha_W^2 M_\chi\right)$ $\quad V_{L+S\,\mathrm{odd}}$

$$\int d^3\mathbf{r} \left| \Psi_f \, (\mathbf{r}) \right|^2 = 1$$

$$H_{L+S\,\mathrm{odd}}^0 \; \Psi_f \left[{}^{2S+1} L_J \right] = E_n \, \Psi_f \left[{}^{2S+1} L_J \right]$$

$SU(2)_L$-symmetric limit

- high-mass limit in which $SU(2)_L$ symmetry approximately unbroken:

$$M_W, M_Z \ll M_\chi \qquad \delta M \to 0$$

- Coulombic limit: $V(r) \to -\frac{\alpha_W}{r}\bar{V}$

- diagonalize the Schrödinger eqtn:

$$-\frac{1}{2\mu}\nabla^2\Psi - \frac{\alpha_W}{r}\bar{V}\Psi = \begin{cases} \frac{p^2}{2\mu}\Psi \\ E_n\Psi \end{cases}$$

$$\Psi(\mathbf{r}) = \sum_i \eta_i\phi_i(\mathbf{r}) \qquad \bar{V}\eta_i = \lambda_i\eta_i$$

- Wino:

$$V_{L+S\text{even}}(r) = -\frac{\alpha_W}{r}\begin{pmatrix} 0 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}$$

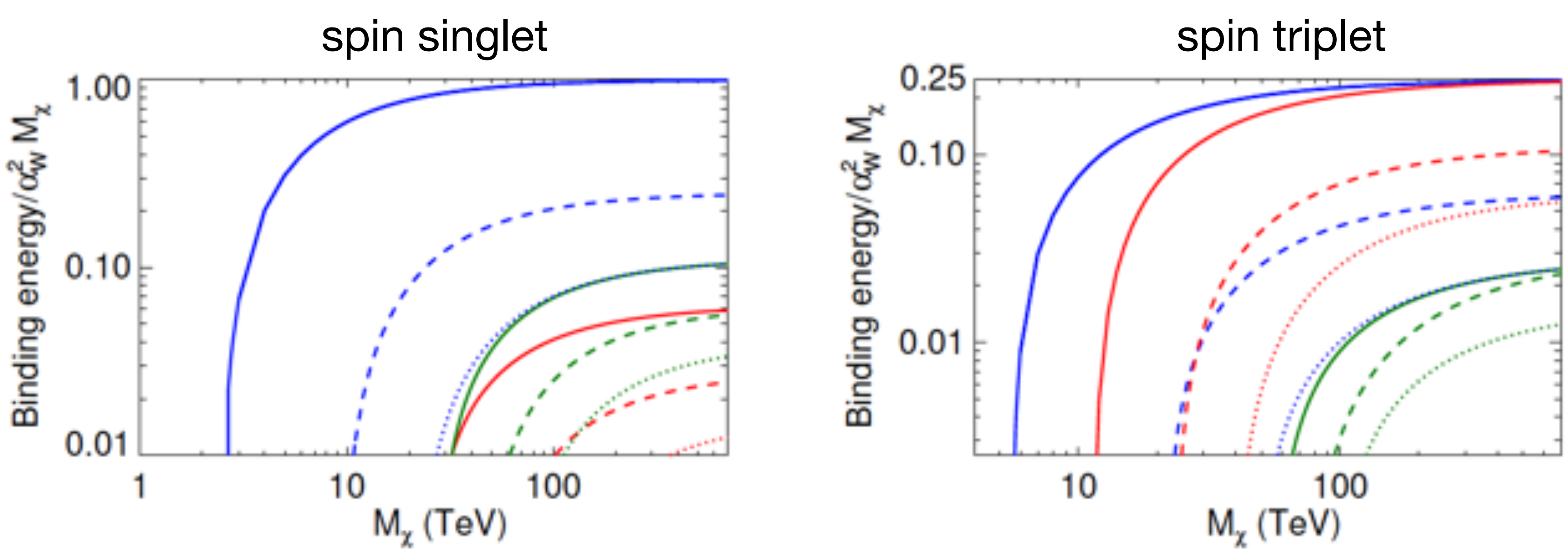
$$\lambda_1 = 2 \to E_n = -\frac{M_\chi(2\alpha_W)^2}{4n^2}$$

$$\lambda_2 = -1$$

$$V_{L+S\text{odd}}(r) = -\frac{\alpha_W}{r}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\lambda = 1 \to E_n = -\frac{M_\chi\alpha_W^2}{4n^2}$$

bound state spectrum



blue, red, green: s, p, d wave. solid, dashed, dotted ranking n from lowest

	Spin-Singlet Spectrum	Spin-Triplet Spectrum
$\lvert\frac{E_n}{M_\chi \alpha_W^2}\rvert = \frac{1}{144}$		6D
$\lvert\frac{E_n}{M_\chi \alpha_W^2}\rvert = \frac{1}{100}$	5P	5D
$\lvert\frac{E_n}{M_\chi \alpha_W^2}\rvert = \frac{1}{64}$	4P	4S 4D
$\lvert\frac{E_n}{M_\chi \alpha_W^2}\rvert = \frac{1}{36}$	3P 6D	3S 3D
$\lvert\frac{E_n}{M_\chi \alpha_W^2}\rvert = \frac{1}{25}$	5D	5P
$\lvert\frac{E_n}{M_\chi \alpha_W^2}\rvert = \frac{1}{16}$	4S 2P 4D	2S 4P
$\lvert\frac{E_n}{M_\chi \alpha_W^2}\rvert = \frac{1}{9}$	3S 3D	3P
$\lvert\frac{E_n}{M_\chi \alpha_W^2}\rvert = \frac{1}{4}$	2S	1S 2P
$\lvert\frac{E_n}{M_\chi \alpha_W^2}\rvert = 1$	1S	

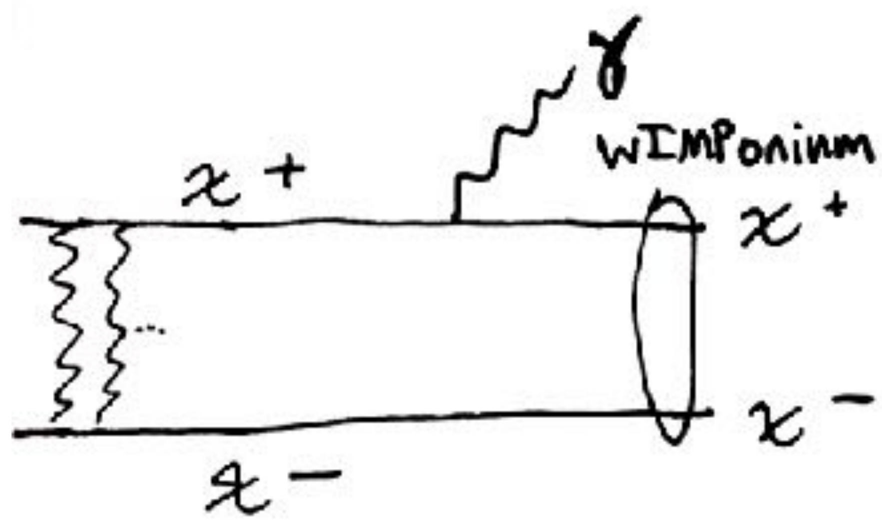
high-mass limit

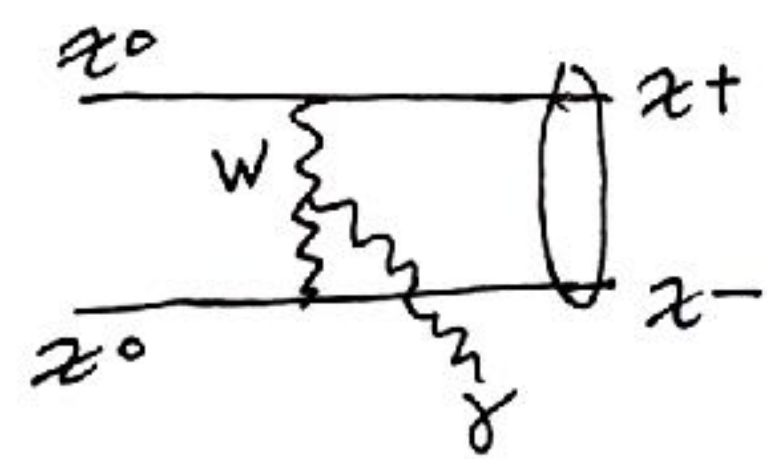
$$E_n = \frac{(2\alpha_W)^2 M_\chi}{4n^2}$$ even L+S

$$E_n = \frac{\alpha_W^2 M_\chi}{4n^2}$$ odd L+S

WIMPonium formation

- initial population of free neutralinos, bound states form via radiative capture





$$(\sigma v_{\rm rel})_{s=1,l=1\to{}^3S_1,n=1} \propto \frac{\alpha\alpha_W^2}{M_\chi^2 v_{\rm rel}} e^{-4n\lambda_i/\lambda_f} = \frac{\alpha\alpha_W^2}{M_\chi^2 v_{\rm rel}} e^{-8}$$

$$(\sigma v_{\rm rel})_{s=0,l=0\to{}^1P_1,n=2,\sum_m} \propto \frac{\alpha\alpha_W^2}{M_\chi^2 v_{\rm rel}} e^{-16}$$

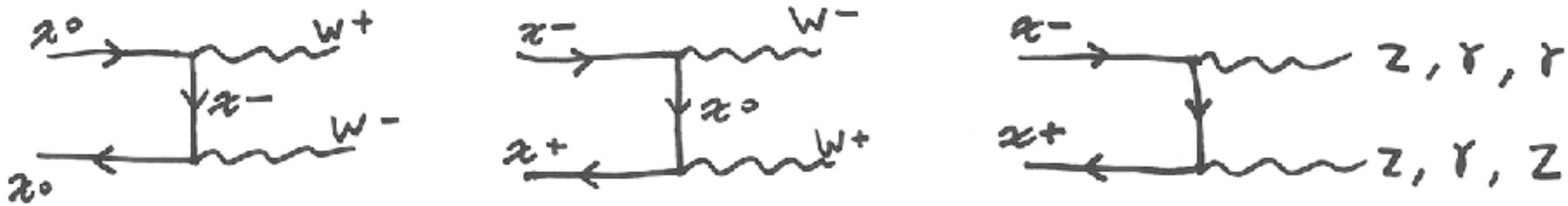
- capture into s=1 (spin-triplet) l=0 n=1 bound state (arising from p-wave part of initial state) dominates capture to 2p states — due mostly to suppression $e^{-4n\lambda_i/\lambda_f} = e^{-8n}$

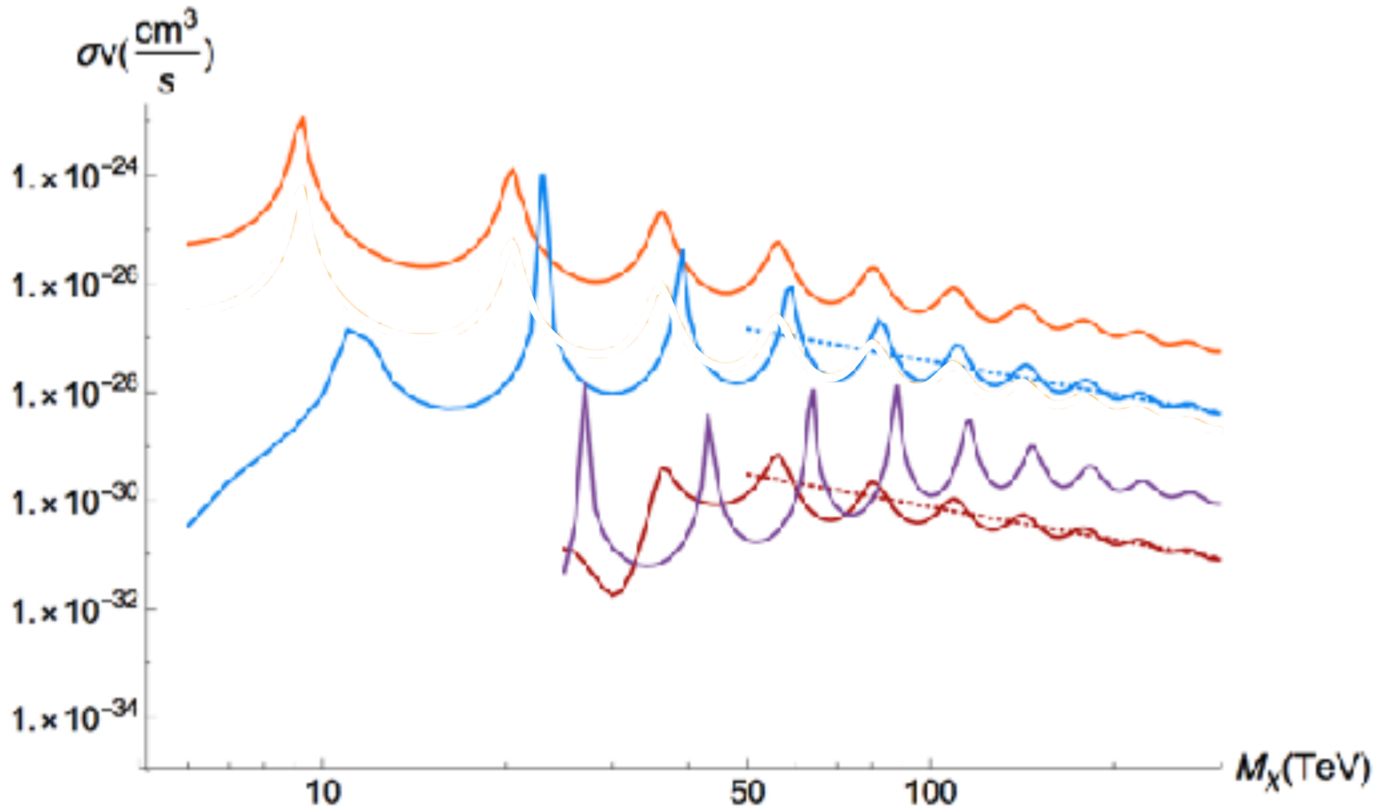
- bound states subsequently decay to lower-energy states or annihilate to SM particles.

- note: detecting photon lines from capture and/or transitions extremely challenging: NFW DM profile $\rho\,(8.5\mathrm{kpc}) = 0.4\mathrm{GeV/cm^3}$, $\sigma_{\rm cap} = 5\times 10^{-29}\mathrm{cm^3/s}$ $\longrightarrow$ $\mathcal{O}\left(10^{-3}\right)\ \mathrm{photons/m^2/yr}$

capture vs. direct annihilation

- leading-order s-wave annihilation into all channels given by diagrams:





dark orange: tree-level inclusive annihilation —> WW, γZ, γγ. *blue*: p−wave —> $^3S_1$ + γ, (lowest) n=1. *purple*: d-wave —> $^1P_1$ + γ (n = 2) . *maroon*: s-wave —> $^1P_1$ + γ, n = 2

$$(\sigma v_{\rm rel})_{\rm da} \propto \frac{\alpha_W^2}{M_\chi^2}\frac{\alpha_W}{v_{\rm rel}}$$

$$\frac{(\sigma v_{\rm rel})_{cap}}{(\sigma v_{\rm rel})_{da}} \propto \frac{\alpha}{\alpha_W} e^{-4\lambda_i/\lambda_f} = \frac{\alpha}{\alpha_W} e^{-8}$$

- direct annihilation dominates the radiative capture for the wino, due to factor e^{-8n}
- in contrast to positronium $\propto e^{-4n}$

conclusions

- due to spin statistics, states with odd vs. even L+S experience different effective potentials and form distinct towers of bound states —> bound spectrum, unsuppressed decay channels different from hydrogen-like atoms.

- wino bound state capture rate subdominant to direct annihilation —> previous calculations of detectability of e.g. high-energy gamma-ray lines from wino DM should not require significant modification.

- detection of low-energy photon lines from radiative capture and transitions between bound states seem very challenging for wino.

- factors which suppress wino-onium cross section not generic — depend sensitively on rep. of DM under the gauge group, and relative masses of DM and force carriers —> formation of bound states cannot be safely ignored in models with non-trivial dark sectors.

—>See e.g. Cirelli et al JCAP 1705 (2017) 036 : DM charged under dark U(1) —> formation and decay of DM bound states have significant effect on radiative signals in indirect detection.

—>See e.g. Mitridate et al. 2017: DM fermionic 5plet of $SU(2)_L$ with zero hypercharge. *bound states reduce the DM thermal abundance by about 30%*, increasing the DM mass that reproduces the cosmological abundance to about 11.5TeV. *significant bound-state corrections to DM indirect detection*, characteristic spectrum of mono-chromatic lines around E (10 80) GeV, with rates of experimental interest.

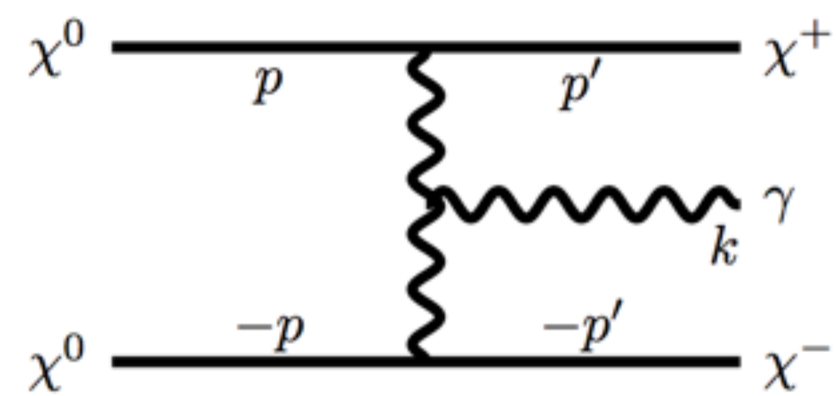
WIMPonium formation, transitions, annihilation

- initial population of free neutralinos, bound states form via radiative capture…subsequently decay to lower-energy states or annihilate to SM particles

- continuum-bound and bound-bound transitions in time-ordered perturbation theory.

$$H = H^0 + V_{\text{rad.}}$$

$$V_{\text{rad.}} = \left(-\sum_n \frac{e_n}{M_\chi} \mathbf{A}(\mathbf{x}_n) \cdot \mathbf{p}_n + \sum_n \frac{e_n^2}{2m_n} \mathbf{A}(\mathbf{x}_n)^2 \right) \mathbb{P}_{CC} + \left(i\sqrt{2}\, e\, \alpha_W \mathbf{A}(0) \cdot \hat{\mathbf{r}}\, e^{-m_W r} \right) \mathbb{P}_{NC}$$



$$S_{i,\,f\gamma} = 2\pi i\, \delta\left[M_\chi v^2/4 - E_n - k - P_{\text{BS}}^2/(4M_\chi)\right] \left(\sum_n \frac{e_n}{M_\chi} \left\langle \Psi_f\left[{}^{2S+1}L_J\right]\, \gamma(k) \right| \mathbf{A}(\mathbf{x}_n) \cdot \mathbf{p}_n \left| \psi_{i,C} \right\rangle - i\sqrt{2}\, e\, \alpha_W \left\langle \Psi_f\left[{}^{2S+1}L_J\right]\, \gamma(k) \right| e^{-m_W r} \mathbf{A}(0) \cdot \hat{\mathbf{r}} \left| \psi_{i,N} \right\rangle \right)$$

$$\begin{matrix} (d\sigma) v_{\text{rel}} \\ d\Gamma \end{matrix} = (2\pi)^2 \mu_f\, k\, |M|^2_{i,\,f\gamma}\, d\Omega_k$$

where $\mu_f = k\, E_{\text{BS}}/(k + E_{\text{BS}}) \approx k$

$k = -E_n + M_\chi v_{\text{rel}}^2/4$ for capture

$k = E_{n1} - E_{n2}$ for decay

annihilation from bound state

- bound states also decay through annihilation to SM final states.

$$|\psi\rangle = \sqrt{\frac{1}{2\mu}}\int \frac{d^3p}{(2\pi)^3}\psi(p)|\mathbf{p},-\mathbf{p}\rangle \quad \text{(distinguishable particles)}$$

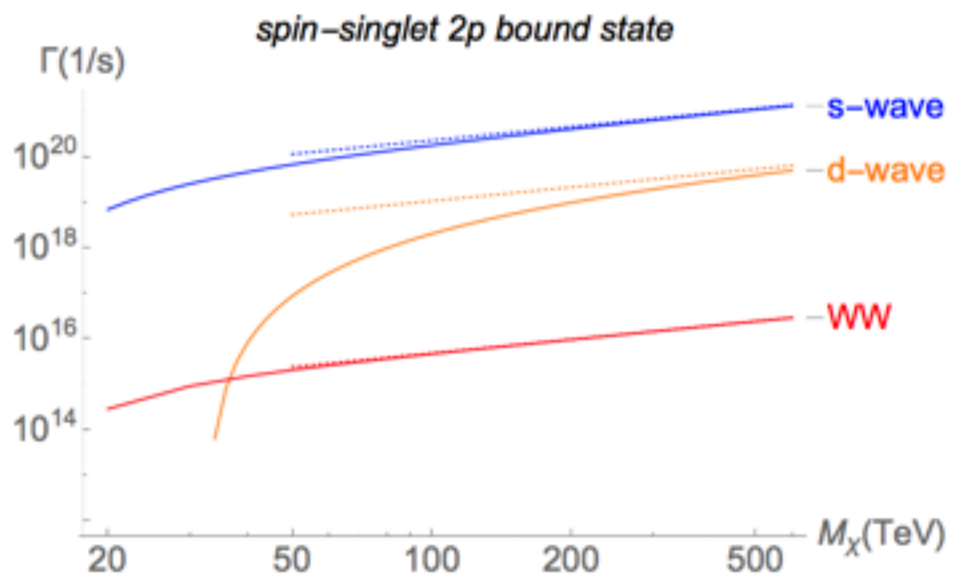
$$\sqrt{\frac{1}{4\mu}}\int \frac{d^3p}{(2\pi)^3}\psi(p)|\mathbf{p},-\mathbf{p}\rangle \quad \text{(identical particles)},$$

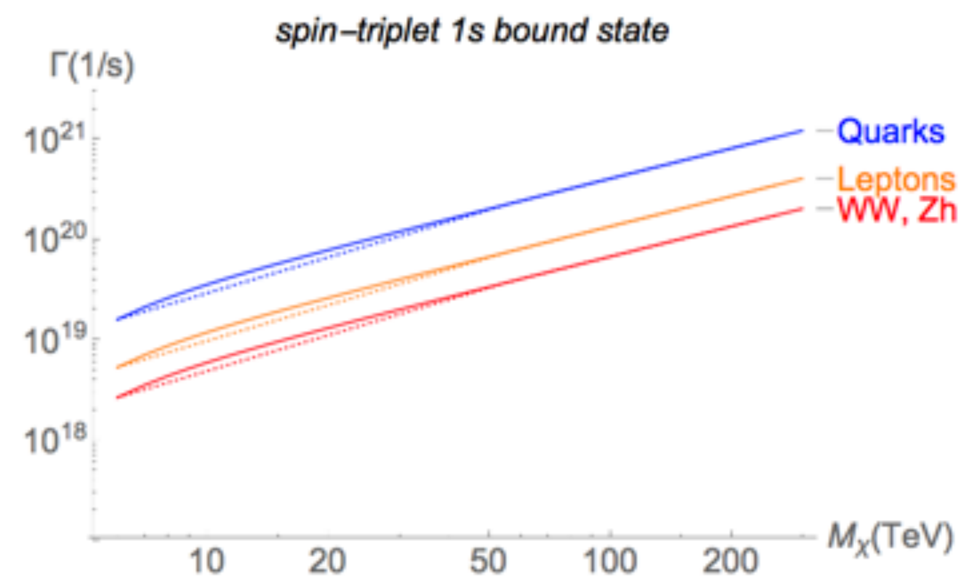
$$\mathcal{M}(B\to f) = \sqrt{\frac{1}{2\mu}}\int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{\sqrt{2}}\psi_N(p)\mathcal{M}(\chi^0(\mathbf{p})\chi^0(-\mathbf{p})\to f) + \psi_C(p)\mathcal{M}(\chi^+(\mathbf{p})\chi^-(-\mathbf{p})\to f)\right]$$

$$\Gamma = \frac{1}{2M_B}\int d\Pi_n\, |\mathcal{M}(B\to f)|^2$$

WIMPonium decays

- decays for lowest-energy bound states:





blue —> 1-3s; yellow —> 3d; red —> W+W-

- SU(2)-symmetric limit: $\Gamma_{\rm dec} \propto \alpha\, \alpha_W^4 M_\chi$ dominate $L > 0$

$$\Gamma_{\rm annih} \propto \alpha_W^{5+2L} M_\chi$$

- dominant capture into spin-triplet 1s

- spin-singlet 2p: annihilation decay rate suppressed relative to ED transitions to lower s and d.

detectability

- photons radiated upon capture/transitions could allow study of the QM numbers of DM…constitute a detectable signal? assuming:

NFW DM profile $\rho\left(8.5\mathrm{kpc}\right) = 0.4\mathrm{GeV}/\mathrm{cm}^3$ $R_s = 20\mathrm{kpc}$

$$\sigma_{\mathrm{cap}} = 5 \times 10^{-29}\mathrm{cm}^3/\mathrm{s}$$

$\mathcal{O}\left(10^{-3}\right)$ photons/m^2/yr at Earth from the Milky Way halo

- from region within 1 degree of Galactic center, rate is instead:

$$\mathrm{few} \times 10^{-5}\mathrm{photons}/\mathrm{m}^2/\mathrm{yr}$$

- rate is prohibitively small for reasonable space-based telescope.
- ground-based gamma-ray telescope with effective areas $\sim 10^{5-6}\mathrm{m}^2$
- however, current and near-future ground based telescopes have low-energy thresholds $10 - 20\mathrm{GeV}$
- need to be lowered by an order of magnitude to observe capture and transition photons from DM $\mathcal{O}\left(10\right)\mathrm{TeV} \rightarrow \mathrm{E_n} \sim 1\mathrm{GeV}$ deepest bound states