Post-Inflationary Dynamics & Non-Standard Expansion Histories

Mustafa Amin & Tom Giblin

after inflation: a few min GAP in our cosmic history

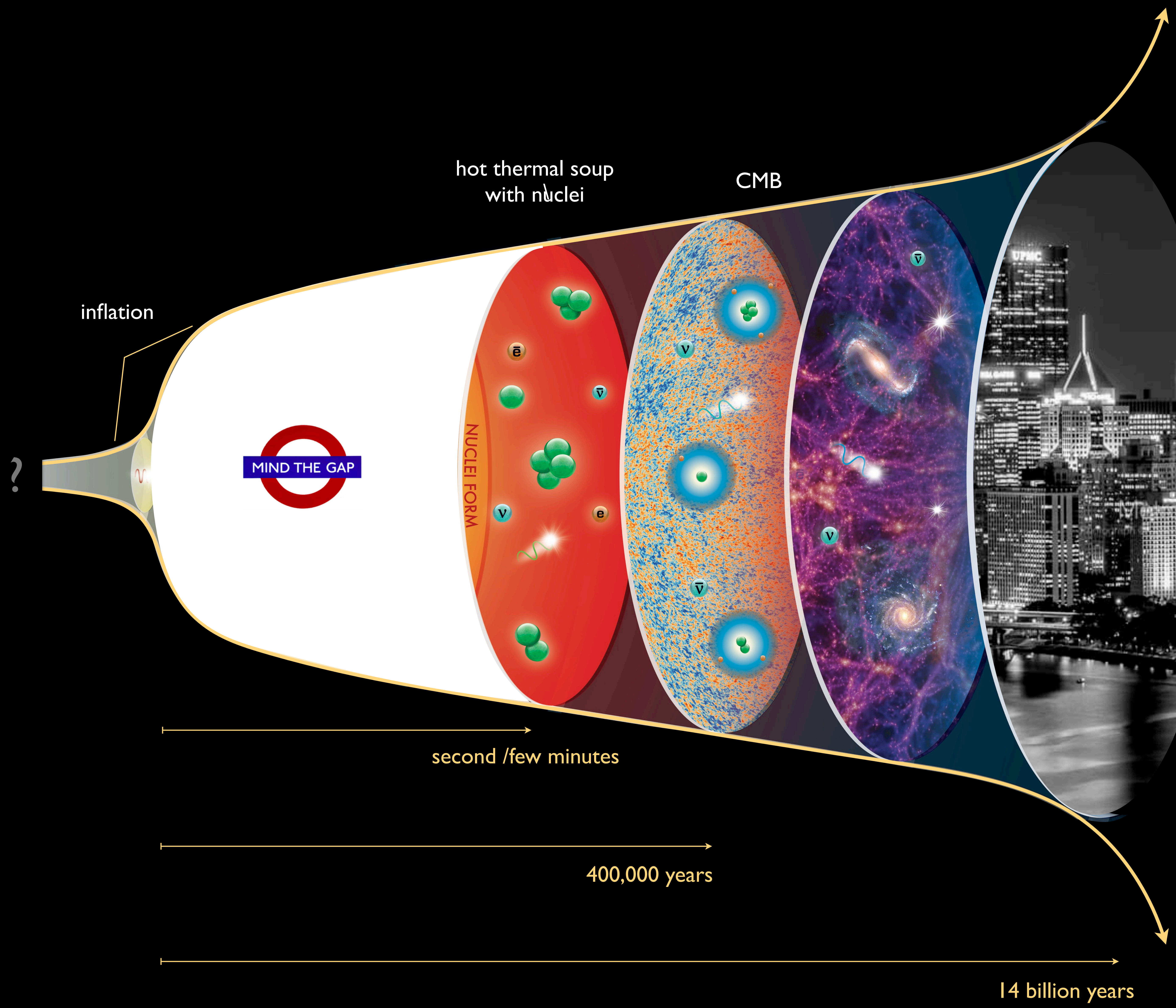
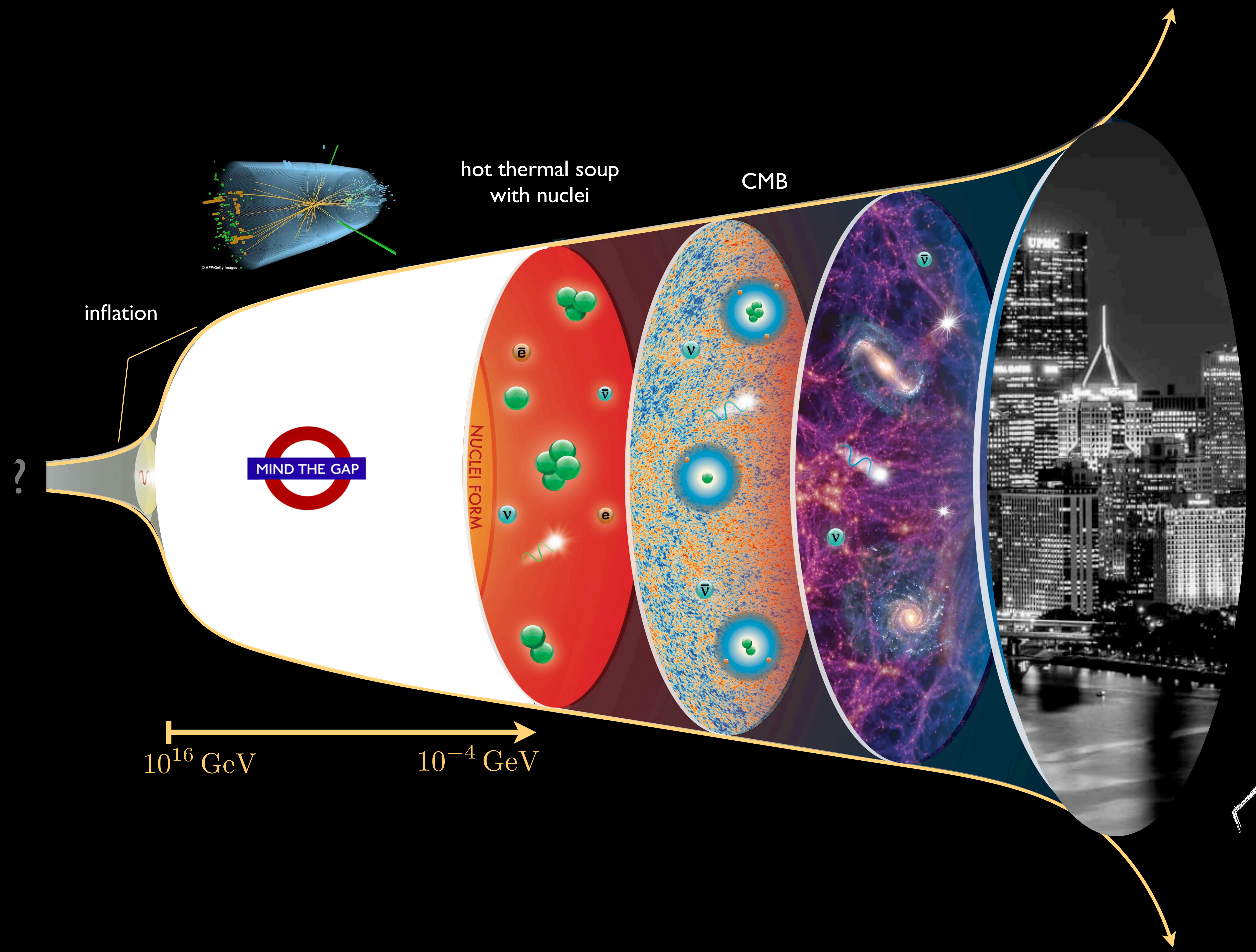
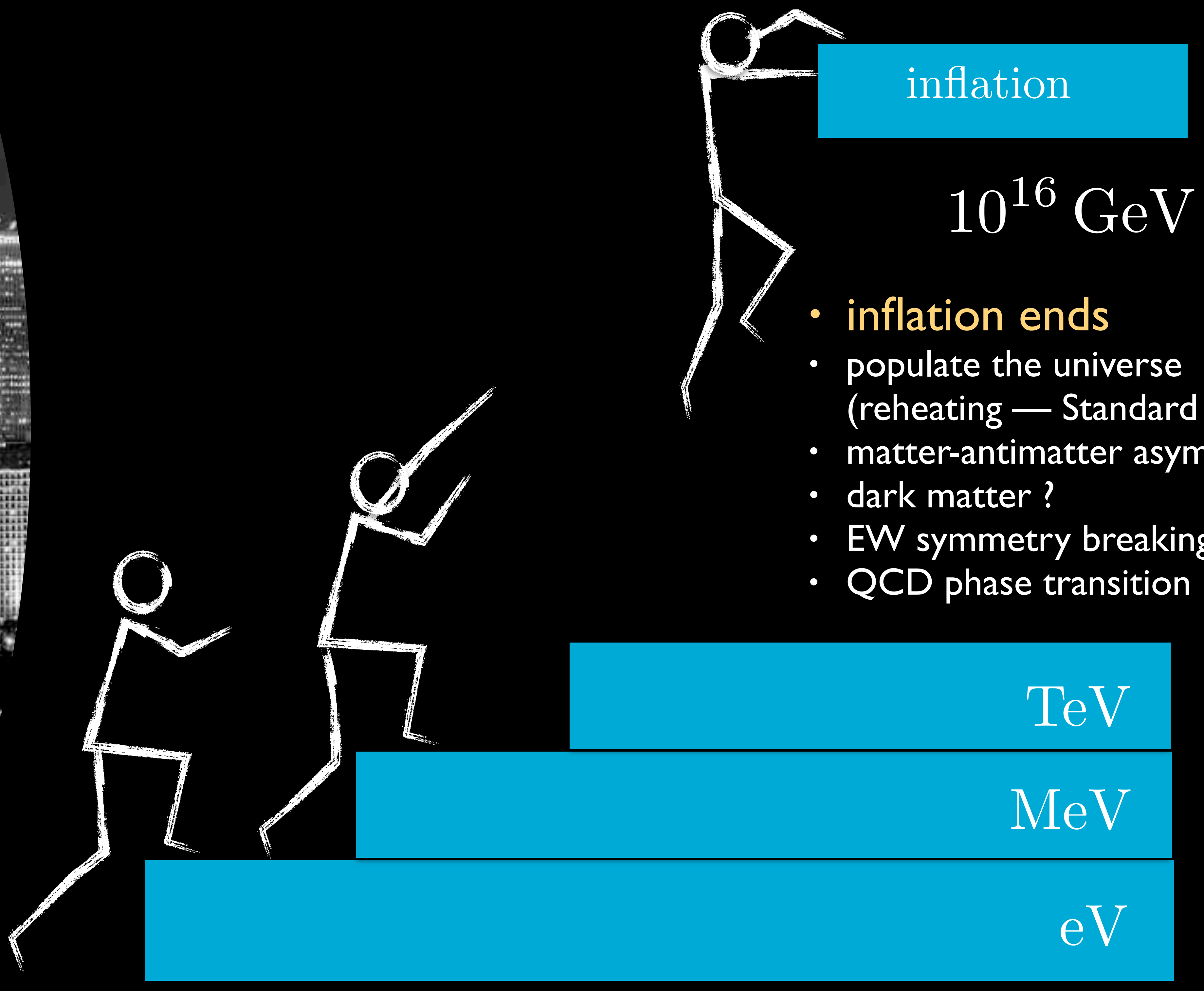


*image is a modification of the one produced by the PDG, 2014

after inflation: a huge energy GAP in our cosmic history

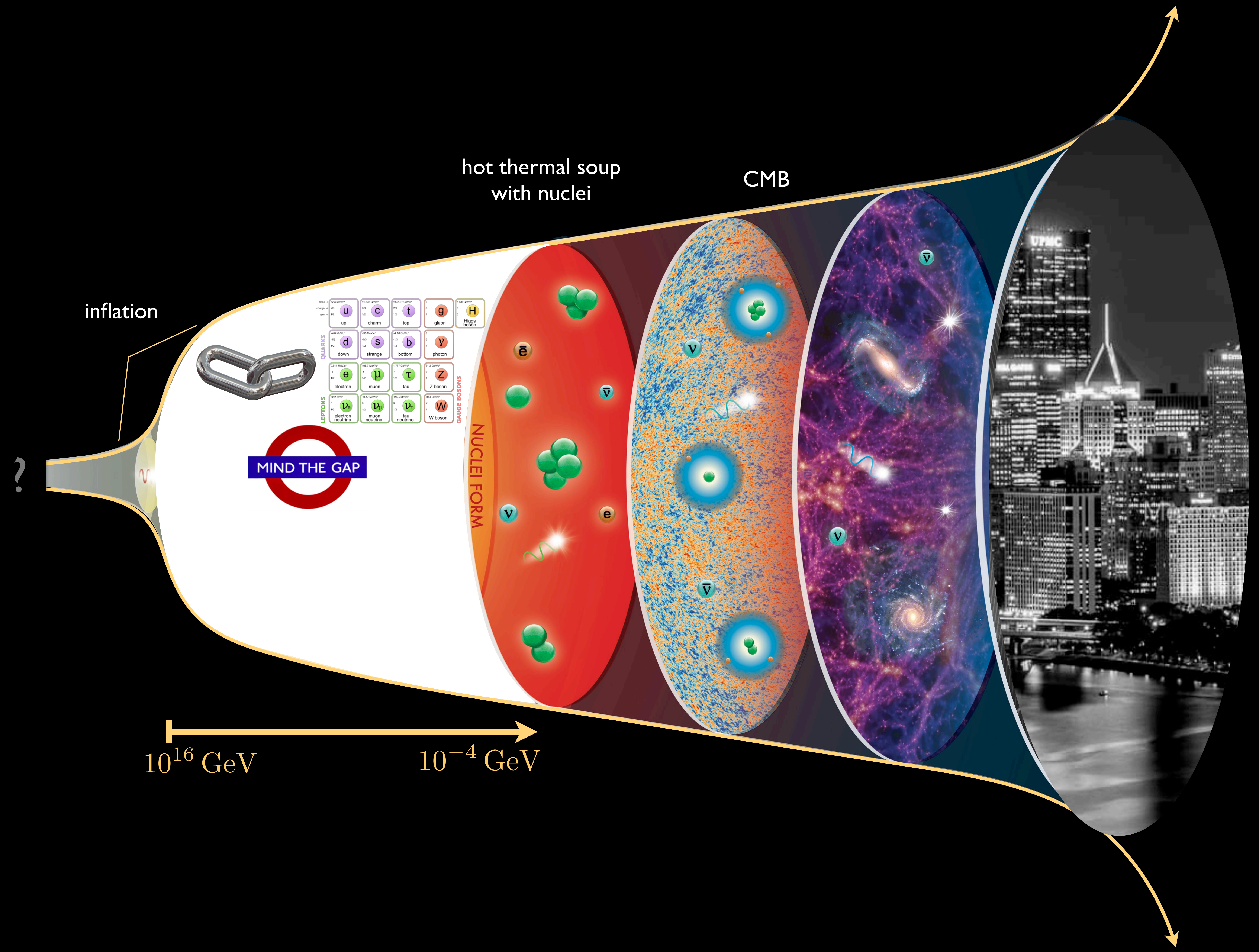




- inflation ends
- populate the universe (reheating — Standard Model) ?
- matter-antimatter asymmetry ?
- dark matter ?
- EW symmetry breaking
- QCD phase transition

*image is my modification of the one produced by the PDG, 2014

after inflation: GAP — consequences ?



focus on: “soon” after the end of inflation, simple models with “universal” dynamics

Last 3 reviews: Allahverdi et. al (2010), MA et. al (2014), Lozanov (2019)

some of the above phenomenology has already been covered earlier

reminder

$$\text{eq. of state } w = \frac{\text{pressure}}{\text{density}}$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$a(t) \propto t^{\frac{2}{3(1+w)}}$$

reminder

eq. of state $w = \frac{\text{pressure}}{\text{density}}$

$G_{\mu\nu} = 8\pi G T_{\mu\nu}$ →

$a(t) \propto t^{\frac{2}{3(1+w)}}$

$N_\star \supset \frac{1}{4}(1-3w)\Delta N_{\rm rad}$

←

inflationary observables
$r(N_\star), n_{\rm s}(N_\star)$

reminder

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(p)reheating observables

$\Omega_\mathrm{gw}(f)$

←

$\Omega_\mathrm{gw}(f) \propto \exp[-\Delta N_\mathrm{rad}(1-3w)]$

$f \propto \exp[-\Delta N_\mathrm{rad}(1-3w)/4]$

reminder

eq. of state $w = \frac{\text{pressure}}{\text{density}}$ $\xrightarrow{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$ $a(t) \propto t^{\frac{2}{3(1+w)}}$

inflationary observables
$r(N_\star), n_s(N_\star)$ ← $N_\star \supset \frac{1}{4}(1-3w)\Delta N_{\rm rad}$

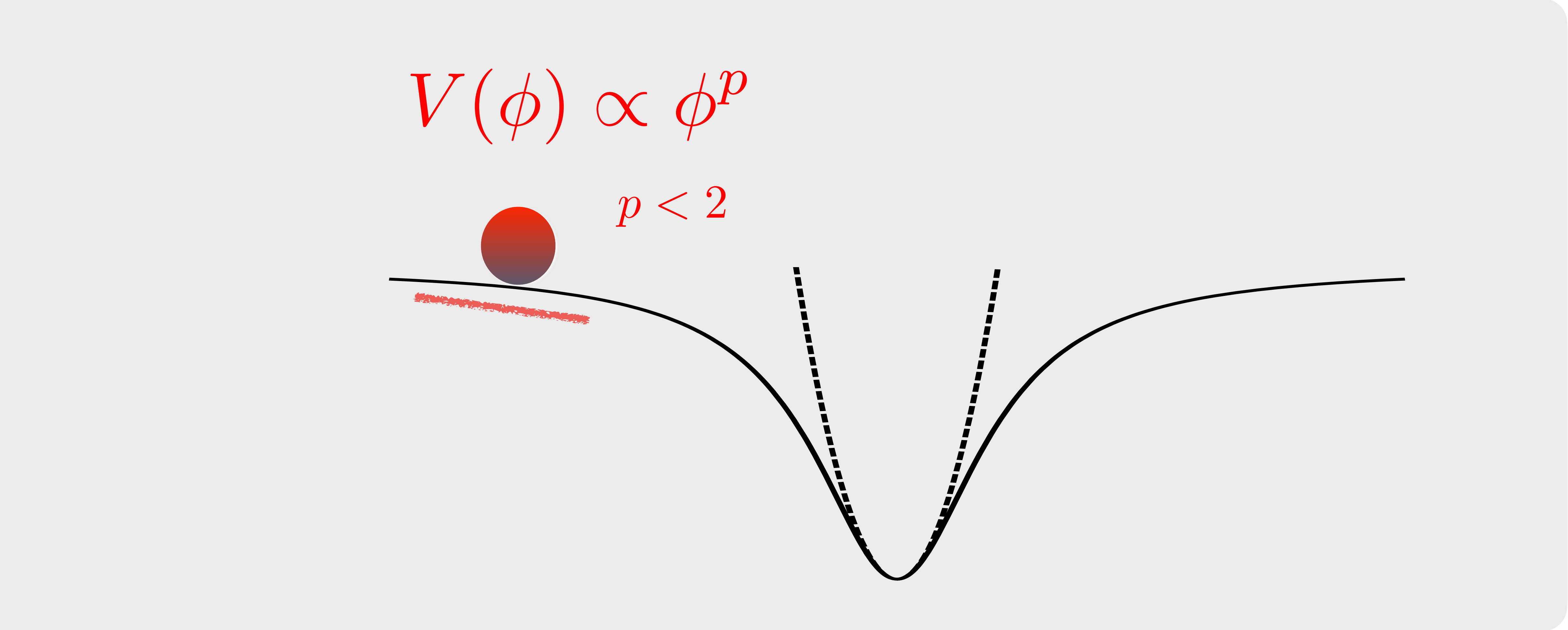
(p)reheating observables
$\Omega_{\rm gw}(f)$ ← $\Omega_{\rm gw}(f) \propto \exp[-\Delta N_{\rm rad}(1-3w)]$
$f \propto \exp[-\Delta N_{\rm rad}(1-3w)/4]$

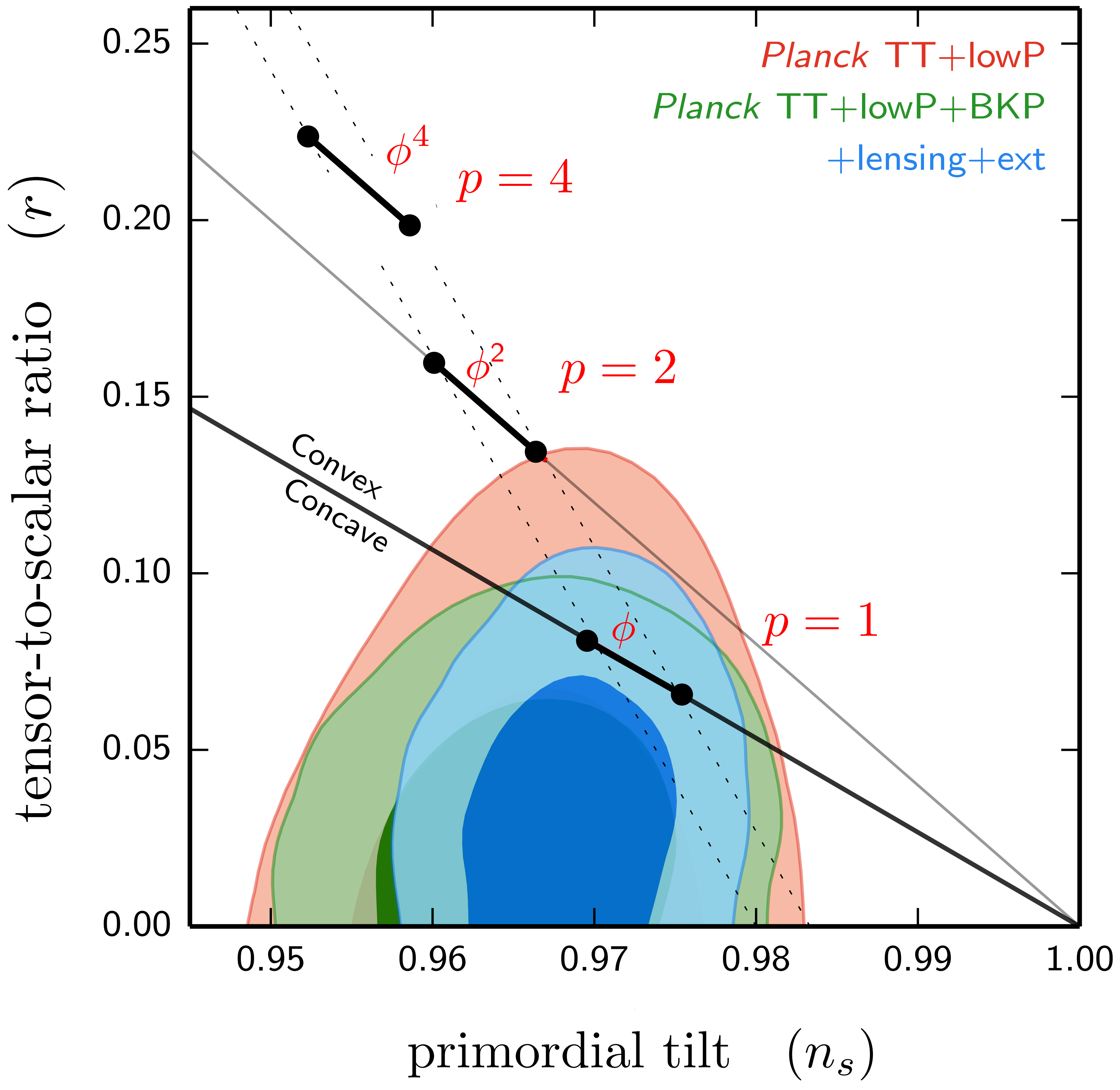
$P_\delta(k), \Omega_{\rm dm} \ldots$ ← $f_i(t, \boldsymbol{x}, \boldsymbol{p})$ + expansion history +

modeling the end of inflation

what we "know" about inflation (simplest case - scalar field driven inflation)
— flattened potentials

$$S = \int d^4x\sqrt{-g}\left[\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

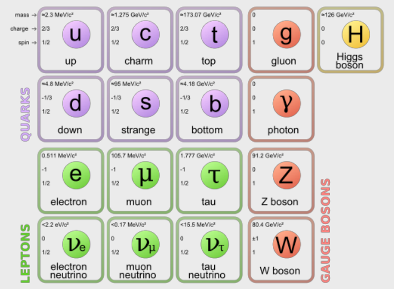




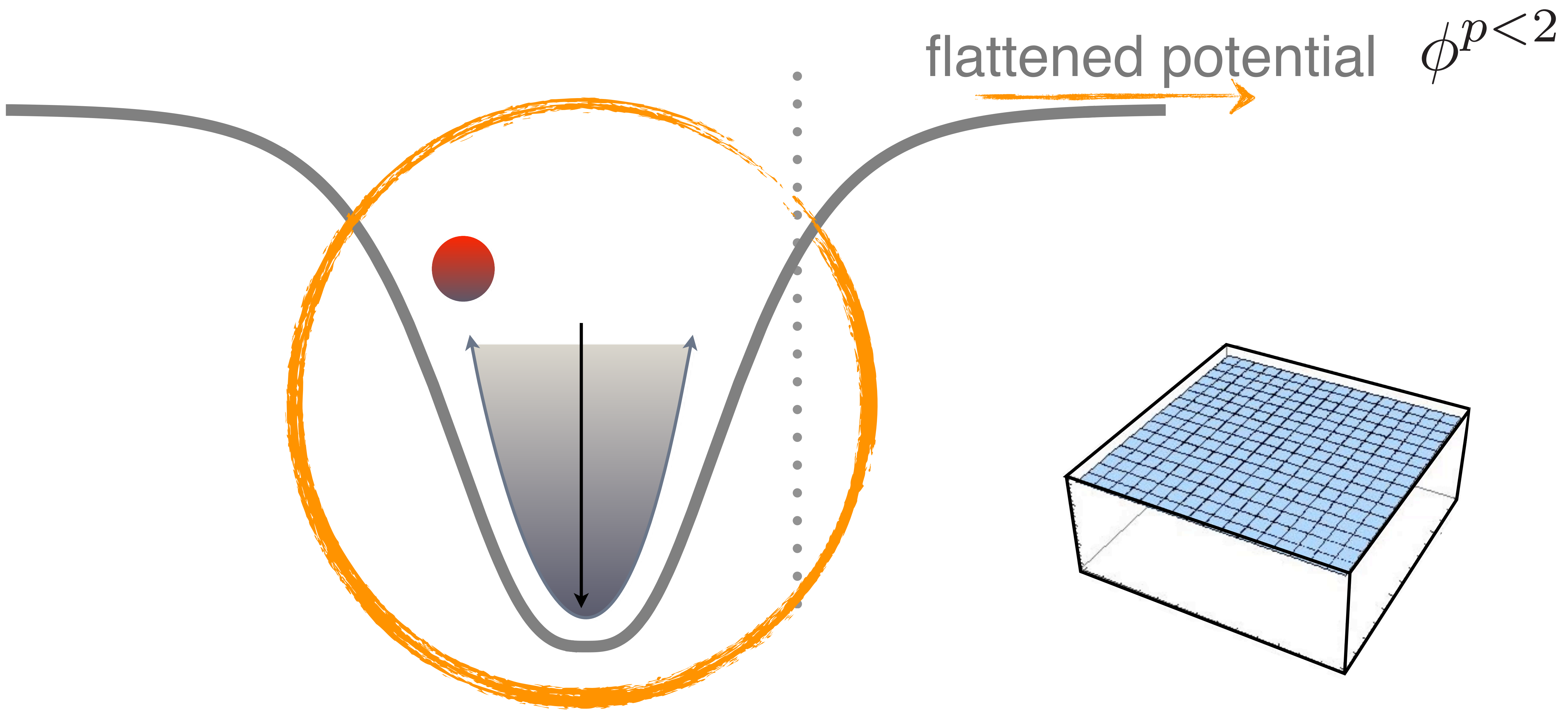
for example:
Starobinsky(1979/80), Nanopolous et. al (1983), Silverstein & Westhpal (2008), Kallosh & Linde (2013), McAllister et. al (2014) …

end of inflation depends on …

- shape of the potential (self-couplings)
- couplings to other fields



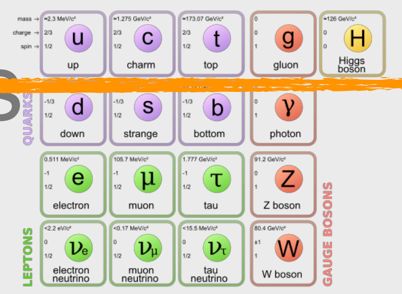
$\chi\ ,\psi\ A_\mu$

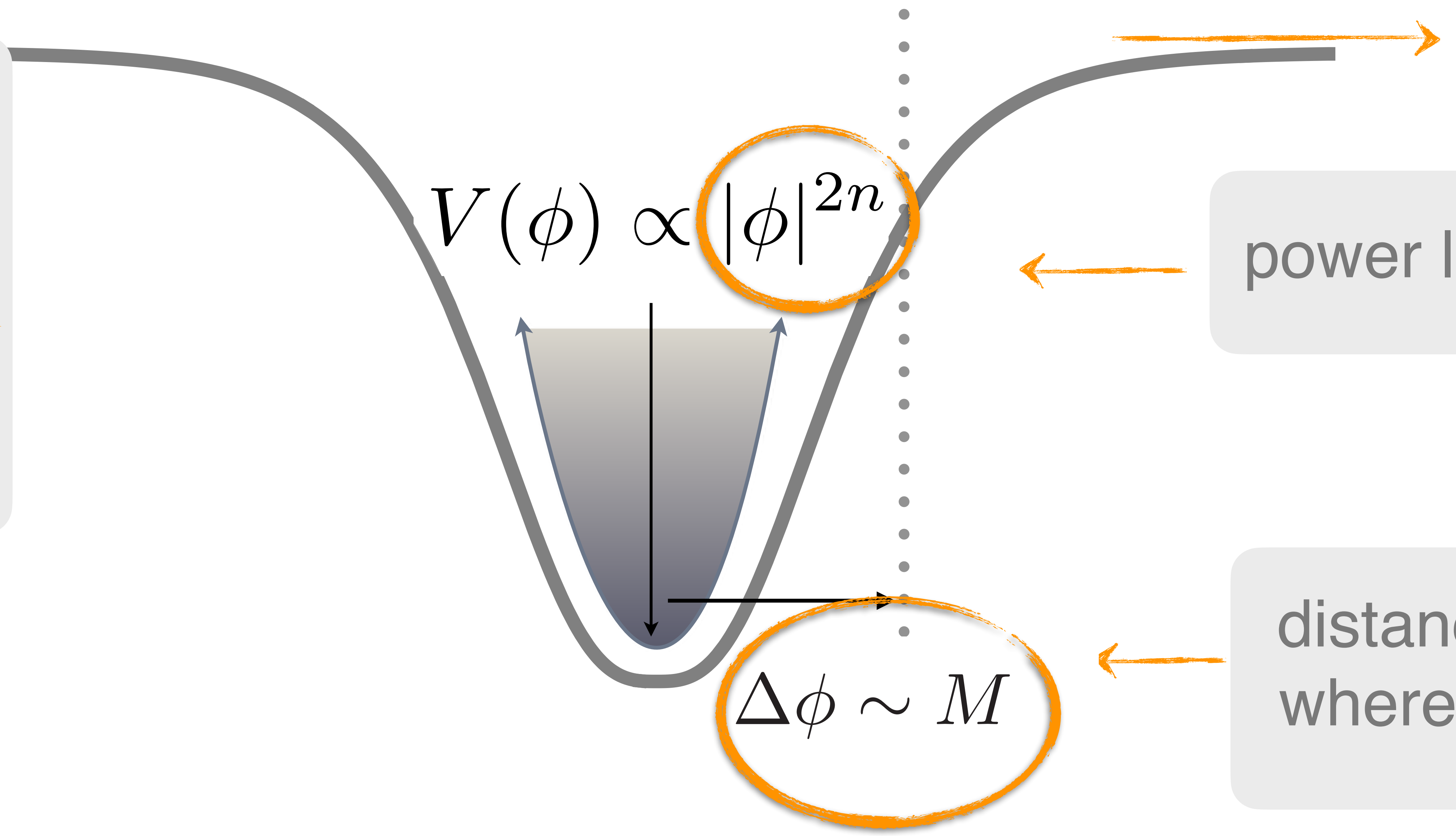


*for “model-independent” attempts see Oszoy et. al (2015)

end of inflation (ignoring couplings to other fields*)

- shape of the potential (self couplings)
- ~~couplings to other fields~~ χ , ψ A_μ





flattened potential

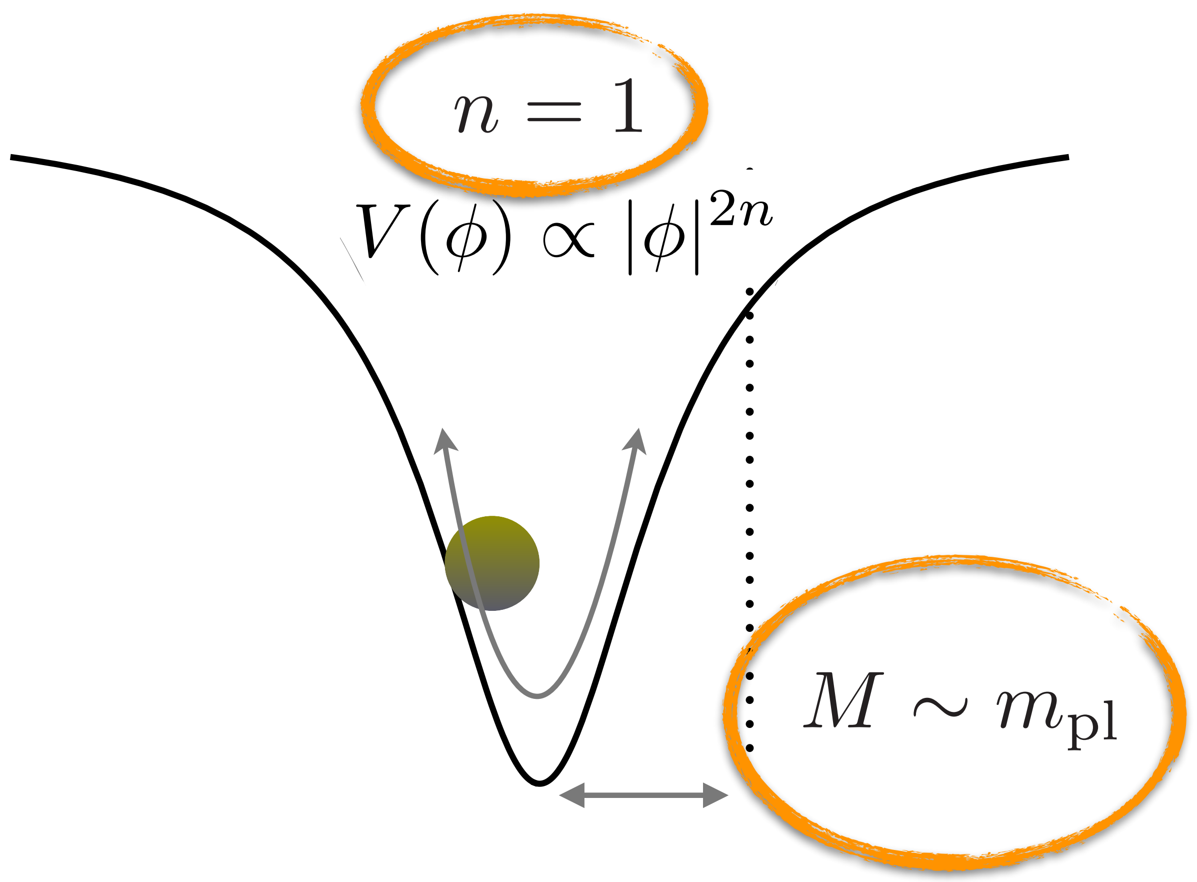
power law at minimum

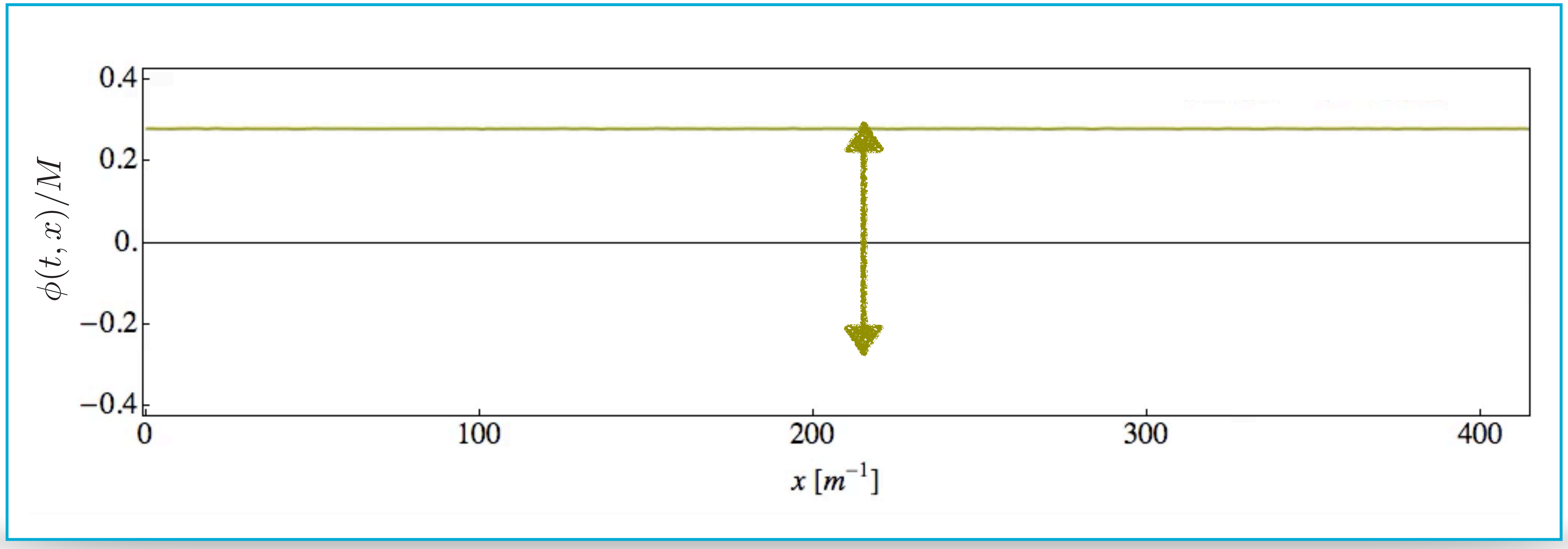
distance from minimum where potential flattens

oscillating "free" scalar field: matter-dominated expansion

$$w \approx \frac{n-1}{n+1} = 0$$

$$\Box\phi \approx m^2\phi$$

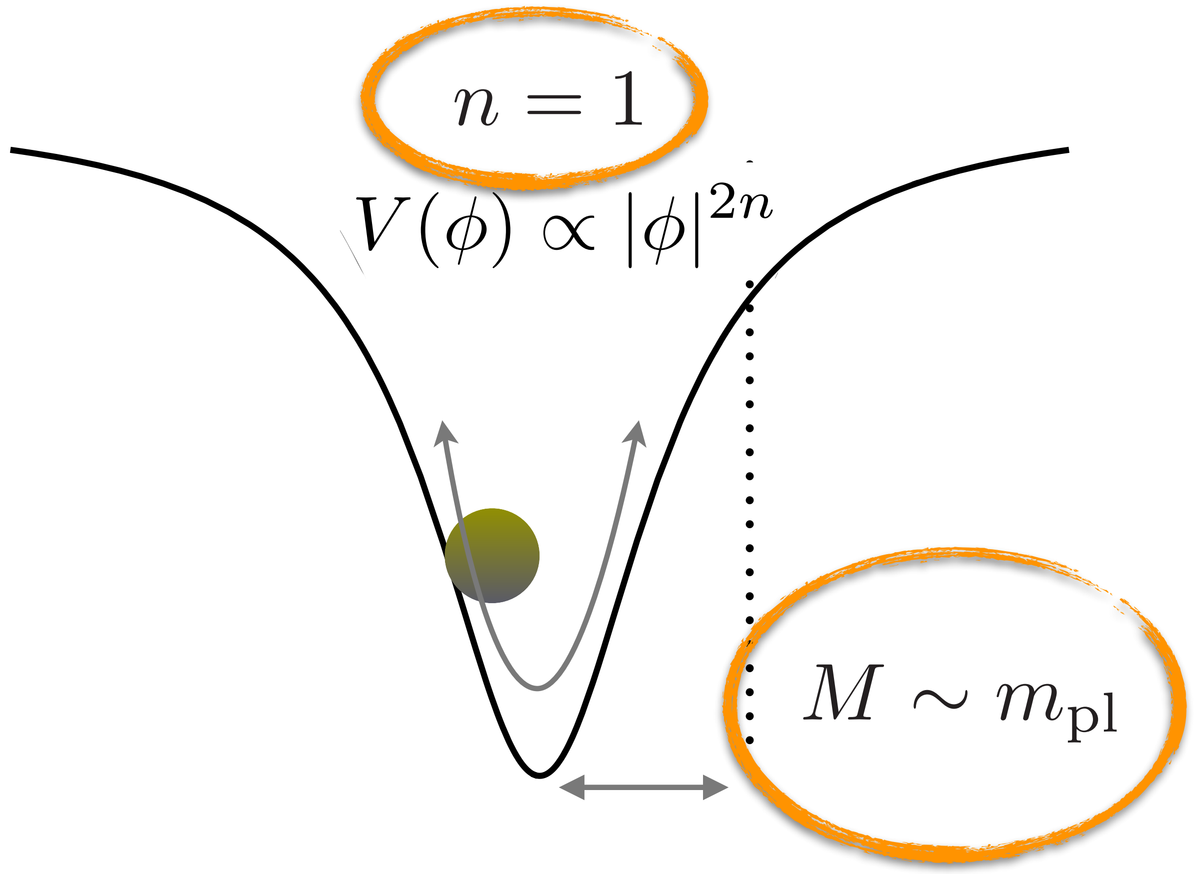


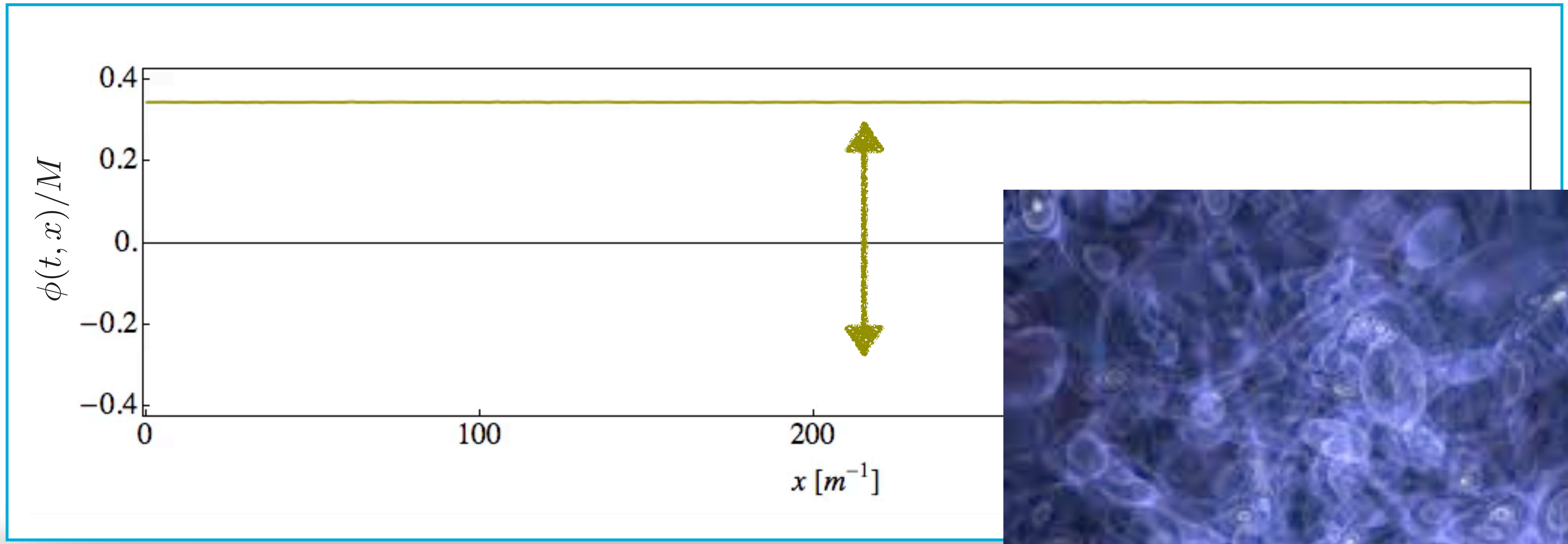


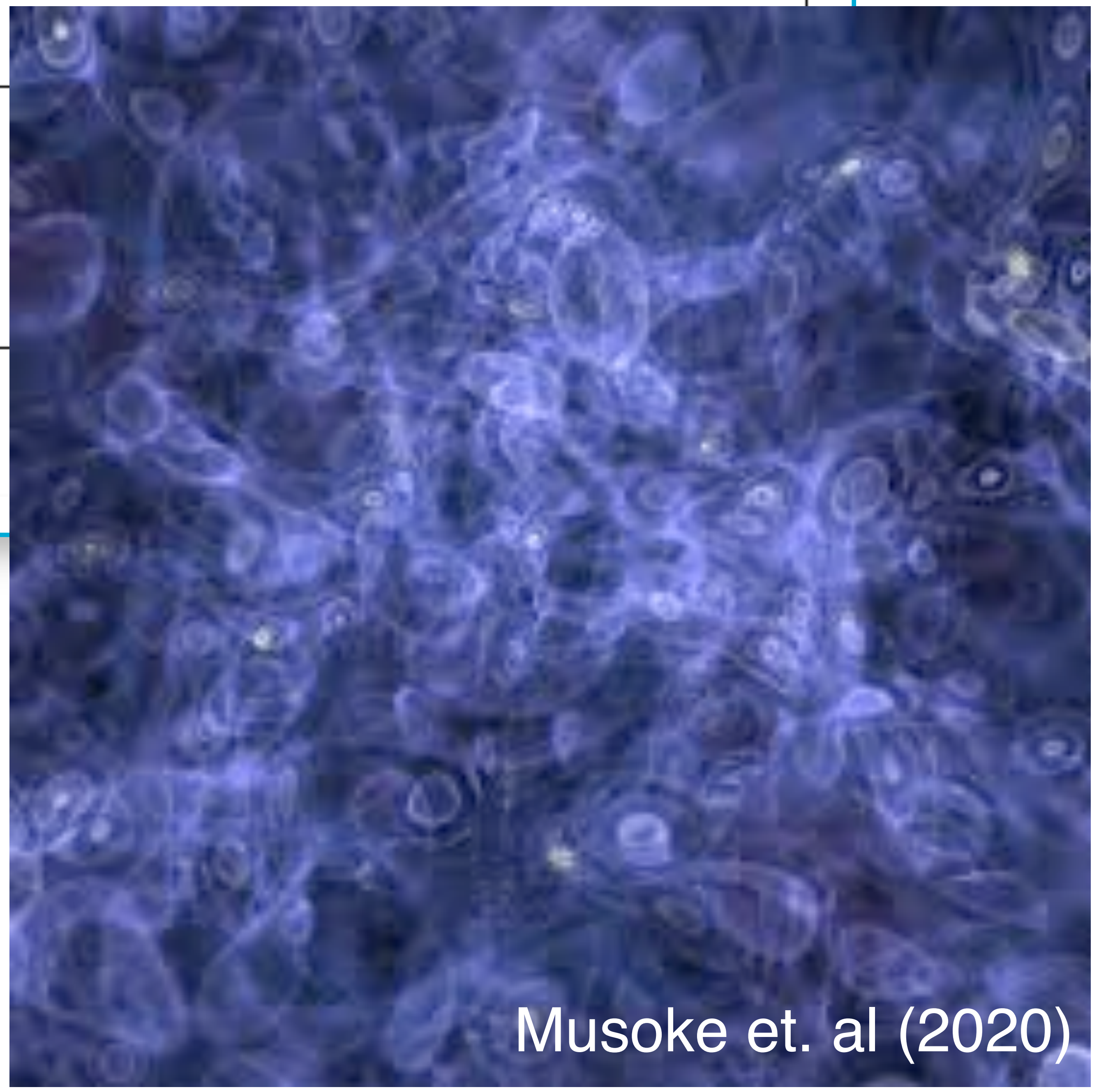
oscillating “free” scalar field: matter-dominated expansion +“slow” gravitational instability

$$w \approx \frac{n-1}{n+1} = 0$$

$$\Box\phi \approx m^2\phi$$

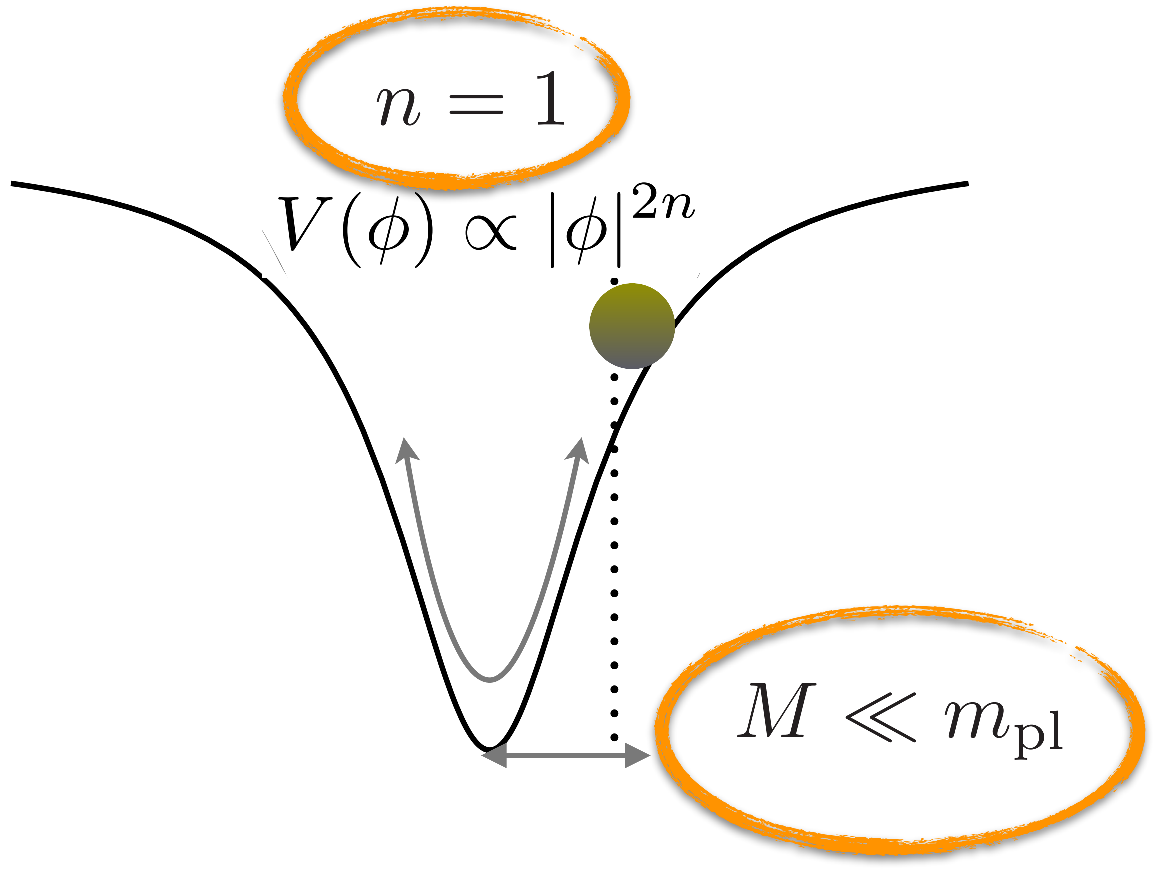




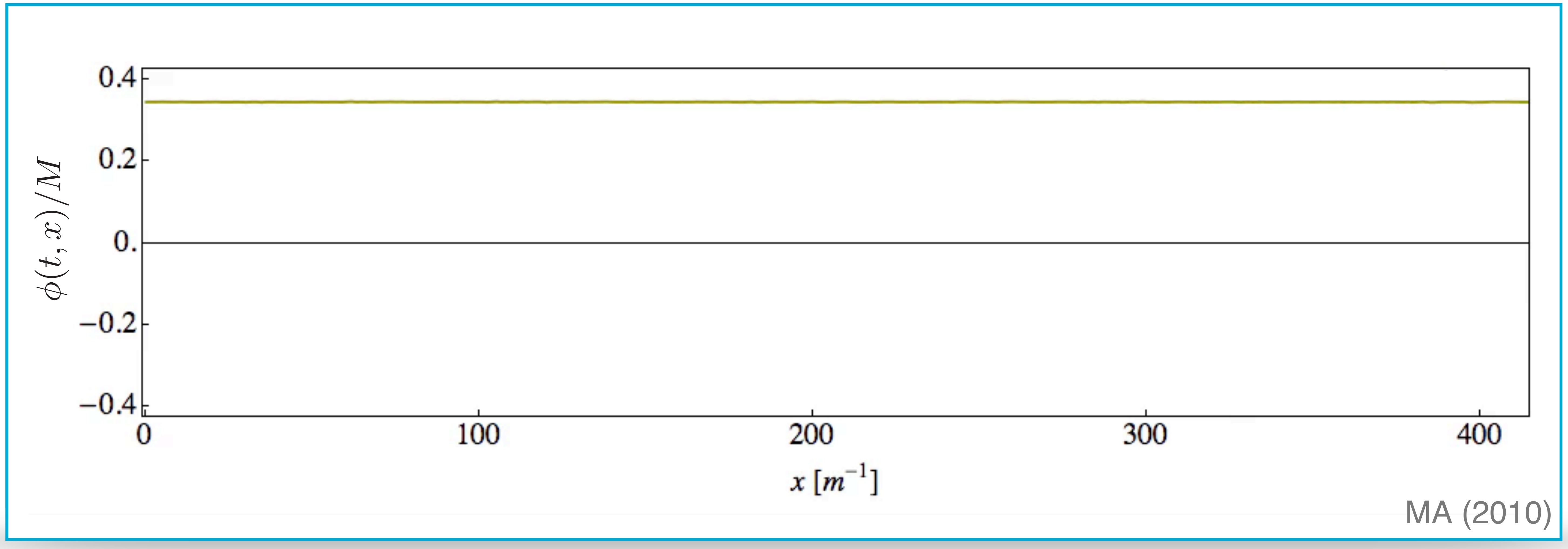


*similar to a late matter dominated universe

oscillating scalar field: self-interaction driven fast instability & "oscillon" formation



$$\Box\phi = V'(\phi)$$



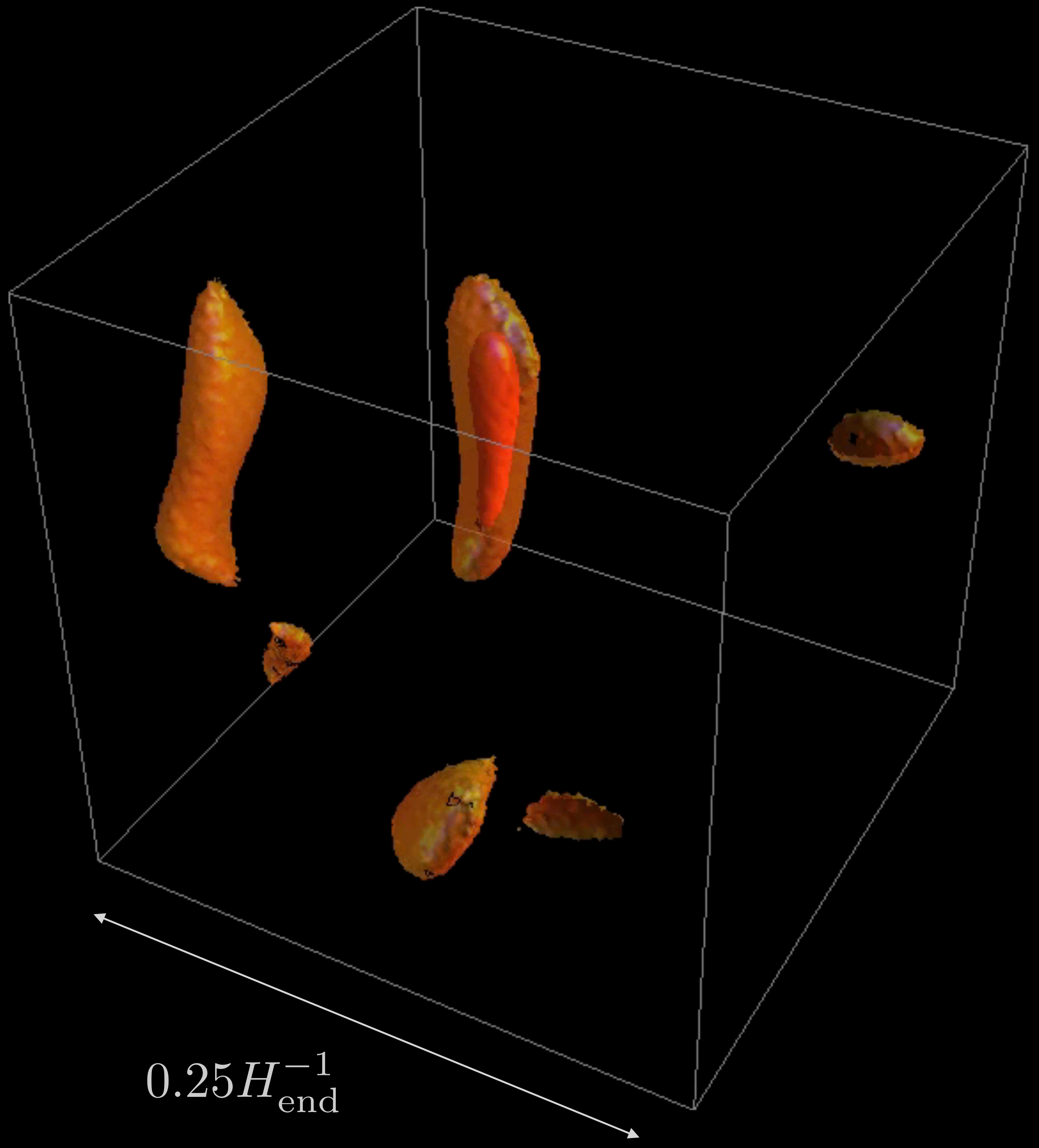
parametric resonance ►

$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} \sim \frac{m_{pl}}{M} \gg 1$$

*without oscillons, but relevant for instabilities, see related (much) earlier work: Khlopov, Malomed & Zeldovich (1985)

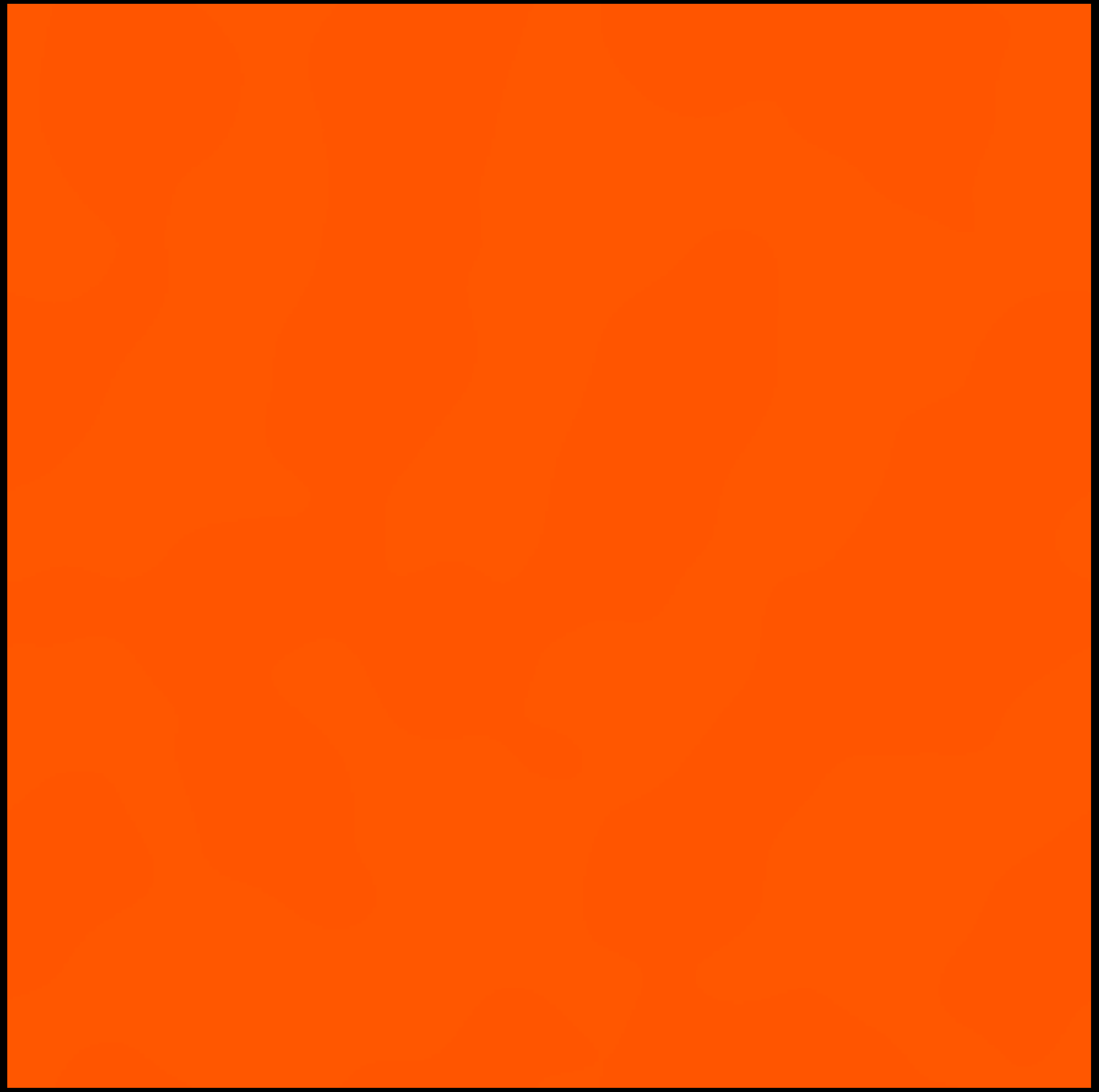
self-interaction driven fast instability & "oscillon" formation + gravitational clustering

expansion ✓
self-interactions ✓
gravitational int. ✗



MA, Easther, Finkel, Flauger & Hertzberg (2011)

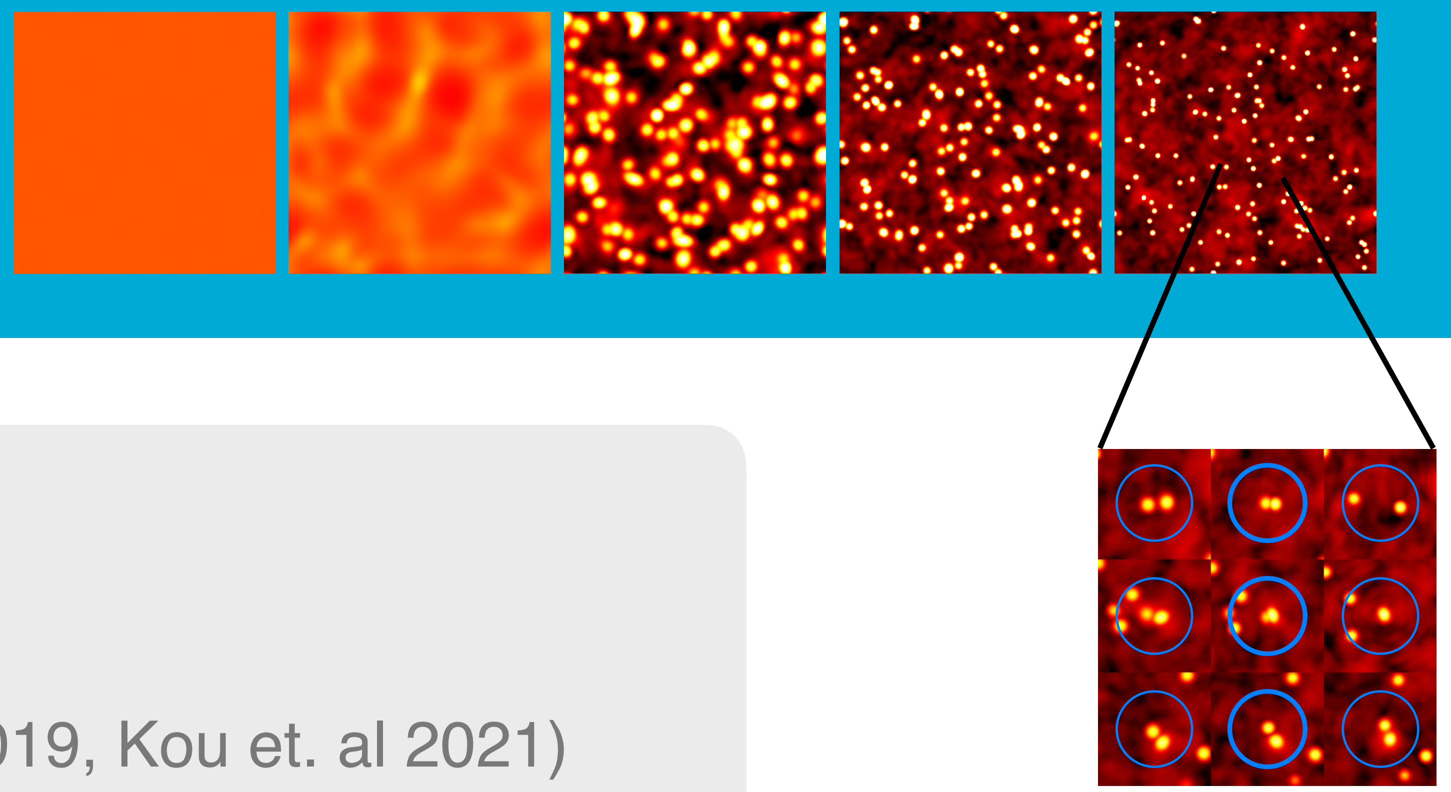
expansion ✓
self-interactions ✓
gravitational int. ✓



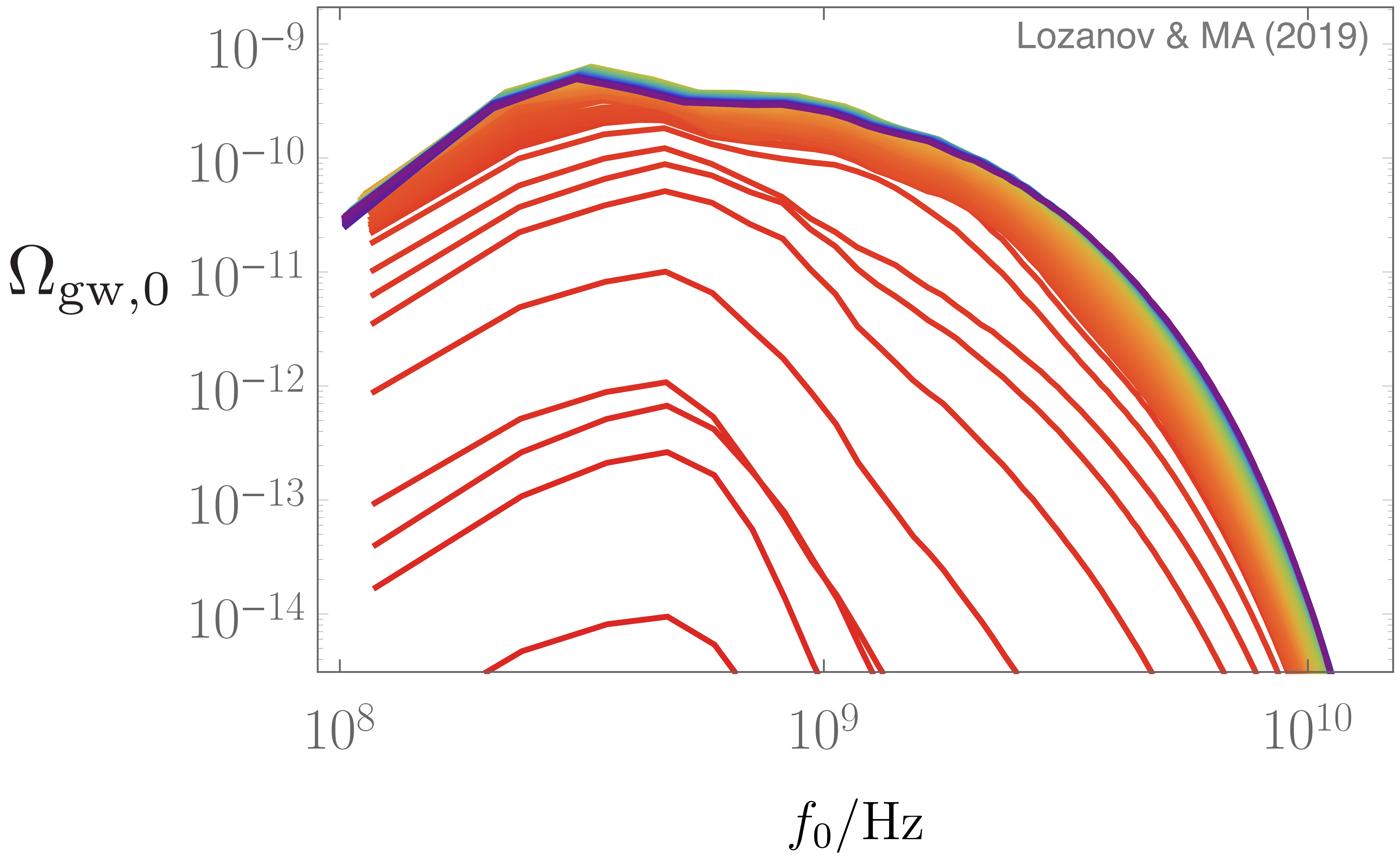
MA & Mocz (2019)

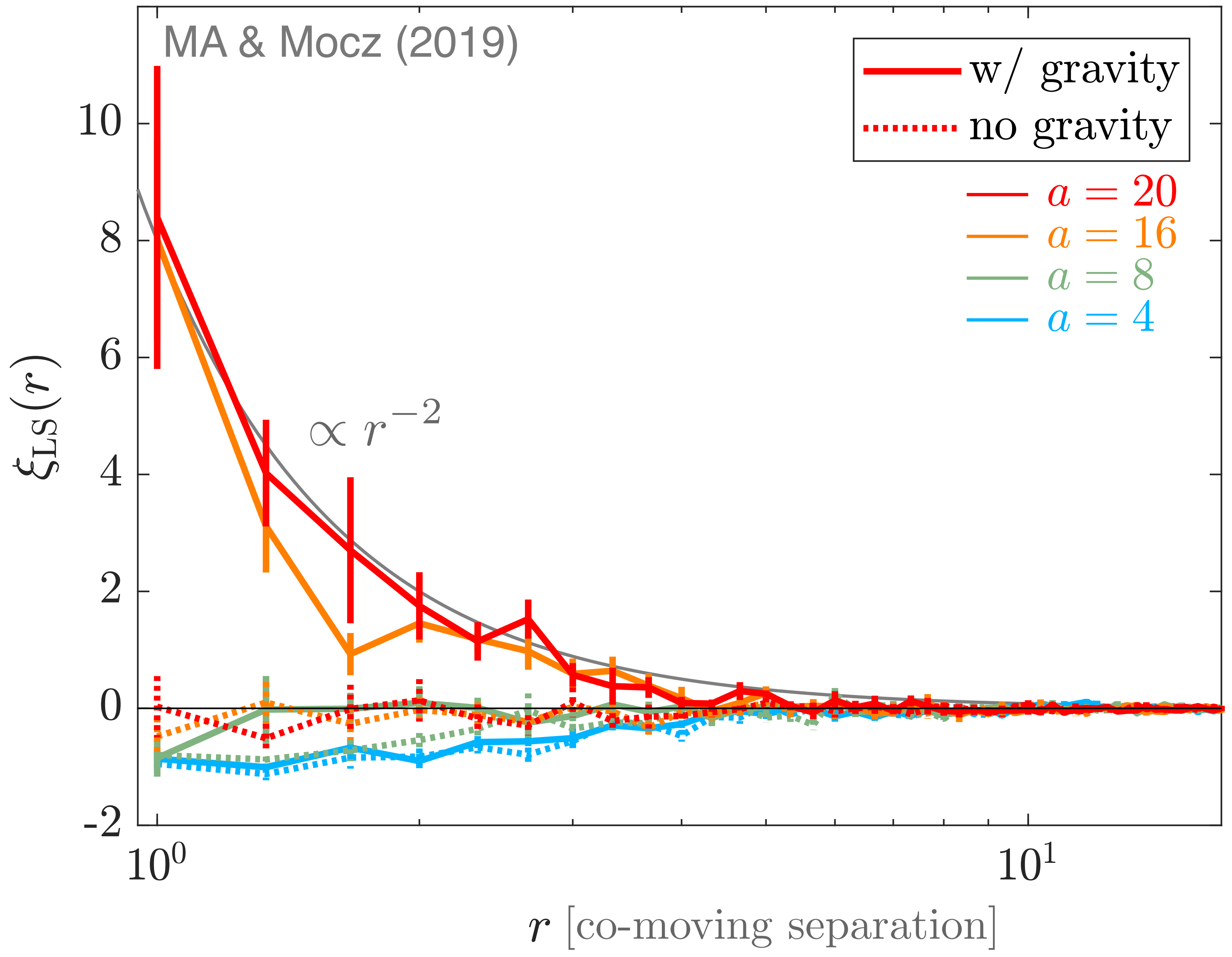
* non-relativistic, Schrodinger-Poisson

gravitational effects



- stochastic gravitational wave-generation (example: Zhou et. al 2013, Kitajima et. al 2018)
- primordial black hole (PBH) formation ? (Cotner et. al 2019, full GR simulations: Giblin & Tishue 2019, Kou et. al 2021)
- For particle DM clustering and effects from reheating (eg. Erickcek and Sigurdsen 2011)

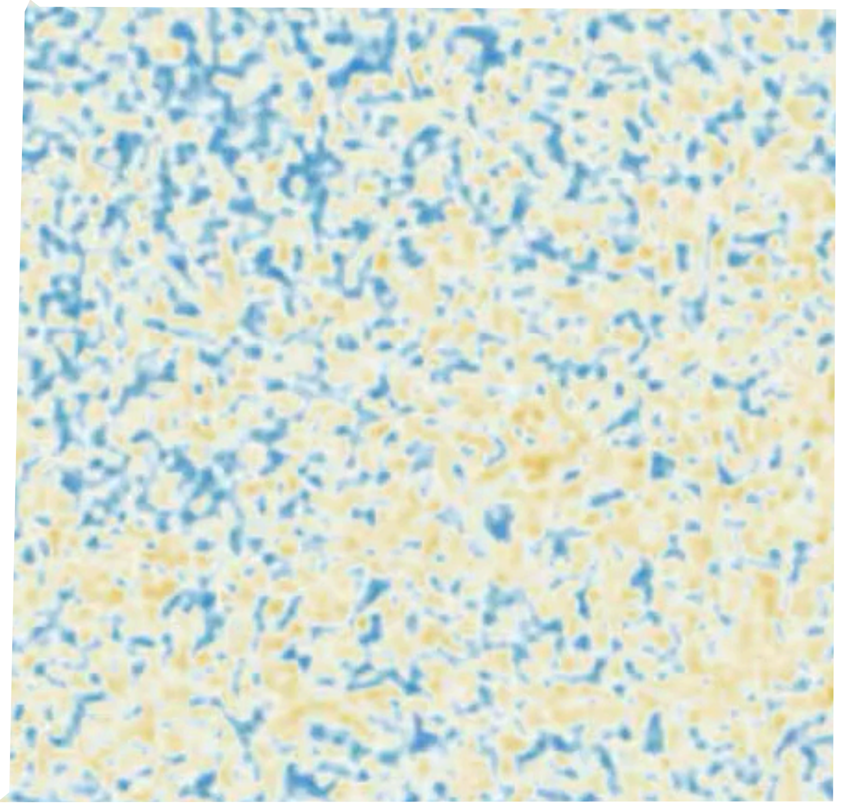




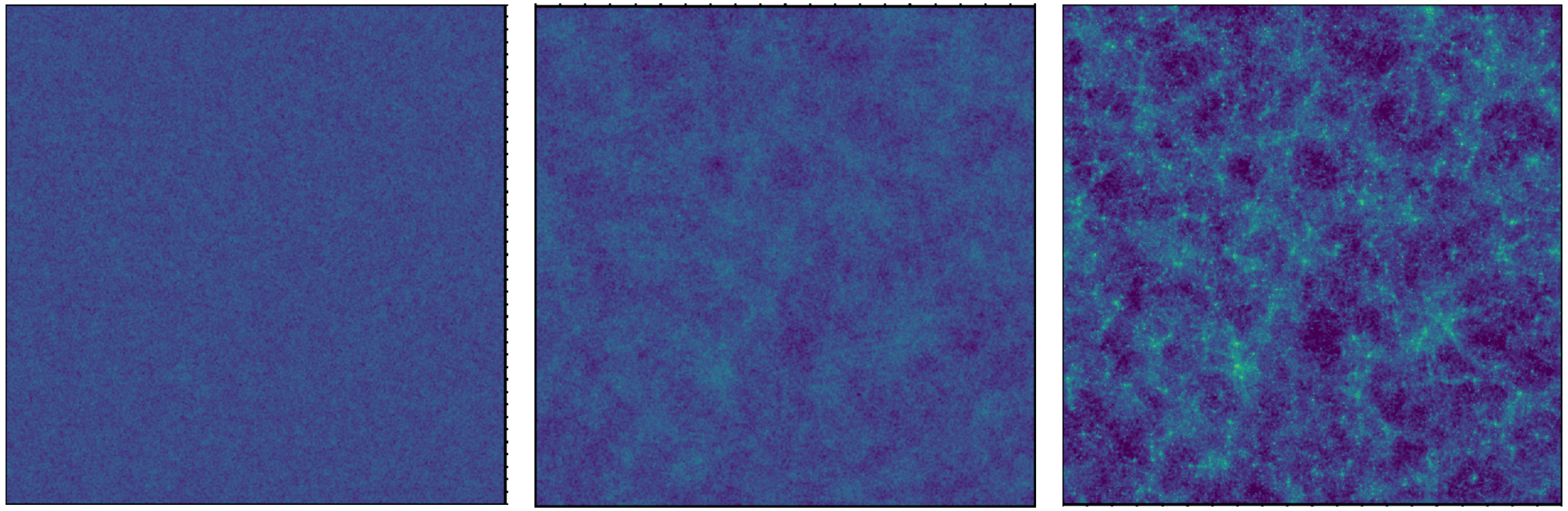
$\Omega_{\mathrm{gw},0} \sim 10^{-6}(M/m_{\mathrm{pl}})^2$ $\quad$ $|\Phi_{\mathrm{max}}| \lesssim 10(M/m_{\mathrm{pl}})^2 \ll 1$

gravitational implications

- gravitational waves (rapid energy density evolution)
- gravitational clustering and free-streaming (warm ICs)
  - likely negligible for inflaton, but relevant for lighter offsprings
- small scale, white-noise isocurvature perturbations



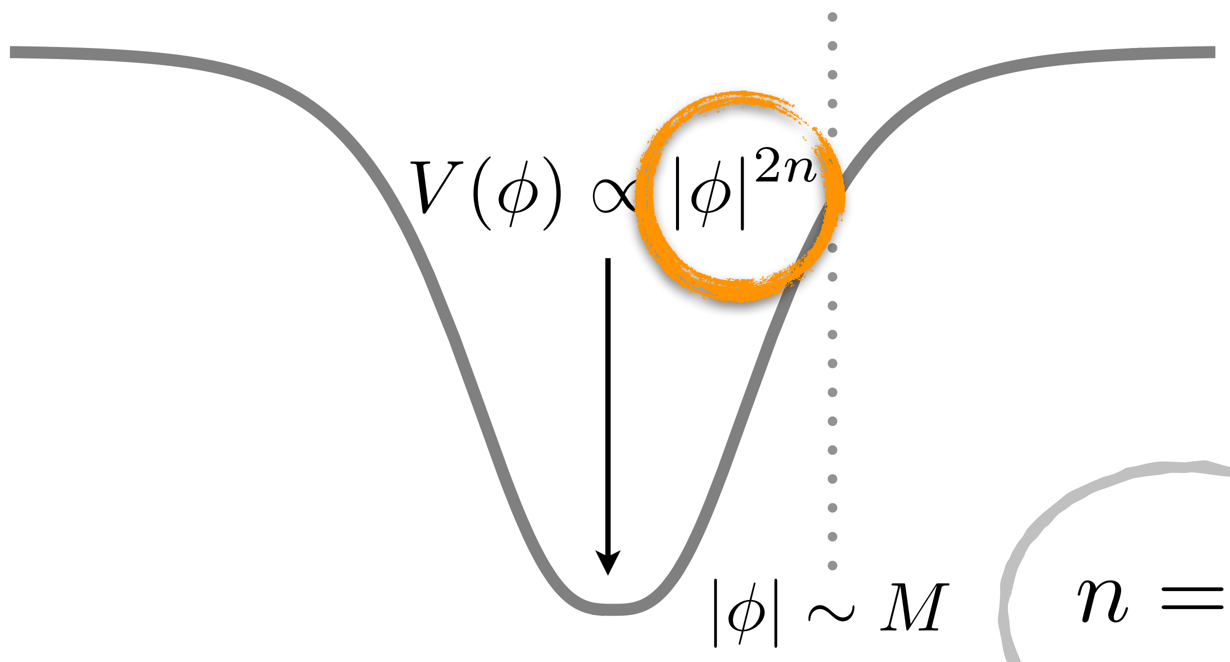
relativistic simulations



non-relativistic simulations (Schrodinger-Poisson)

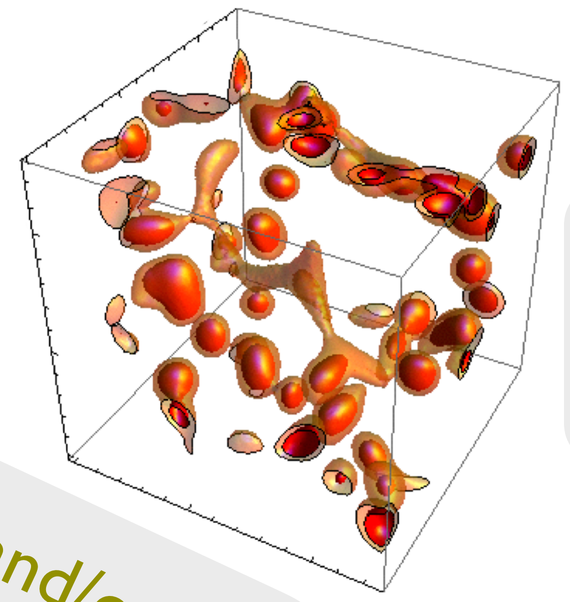
summary: dynamics in quadratic power law minima + wings

inflaton potential



$n = 1$

field fragments into solitons and/or just gravitationally clusters



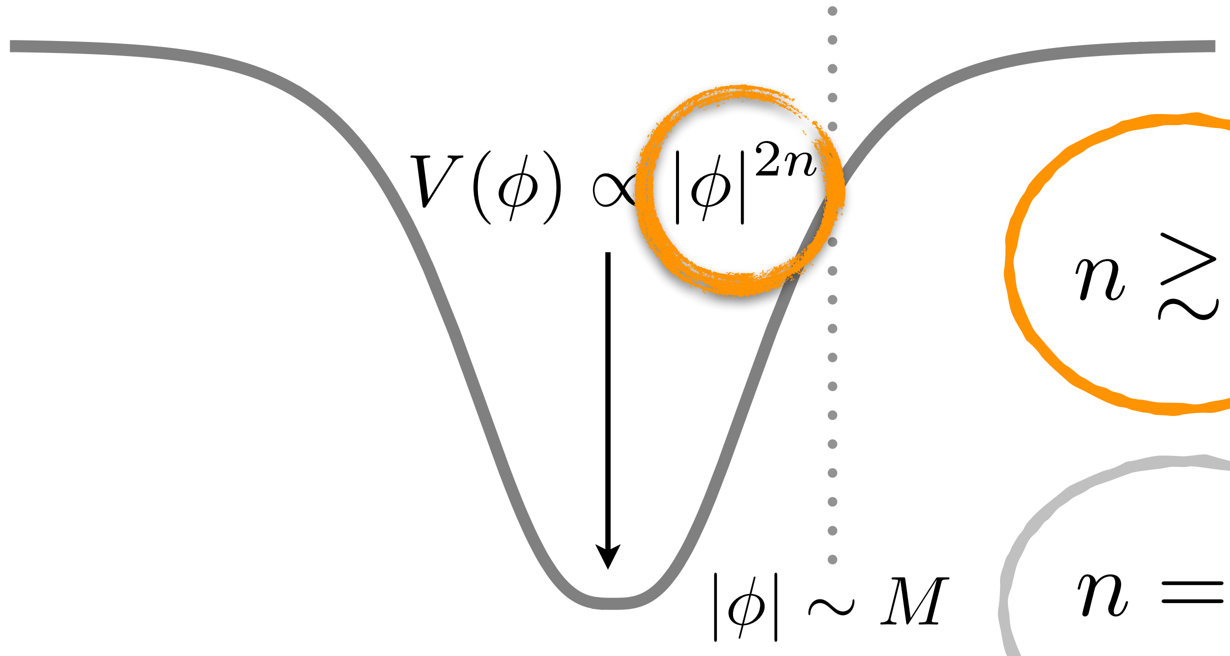
eq. of state

$w \to 0$

matter domination

dynamics in different power law minima + wings

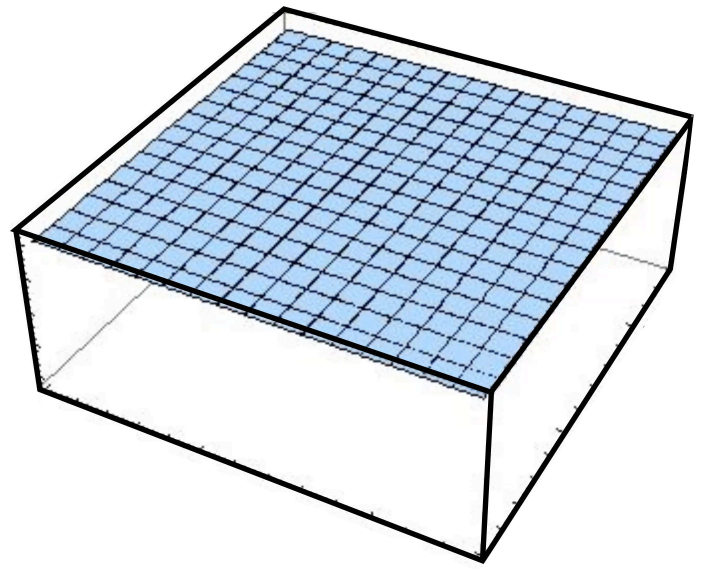
inflaton potential



homogeneous oscillations

$$w = \frac{n-1}{n+1}$$

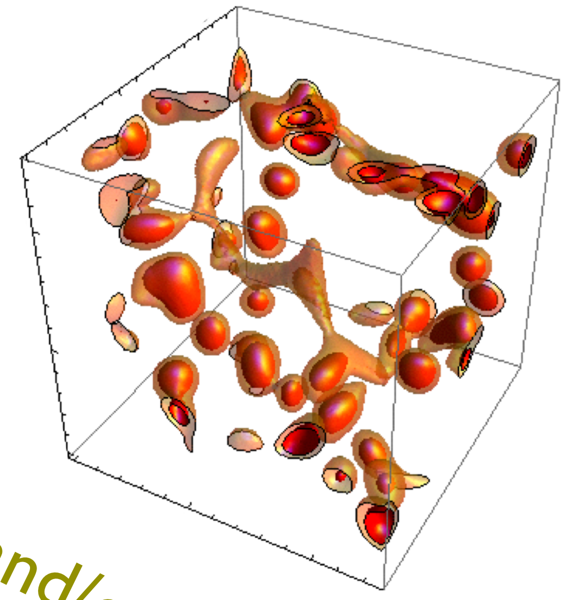
Turner (1983)



$n \gtrsim 1$

$n = 1$

field fragments into solitons and/or just gravitationally clusters



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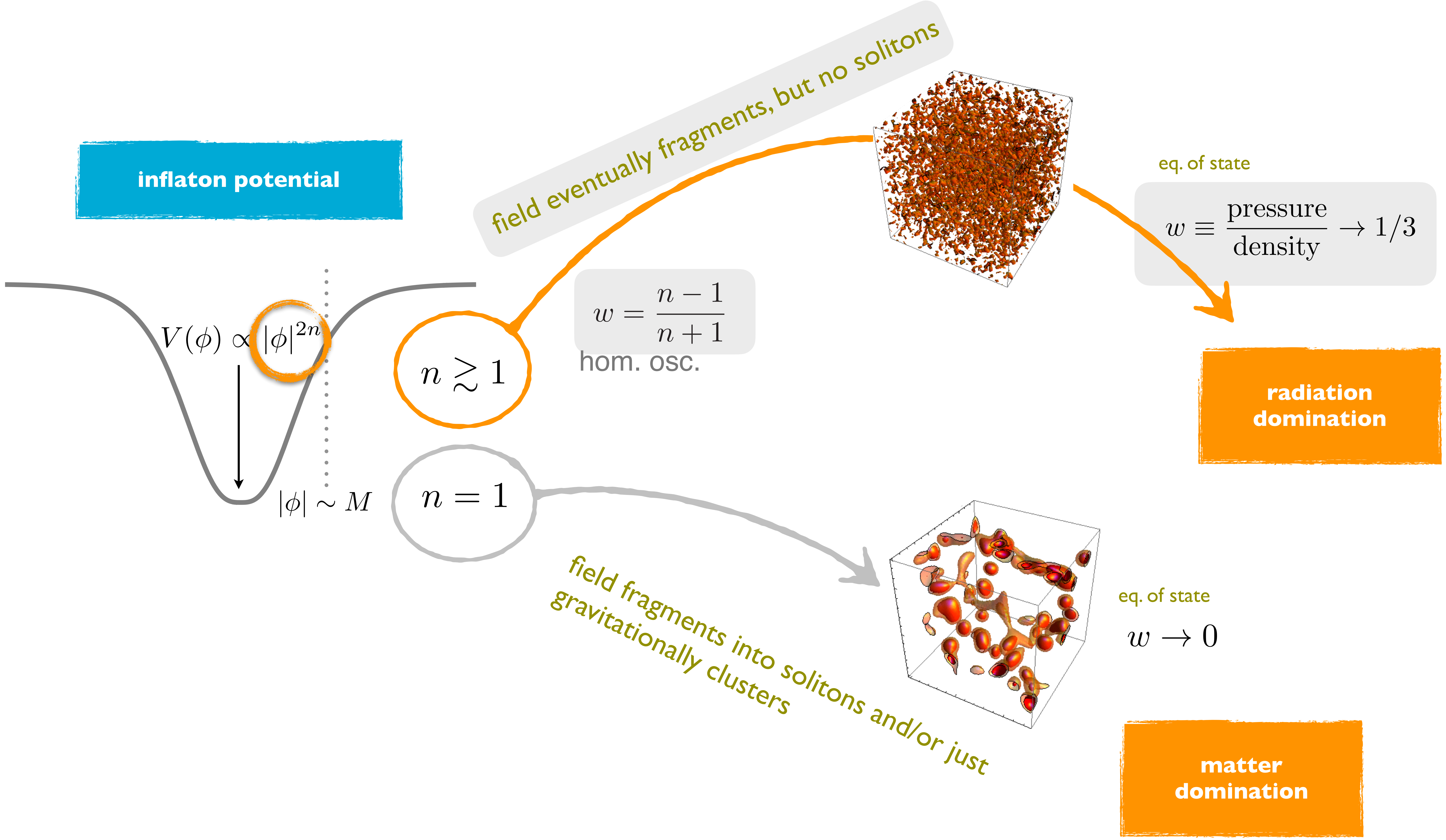
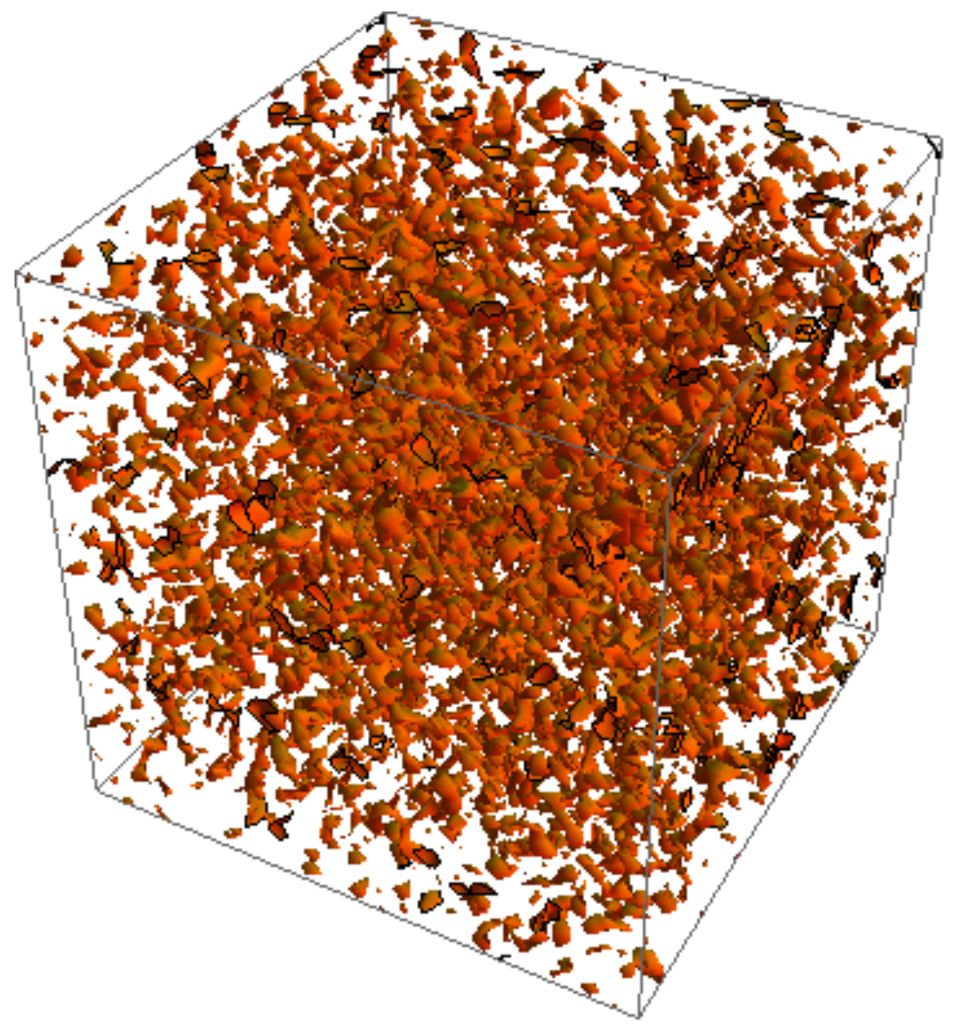
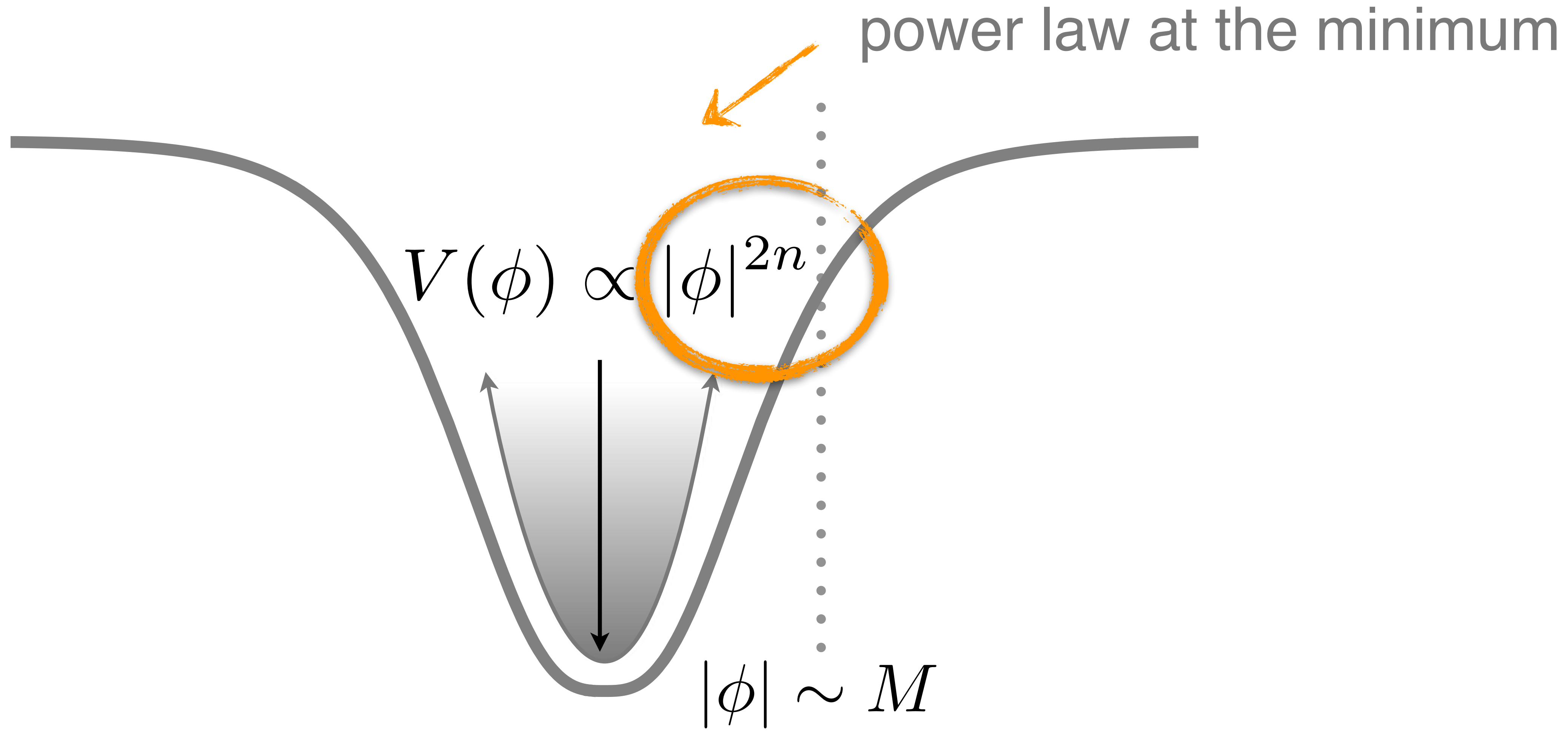


Lozanov & MA (2016/17)

why the universality ?

- **($n > 1$) non-quadratic minima $w = 1/3$** (after sufficient time)





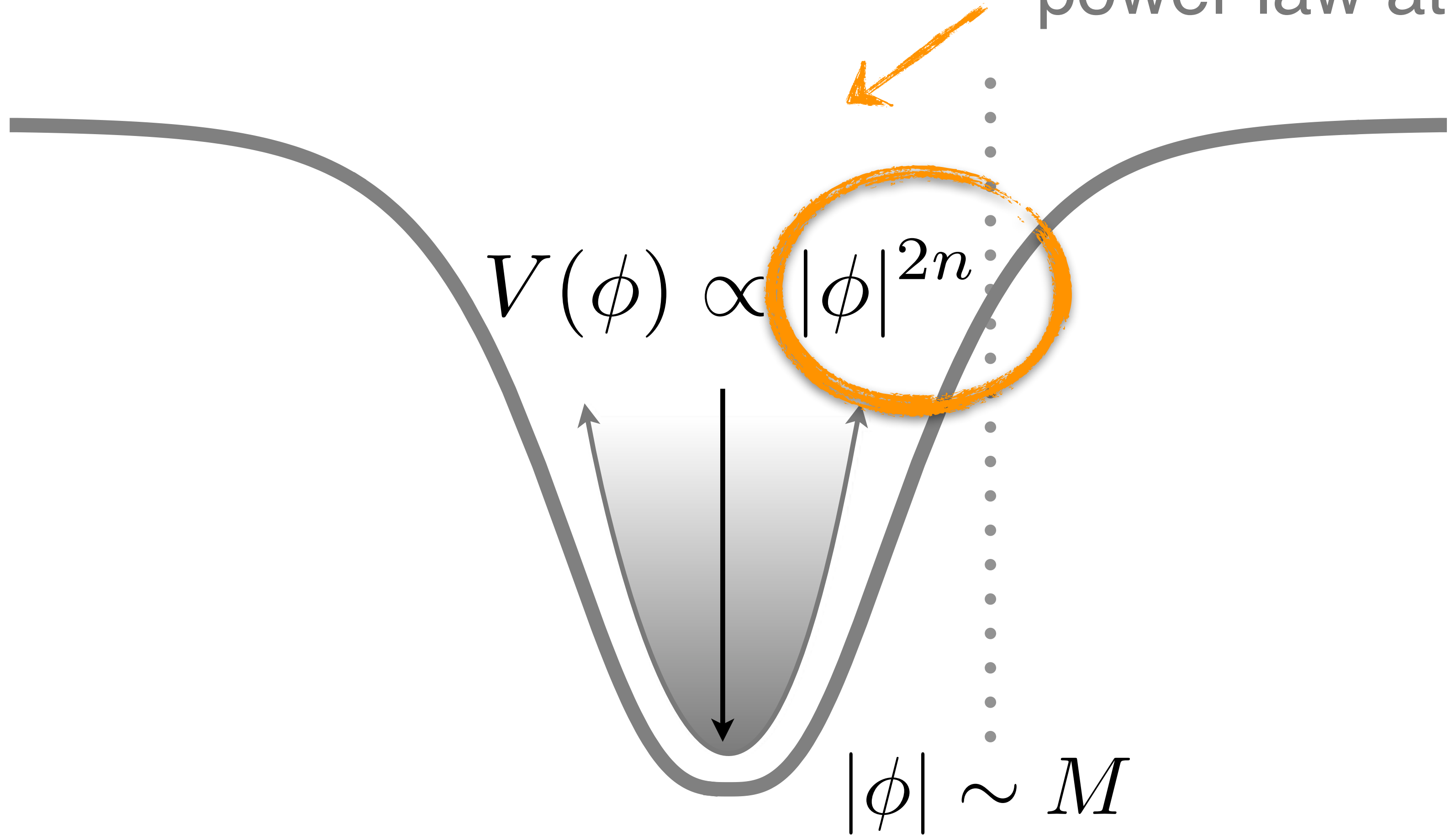
fragmentation is inevitable

$$n \gtrsim 1 \qquad \frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} \propto 1/\phi$$

+perturbations are effectively massless

e-folds to radiation domination?

- ($n > 1$) non-quadratic minima $w = 1/3$ (after sufficient time)

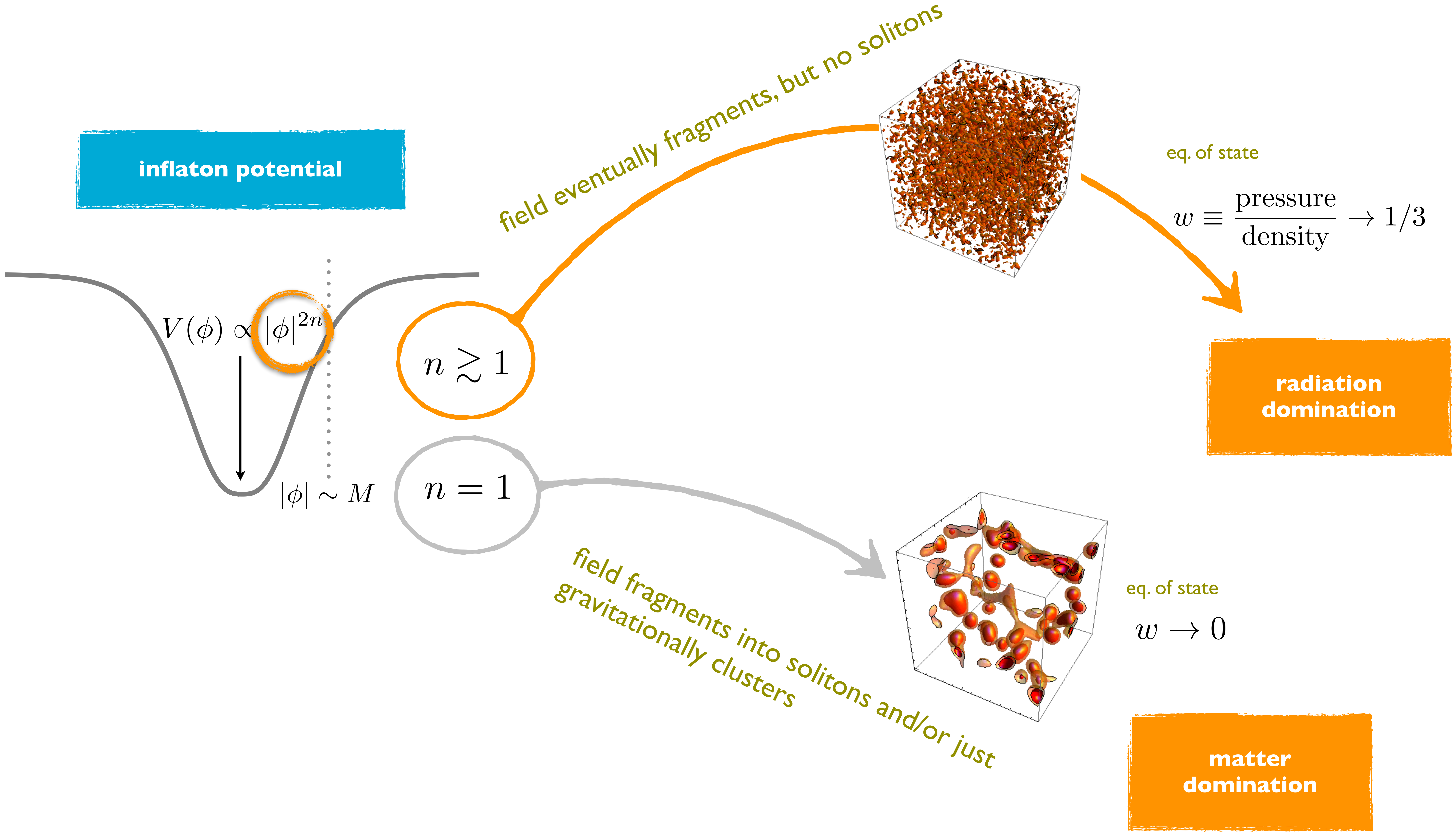


How many e-folds to radiation domination?

$$\Delta N_{\rm rad} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\rm Pl}\,, \\ \dfrac{n+1}{3} \ln\left(\dfrac{\kappa}{\Delta\kappa}\dfrac{10M}{m_{\rm Pl}}\right) & M \gtrsim 10^{-2} m_{\rm Pl}\,. \end{cases}$$

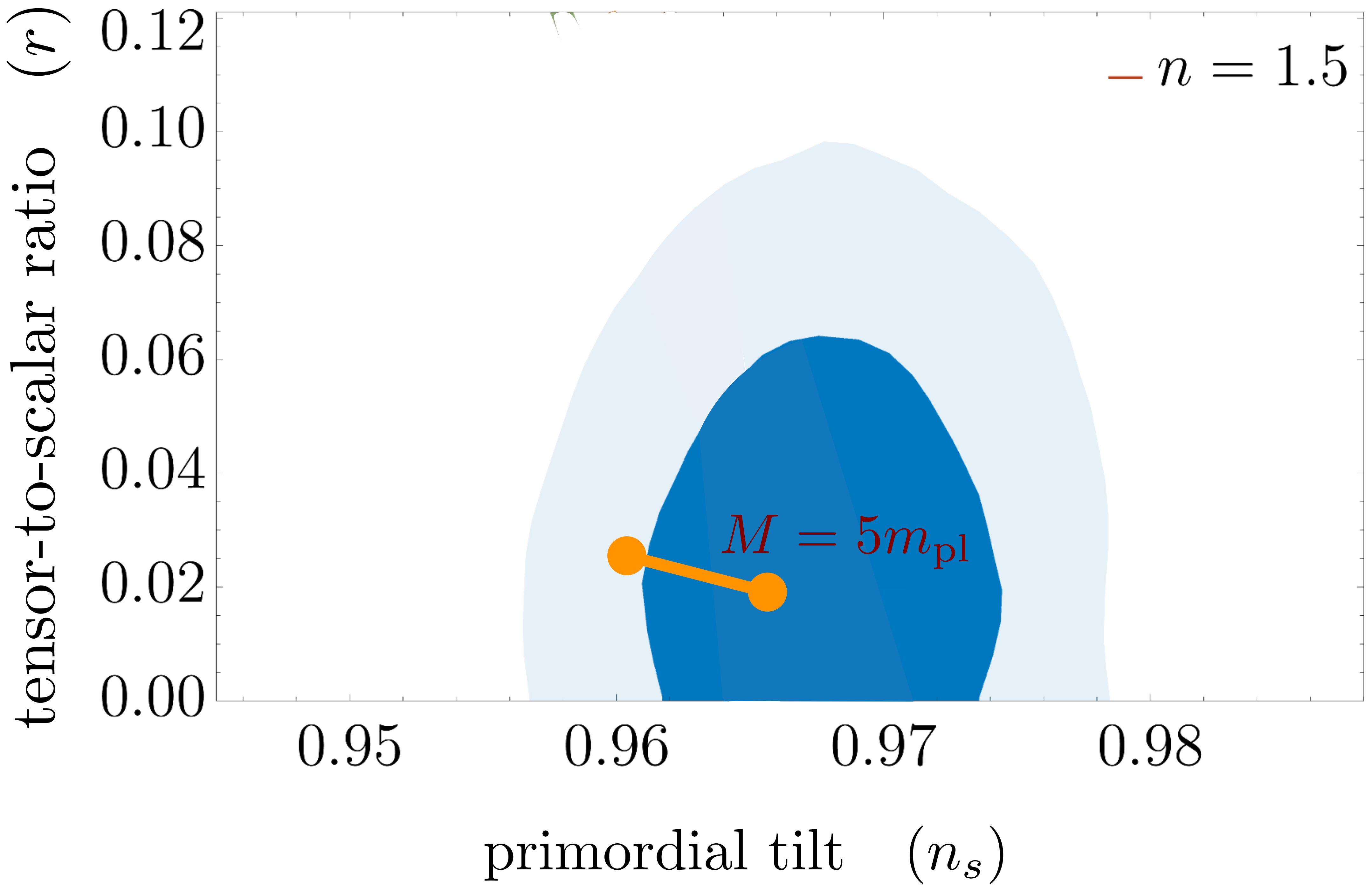
Lozanov & MA (2016/17)

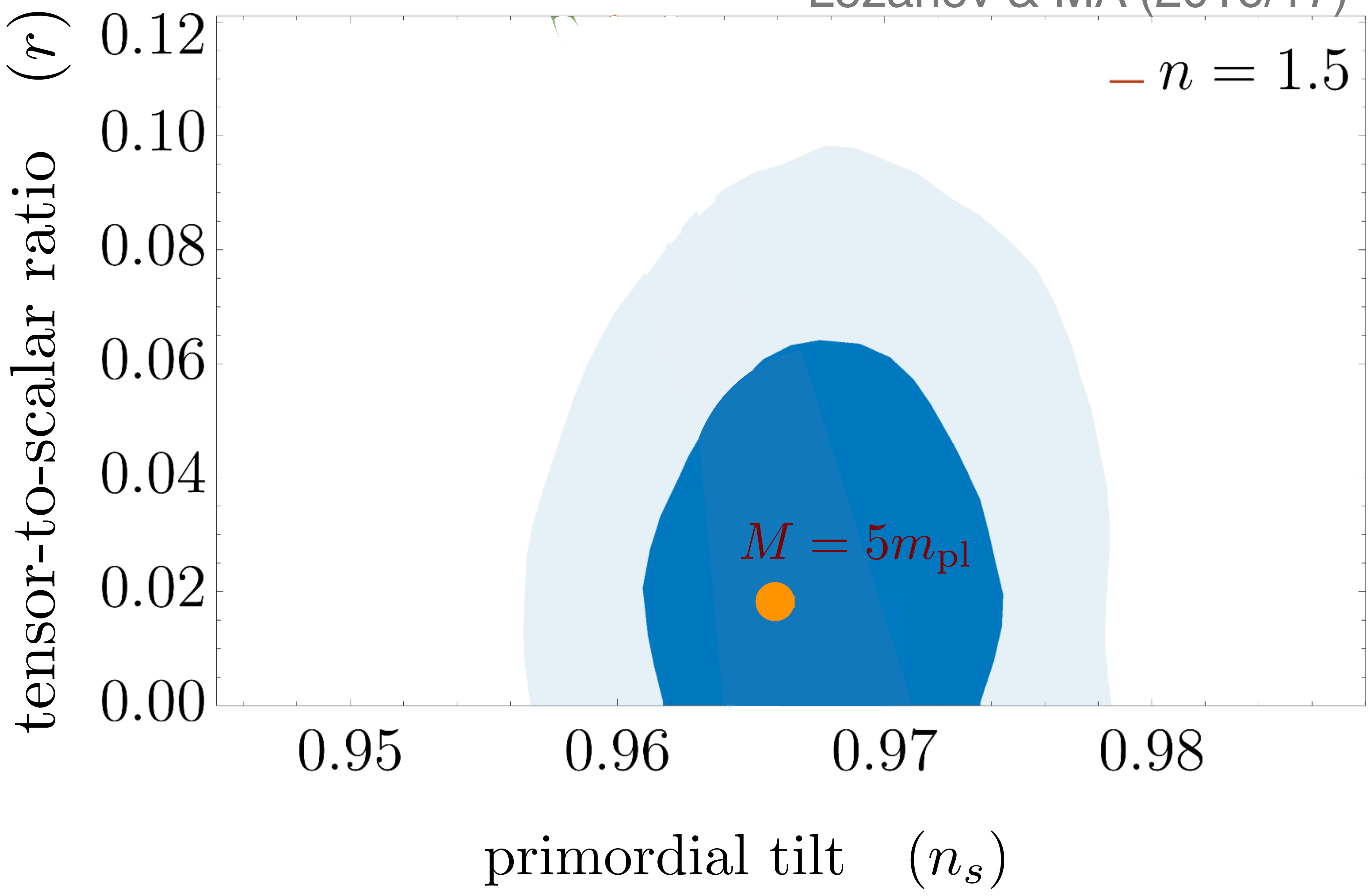
dynamics in different power law minima + wings



upper bound on duration to radiation domination (*n* >1)

Lozanov & MA (2016/17)





$$N_\star \supset \frac{1}{4}(1-3w)\Delta N_{\rm rad}$$

inflationary observables

$r(N_\star), n_{\rm s}(N_\star)$

* addition of other light fields, see Antusch, Figueroa, Marschall, Torrenti (2020)

* implications of CMB observations for/on reheating (Martin & Ringeval 2010, Cook et. al 2015, Munoz and Kamionkowski 2015)

caution

time when we have radiation-like equation of state

$\neq$ transfer of energy to SM species

$\neq$ thermalized SM universe

$\neq$ not necessarily

expansion history + fragmented vs. non-fragmented field

Reheating (to SM) temperature larger by 10^7 if fragmentation is ignored

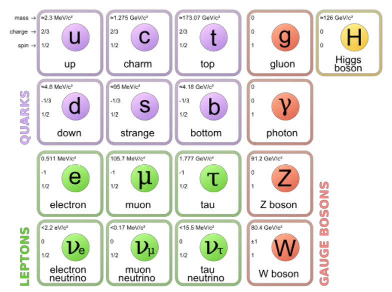
*model dependent

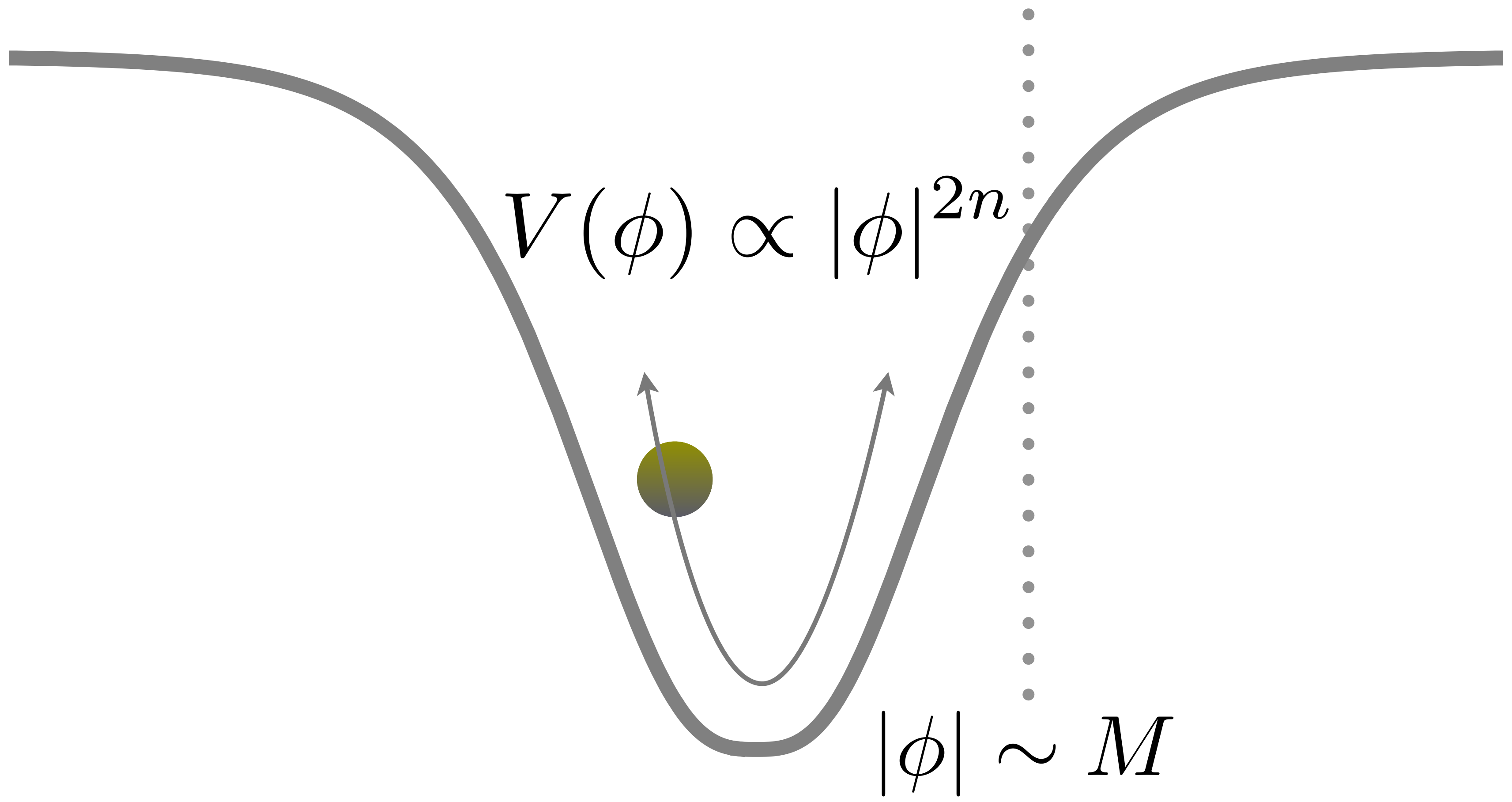
Garcia & Pierre (2023)

DM abundance/mass might be affected by initial state of the field+expansion

time to bring in couplings to other fields …

$$S = \int d^4x \sqrt{-g} \left[\frac{m_{\rm pl}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

- shape of the potential (self couplings)
- couplings to other fields  χ , ψ A_μ



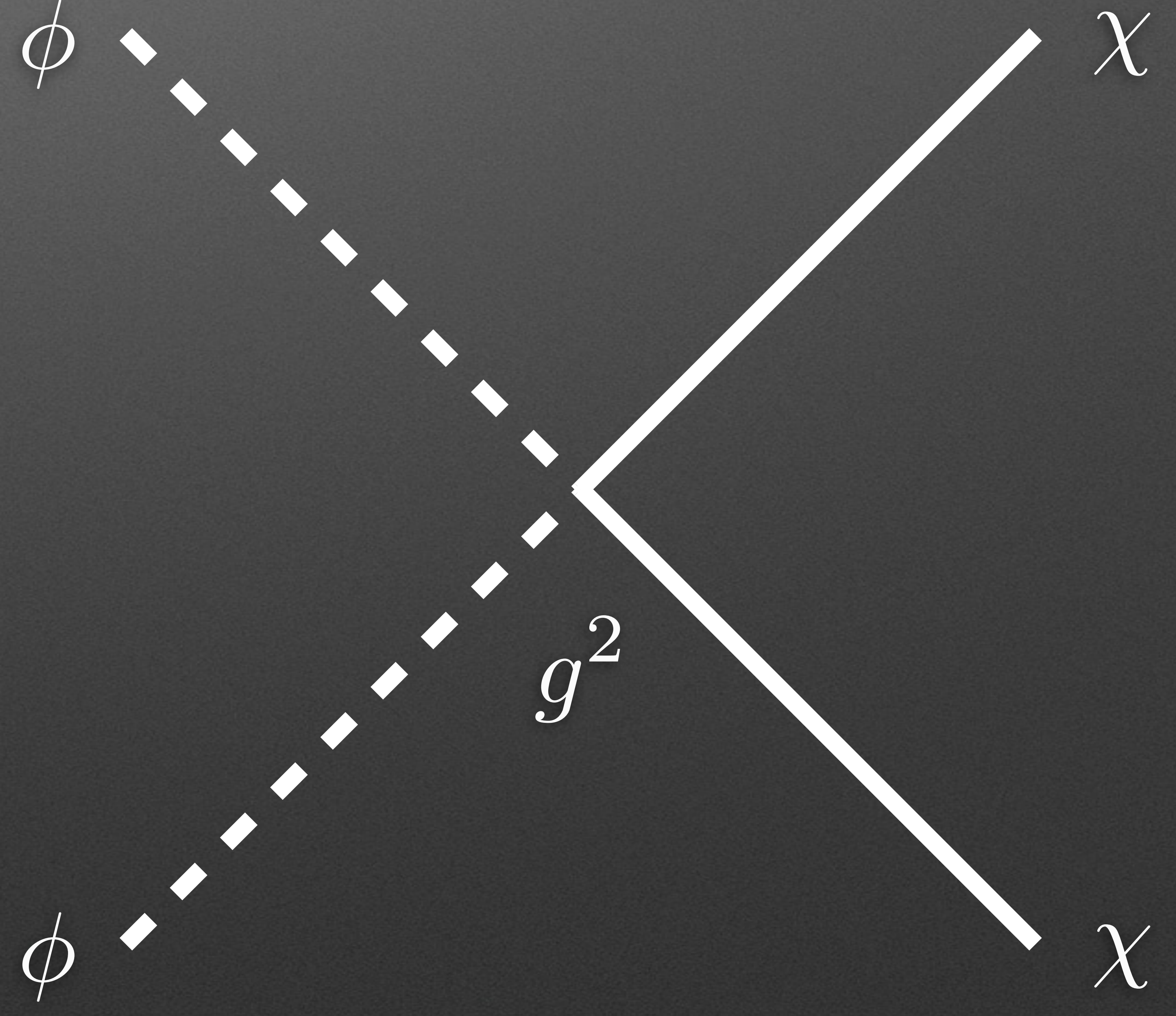
Ending Inflation with More than One Field

- In order to protect the inflation potential, *direct couplings* to other degrees of freedom have to be small
- Perturbative decay (the ‘old theory of reheating’) can sometimes take too long*
- This is/was a problem for fans of extremely low-scale inflation

Vanilla Preheating

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}m^2\phi^2 - \frac{g^2}{2}\phi^2\chi^2$$

- if g is too big - it messes up inflation
- if go s too small - the inflation never decays



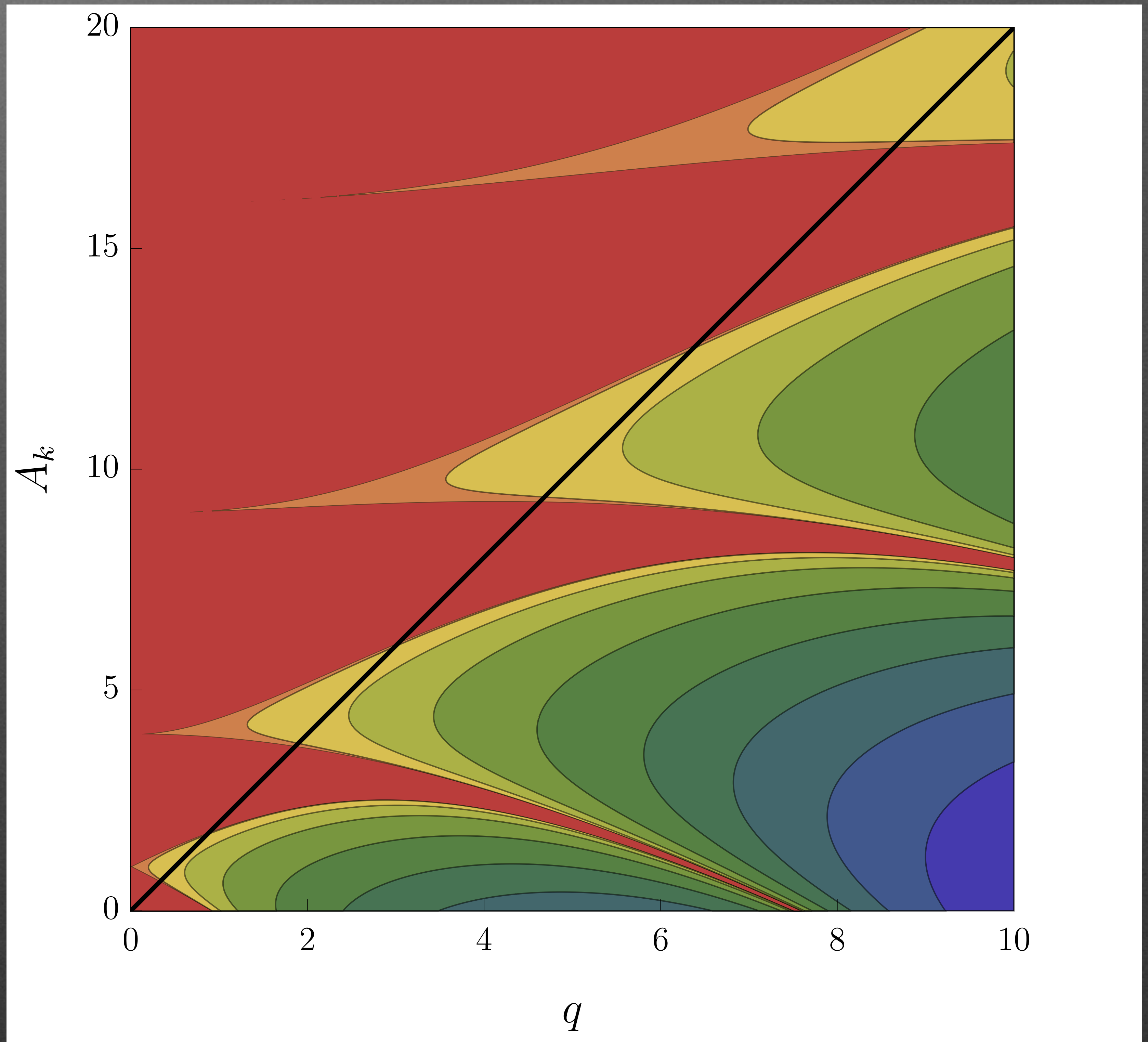
What can we see from this?

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \frac{k^2}{a^2}\phi_k = -\frac{\partial V}{\partial \phi}$$

$$g^2 \langle \phi^2 \rangle \chi$$

time dependent mass

parametric resonance



What about gravity?

What we have to do…

- Luckily there are a set of new approaches. We use the most common of these: the BSSN formalism.

- It is based on the ADM metric decomposition

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \gamma_{lk}\beta^l\beta^k & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix}$$

- We we introduce more parameters than (minimally) necessary so that the equations are easier to solve

Importantly

These variables have well-behaved differential equations and *are a complete description of GR without dimensional reductions or simplifications*

$$\partial_t \phi = -\frac{1}{6}\alpha K + \beta^i \partial_i \phi + \frac{1}{6}\partial_i \beta^i$$

$$\partial_t \bar{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} + \beta^k \partial_k \bar{\gamma}_{ij} + \bar{\gamma}_{ij}\partial_j \beta^k + \bar{\gamma}_{ij}\partial_i \beta^k - \frac{2}{3}\bar{\gamma}_{ij}\partial_k \beta^k$$

$$\partial_t K = \gamma^{ij} D_j D_i \alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2) + 4\pi\alpha(\rho + S) + \beta^i \partial_i K$$

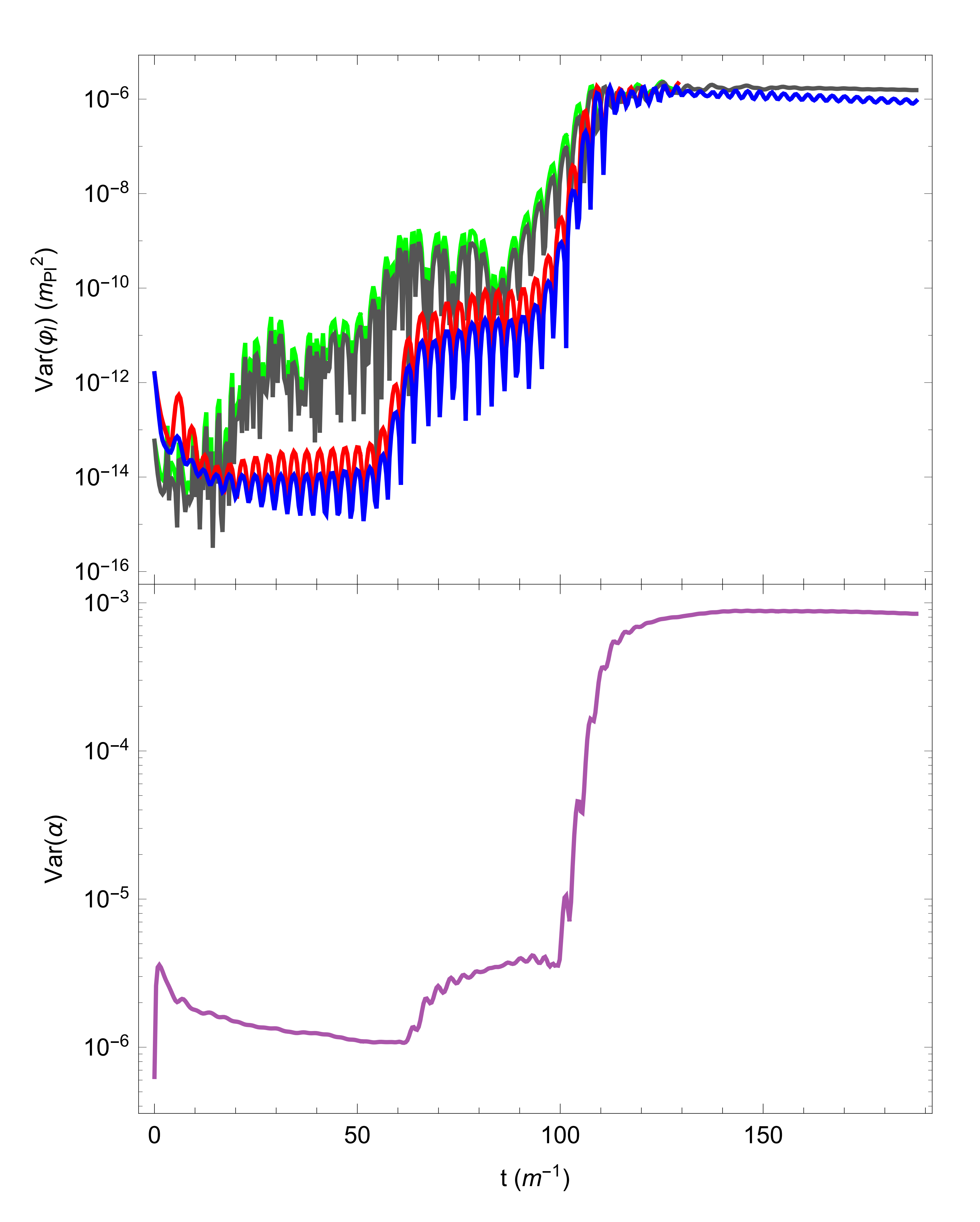
$$\partial_t \tilde{A}_{ij} = e^{-4\phi}\left(-(D_i D_j \alpha)^{TF} + \alpha(R_{ij}^{TF} - 8\pi S_{ij}^{TF})\right) + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l{}_j$$

$$+ \beta^k \partial_k \tilde{A}_{ij} + \tilde{A}_{ik}\partial_j \beta^k + \tilde{A}_{kj}\partial_i \beta^k - \frac{2}{3}\tilde{A}_{ij}\partial_k \beta^k$$

$$\partial_t \bar{\Gamma}^i = -2\tilde{A}^{ij}\partial_j \alpha + 2\alpha\left(\bar{\Gamma}^i_{jk}\tilde{A}^{kj} - \frac{2}{3}\bar{\gamma}^{ij}\partial_j K - 8\pi\bar{\gamma}^{ij}S_j + 6\tilde{A}^{ij}\partial_j \phi\right)$$

$$+ \beta^j \partial_j \bar{\Gamma}^j \partial_j \beta^i + \frac{2}{3}\bar{\Gamma}^i \partial_j \beta^j + \frac{1}{3}\bar{\gamma}^{il}\partial_l \partial_j \beta^j + \bar{\gamma}^{lj}\partial_j \partial_l \beta^i$$

Does Gravity Matter in Preheating?

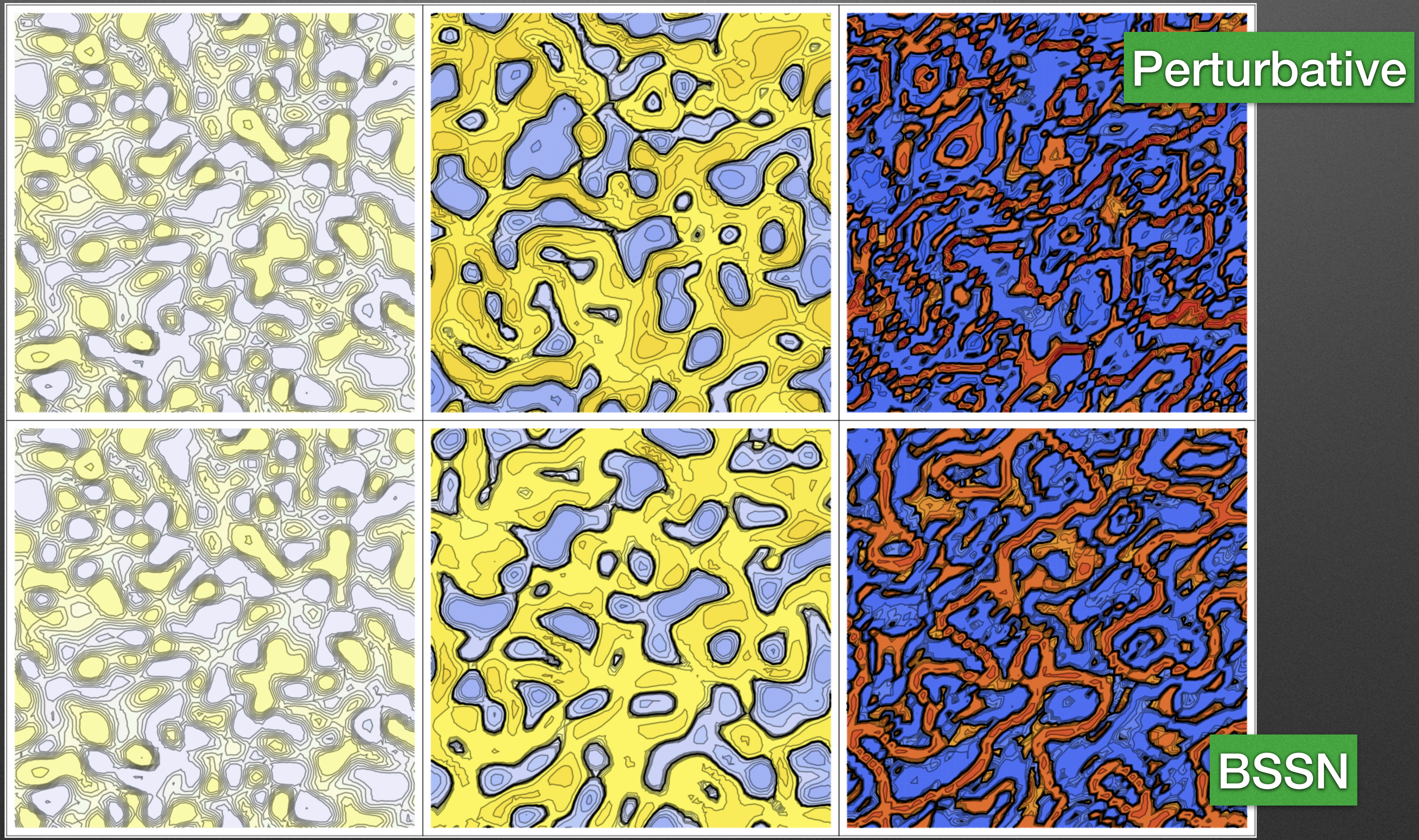


Red: inflaton Perturbative
Blue: inflaton BSSN

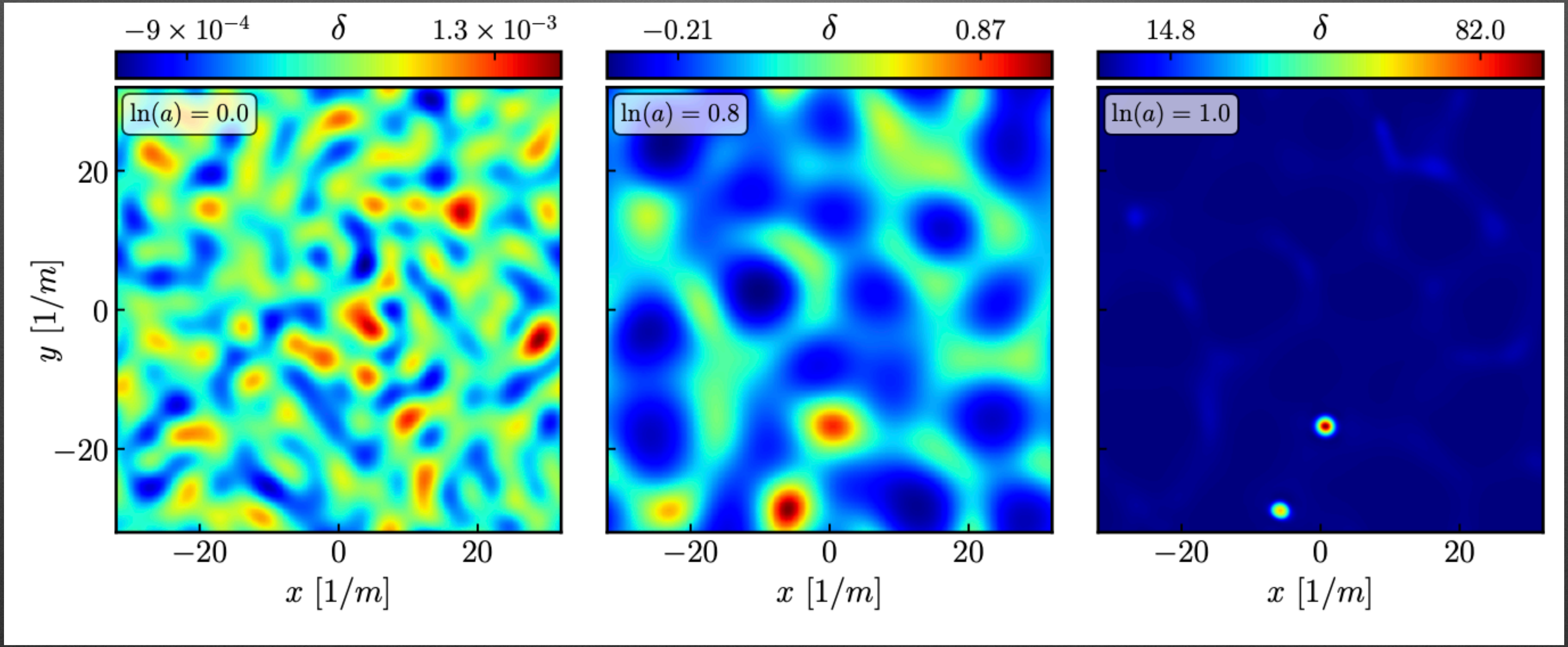
Green: decay field Perturbative
Black: decay field BSSN

The *variance* of the lapse does not show departures from homogeneity that indicate strong gravity is important
Even though the linearized Einstein Equations are violated

How do they look



Numerical Relativity and Oscillons



https://arxiv.org/abs/2304.01673

Beyond Scalar Fields

...beyond $w=1/3$

Gauge-Preheating

- There is a history of incorporating couplings of the inflation to gauge fields, generally with *charged* inflation fields (often in the context of Higgs inflation)
    - Coupling to U(1) fields by A. Rajantie , E. J. Copeland, and S. Pascal
    - Coupling to SU(2) fields by J. Garcia-Bellido et. al., Saffin et al.
- However using uncharged *scalar (or pseudo-scalar)* degrees of freedom were technically a bit more challenging

Gauge-Preheating

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{W(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{X(\phi)}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- The “normal” Maxwell Stress-Tensor
- (but not for “normal” E/M)

Gauge-Preheating

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{W(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{X(\phi)}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$W(\phi) = e^{-\phi/M}$$

$$X(\phi) = 0$$

- W is a *dilatonic* coupling that vanishes as the inflation decays to zero
- Possible generation of long-wavelength magnetic fields during inflation, e.g. Caldwell, Motta, Kamionkowski Phys. Rev. D 84, 123525 (2011).

Gauge-Preheating

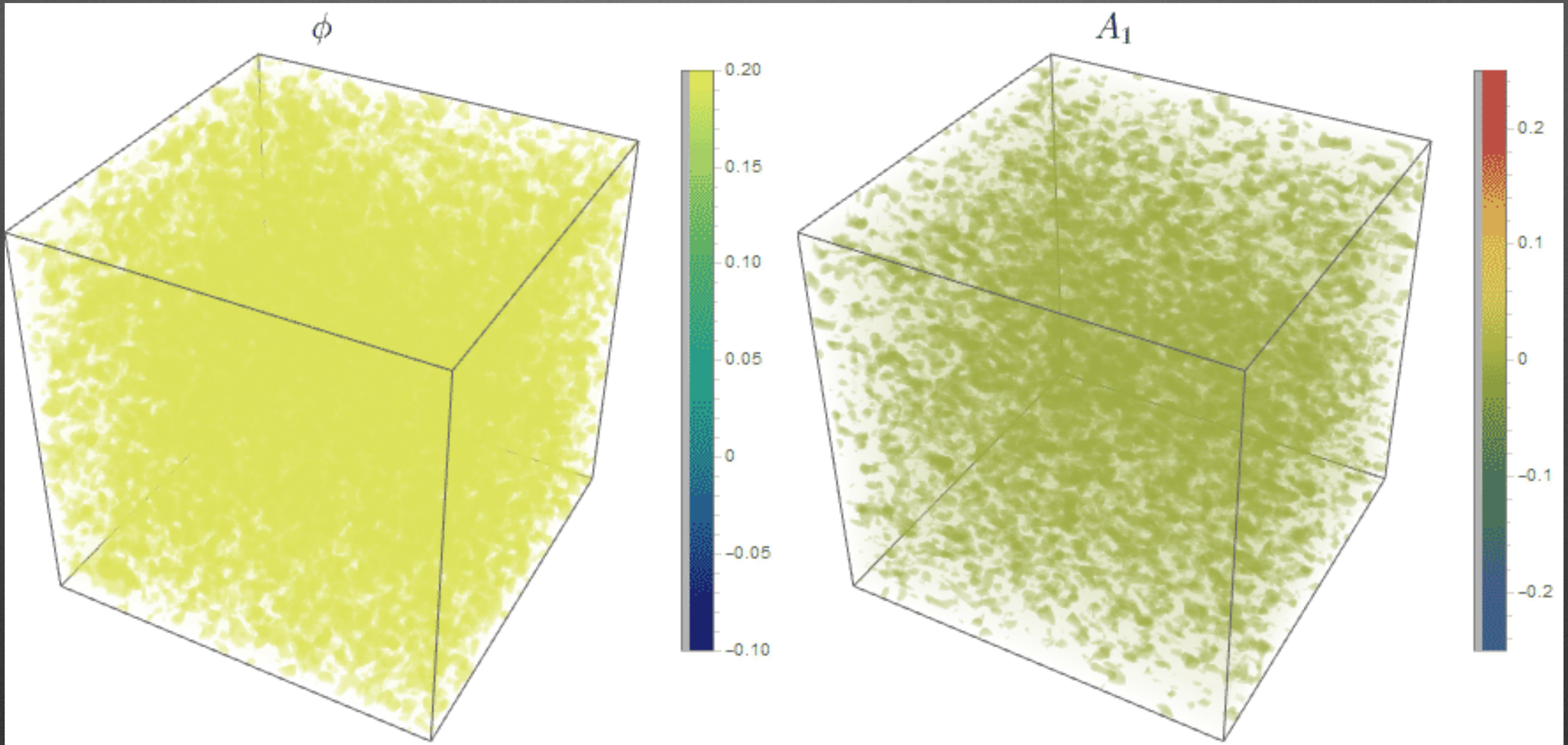
$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{W(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{X(\phi)}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$W(\phi) = 1$$

$$X = \frac{\alpha_g}{f}\phi$$

- X is a *Chern Simons* coupling that couples the inflation to the curl of the vector field
- A coupling consistent with a shift-symmetric inflaton
- Also possible generation of polarized magnetic fields during inflation, e.g. Garretson, Field and Carroll, Phys. Rev. D 46 5346 (1992)

But… we get structure



$$W(\phi) = e^{-\phi/M}$$

Is Gravity Important Here?

We can write down a set of evolution equations,

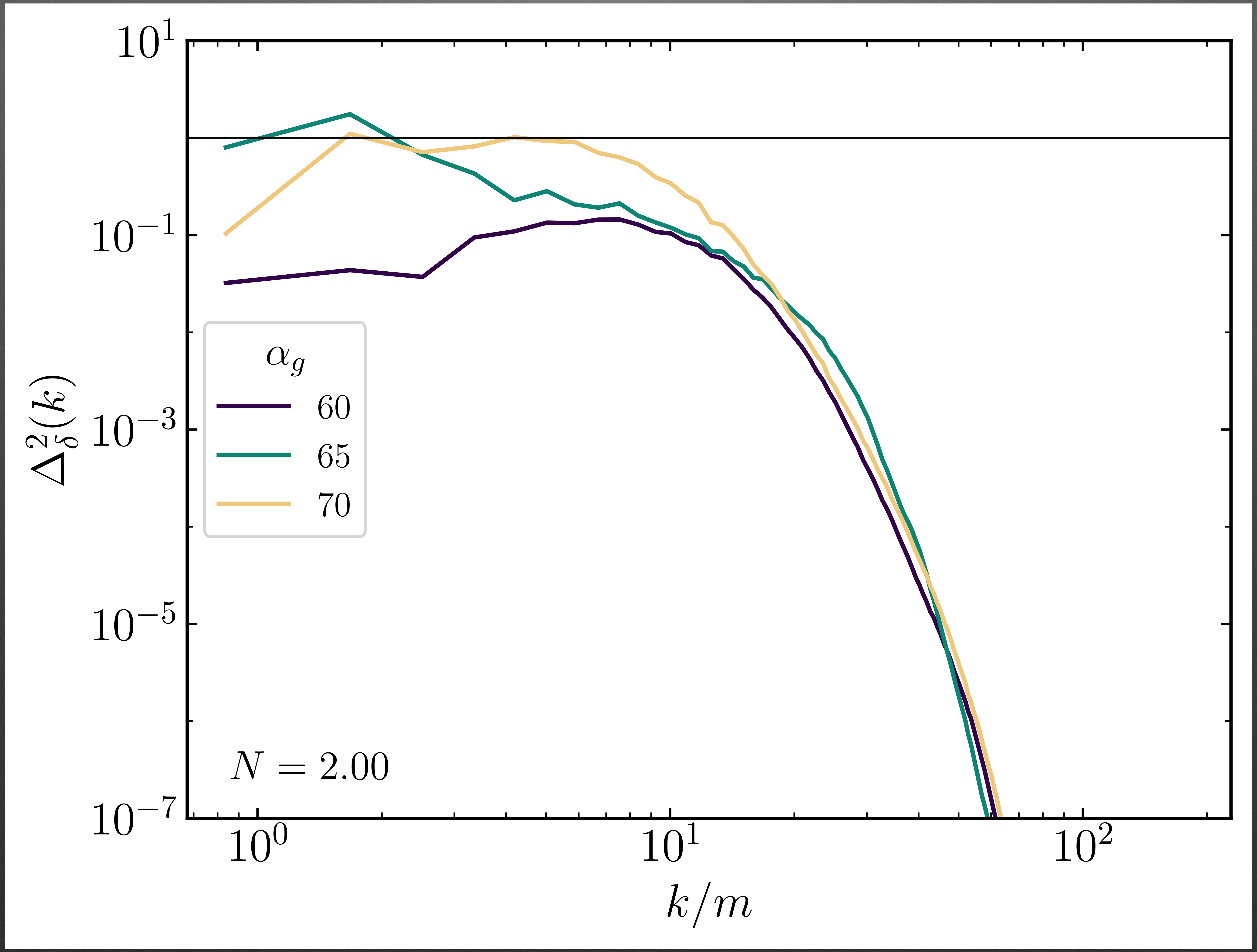
$$\partial_t E^m = \beta^o \partial_o E^m - E^o \partial_o \beta^m + \alpha \left(K E^m + \epsilon^{mno} D_n B_o - \mathcal{J}^m \right)$$
$$+ \epsilon^{mno} D_n \alpha B_o$$
$$\partial_t \mathcal{A}_m = \beta^o \partial_o \mathcal{A}_m + \mathcal{A}_o \partial_m \beta^o - \alpha \left(E_m + D_m \mathcal{A} \right) - \mathcal{A} D_m \alpha$$
$$\partial_t \mathcal{A} = \beta^o D_o \mathcal{A} + \alpha \left(K \mathcal{A} - D^m \mathcal{A}_m \right) - \mathcal{A}^m D_m \alpha$$

with…

$$\mathcal{J} = -\frac{1}{W(\varphi)} \left(W'(\varphi) E^m D_m \varphi - X'(\varphi) B^m D_m \varphi \right)$$
$$\mathcal{J}^m = \frac{1}{W(\varphi)} \left(W'(\varphi) \left[\Pi E^m - \epsilon^{mno} D_n \varphi B_o \right] - X'(\varphi) \left[\Pi B^m + \epsilon^{mno} D_n \varphi E_o \right] \right)$$

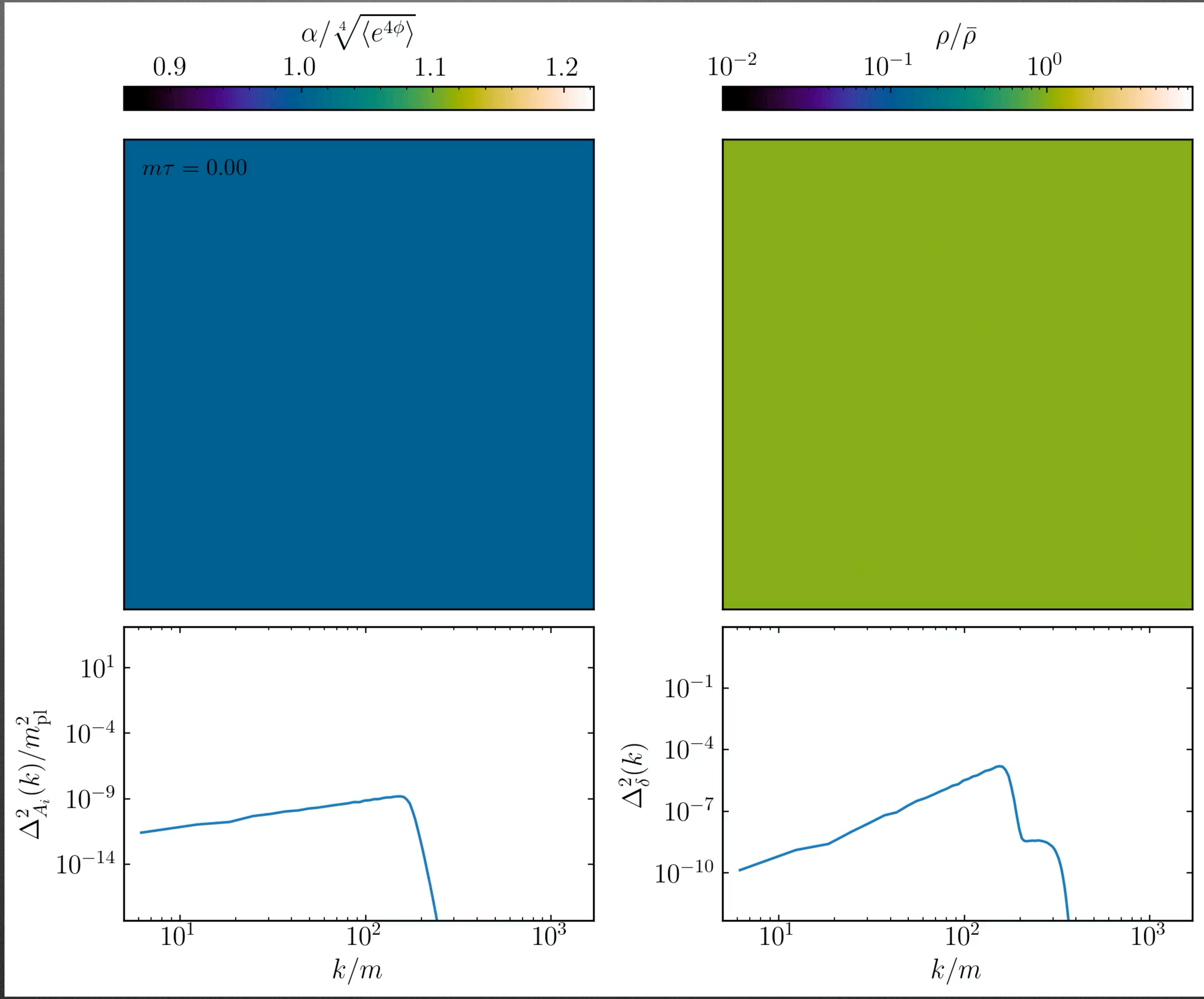
$$B^m = \epsilon^{mno} D_n \mathcal{A}_o = \epsilon^{mno} \partial_n \mathcal{A}_o, = e^{-6\phi} \varepsilon^{mno} \partial_n \mathcal{A}_o$$

We see very BIG density contrasts!

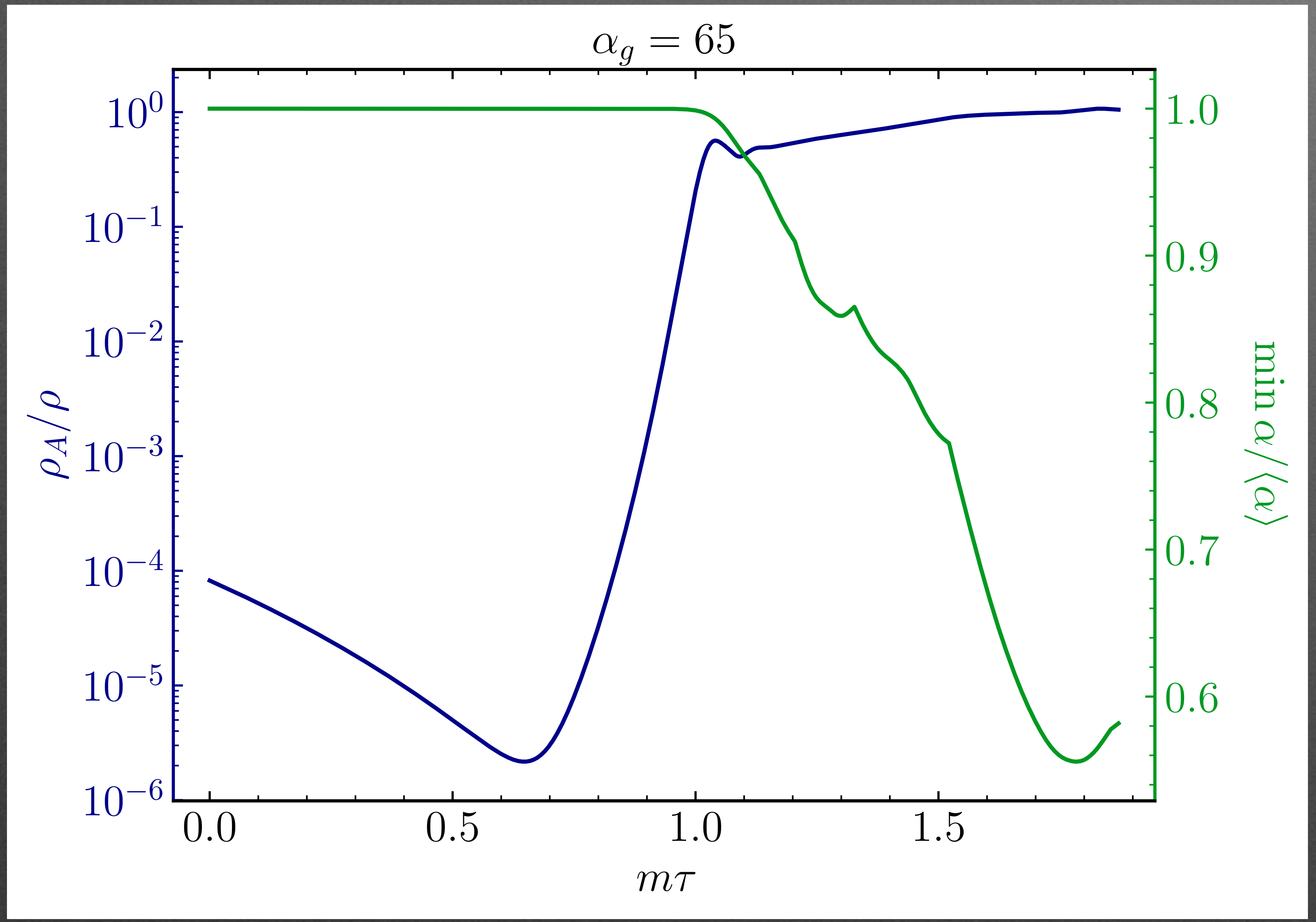


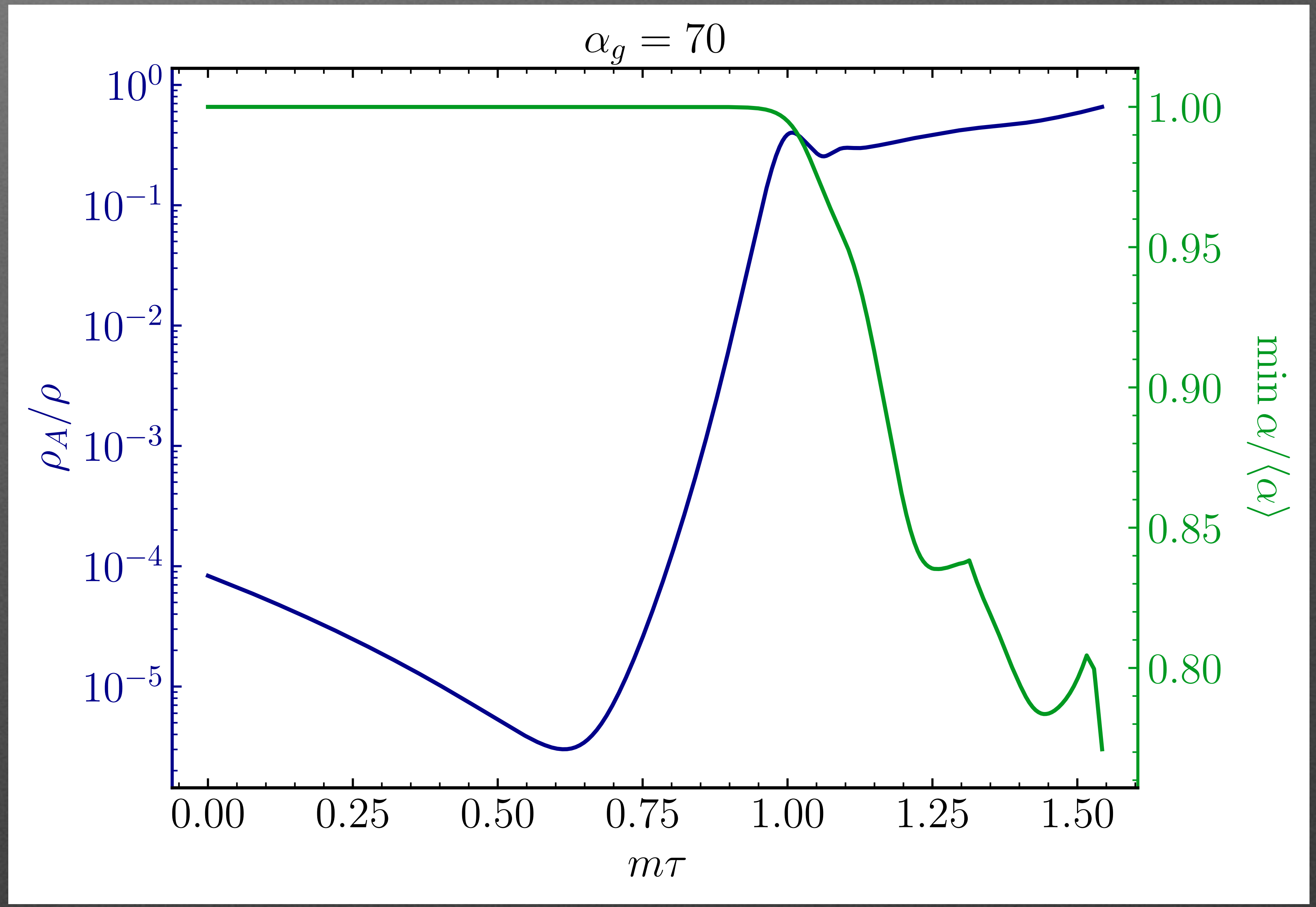
For *exciting* couplings

$$\frac{\alpha_g}{f} = 70\, m_{\rm pl}^{-1}$$



But ... there are no PBH

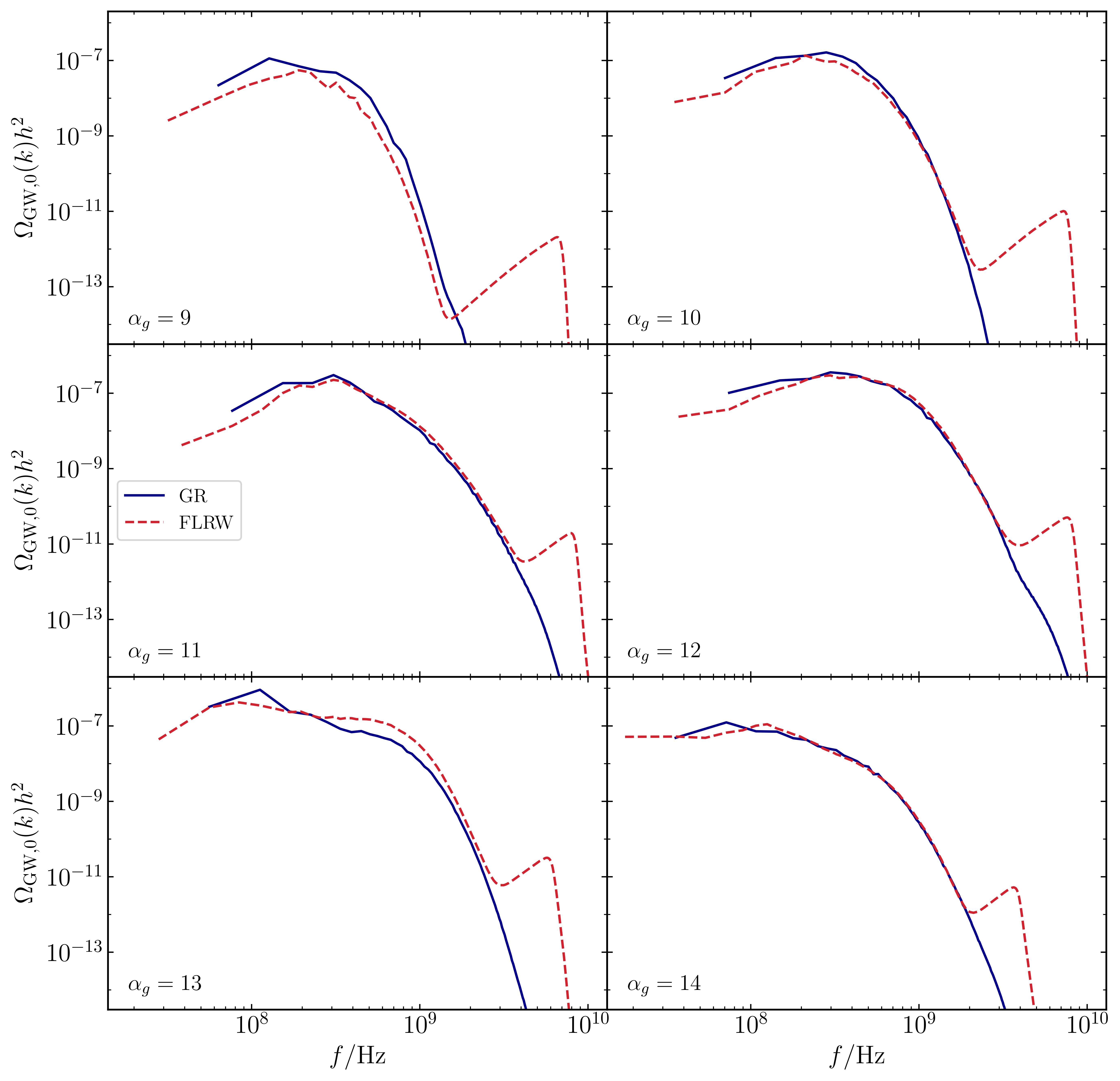




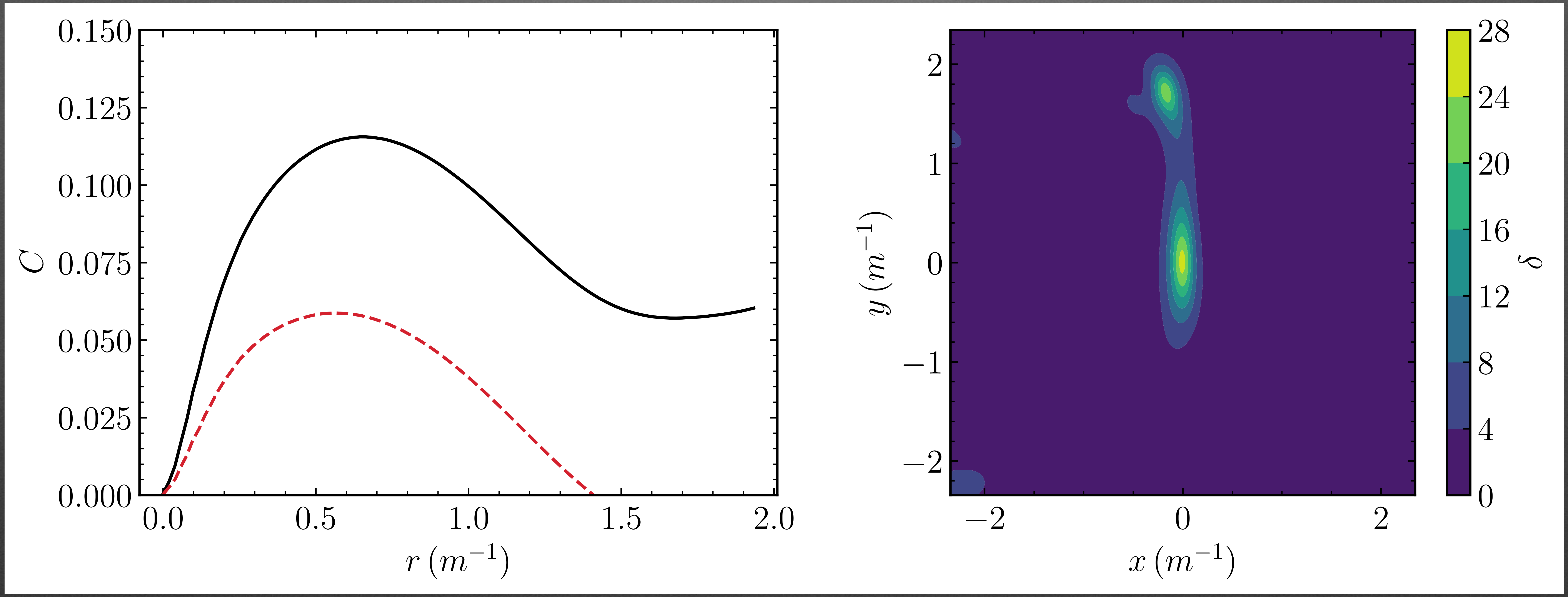
The minimum value of the lapse throughout the grid doesn't approach zero

But the GW

- Rapid-preheat models lead to extremely efficient gravitational wave production
- Which can be ruled out via Neff constraints

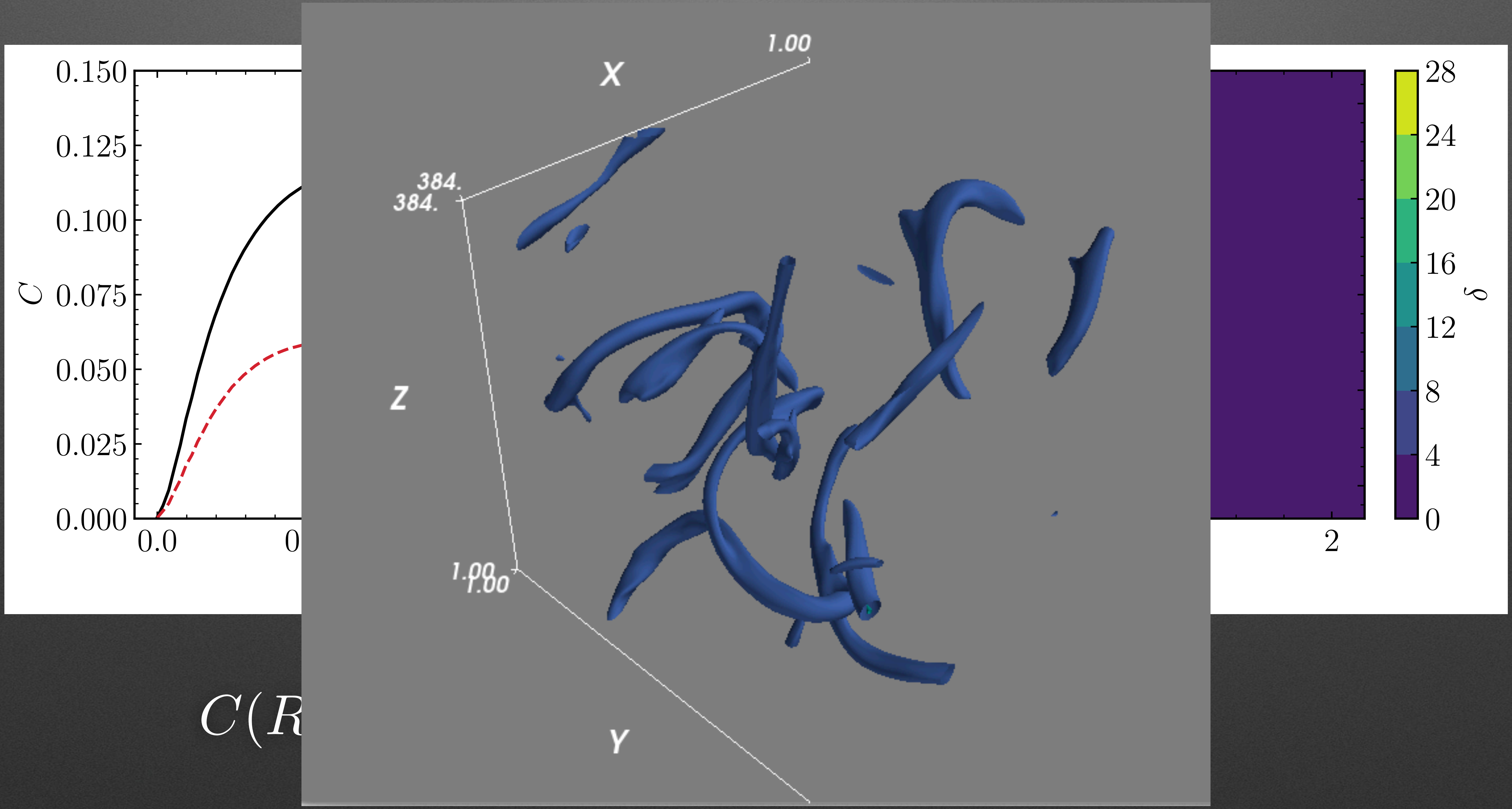


What does it look like?



$$C(R) = \frac{G\delta M}{R}$$

What does it look like?

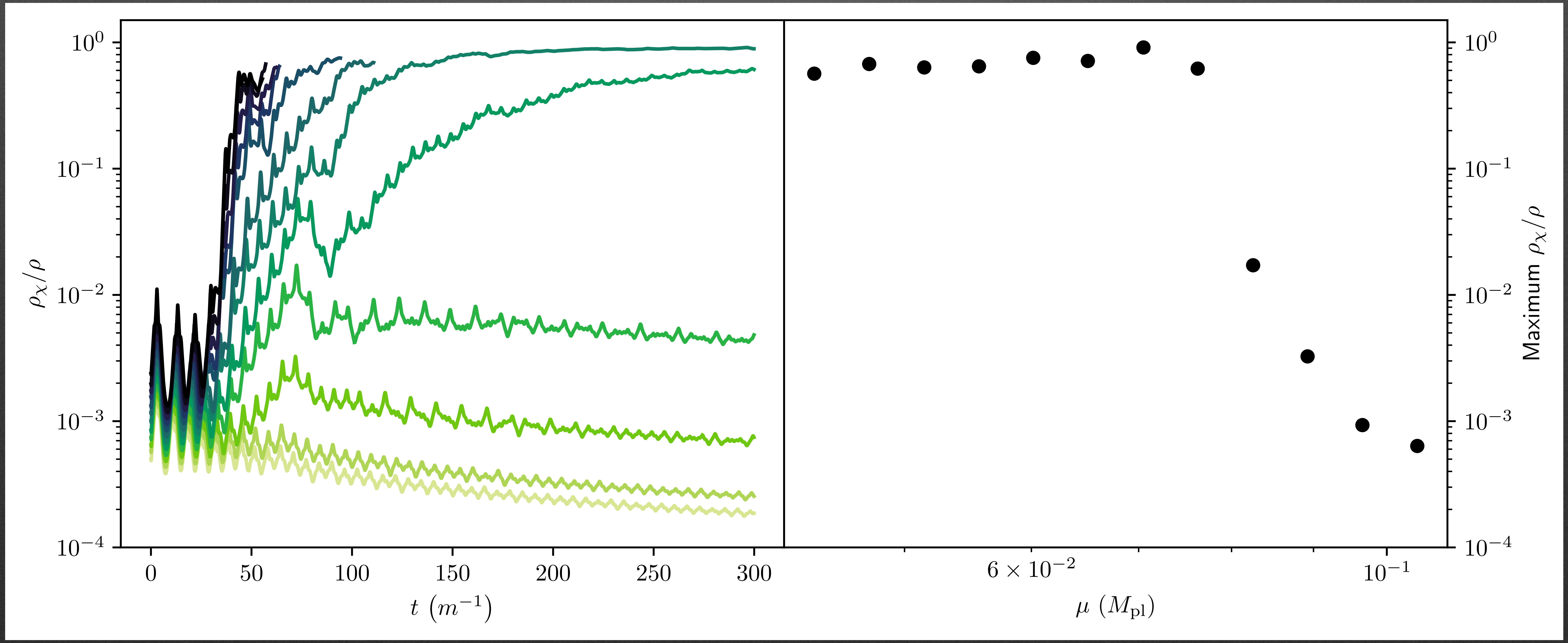


$C(R$

Reheating with Alpha-Attractors

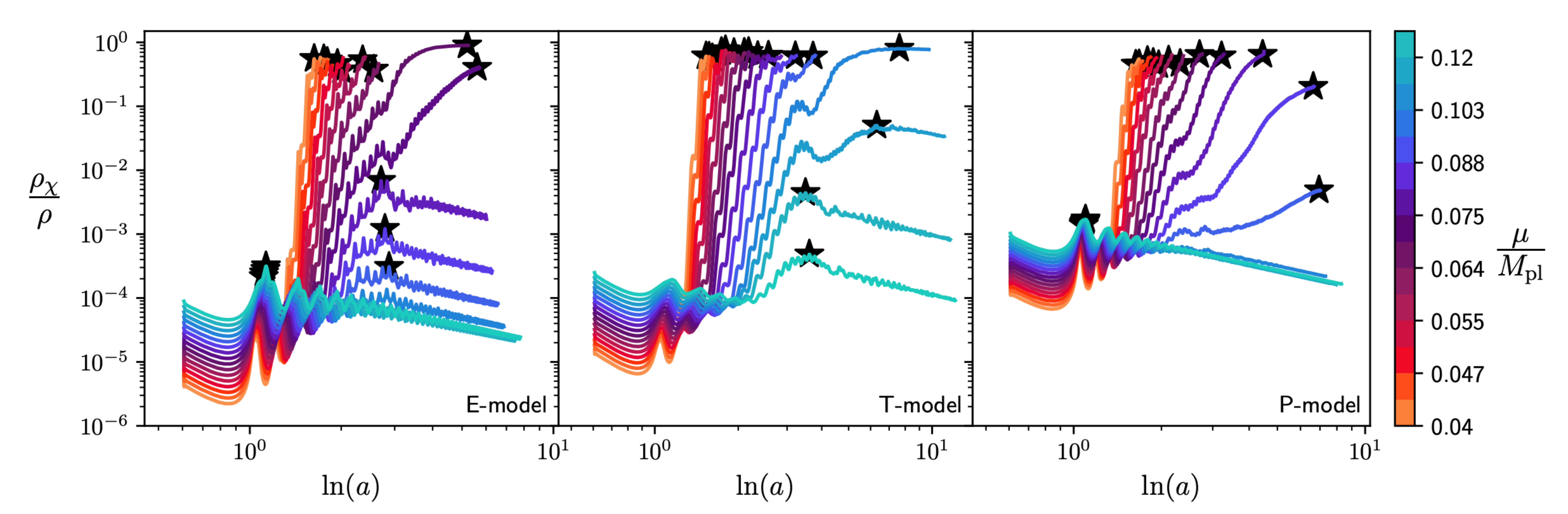
Alpha-attractors are a possibility (a la 2311.17237):

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{e^{2\phi/\mu}}{2}(\partial\chi)^2 - \frac{m^2\mu^2}{2}\left(1 - e^{-\phi/\mu}\right)^2$$

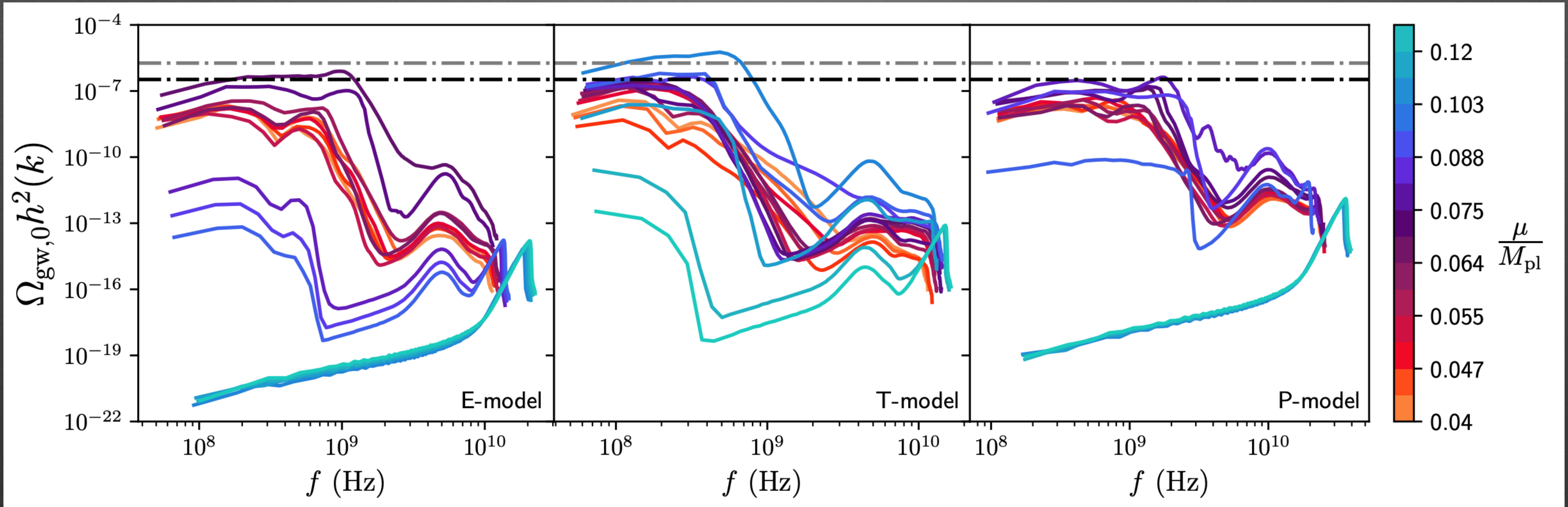


Which is only weakly model dependent

$$V = \frac{m^2\mu^2}{2}\left(1 - e^{-\frac{\phi}{\mu}}\right)^2 \qquad V = \frac{m^2\mu^2}{2}\tanh^2\left(\frac{\phi}{\mu}\right) \qquad V = \frac{m^2\mu^2}{2}\frac{\phi^2}{\phi^2+\mu^2}$$



They have lots of Gravitational Waves!



And also implications for matter-dominated eras

A modulus-dominated era might be dramatically concluded by a diatonic coupling to an anxion

$$S \simeq \int d^4x \sqrt{-g} \left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu a\partial^\mu a + \frac{c}{m_{\rm pl}}\phi\partial_\mu a\partial^\mu a - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_a^2 a^2 \right)$$

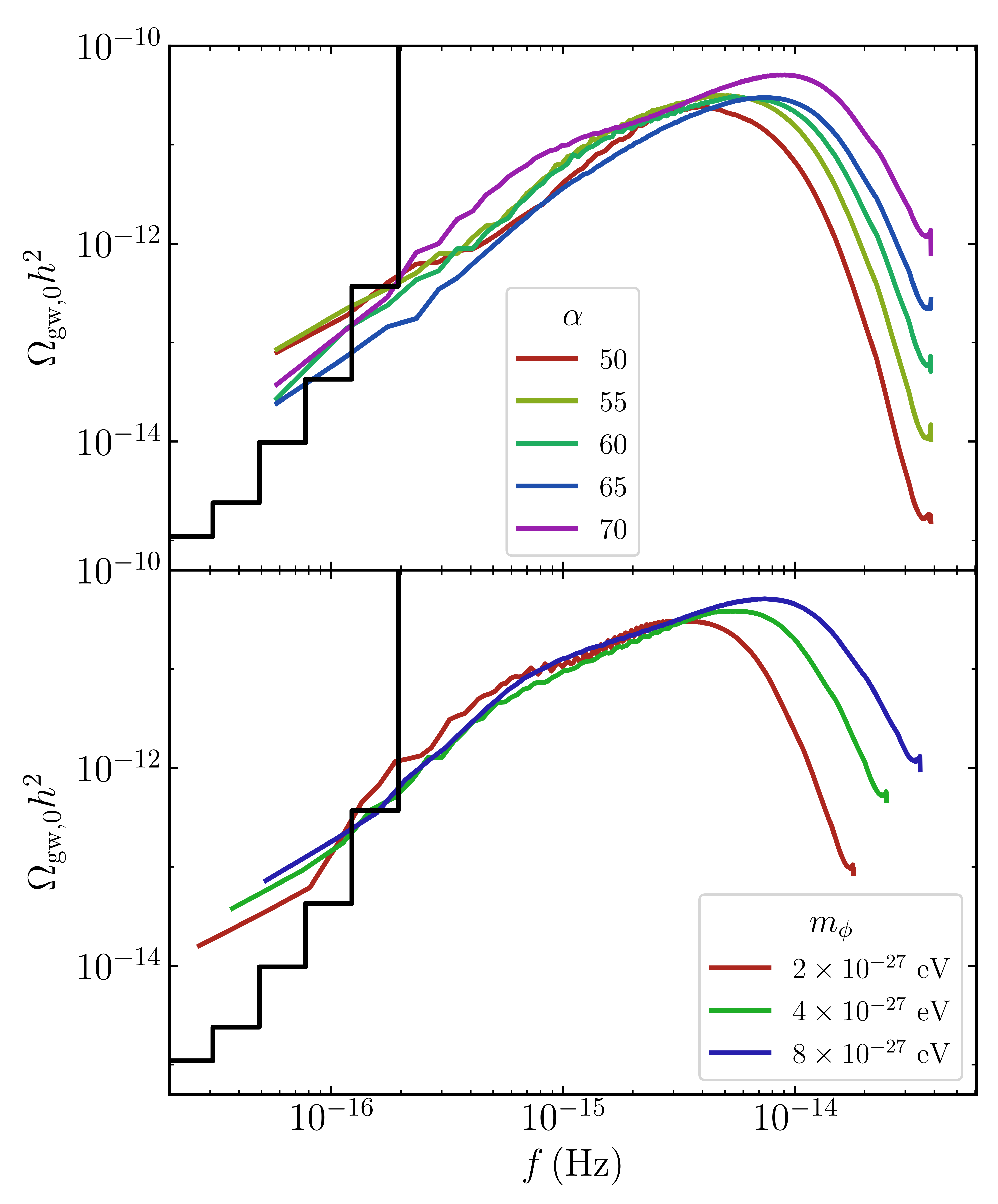
$$\sim \frac{e^{2c\phi/m_{\rm pl}}}{2}\partial_\mu a\partial^\mu a$$

With Fred Adams, Leia Barrowes, Robert Wiley Deal, Kuver Sinha, and Scott Watson

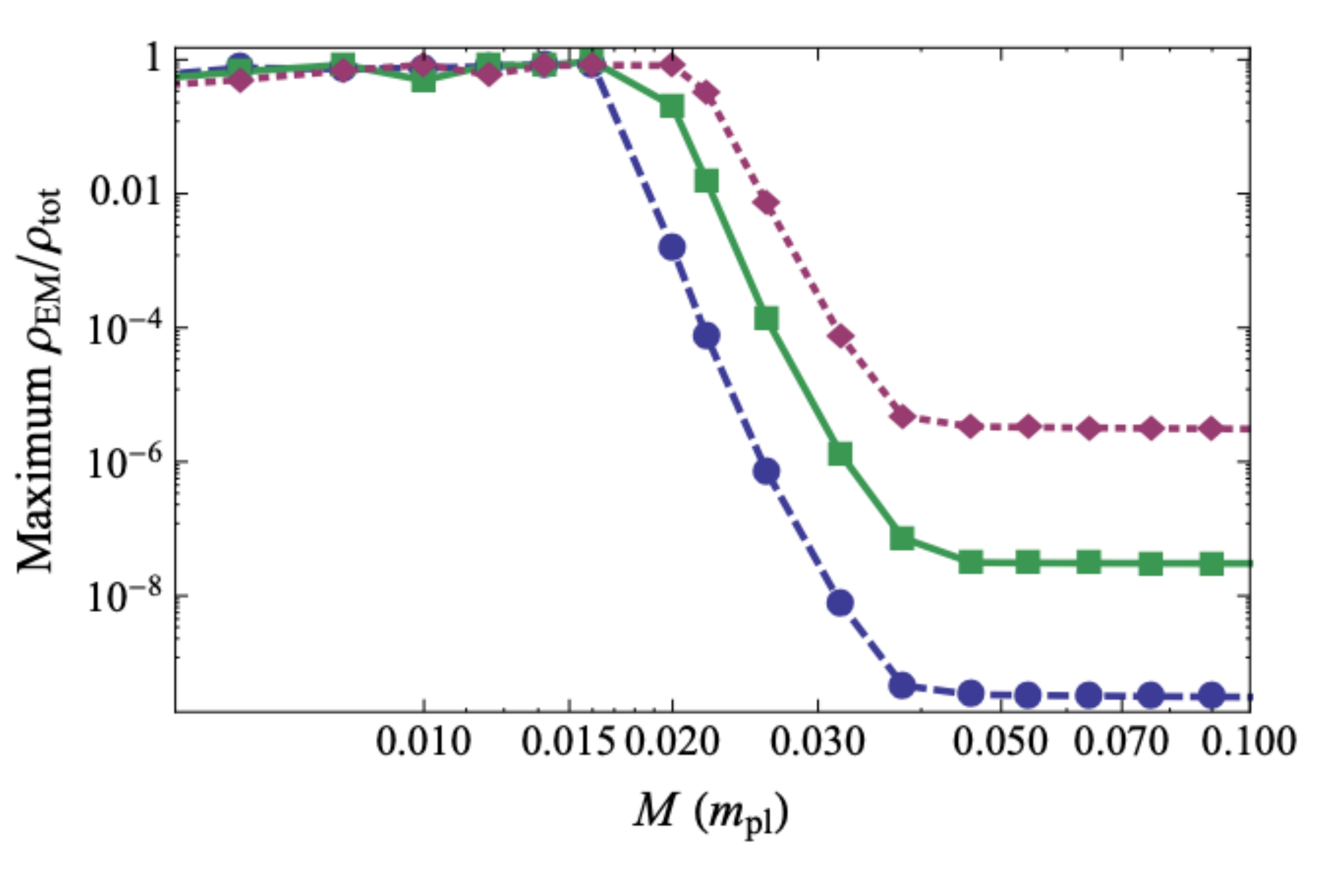
Or Early Dark Energy

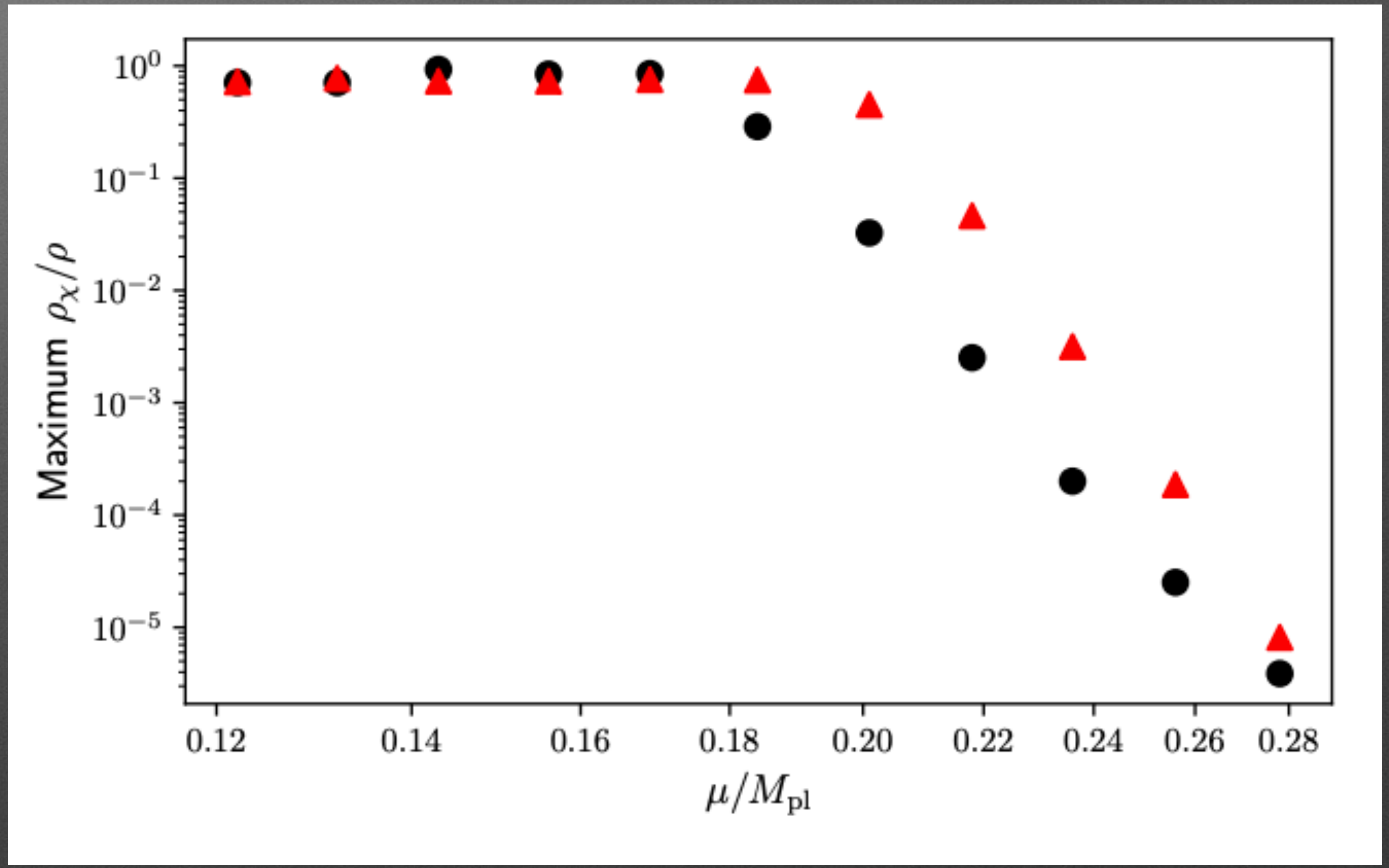
$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - m_\phi^2 f^2\left(1-\cos\frac{\phi}{f}\right)$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- If you're looking to get rid of EDE in a quick and resonant way



Gauge vs. Scalar Decay





$W = e^{\phi/M}$

$W = e^{2\phi/\mu}$

$$M_{crit} = 0.02\, m_{pl} = 0.1\, M_{pl}$$

$$M_{crit} = \frac{\mu}{2} = \frac{0.2\, M_{pr}}{2} = 0.1\, M_{pl}$$

Things we didn’t talk about

- Kination-dominated (p)reheating
- Tachyonic-preheating from 3-leg interactions
- (P)reheating from non minimally coupled field(s)

discussion topics

More discussion topics

1. Expansion history & scale dependence of signatures (eg. g-wave spectrum)
2. thermal vs. non-thermal initial conditions (and inhomogeneous), for DM production — freestreaming, isocurvature, clustering
3. coarse grained parameters (equation of state, Neff) vs. scale-dependent observables — PS, GW spectrum
4. Are there any generic expectations for potentials and couplings for end of inflation ?

Ask the experts in the audience

1. Andrew for GPP — DM abundance and expansion history
2. Adrienne — ask about fragmentation during Kination after inflation
3. Kim — isocurvature constraints small scale
4. somebody — how non-gaussianity is expected to be affected
5. Scott/Keith — non-inflationary “heating”