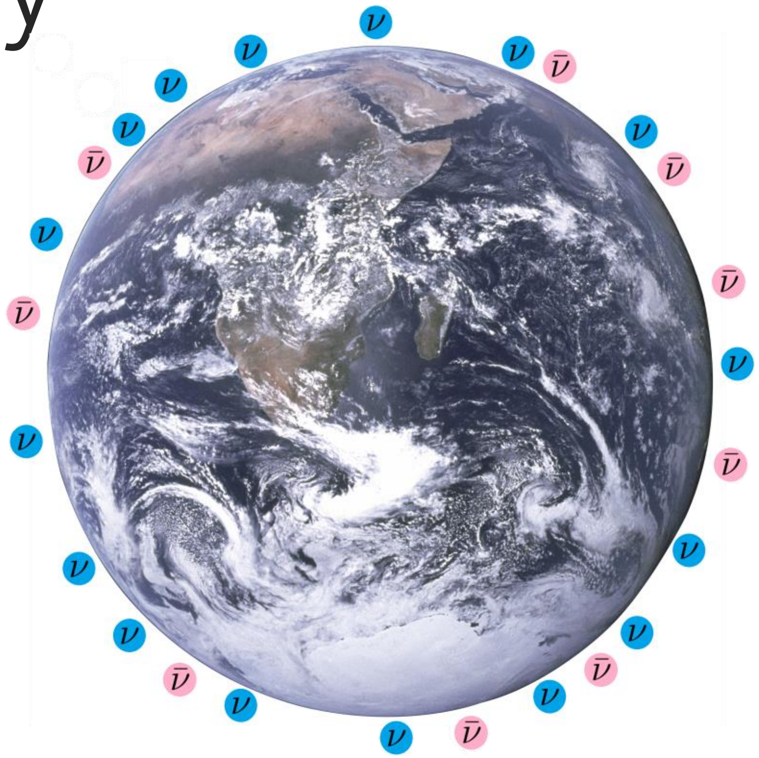
Tunneling away the relic neutrino asymmetry



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Based on arXiv:2404.11664

Introduction

- Cosmic neutrino background (CvB): relic neutrinos with $T_\nu \sim 2\,\mathrm{K}$
- Contains information about early universe and neutrino properties
- Difficult to detect because scattering/absorption are $\mathcal{O}(G_F^2)$
- All $\mathcal{O}(G_F)$ effects, e.g. torque on spins, depend on *asymmetry* $\Delta = \frac{n_\nu - n_{\bar{\nu}}}{n_0}$
- Arvanitaki and Dimopoulos ’23 argued that Earth creates a large asymmetry!
  - Approximated Earth as flat
- I will show that round Earth has no large asymmetry

Neutrinos in matter

- Neutrinos/antineutrinos feel a potential in matter

$$|U| \sim 10^{-14}\,\mathrm{eV} \cdot \left(\frac{n_{\mathrm{matter}}}{10^{22}\,\mathrm{cm}^{-3}}\right)$$

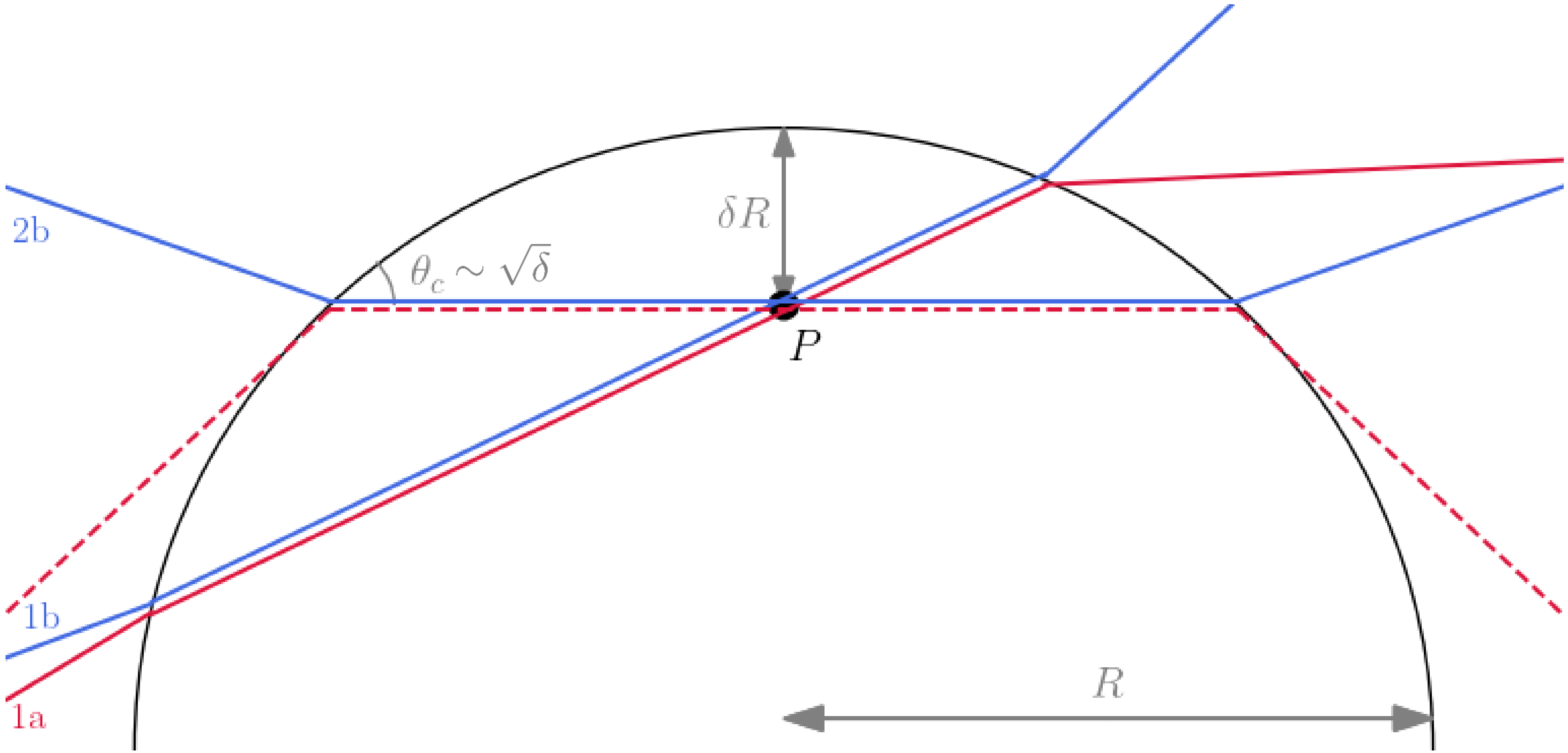
- Leads to index of refraction which differs from vacuum by

$$\delta = -\frac{m_\nu U}{k^2} \sim \pm 10^{-8} \cdot \left(\frac{n_{\mathrm{matter}}}{10^{22}\,\mathrm{cm}^{-3}}\right) \cdot \left(\frac{m_\nu}{0.1\,\mathrm{eV}}\right) \cdot \left(\frac{T_\nu}{k}\right)^2$$

– for neutrinos, + for antineutrinos → refract differently near Earth's surface

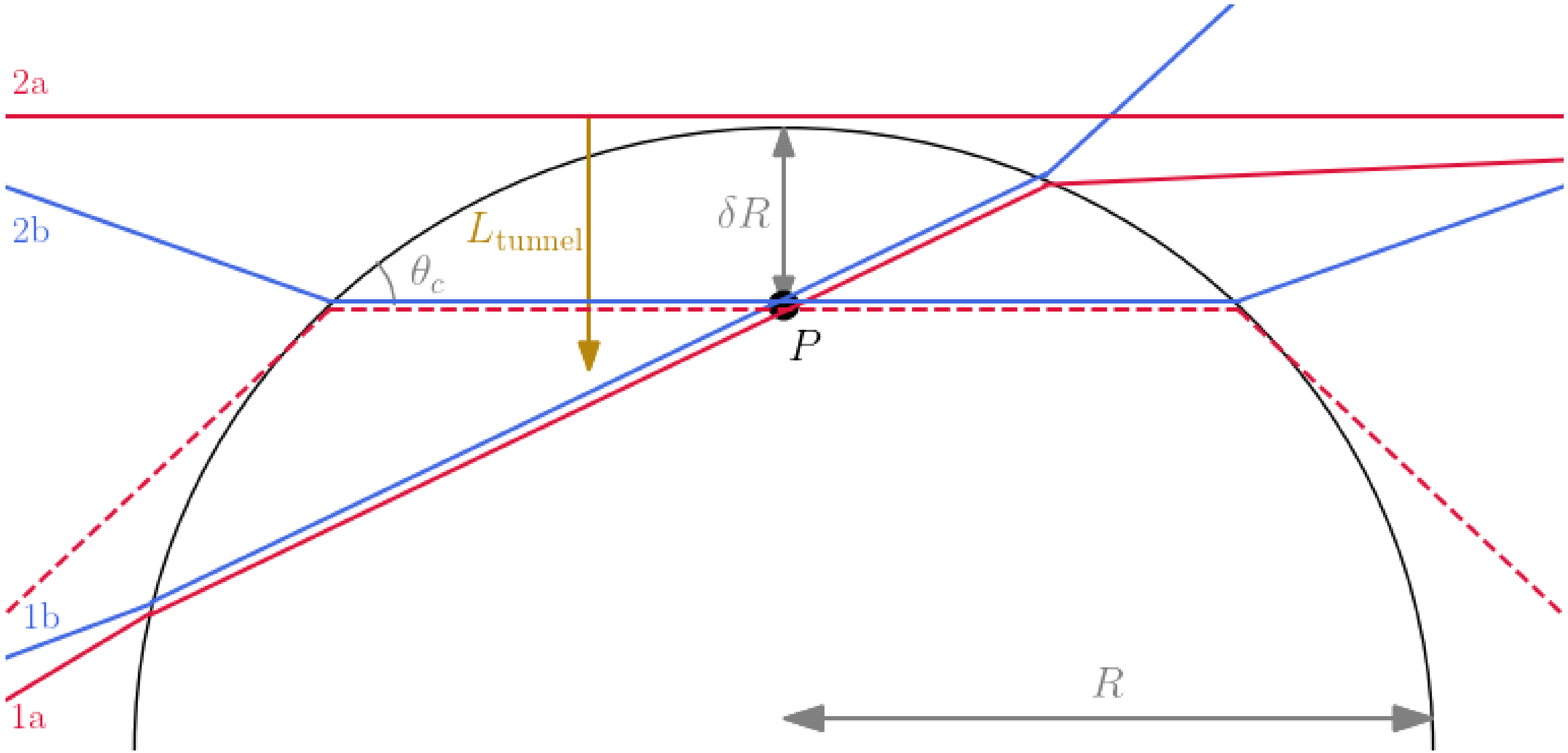
- Arvanitaki and Dimopoulos claimed this led to $\Delta = \mathcal{O}(\sqrt{\delta})$
- I will show that because $\delta^{3/2} kR \ll 1$, then $\Delta = \mathcal{O}(\delta)$, where R is Earth's radius

Heuristic picture



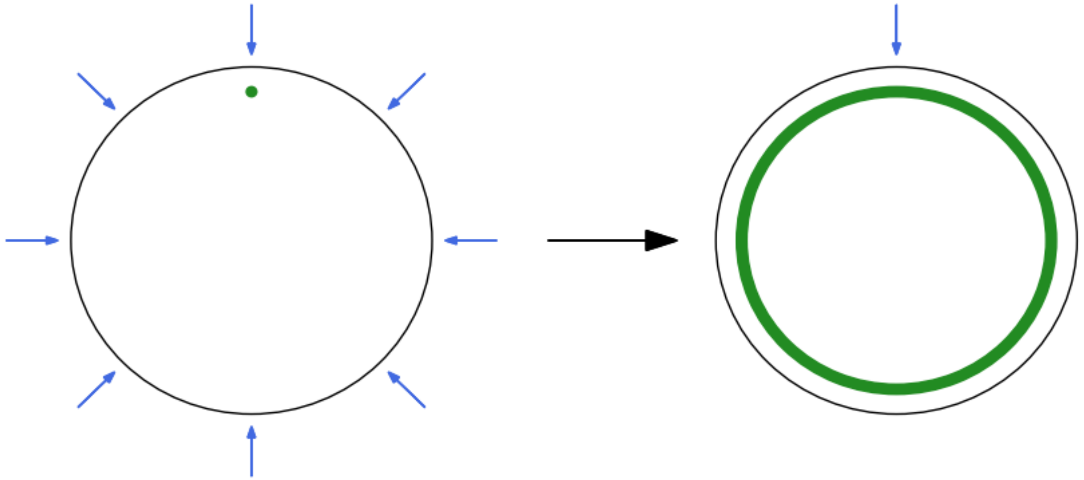
Heuristic picture

$$L_{\text{tunnel}} \gg \delta R \iff \delta^{3/2} k R \ll 1$$



Spherical calculation

- Assume monochromatic, isotropic CvB and spherical, uniform Earth
- Average over CvB momentum → average over sphere

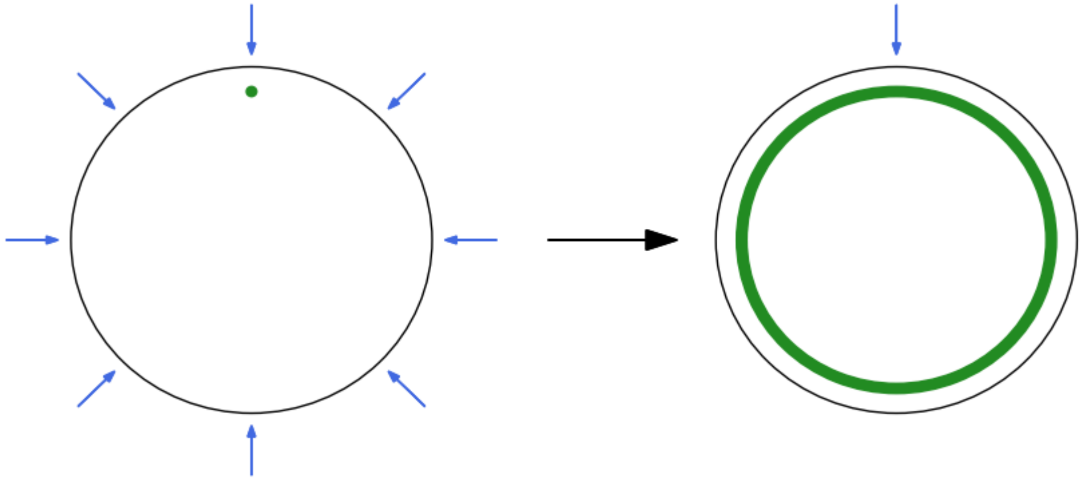


Spherical calculation

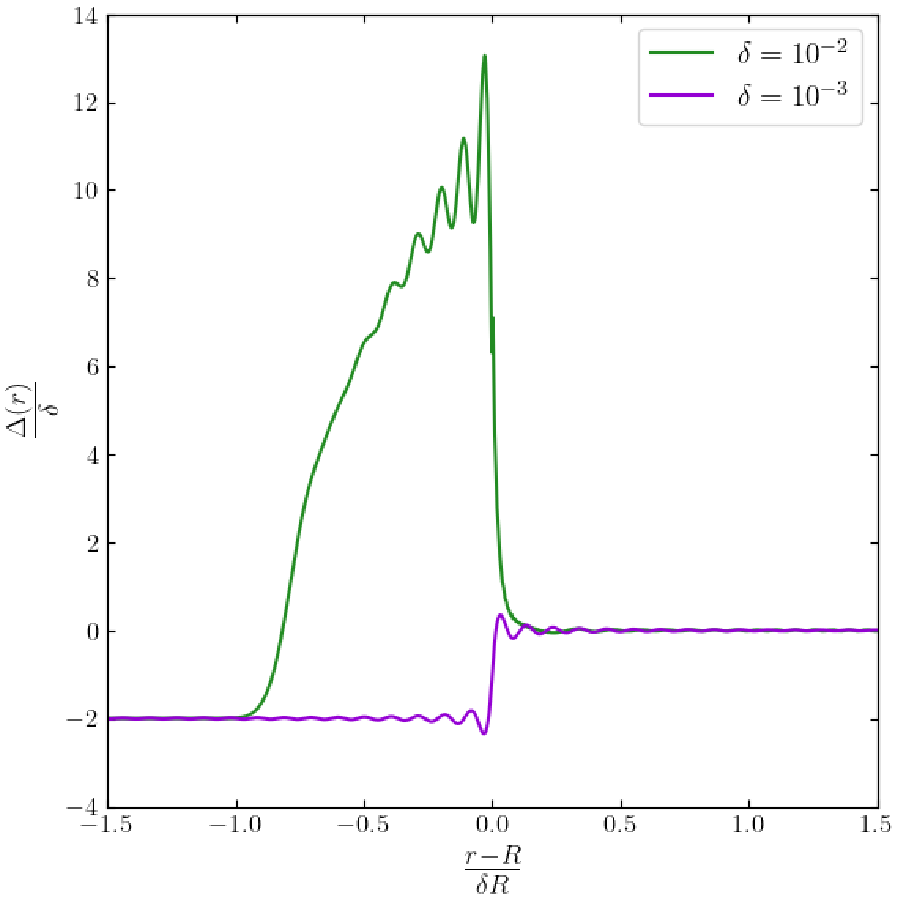
- Assume monochromatic, isotropic CvB and spherical, uniform Earth
- Average over CvB momentum → average over sphere
- Spherical harmonic decomposition

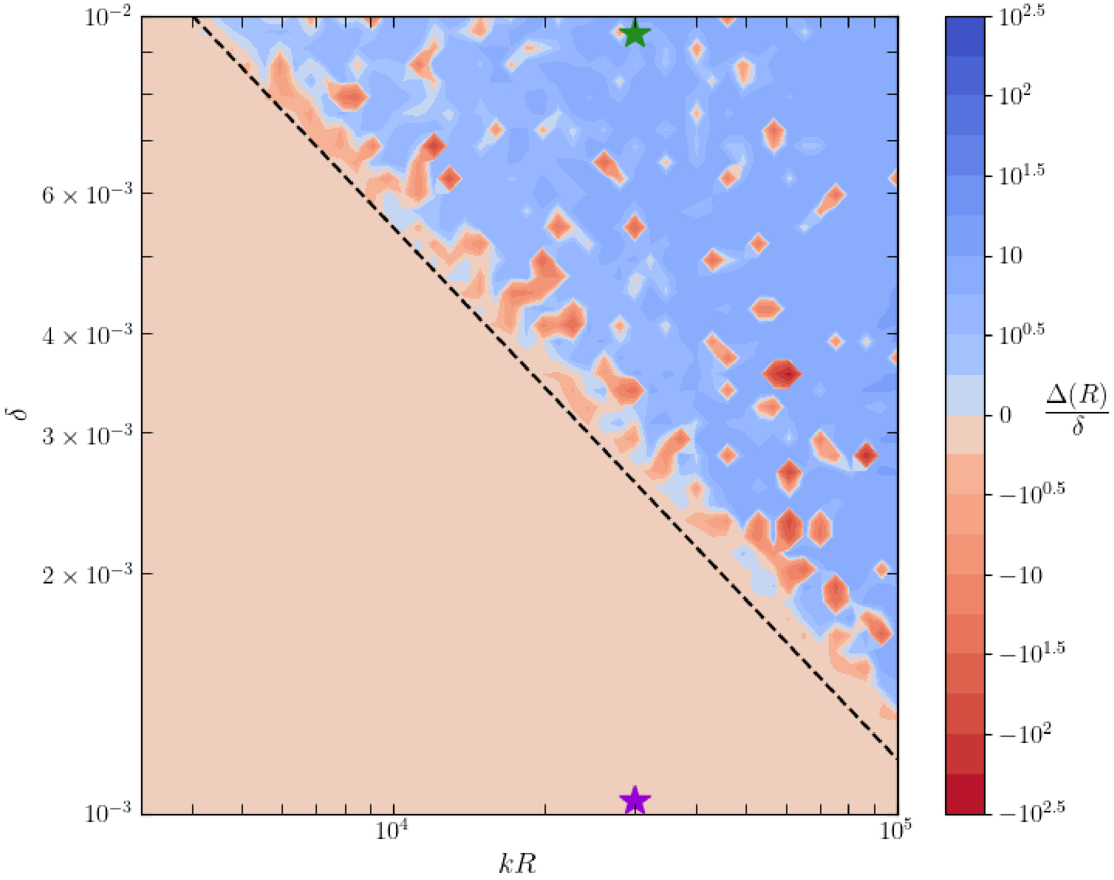
$$\psi(r,\Omega) = \sum_{\ell=0}^{\infty} \psi_\ell(r) Y_{\ell 0}(\Omega)$$

- Solve Schrodinger equation for each mode
- Number density $n_{\nu,\bar{\nu}}(r) = \frac{n_0}{4\pi} \sum_{\ell=0}^{\infty} |\psi_\ell(r)|^2$



Numerical results



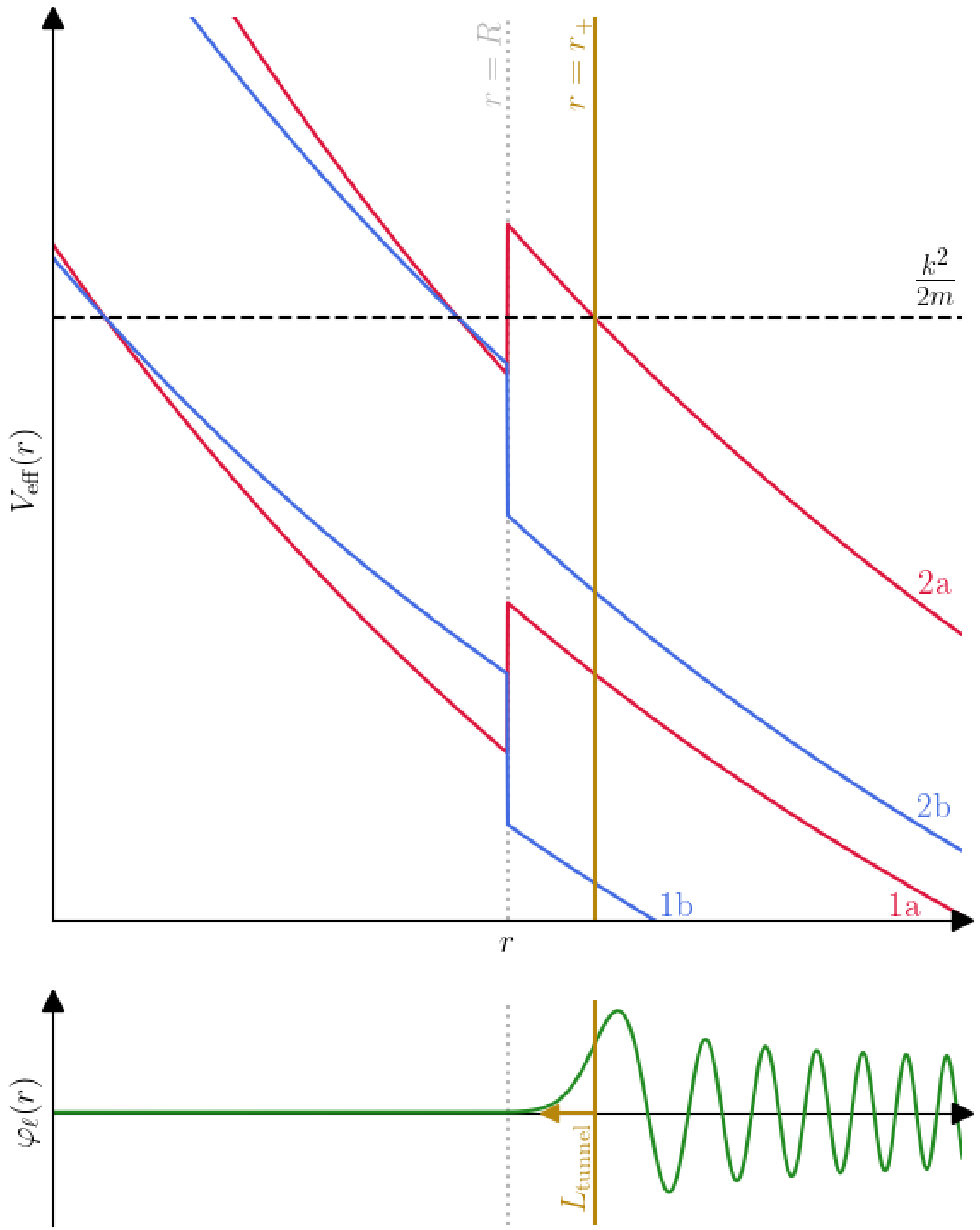


Effective potential

- Change of variables $\varphi_\ell = r\psi_\ell$
- Schrodinger equation becomes

$$-\frac{1}{2m}\partial_r^2\varphi_\ell + V_{\text{eff}}(r)\varphi_\ell = \frac{k^2}{2m}\varphi_\ell,$$

$$V_{\text{eff}}(r) \equiv \frac{\ell(\ell+1)}{2mr^2} + U\cdot\Theta(R-r)$$



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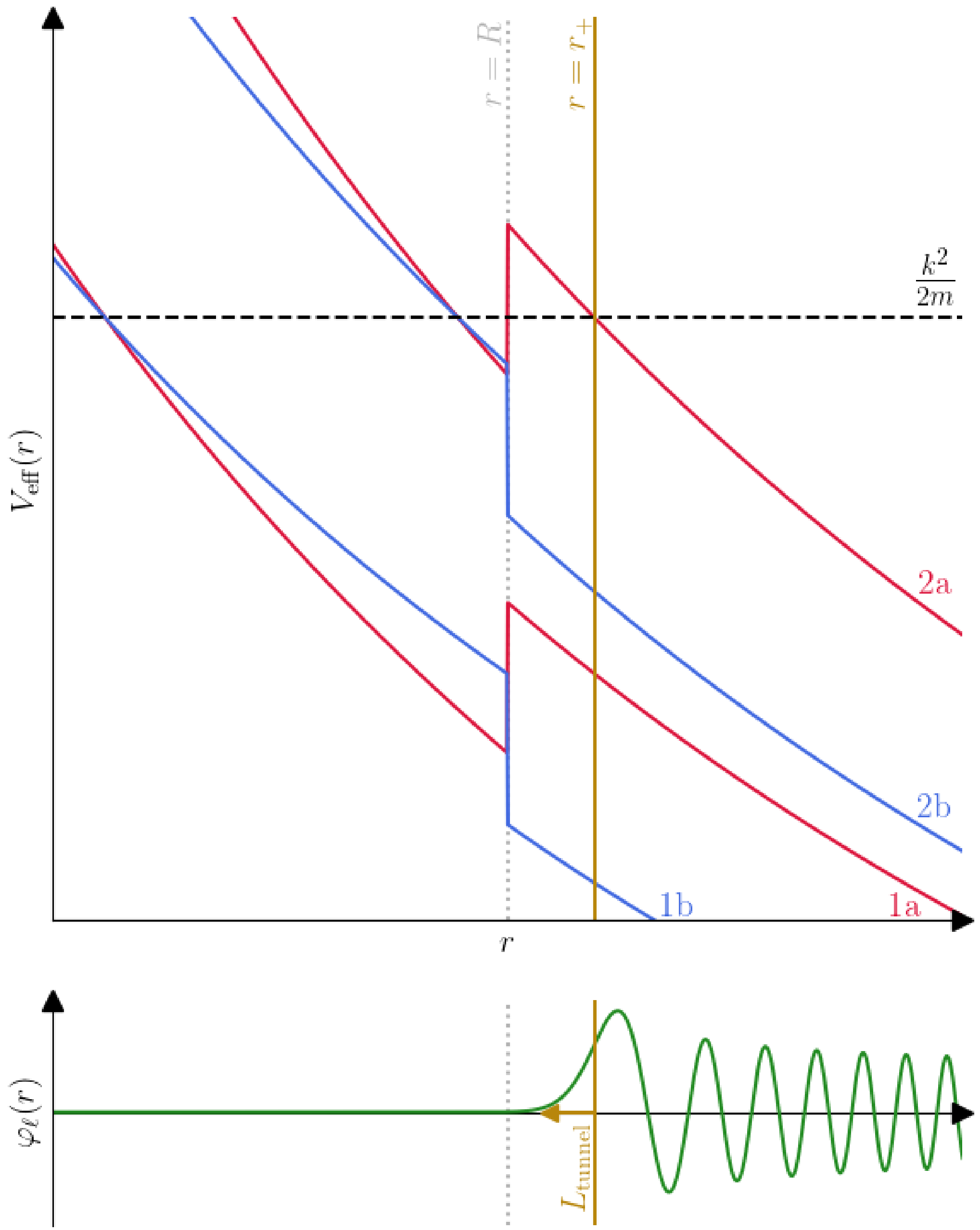
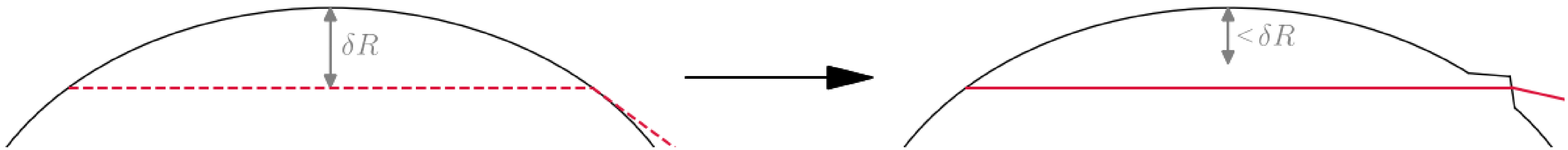
- WKB approximation: $L_{\text{tunnel}} \sim (kR)^{1/3}/k$
- $L_{\text{tunnel}} > \delta R \rightarrow$ asymmetry washed out



Asphericity caveat

- Argument assumes Earth is perfectly spherical
- Relevant scales: $L_{\text{tunnel}} \sim 1\,\text{m}, \delta R \sim 1\,\text{cm}$ → not spherical!
- L_{tunnel} does not depend on presence of Earth
- Asphericity should generically *reduce* inaccessible region
- More likely for asymmetry to get washed out



Conclusion

- Earth can affect local CvB asymmetry → enhancement for flat Earth
- Flat-Earth result requires $\delta^{3/2} k R \gg 1$, which physical params don't satisfy
- Required to prevent antineutrinos to tunnel onto inaccessible trajectories
- While argument assumes perfect sphericity, can argue necessary in general

Backup Slides

Ray tunneling

- Conservation of energy:

$$\frac{m_\nu v_1^2}{2} = \frac{m_\nu v_2^2}{2} - U$$

- Conservation of momentum:

$$m_\nu v_1 r_1 = m_\nu v_2 r_2$$

- Tunneling distance:

$$r_2 - r_1 = \frac{r_1}{\sqrt{1 + \frac{2U}{m_\nu v_1^2}}} - r_1 \approx \delta r_1$$

Spherical calculation

- Radial Schrodinger equation for ψ_ℓ:

$$\partial_r^2 \psi_\ell + \frac{2}{r}\partial_r \psi_\ell + \left(k^2 - 2mU \cdot \Theta(R-r) - \frac{\ell(\ell+1)}{r^2}\right)\psi_\ell = 0$$

- Plane wave decomposition:

$$k' \equiv k\sqrt{1+2\delta}$$

$$e^{ikz} = \sum_{\ell=0}^{\infty} i^\ell \sqrt{4\pi(2\ell+1)}\, j_\ell(kr) Y_{\ell 0}(\Omega)$$

- Solution:

$$\psi_\ell(r) = \begin{cases} i^\ell \sqrt{4\pi(2\ell+1)}\, j_\ell(kr) + B_\ell h_\ell^{(1)}(kr), & r > R \\ C_\ell j_\ell(k'r), & r < R, \end{cases}$$

Spherical calculation (cont.)

- Boundary conditions:

$$i^\ell \sqrt{4\pi(2\ell+1)} j_\ell(kR) + B_\ell h_\ell^{(1)}(kR) = C_\ell j_\ell(k'R)$$

$$i^\ell \sqrt{4\pi(2\ell+1)} k j_\ell'(kR) + k B_\ell h_\ell^{(1)\prime}(kR) = k' C_\ell j_\ell'(k'R)$$

- When $\ell < k'r$,

$$\langle |C_\ell|^2 \rangle \approx 4\pi(2\ell+1)(1+\delta),$$

where $\langle \cdot \rangle$ is average over ℓ modes

- Therefore, for $r < (1-\delta)R$,

$$n_{\nu,\bar{\nu}}(r) \approx (1+\delta) n_0 \sum_{\ell=0}^{\infty} (2\ell+1) j_\ell(k'r)^2 = (1+\delta) n_0$$

WKB Approximation

- WKB phase:

$$\int_{r_+-L}^{r_+} \sqrt{2m\left(V_{\text{eff}}(r) - \frac{k^2}{2m}\right)} dr \approx \int_{r_+-L}^{r_+} \sqrt{(r_+ - r) \cdot \frac{2\ell(\ell+1)}{r_+^3}} dr$$

$$= \frac{2}{3}\sqrt{L^3 \cdot \frac{2\ell(\ell+1)}{r_+^3}}$$

- Phase $\mathcal{O}(1)$ when:

$$L_{\text{tunnel}} \sim \frac{r_+}{\ell^{2/3}} \sim \frac{(kR)^{1/3}}{k}$$