Discussion: “Connections to Fundamental Theory”

Gary Shiu

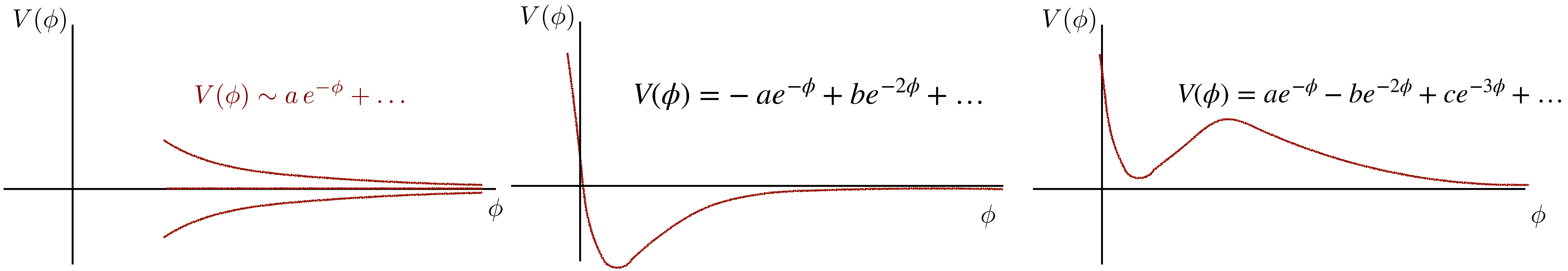
Cosmic Expansion and Fundamental Theory

In realizing cosmic expansion, having a fundamental theory is both a blessing and a curse:

- 😀 Cosmic acceleration is often driven by scalars (inflaton, quintessence,…). Fundamental theory such as string theory has many scalars (and axions, hence axiverse).
- ☹️ String theory has too many scalars: if not stabilized, they could lead to varying coupling constants, 5-th force, and mess up BBN (unless $m \gtrsim 30\ \mathrm{TeV}$).
- 😀 These scalars can also alter cosmic expansion histories (moduli dominated non-thermal history, EMD, stasis, …). Cosmology before BBN is the Wild West!
- ☹️ Why most scalars are stabilized while one (or a few) is dynamical? Who order the mass hierarchy? Naturalness? Does string theory suggest a departure from the simple single field scenarios?
- 😀 String theory provides a UV complete framework to address various naturalness problems: Snowmass white papers: 2203.07629 [hep-th], 2204.01742 [hep-th], Review: 2401.01939 [hep-th].
- ☹️ **Scalars in string theory have a second role: Dine-Seiberg problem.**

Dine-Seiberg Problem

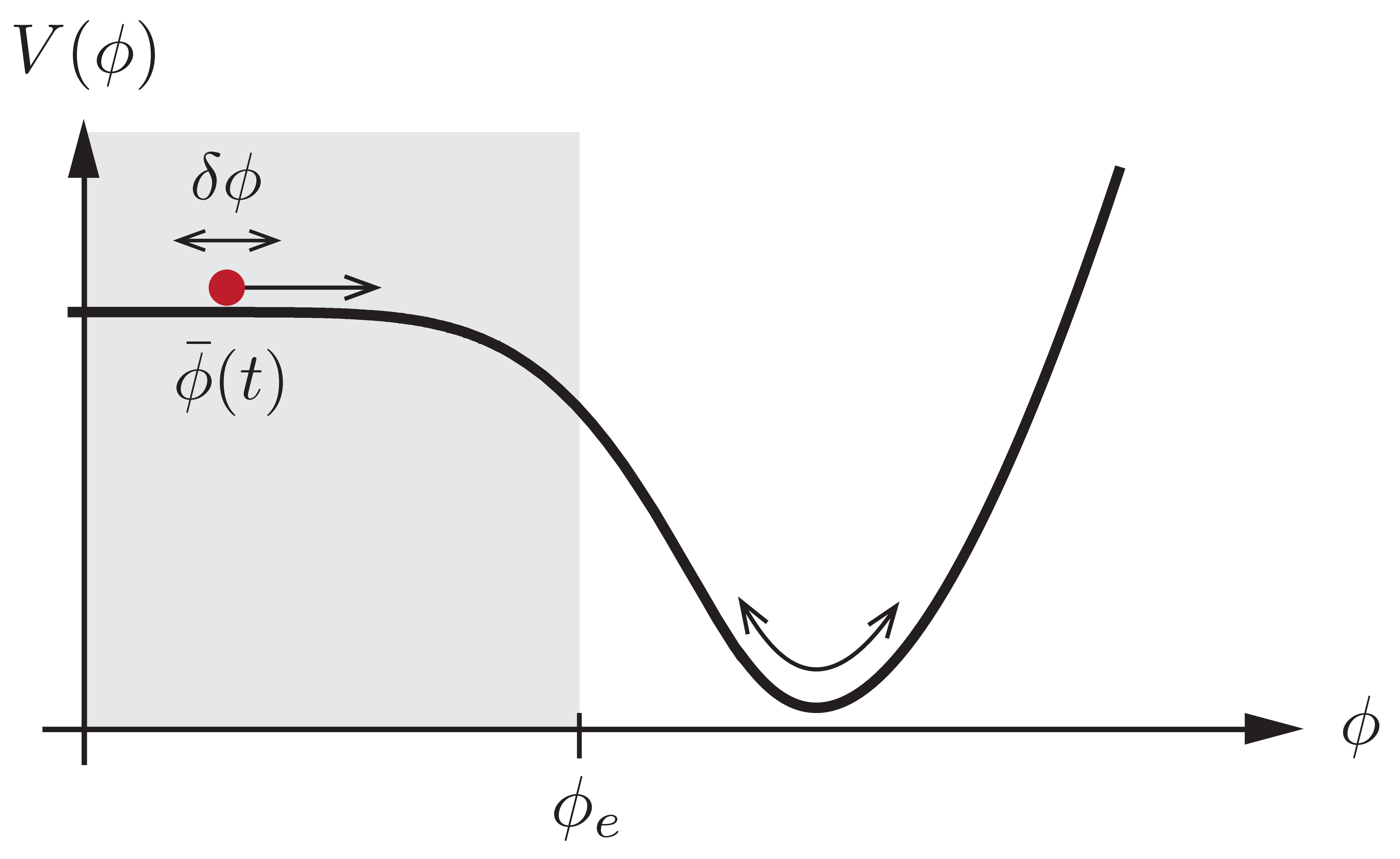
- There are no free parameters in string theory: coupling constants are vevs of scalar fields.



- The vevs of scalar fields are the perturbative expansion parameter, e.g., $g \sim \langle \exp(-\phi) \rangle$.
- A vacuum exists only if terms of different order compete. A de Sitter vacuum requires at least 3 competing terms.
- If different order terms compete to give a minimum, why aren't higher order terms important?
- The Dine-Seiberg problem: difficulty in finding parametrically controlled vacua. (LVS? KKLT? DGKT?).

Cosmic Acceleration

- I'll focus on **cosmic acceleration**, leaving alternative cosmic expansion histories to Jim.
- Observations suggest two accelerating phases: **inflation** (early), & **dark energy** (now).
- Hurdles to embed these two accelerating phases into string theory are somewhat different.
- **Inflation:** Assuming that other than the inflaton, all moduli are stabilized, are we done?



Slow roll conditions:

$$\epsilon_V = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1, \; |\eta_V| = |M_P^2 \frac{V''}{V}| \ll 1$$

Dimension 6 operators

$$\mathcal{O}_6 = c_1 \frac{V(\phi)}{\Lambda_{UV}^2} \quad \rightarrow \quad \Delta\eta \sim \mathcal{O}\left(\frac{M_P}{\Lambda_{UV}}\right)^2,$$

Λ_{UV} = KK scale, string scale, Planck scale,…

Primordial Gravitational Waves

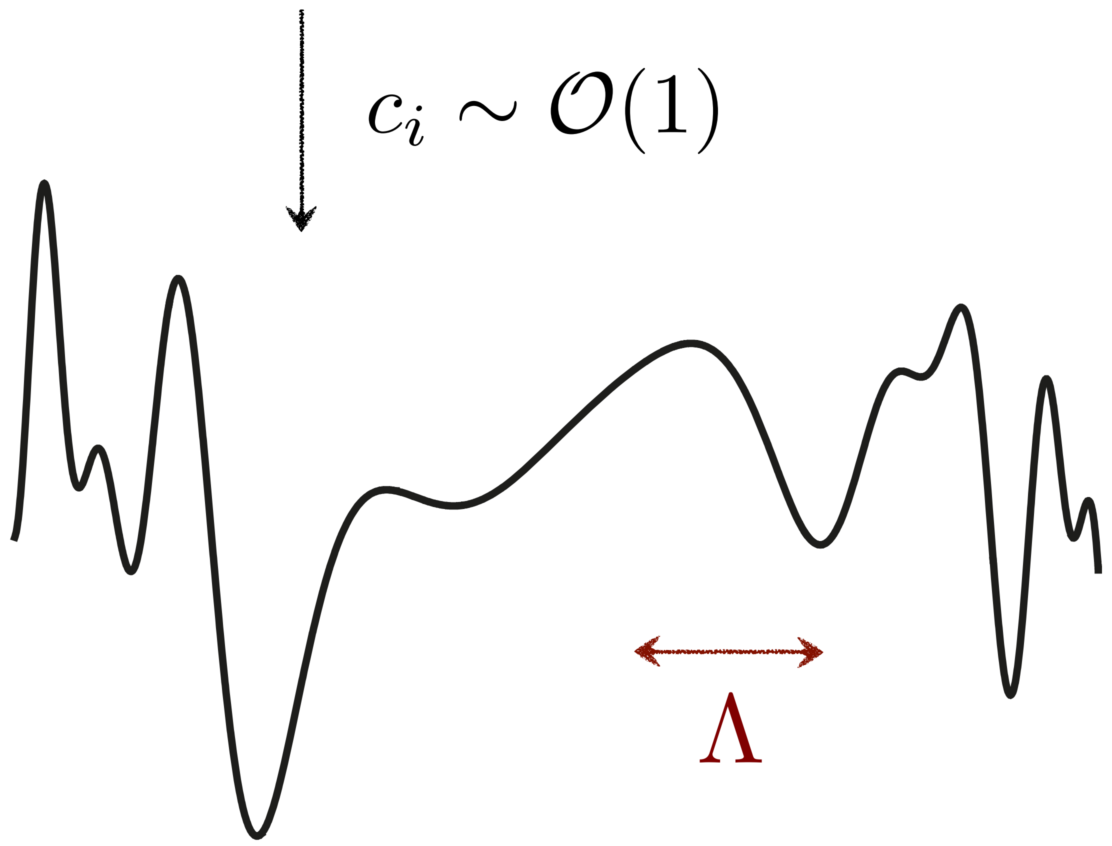
- Models of inflation that generate detectable gravitational waves require $V(\phi)$ to be nearly flat over a super-Planckian field range:

$$\Delta\phi \gtrsim \left(\frac{r}{0.01}\right)^{1/2} M_{\mathrm{Pl}}$$

[Lyth '96]

- Near future experiments e.g. CMB-S4, Simons Observatory, LiteBIRD are reaching the 10^{-3} level.

$$\mathcal{L}_{\mathrm{eff}}[\phi] = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2\left(1 + \sum_{i=1}^{\infty} c_i \frac{\phi^{2i}}{\Lambda^{2i}} + \cdots\right)$$



Dark Energy

Nobel Prize 2011

Photo: Roy Kaltschmidt. Courtesy: Lawrence Berkeley National Laboratory

Saul Perlmutter

Photo: Belinda Pratten, Australian National University

Brian P. Schmidt

Photo: Homewood Photography

Adam G. Riess

But Riess suspects that the mystery can't be solved by observations alone. "We won't really resolve it until some brilliant person, the next Einstein-like person, is able to get the idea of what's going on," he said.

So he issued **a plea to the theorists**: "Keep working," he said. "We need your help. ... It's a very juicy problem, it's hard, and **you'll win a Nobel Prize if you figure it out. In fact, I'll give you mine.**"

Oct 4, 2011 https://www.nbcnews.com/science/cosmic-log/physics-prize-highlights-puzzles-flna6c10402772

To roll or not to roll?

Current cosmic acceleration can be realized by:

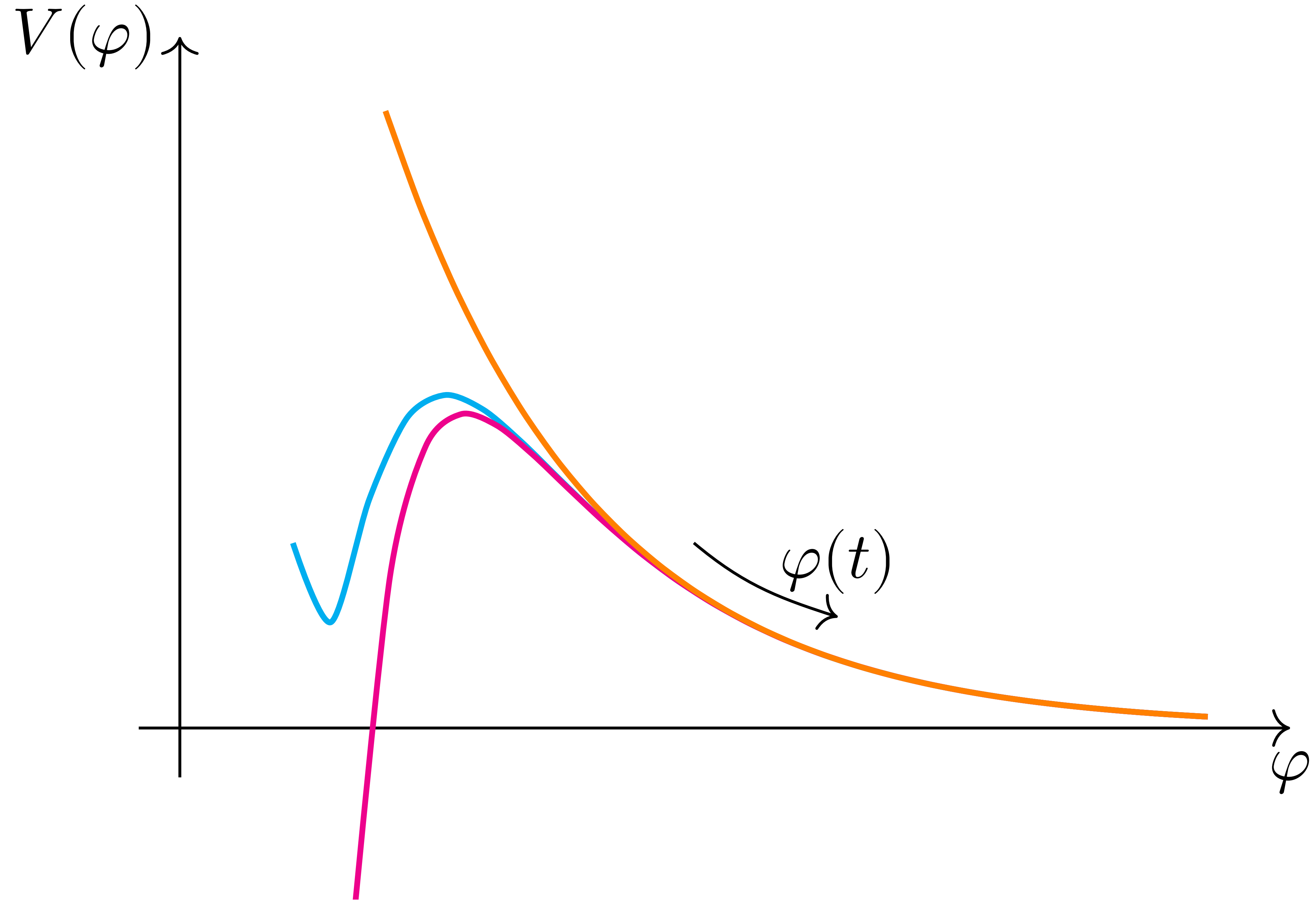
- a de Sitter minimum,
- a de Sitter maximum, or
- a runaway potential with $\epsilon \equiv -\dfrac{\dot{H}}{H^2} < 1$

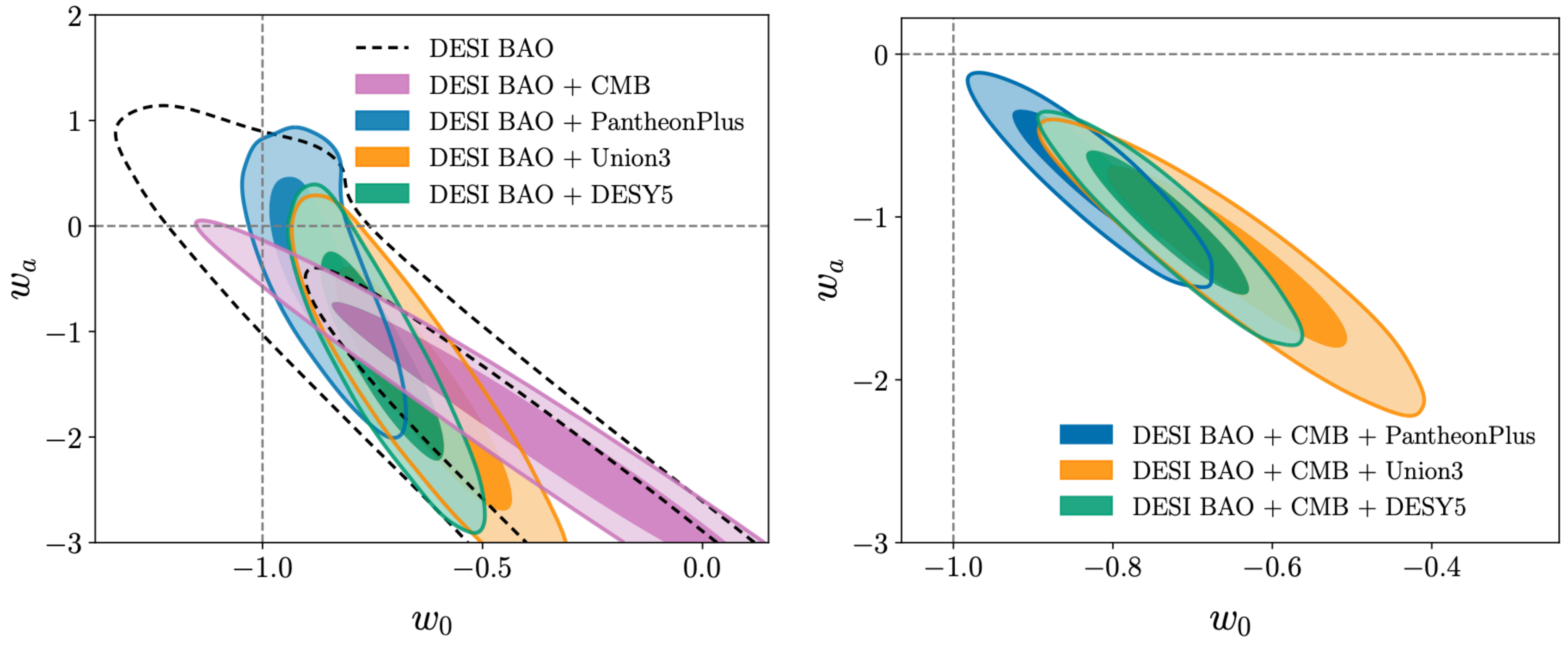
Unlike inflation which needs to last 60 e-folds to solve the flatness & horizon problems, the current acceleration may last only an e-fold or less.

If the universe underwent a rolling phase before, why not again? (main hurdle: 5-th force constraint)

Recent DESI results gave a tantalizing hint of varying dark energy, though it is too early to tell.

Generally $\epsilon \neq \epsilon_V$ due to non-negligible kinetic energy. How do we bound ϵ w/o knowing on-shell solutions?





Bounds on late-time acceleration and cosmological attractors

Flavio Tonioni
UW-Madison Physics → KU Leuven

Hung V. Tran
UW-Madison Math

[STT1]: ``Accelerating universe at the end of time,'' PRD **108**, no.6, 063527 (2023) [2303.03418].

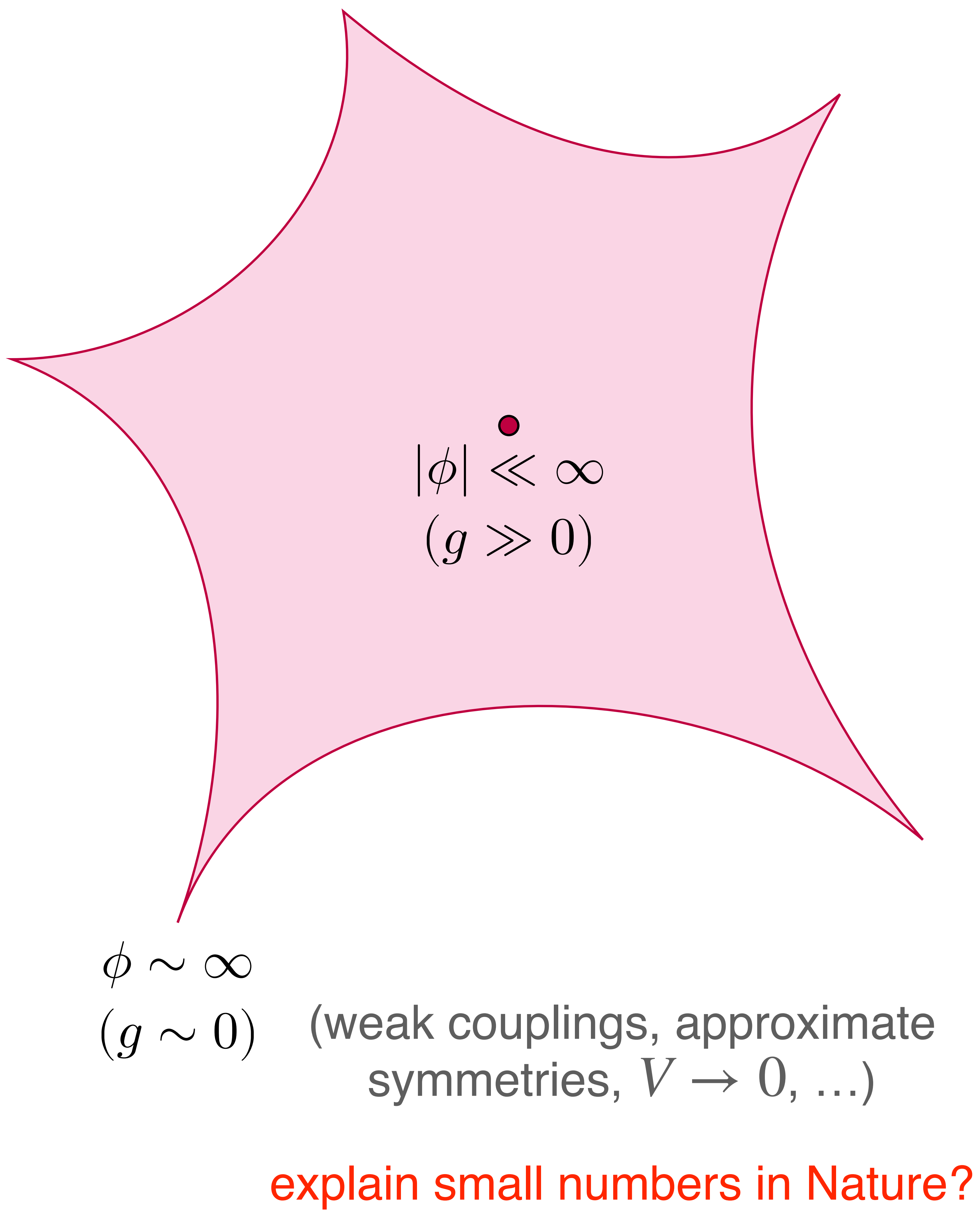
[STT2]: ``Late-time attractors and cosmic acceleration,” PRD **108**, no.6, 063528 (2023) [2306.07327].

[STT3]: ``Collapsing universe before time,” JCAP **05**, 124 (2024) [2312.06772].

[STT4]: ``Analytic bounds on late-time axion-scalar cosmologies,” [2406.17030].

Asymptotic Dark Energy

- Could the current acceleration be realized by rolling towards the asymptotic regions of the landscape?
- Does not require terms of different order to compete, in contrast to the Dine-Seiberg problem for vacua.
- A tower of states becomes light as we approach the asymptotic. Entropy bound suggest that the potential has an exponential falloff [Ooguri, Palti, GS, Vafa].
- But solving multi-field dynamics is much more difficult than taking derivatives of potential!
- As in many dynamical systems, the late-time regime exhibits some universal behaviors. This allows us to prove bounds on acceleration [GS, Tonioni, Tran].



Multi-field Quintessence

- String theoretical potentials generically take the form (also argument by [Ooguri, Palti, GS, Vafa]):

$$V = \sum_{i=1}^{m} \Lambda_i \, \mathrm{e}^{-\kappa_d \gamma_{ia} \phi^a} .$$

after canonically normalizing the scalar fields to ϕ^a, $a = 1,\ldots,n$.

- Λ_i, γ_{ia} depend on the microscopic origin of V_i, κ_d = d-dim. gravitational coupling. Potentials from e.g. internal curvature, fluxes, branes/O-planes, Casimir-energy, etc take this form.

- Given a multi field quintessence model, how do we diagnose if it can support acceleration without solving for the time-dependent solutions? ([STT1, STT2]).

- We consider scalars rolling towards the field space boundary: axions with a compact field space are assumed to be stabilized above. The saxions can then be canonically normalized.

- In the presence of dynamical axions, the field space metric is curved but in certain classes of models, the bounds we derived continue to apply [STT4].

Summary of Results

- Treating the universe as a dynamical system, we bound the rate of time variation of the Hubble parameter at late time [STT1]. The bound provides a useful diagnostic for dark energy models.
- Our bound when applied to string theoretic constructions identifies a generic obstacle to acceleration if the d-dim. dilation is one of the rolling fields. We also suggest several ways out.
- We prove conditions under which scaling solutions are **late-time attractors**. Moreover, we prove that scaling solutions **saturate** our bound on ϵ [STT2].
- Our results apply irrespective of whether the potential is generated classically or quantum mechanically, whether the kinetic term is negligible, & whether some potential term dominates.
- This program can be extended to quintessence models with dynamical axions as well [STT4].
- As a spinoff, we derived analogous bounds on ekpyrosis [STT3].

Cosmological Equations

- Non-compact d-dim. spacetime is characterized by the FLRW metric:

$$d\tilde{s}_d^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}l^2_{\mathbb{R}^{d-1}}\,,$$

- Hubble parameter: $H \equiv \dfrac{\dot{a}}{a}$. The proper diagnostic for cosmic **acceleration** is $\epsilon \equiv -\dfrac{\dot{H}}{H^2} < 1$ to be **distinguished from the slow-roll parameter** $\epsilon_V = \dfrac{d-2}{4}\kappa_d^2\left(\dfrac{\nabla V}{V}\right)^2$.

- Scalar field equations and Friedmann equations:

$$\ddot{\phi}^a + (d-1)H\dot{\phi}^a + \frac{\partial V}{\partial \phi_a} = 0,$$

$$\frac{(d-1)(d-2)}{2}\,H^2 - \kappa_d^2\left[\frac{1}{2}\,\dot{\phi}_a\dot{\phi}^a + V\right] = 0,$$

$$\dot{H} = -\frac{\kappa_d^2}{d-2}\left[\frac{1}{2}\,\dot{\phi}_a\dot{\phi}^a - V\right] - \frac{d-1}{2}H^2,$$

Cosmology as a Dynamical System

- It is convenient to work with the rescaled variables:

$$x^a = \frac{\kappa_d}{\sqrt{d-1}\sqrt{d-2}}\frac{\dot{\phi}^a}{H}, \; y_i = \frac{\kappa_d\sqrt{2}}{\sqrt{d-1}\sqrt{d-2}}\frac{\sqrt{V_i}}{H}$$

- The cosmological equations can be formulated in terms of an autonomous system of ODEs given schematically as follows:

$$\frac{d\vec{z}}{dt} = g(\vec{z}) \; , \qquad \text{where } \vec{z} \equiv (x^1, \ldots, x^n, y^1, \ldots, y^m, H)$$

- Among the above ODEs is $\epsilon = -\dot{H}/H^2 = (d-1)x^2$; strategy is to bound the kinetic energy.

- Friedmann equation also takes a simple form:

$$(x)^2 + (y)^2 = 1$$

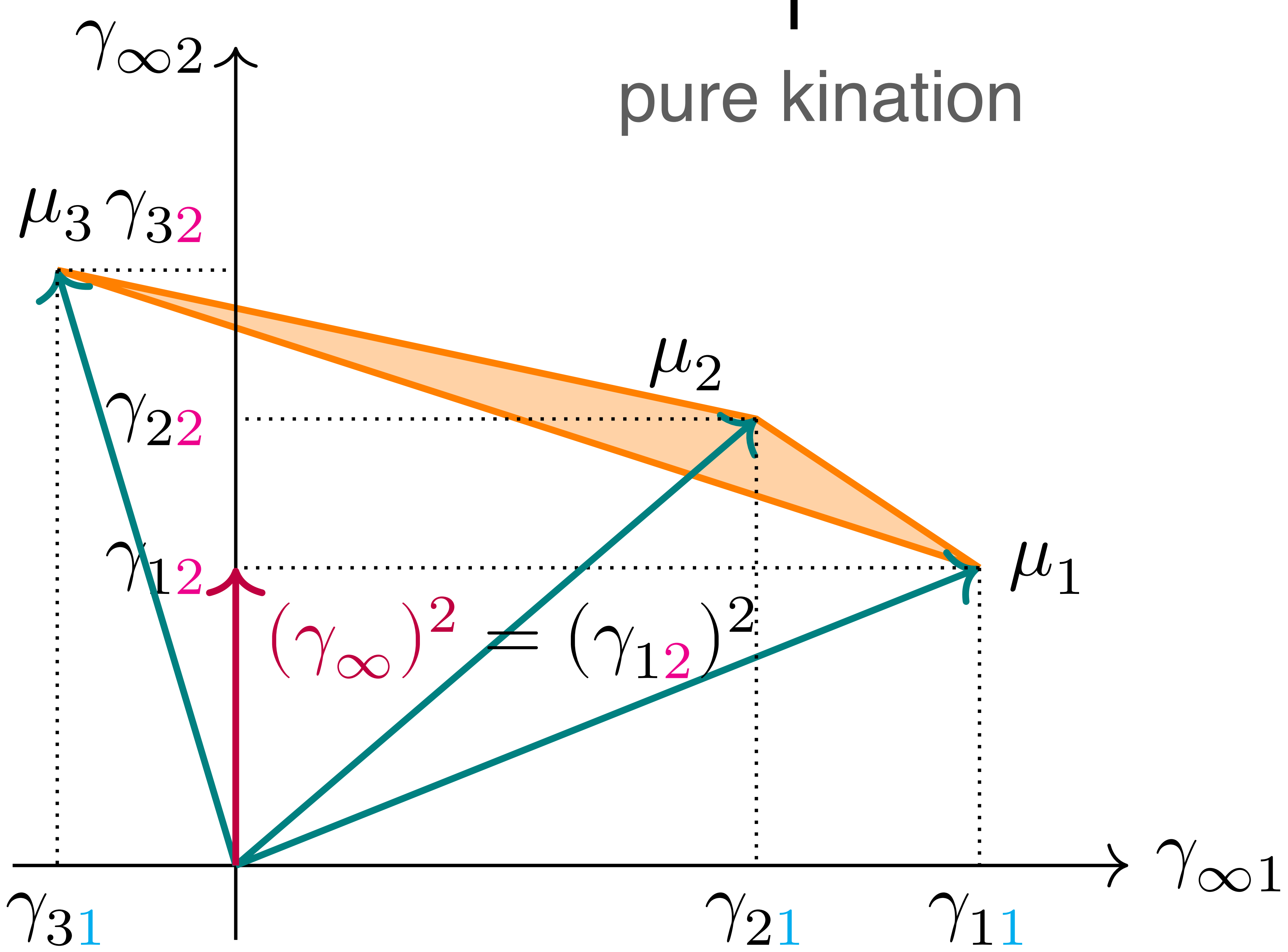
Geometric Bound on Cosmic Acceleration

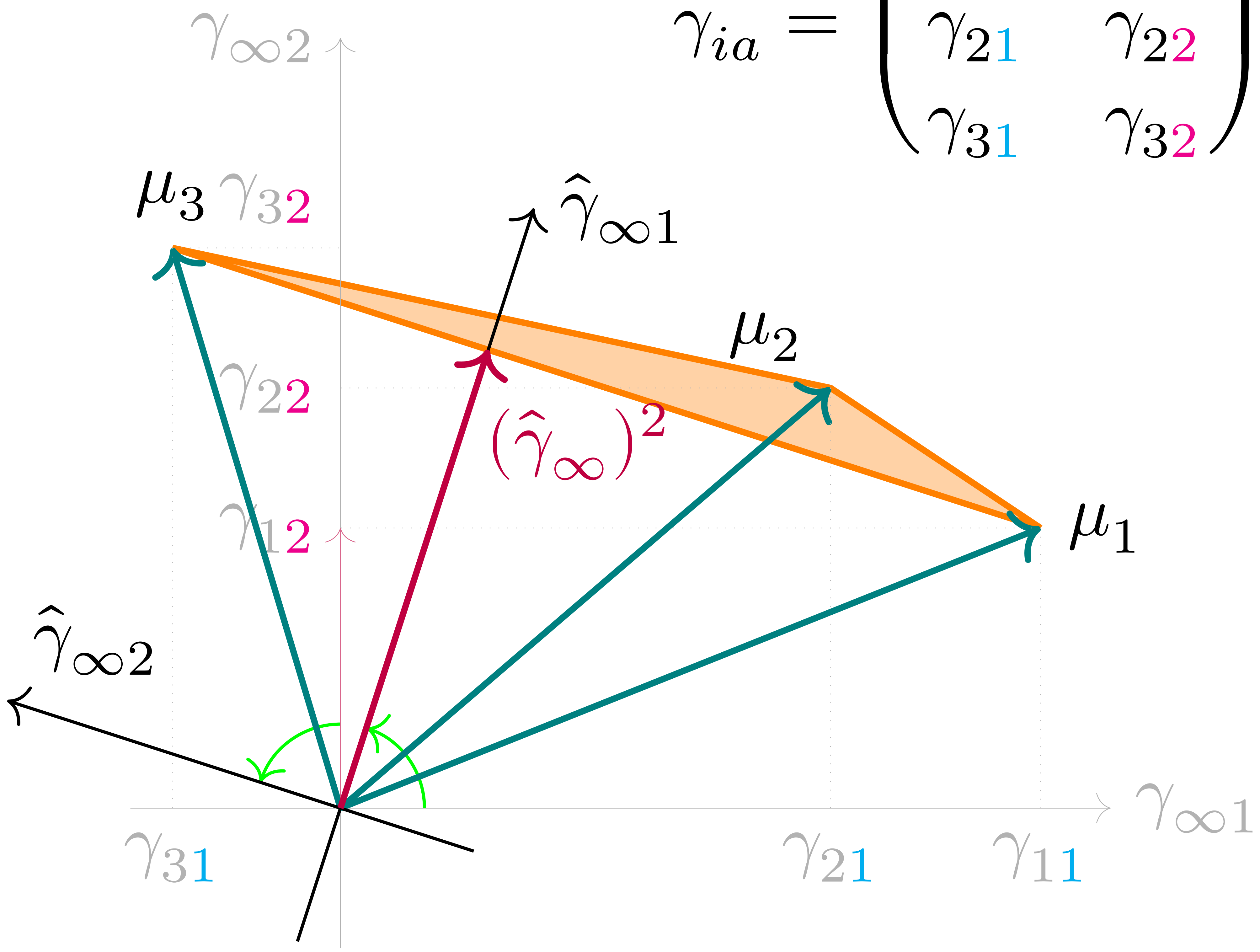
- Define m vectors μ_i, one for each potential term with components $(\mu_i)_a = \gamma_{ia}$

$$d - 1 \geq \epsilon \geq \frac{d-2}{4} \max_{R \in O(n)} [\gamma_\infty(R)]^2$$

↑ pure kination

$$\gamma_{ia} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \\ \gamma_{31} & \gamma_{32} \end{pmatrix}$$





Obstruction by the Dilaton

- String-theoretical potentials take the form:

$$S = -\int_{\mathrm{X}_{1,9}} [A_r \wedge \star_{1,9} A_r]\, \Lambda_{10,r}\, \mathrm{e}^{-k\sigma - \chi_{\mathrm{E}}\Phi} = -\int_{\mathrm{X}_{1,d-1}} \tilde{*}_{1,d-1} \Lambda\, \mathrm{e}^{\kappa_d[\gamma_{\tilde{\delta}}(\chi_{\mathrm{E}})\tilde{\delta} - \gamma_{\tilde{\sigma}}(\chi_{\mathrm{E}}, r, k)\tilde{\sigma}]}$$

RR fields are not weighed by $e^{-\chi_E \Phi}$ (effectively set $\chi_E = 0$) but would not affect our argument.

- The d-dim. dilaton $\tilde{\delta}$ is a linear combination of the 10d dilaton Φ and Einstein frame volume.

- While the field basis choice is not unique, d-dimensional dilaton $\tilde{\delta}$ has **universal properties**:

$$\gamma_{\tilde{\delta}} = \frac{d}{\sqrt{d-2}} - \frac{1}{2}\chi_{\mathrm{E}}\sqrt{d-2} \geq \frac{2}{\sqrt{d-2}} \quad \Rightarrow \quad \epsilon \geq \frac{d-2}{4}(\gamma_\infty)^2 \geq \frac{d-2}{4}\gamma_{\tilde{\delta}}^2 \geq 1$$

- Ways out: 1) $\tilde{\delta}$ is stabilized; 2) $\tilde{\delta}$ is rolling but not in the asymptotic regions; 3) V contains at least three terms, not all of the same sign (e.g., from loop corrections).

- Non-universal couplings for other moduli: can use our bound to **constrain compactifications**.

Scaling Solutions

- The cosmological autonomous system admits **scaling solutions** (ϵ = constant > 0):
  - scale factor takes a power law form: $a(t) \sim t^p$
  - critical points of the autonomous system: $\dot{x}^a = 0$
- **Analytic solution:** for $\mathrm{rank}\, \gamma_{ia} = m$
  - field space trajectory: $$\phi_*^a(t) = \phi_0^a + \frac{2}{\kappa_d}\left[\sum_{i=1}^{m}\sum_{j=1}^{m}\gamma_i{}^a (M^{-1})^{ij}\right]\ln\frac{t}{t_0}, \qquad M_{ij} = \gamma_{ia}\gamma_j{}^a$$
  - scale factor: $$p = \frac{4}{d-2}\sum_{i=1}^{m}\sum_{j=1}^{m}(M^{-1})^{ij}.$$

[Copeland, Liddle, Wands, '97]
[Collinucci, Nielsen, Van Riet, '04]

- The kinetic term & every potential term have the same parametric dependence in time:

No slow-roll: $$T(t) = T(t_0)\left(\frac{t_0}{t}\right)^2, \quad V_i(t) = V_i(t_0)\left(\frac{t_0}{t}\right)^2$$

Late-time attractor behavior **proved** in [STT2, STT4], going beyond earlier analysis of linear stability.