Generalising flavoured potentials and their minima

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based on work with J Talbert (LANL), I Varzielas (CFTP), A Sengupta (SUNY)

Motivation

Flavour puzzle and dynamic solutions

What are flavour puzzles?

→ Explain the pattern of fermionic mass, mixing, CP violation…

If we can reduce the number of free parameters in SM that would be great!

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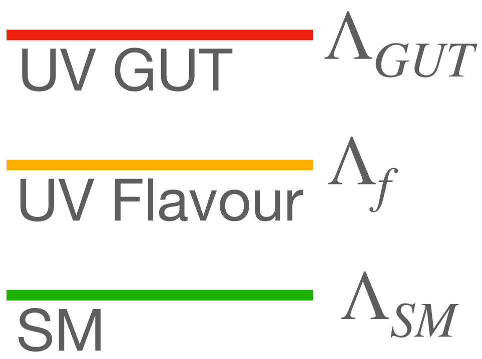

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<u>Do they work?</u> ⟵ cf. Micah Mellors' talk on FN

The Flavon Model

Backgrounds on Universal Texture Zero (UTZ)

STEP 0 — Define $\mathcal{G}_f = \Delta_{27} = \left(Z_3 \times Z_3\right) \rtimes Z_3$

STEP 1 — INGREDIENTS

BSM scalar fields **charged** by Δ(27), e.g., θ (***flavon***)

fermions **anti-charged** by Δ(27), e.g., q_i

STEP 2 — COUPLINGS

form interaction terms $\theta q_i \theta q_j$

break flavour symmetry $\theta q_i \theta q_j \to v^2 q_i q_j$,

get Yukawa terms and <u>**CHECK**</u> e.g., CKM, PMNS…

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BUT DO THEY EXIST?

$$\begin{cases} \langle \theta_3 \rangle = (0,0,1) \\ \langle \theta_{123} \rangle = (1,1,-1) \\ \langle \theta_{23} \rangle = (0,1,1) \end{cases}$$

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d=6 term could destabilise

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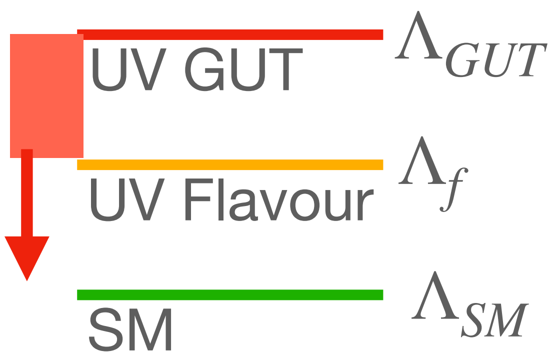

Challenges

Backgrounds on Universal Texture Zero (UTZ)

What are the $d = 6$ contractions?

1. Discrete symmetries and Efficient Counting of Operators
   Hilbert series based [Calò, Marinissen, Rahn 2023]
2. Enumerate via the hypercharge
   [new] use tree isomorphism to count degeneracy

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2. Enumerate via the hypercharge

   [new] use tree isomorphism to count degeneracy

[new] How do these invariants affect the stability of the solutions?

1. Perturb from the renormalizable alignment, or
2. Find new solutions!

How many invariants are there?
Hilbert Series approach to group contraction

DECO - **D**iscrete symmetries and **E**fficient **C**ounting of **O**perators

[Simon Calo, Coenraad Marinissen and Rudi Rahn, 2022]

Enumerates the number of terms for an effective theory for

- Arbitrary dimension
- Arbitrary field/symmetries
- Includes $S_4, A_4, Z_n, U(1)_R$ etc…

[Lehman and Martin 2015]

based on form and powered by Hilbert Series…

[new] DECO 1.1 includes $\Delta_{27} = \left(Z_3 \times Z_3\right) \rtimes Z_3$

[new] What are the invariants?

Discrete non-Abelian singlet of N flavons

Given two flavons θ, θ' that are 3D fundamental representation

The singlets transform as $\mathbf{1}_{r,s} \to \omega^r \mathbf{1}_{r,s}$ or $\omega^s \mathbf{1}_{r,s}$

STEP 1 – Partition of $d = 6 = 2 + 2 + 2 = 2 + 4 = 3 + 3 = 6$

The partitions are the dimension of each singlets that makes up the overall singlet

STEP 2 – Enumerate the triplets it takes to form that singlet

STEP 3 – Enumerate the r, s within each partition

e.g., $6 = 2 + 4$ represents $\mathbf{1}_{r,s} \cdot \mathbf{1}_{r',s'} = \mathbf{1}_{r+r',s+s'}$

Need $r + r' = s + s' = 0$ mod 3 ….

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How to keep track?

[new] What are the invariants?

What's the problem?

STEP 2 – Enumerate the triplets it takes to form that singlet

How to keep track?

How do we know we have exhausted the ways to form singlet/triplet?

e.g., to form a $d = 4$ singlet we can either carry out

$$\left[(\theta \times \theta \times \theta) \cdot \theta\right]_{r,s} \text{ or } \left[(\theta \times \theta) \cdot (\theta \times \theta)\right]_{r,s}$$

it is not obvious if we exhausted all the ways to form a singlet…

What do we do?

[new] What are the invariants?

The contraction graph – definition

Definition (Contraction graph). A *contraction graph*[5] is a 2-coloring tree G with V vertices and E edges, where each vertex $i = 1 \ldots N$ has degree e_i, so $E = \frac{1}{2} \sum_i e_i$.

1. The graph is 2-colouring, so no two vertices share the same colour if they are connected by an edge, *i.e.*, no ●——● or ○——○.
2. The number of leaves is the dimension of the term the graph represents.
3. Each vertices have degree 1, 2, or 3, and are either *active* with an even degree or *inactive* with an odd degree. An active vertex is a triplet that has not been contracted.
4. New connections can only be made with an active vertex.
5. A line represents triplet multiplication and a dashed line represents singlet contraction.

To convert a contraction graph back to their symbolic form, first write down the number of fields represented by the leaf edges, then insert $\times$ if two leaves n_1, n_2 are connected via another vertex (distance $= 2$), or $\circ$ if connected by an edge (distance $= 1$).

[new] What are the invariants?

The contraction graph – example

An example of $d = 6 = 6$ contraction

+ means triplet, - means anti-triplet

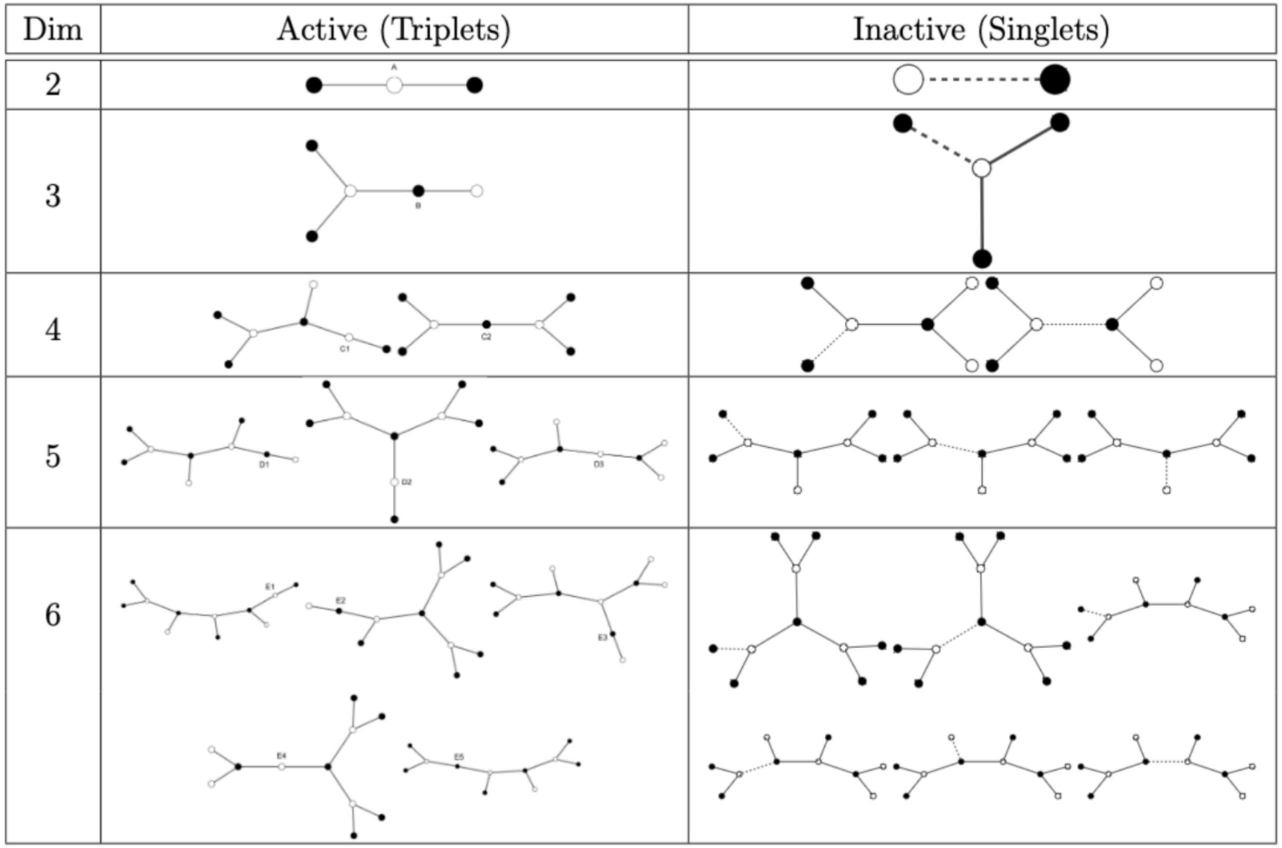
$$\big(\underbrace{\underbrace{\underset{+}{\theta} \times \underset{+}{\theta}}_{-} \times \underset{-}{\theta}}_{+}\big) \circ \big(\underbrace{\underbrace{\underset{-}{\theta} \times \underset{-}{\theta}}_{+} \times \underset{+}{\theta}}_{-}\big) =$$

We can convert the expression to graph and vice versa.

Each $\circ$ comes with indices $\{r, s\}$ and $\times$ comes with indices $q = 0,1,2$

[new] What are the invariants?

The contraction graph of $d \leq 6$

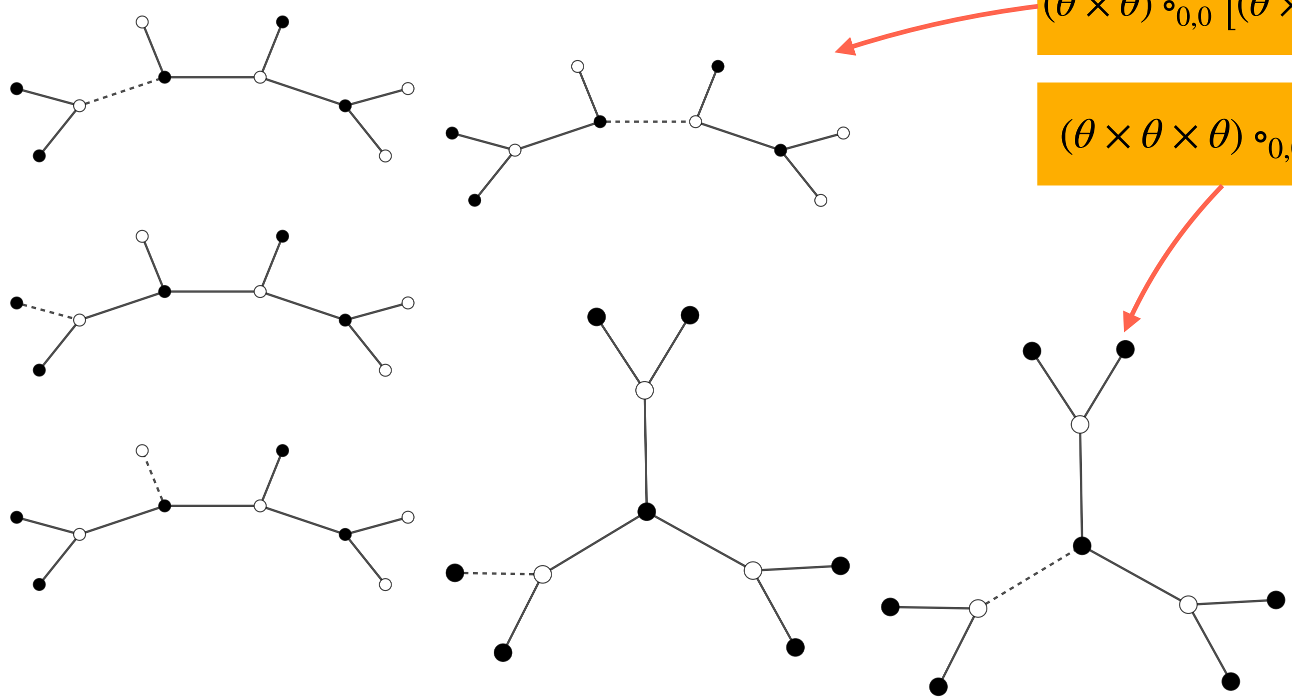


[new] What are the invariants?

The contractions of $d = 6$

$$(\theta \times \theta) \circ_{0,0} \left[(\theta \times \theta) \times (\theta \times \theta)\right]$$

$$(\theta \times \theta \times \theta) \circ_{0,0} (\theta \times \theta \times \theta)$$



6 contraction types!

[new] What are the invariants?

The contractions of $d = 6 = 2 + 4 = 3 + 3 = 2 + 2 + 2$

Examples of $\theta = (\theta_1, \theta_2, \theta_3)$ invariants associated with each partition

There are 39 forms of $6 = 6$ invariants, e.g.,

$$|\theta_1|^6 + |\theta_2|^6 + |\theta_3|^6, \quad \theta_1\theta_2\theta_3\left(\theta_1^3 + \theta_2^3 + \theta_3^3\right), \quad \left(\theta_2^3 + \theta_3^3\right)\theta_1^3 + \theta_2^3\theta_3^3 .$$

There are 30 forms of $6 = 3 + 3$ invariants.

There are 48 forms of $6 = 2 + 4$ invariants.

There are 14 forms of $6 = 2 + 2 + 2$ invariants…

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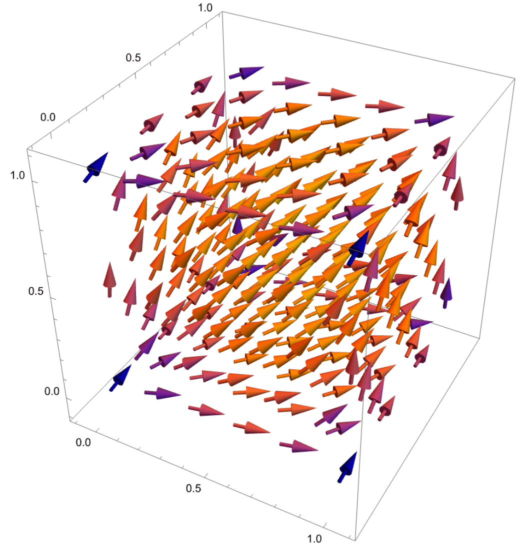
<u>Any of them could destabilise the alignment</u>

[new] Stability analysis $N_f = 1$

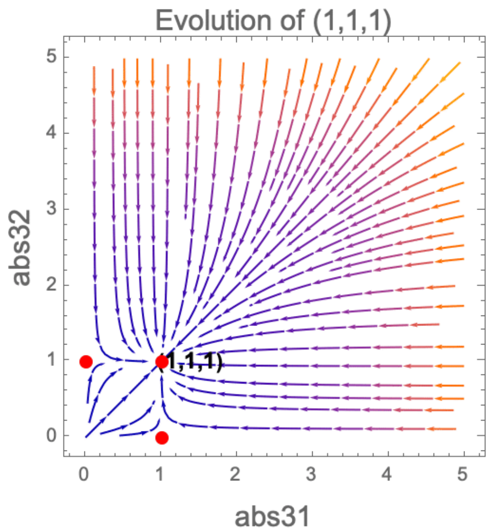
Observations of pure $d = 6$ influence on $\langle\theta_3\rangle$

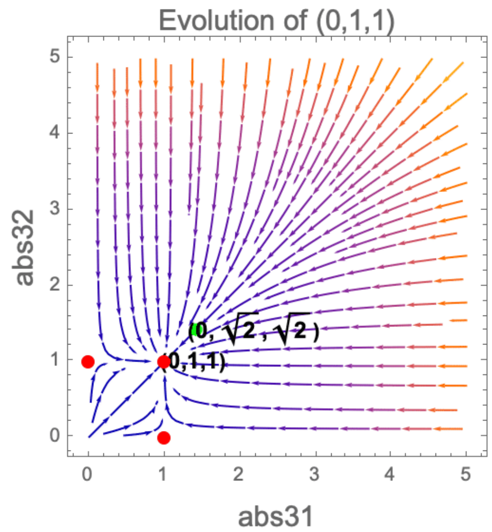


$\langle\theta_3\rangle = (0,0,1), (1,1,1), \ldots$ are stable minimum potential when $V \supset V_{d\leq 4} = m_3^2 \,|\, \theta_3^2 \,|\, + \lambda_3\, \theta_3^2 \theta_3^{\dagger\, 2}$

Introduce non-renormalisable contribution

$$V \supset V_6 = k_3 \left(\theta_3 \theta_3^\dagger\right)_{0,0} \left(\theta_3 \theta_3^\dagger\right)_{0,1} \left(\theta_3 \theta_3^\dagger\right)_{0,2}$$





Most directions are either

1. Preserved $\mathbf{v} \to \mathbf{v}$
2. Scaled $\mathbf{v} \to a\mathbf{v}$
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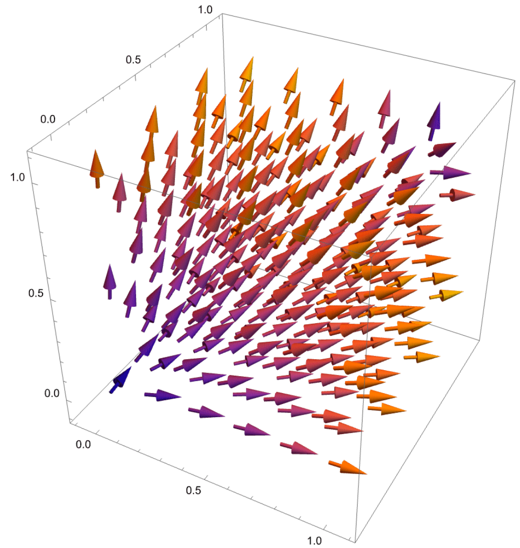

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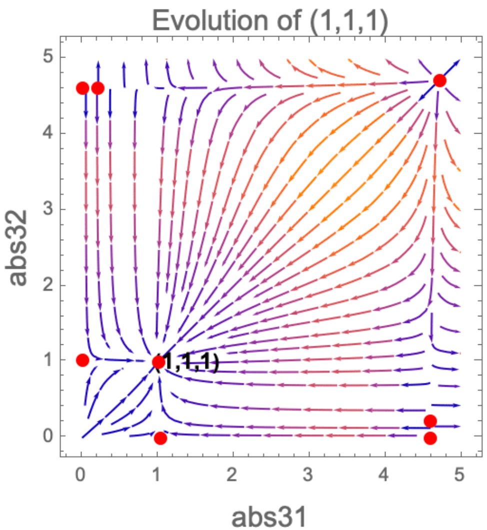

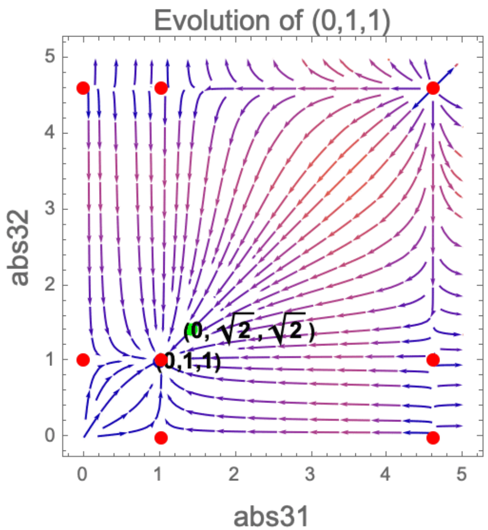

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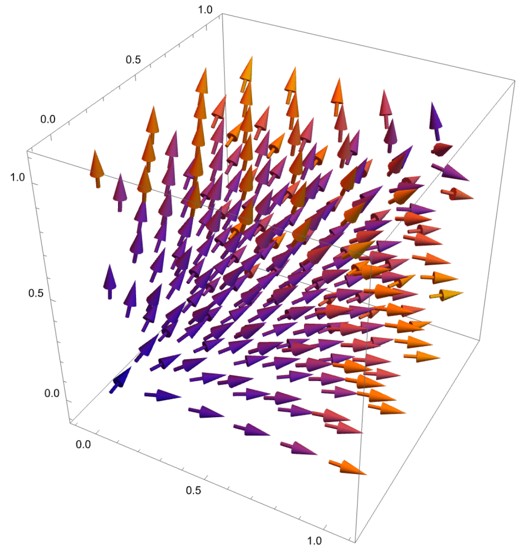

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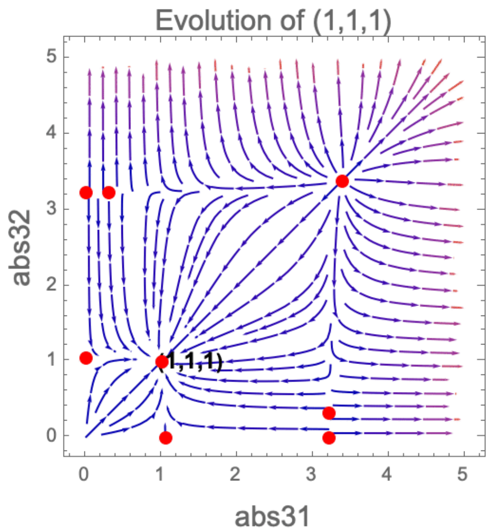

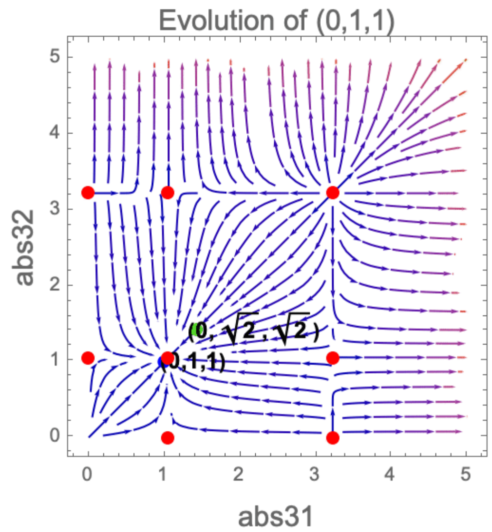

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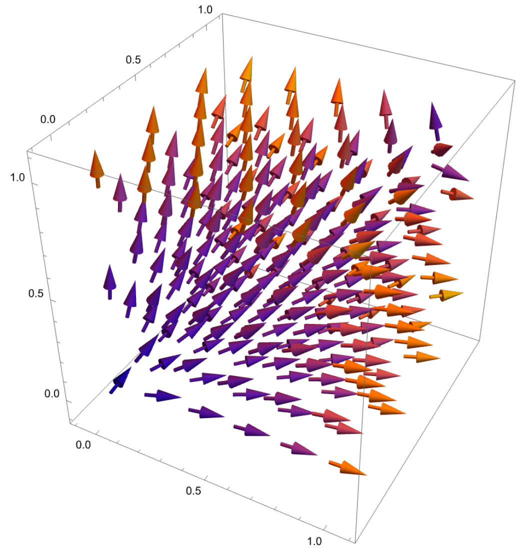

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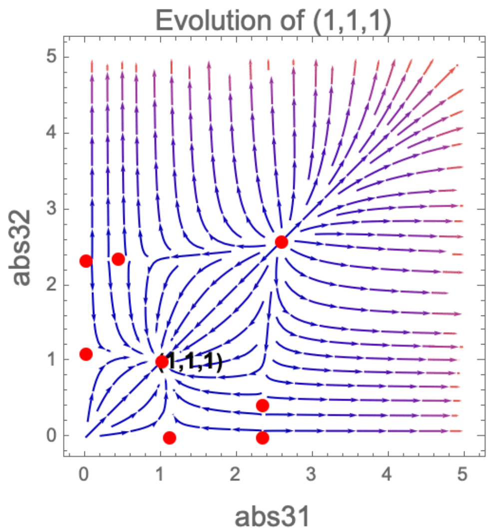

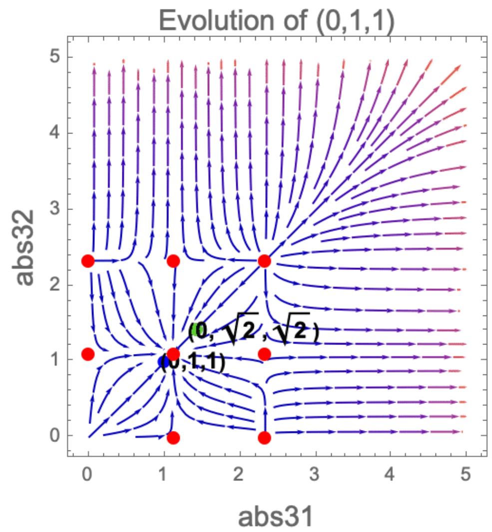

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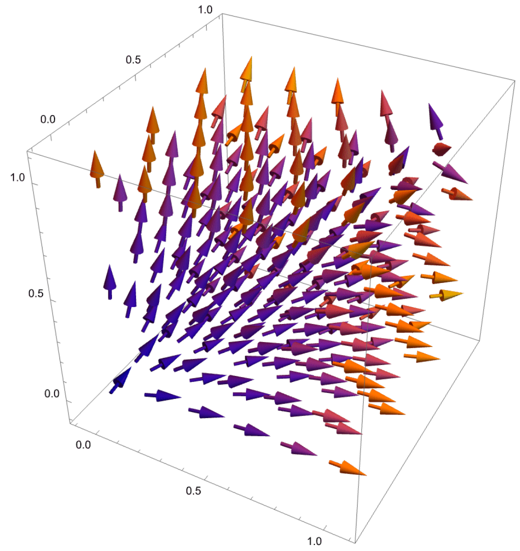

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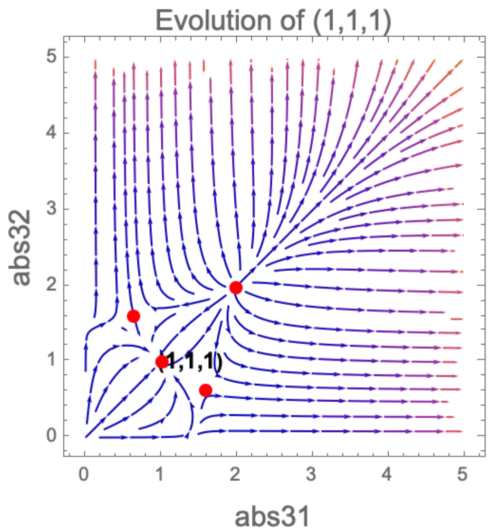

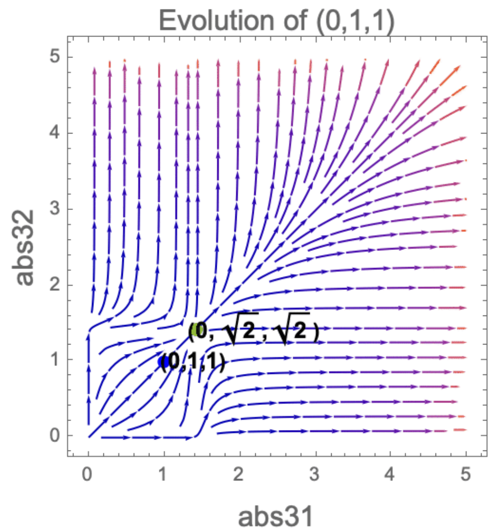

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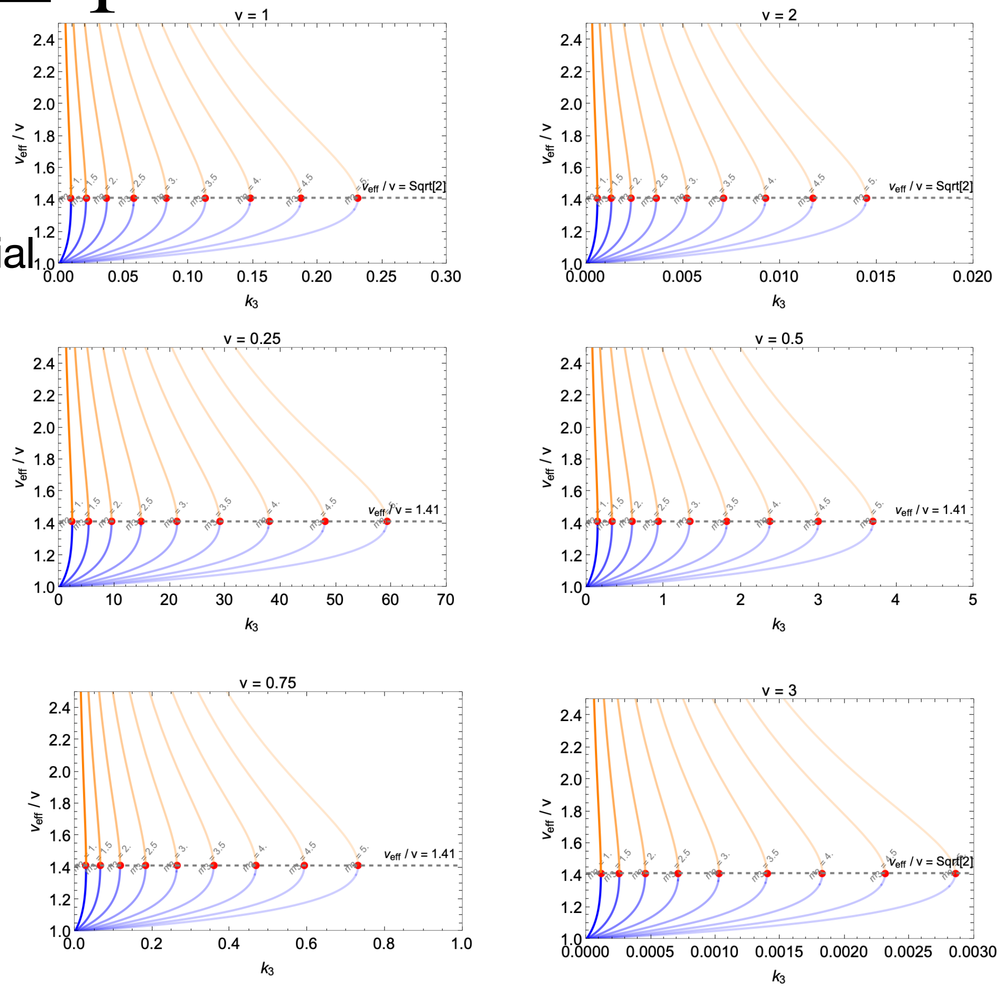

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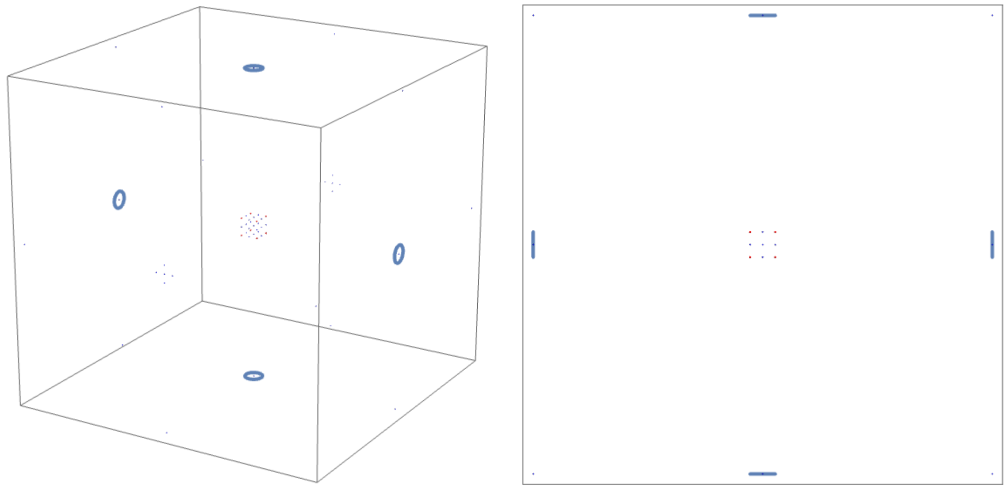
Scaling of pure $d = 6$ contribution to $\langle \theta_3 \rangle$

Minimisation of the scalar sector produced minimum candidates

→ eliminate and select candidates using minimum condition $\nabla^2 V|_{V_0} = 0$

→ different minimum are present in different regions of the parameter



$$(0,0,\frac{m_3}{\sqrt{2\lambda_3}}) \to \left(0,0,\sqrt{\frac{\lambda_3 \pm \sqrt{\lambda_3^2 - 3k_3 m_3^2}}{3k_3}}\right) \quad \text{min. if } 0 < k_3 \leq \frac{\lambda_3^2}{3m_3^2},$$

$$\frac{m_3}{\sqrt{2\lambda_3}}(0,1,1) \to \begin{cases} \sqrt{\frac{\lambda_3 + \sqrt{\lambda_3^2 - 3k_3 m_3^2}}{3k_3}}\,(0,1,1) & \text{min. if } 0 < k_3 \leq \frac{\lambda_3^2}{3m_3^2}\,. \\ \sqrt{\frac{\lambda_3 - \sqrt{\lambda_3^2 - 3k_3 m_3^2}}{3k_3}}\,(0,1,1) & \text{min. if } k_3 = \frac{\lambda_3^2}{3m_3^2}\,. \end{cases}$$

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Fix glitchy animation

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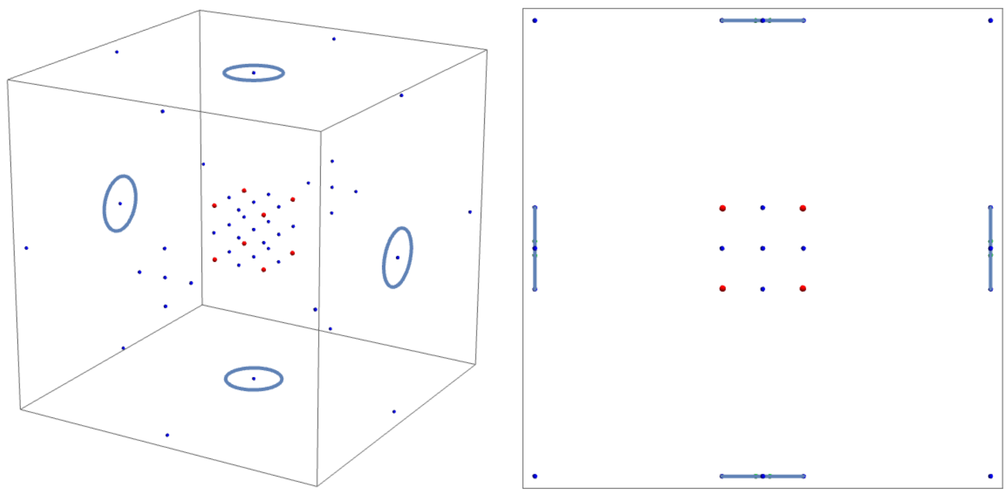
→ different minimum are present in different regions of the parameter



$$(0,0,\frac{m_3}{\sqrt{2\lambda_3}}) \to \left(0,0,\sqrt{\frac{\lambda_3 \pm \sqrt{\lambda_3^2 - 3k_3 m_3^2}}{3k_3}}\right) \quad \text{min. if } 0 < k_3 \leq \frac{\lambda_3^2}{3m_3^2}, \qquad \frac{m_3}{\sqrt{2\lambda_3}}(0,1,1) \to \begin{cases} \sqrt{\frac{\lambda_3 + \sqrt{\lambda_3^2 - 3k_3 m_3^2}}{3k_3}}\,(0,1,1) & \text{min. if } 0 < k_3 \leq \frac{\lambda_3^2}{3m_3^2}\,. \\ \sqrt{\frac{\lambda_3 - \sqrt{\lambda_3^2 - 3k_3 m_3^2}}{3k_3}}\,(0,1,1) & \text{min. if } k_3 = \frac{\lambda_3^2}{3m_3^2}\,. \end{cases}$$

[new] Stability analysis $N_f = 1$

Scaling of pure $d = 6$ contribution to $\langle \theta_3 \rangle$

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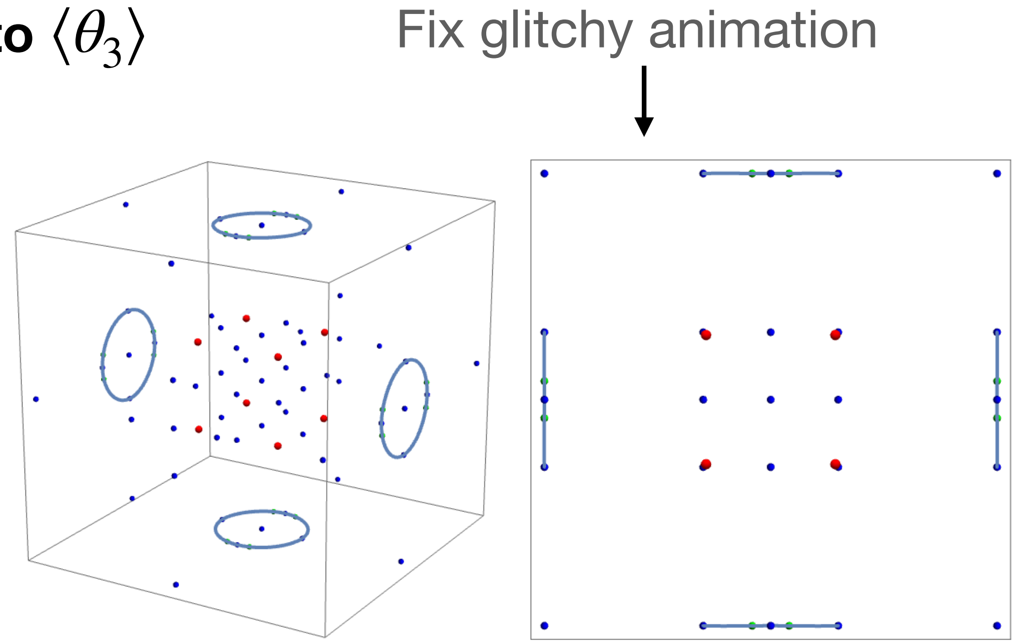

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Fix glitchy animation

↓

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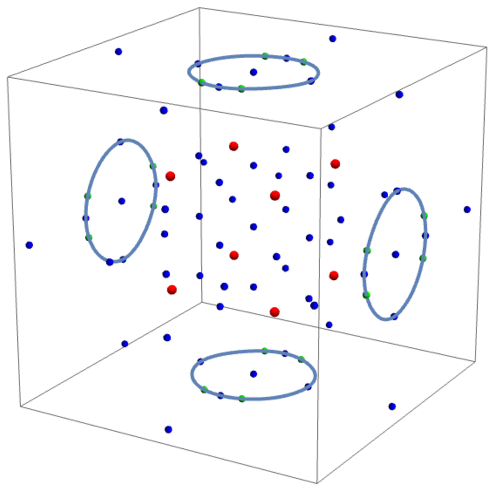

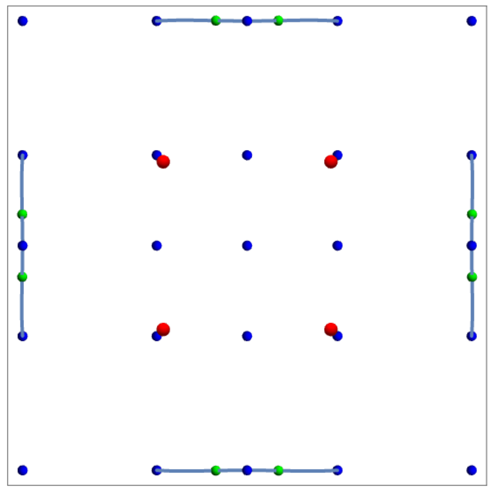

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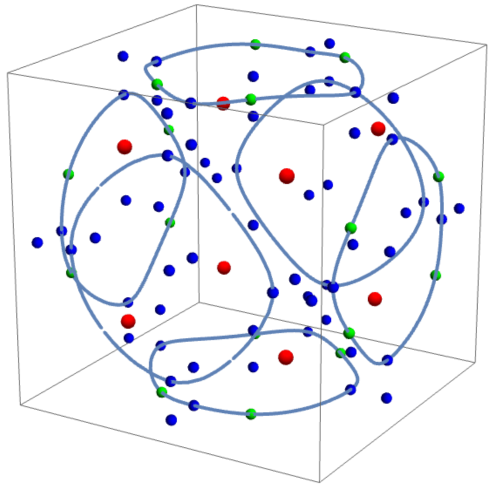

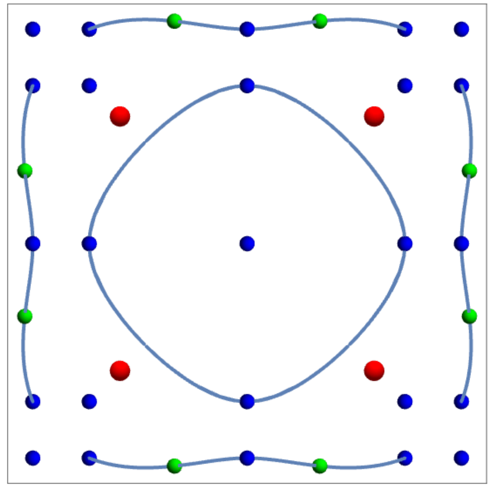

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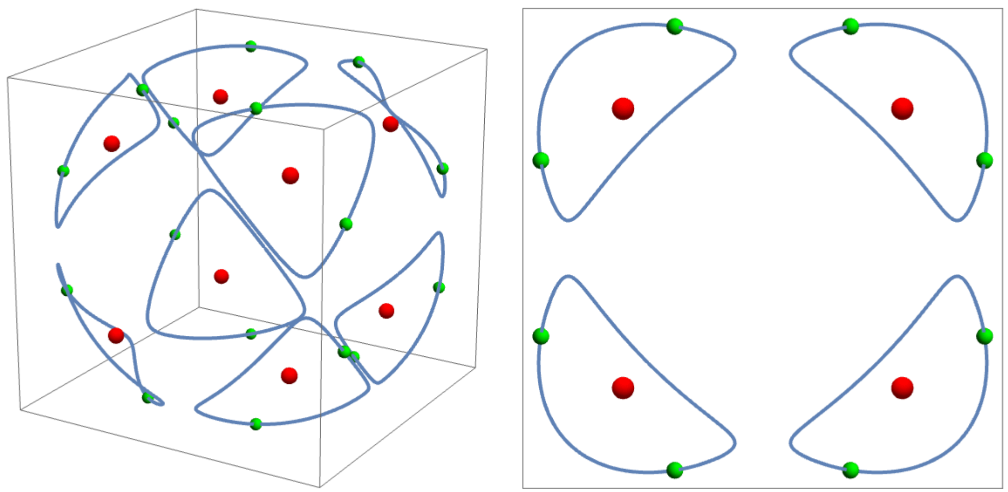

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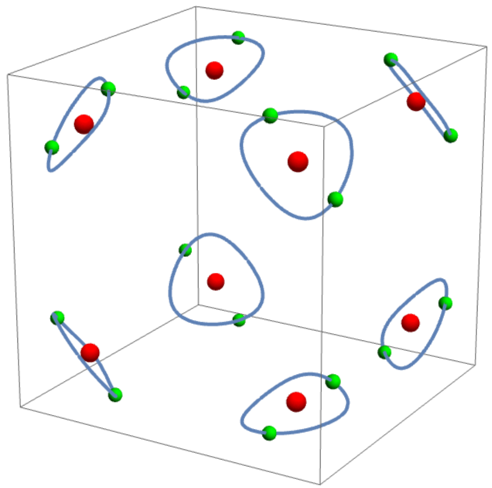

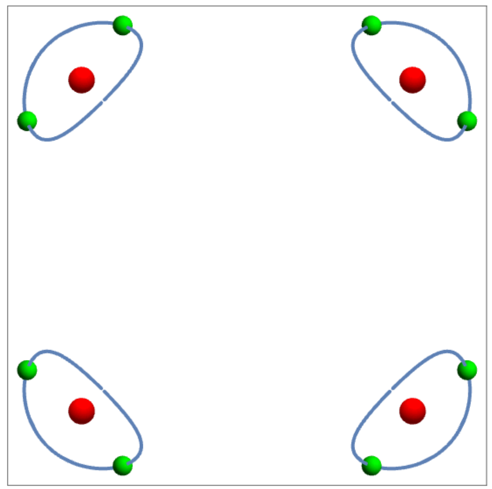

$$\left(0,0,\frac{m_3}{\sqrt{2\lambda_3}}\right) \rightarrow \left(0,0,\sqrt{\frac{\lambda_3 \pm \sqrt{\lambda_3^2 - 3k_3 m_3^2}}{3k_3}}\right) \quad \text{min. if } 0 < k_3 \leq \frac{\lambda_3^2}{3m_3^2},$$

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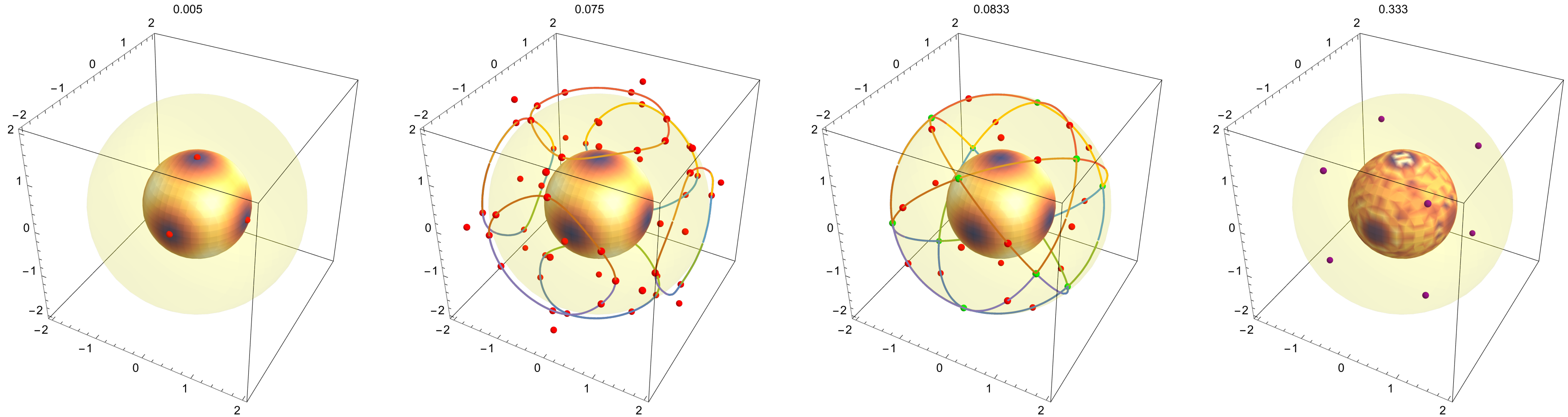

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<u>**The collection of available alignments changes according to k_3**</u>

[new] Stability analysis $N_f = 2$

Mixed flavon case stability - setup

Now we have $\theta_i = (\theta_{i,1}, \theta_{i,1}, \theta_{i,1}), \quad i = 3{,}123$

The invariants according the contraction graphs are

$$V_{\text{mixed}} = k\left(\left[\theta_{123} \times_0 \theta_{123}^\dagger\right] \cdot \left[\theta_{123} \times_0 \theta_{123}^\dagger\right]\right)_{0,0} \left(\theta_3 \cdot \theta_3^\dagger\right)_{0,0} \text{ or } k\left(\theta_3 \cdot \theta_3^\dagger\right)_{0,0} \left(\theta_{123} \cdot \theta_{123}^\dagger\right)_{0,0}^2$$

$$\text{or } k\left(\theta_{123} \cdot \theta_{123}^\dagger\right)_{0,0} \left(\theta_3 \cdot \theta_3^\dagger\right)_{0,0}^2 \text{ or } k\left(\theta_{123} \cdot \theta_{123}^\dagger\right)_{0,0} \left(\left[\theta_3 \times_0 \theta_3^\dagger\right] \cdot \left[\theta_3 \times_0 \theta_3^\dagger\right]\right)_{0,0}$$

$$\downarrow$$

$$|\theta_3|^2 |\theta_{123}|^4, \quad |\theta_3|^2 \theta_{123}^2 \theta_{123}^{\dagger\,2}, \quad |\theta_{123}|^2 |\theta_3|^4, \quad |\theta_{123}|^2 \theta_3^2 \theta_3^{\dagger\,2}$$

The task becomes minimising $V \supset V_0 = \sum_{3,123} m_i^2 |\theta_i^2| + \lambda_i \theta_i^2 \theta_i^{\dagger\,2} + V_{\text{mixed}}$

[new] Stability analysis $N_f = 2$

Mixed flavon case stability - existence

The alignment of $(0,0,1)$ and $(1,1,-1)$ under

$V_1 = m_3^2|\theta_3|^2 + \lambda_3\theta_3^2\theta_3^{\dagger\,2} + m_{123}^2|\theta_{123}|^2 + \lambda_{123}\theta_{123}^2\theta_{123}^{\dagger\,2} + k(|\theta_{123}|^2|\theta_3|^4$ or $|\theta_{123}|^2\theta_3^2\theta_3^{\dagger\,2})$

are transformed by scaling.

$$\begin{cases} \langle\theta_3\rangle = \mathrm{v}_3(0,0,1) \to \overline{\mathrm{v}}_3\,(0,0,1), & \overline{\mathrm{v}}_3 = \sqrt{\dfrac{m_3^2}{2(k|\overline{\mathrm{v}}_{123}|^2 + \lambda_3)}}, \\ \langle\theta_{123}\rangle = \mathrm{v}_{123}(1,1,-1) \to \overline{\mathrm{v}}_{123}\,(1,1,-1), & \overline{\mathrm{v}}_{123} = \sqrt{\dfrac{m_{123}^2 - k|\overline{\mathrm{v}}_3|^2}{2h_{123}}}, \end{cases}$$

and they are **STABLE** if their 6 dimensional Hessian eigenvalues $> 0\ldots$

[new] Stability analysis $N_f = 2$

Mixed flavon case stability - stability

and they are **STABLE** if their 6 dimensional Hessian eigenvalues $> 0 \ldots$

$$H = \begin{pmatrix} A & 0 & 0 & 0 & 0 & 0 \\ 0 & A & 0 & 0 & 0 & 0 \\ 0 & 0 & D & B & B & -B \\ 0 & 0 & B & C & 0 & 0 \\ 0 & 0 & B & 0 & C & 0 \\ 0 & 0 & -B & 0 & 0 & C \end{pmatrix}, \quad \begin{cases} A = \dfrac{6k|\overline{v}_3|^2\left(m_{123}^2 - k|\overline{v}_3|^4\right)}{h_{123}} - 2m_3^2, \\ B = \dfrac{4\sqrt{2}k|\overline{v}_3|^3\sqrt{m_{123}^2 - k|\overline{v}_3|^4}}{\sqrt{h_{123}}} \\ C = 6\left(m_{123}^2 - k|\overline{v}_3|^4\right) + 2k|\overline{v}_3|^4 - 2m_{123}^2, \\ D = 3A + 12h_3|\overline{v}_3|^2 + 4m_3^2 \end{cases} .$$

and their stability regions can be categorised by their renormalisable relation

[new] Stability analysis $N_f = 2$

Mixed flavon case stability - stability

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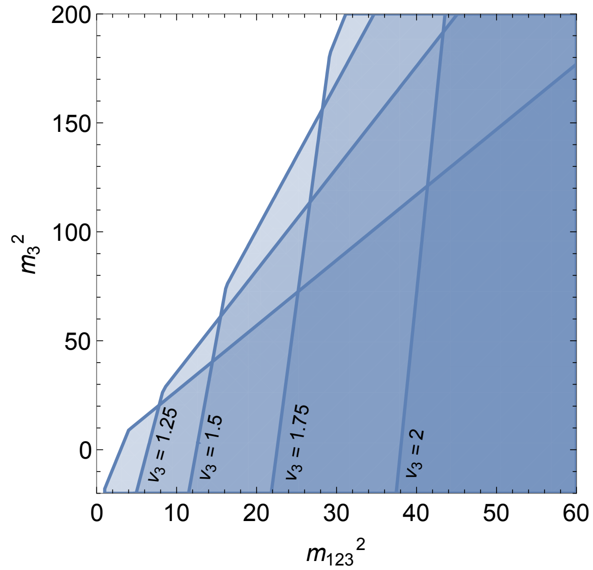
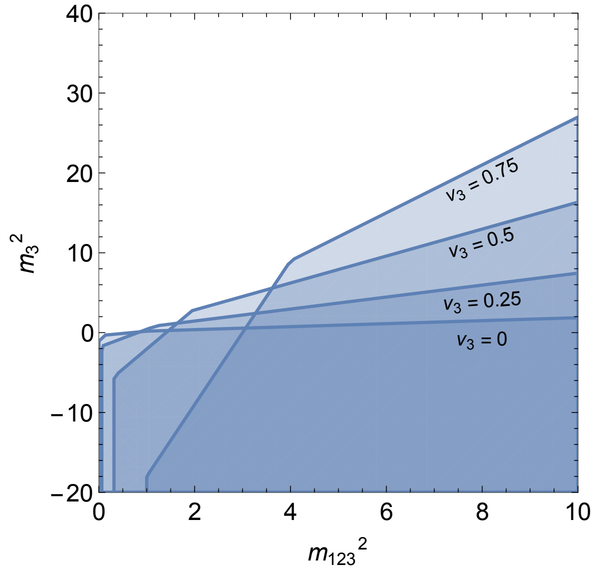
$$\begin{cases} h_3 < 0 \text{ and } m_3^2 < \dfrac{6h_{123}h_3\overline{v}_3^2 - 21k^2\overline{v}_3^6 + 9km_{123}\overline{v}_3^2}{h_{123}} & k\overline{v}_3^4 < m_{123}^2 \leq 3k\overline{v}_3^4 \\ \begin{cases} h_3 \leq \dfrac{3k^2\overline{v}_3^4 - km_{123}}{h_{123}} \text{ and } m_3^2 < \dfrac{6h_{123}h_3\overline{v}_3^2 - 21k^2\overline{v}_3^6 + 9km_{123}\overline{v}_3^2}{h_{123}} \\ \dfrac{3k^2\overline{v}_3^4 - km_{123}}{h_{123}} < h_3 < 0 \text{ and } m_3^2 < \dfrac{3km_{123}\overline{v}_3^2 - 3k^2\overline{v}_3^6}{h_{123}} \end{cases} & m_{123}^2 > 3k\overline{v}_3^4 \end{cases}$$

The eigenvalue >0 conditions competes

→ jagged parameter space range





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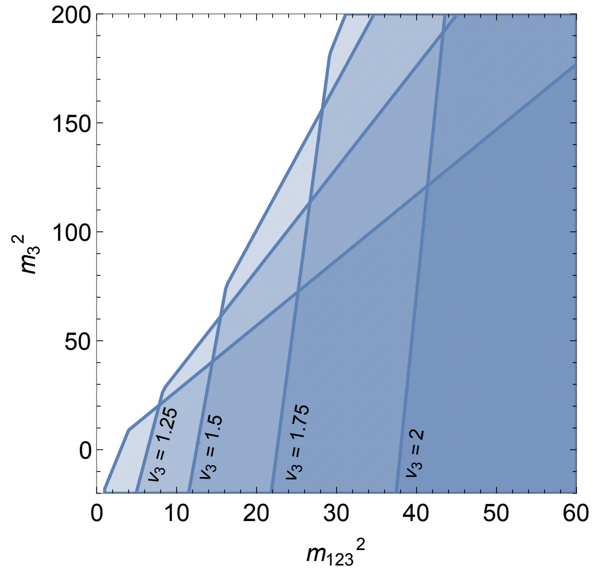

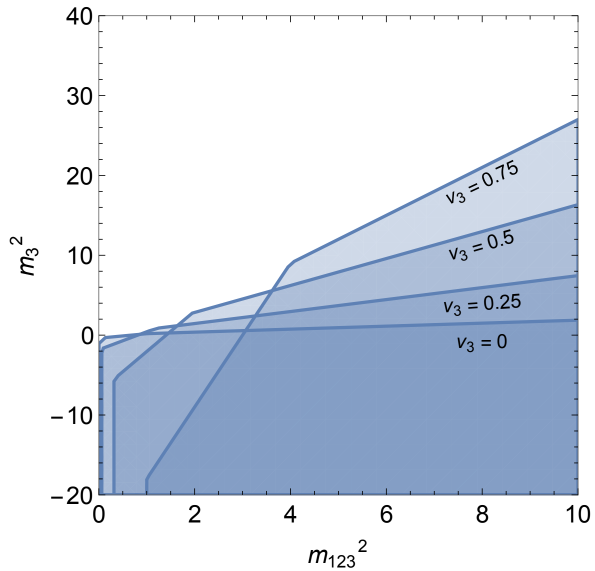

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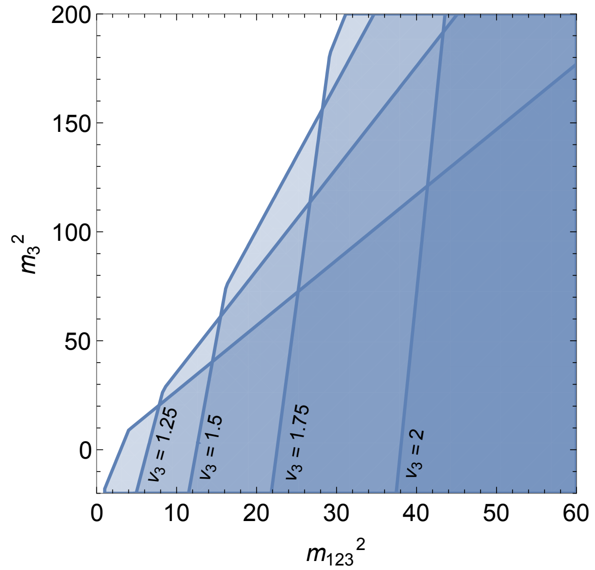
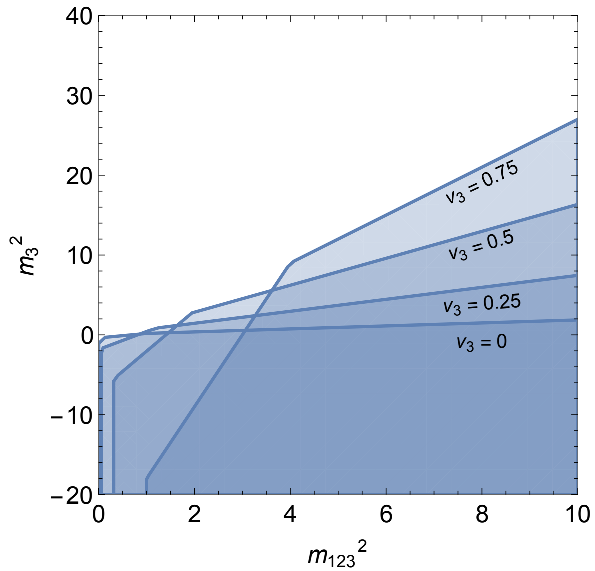
“Surely you can solve this exactly?”

Well…. technically yes





[new] Stability analysis $N_f = 2$

Mixed flavon case stability - complete solution

DON'T READ THESE

$$|\overline{v}_3|^2 = \frac{\sqrt[3]{\sqrt{729h_{123}^2k^8m_3^4+4\left(-6h_{123}\lambda_3k^2-3k^3m_{123}^2\right)^3}-27h_{123}k^4m_3^2}}{3\sqrt[3]{2}k^2} - \frac{\sqrt[3]{2}\left(-6h_{123}\lambda_3k^2-3k^3m_{123}^2\right)}{3k^2\sqrt[3]{\sqrt{729h_{123}^2k^8m_3^4+4\left(-6h_{123}\lambda_3k^2-3k^3m_{123}^2\right)^3}-27h_{123}k^4m_3^2}}$$

$$\begin{aligned}|\overline{v}_{123}|^2 = -\Bigg(&\frac{-16h_{123}^3k^3\lambda_3^3+27h_{123}^2k^5m_3^4-24h_{123}^2k^4\lambda_3^2m_{123}^2-12h_{123}k^5\lambda_3m_{123}^4-2k^6m_{123}^6}{432h_{123}^3k^6}\\ &+\frac{3\sqrt{3}\sqrt{-32h_{123}^5\lambda_3^3k^8m_3^4+27h_{123}^4k^{10}m_3^8-48h_{123}^4\lambda_3^2k^9m_{123}^2m_3^4-24h_{123}^3\lambda_3k^{10}m_{123}^4m_3^4-4h_{123}^2k^{11}m_{123}^6m_3^4}}{432h_{123}^3k^6}\Bigg)^{1/3}\\ &+(-64h_{123}^2k^2\lambda_3^2-64h_{123}k^3\lambda_3m_{123}^2-16k^4m_{123}^4)\Big[48\ 2^{2/3}h_{123}k^2(-16h_{123}^3k^3\lambda_3^3+27h_{123}^2k^5m_3^4-24h_{123}^2k^4\lambda_3^2m_{123}^2\\ &+3\sqrt{3}\sqrt{-32h_{123}^5\lambda_3^3k^8m_3^4+27h_{123}^4k^{10}m_3^8-48h_{123}^4\lambda_3^2k^9m_{123}^2m_3^4-24h_{123}^3\lambda_3k^{10}m_{123}^4m_3^4-4h_{123}^2k^{11}m_{123}^6m_3^4}\\ &-12h_{123}k^5\lambda_3m_{123}^4-2k^6m_{123}^6)^{1/3}\Big]^{-1}+\frac{k^2m_{123}^2-4h_{123}k\lambda_3}{6h_{123}k^2}\end{aligned}$$

[new] Stability analysis $N_f = 2$

Mixed flavon case stability - complete solution

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$$|\bar{v}_3|^2 = \frac{\sqrt[3]{\sqrt{729h_{123}^2k^8m_3^4 + 4\left(-6h_{123}\lambda_3k^2 - 3k^3m_{123}^2\right)^3} - 27h_{123}k^4m_3^2}}{3\sqrt[3]{2}k^2} - \frac{\sqrt[3]{2}\left(-6h_{123}\lambda_3k^2 - 3k^3m_{123}^2\right)}{3k^2\sqrt[3]{\sqrt{729h_{123}^2k^8m_3^4 + 4\left(-6h_{123}\lambda_3k^2 - 3k^3m_{123}^2\right)^3} - 27h_{123}k^4m_3^2}}$$

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→ use the coupled solution ***for analyticity…***

[new] Stability analysis $N_f = 2$

Mixed flavon case stability - complete solution

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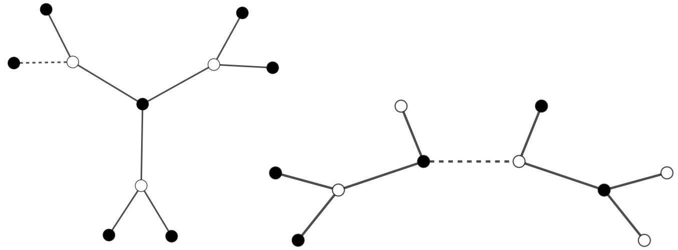
$$\begin{aligned}
|\overline{\mathrm{v}}_{123}|^2 = -\Bigg(&\frac{-16h_{123}^3k^3\lambda_3^3 + 27h_{123}^2k^5m_3^4 - 24h_{123}^2k^4\lambda_3^2m_{123}^2 - 12h_{123}k^5\lambda_3m_{123}^4 - 2k^6m_{123}^6}{432h_{123}^3k^6} \\
&+\frac{3\sqrt{3}\sqrt{-32h_{123}^5\lambda_3^3k^8m_3^4 + 27h_{123}^4k^{10}m_3^8 - 48h_{123}^4\lambda_3^2k^9m_{123}^2m_3^4 - 24h_{123}^3\lambda_3k^{10}m_{123}^4m_3^4 - 4h_{123}^2k^{11}m_{123}^6m_3^4}}{432h_{123}^3k^6}\Bigg)^{1/3} \\
&+(-64h_{123}^2k^2\lambda_3^2 - 64h_{123}k^3\lambda_3m_{123}^2 - 16k^4m_{123}^4)\Big[48\ 2^{2/3}h_{123}k^2(-16h_{123}^3k^3\lambda_3^3 + 27h_{123}^2k^5m_3^4 - 24h_{123}^2k^4\lambda_3^2m_{123}^2 \\
&+3\sqrt{3}\sqrt{-32h_{123}^5\lambda_3^3k^8m_3^4 + 27h_{123}^4k^{10}m_3^8 - 48h_{123}^4\lambda_3^2k^9m_{123}^2m_3^4 - 24h_{123}^3\lambda_3k^{10}m_{123}^4m_3^4 - 4h_{123}^2k^{11}m_{123}^6m_3^4} \\
&-12h_{123}k^5\lambda_3m_{123}^4 - 2k^6m_{123}^6)^{1/3}\Big]^{-1} + \frac{k^2m_{123}^2 - 4h_{123}k\lambda_3}{6h_{123}k^2}
\end{aligned}$$

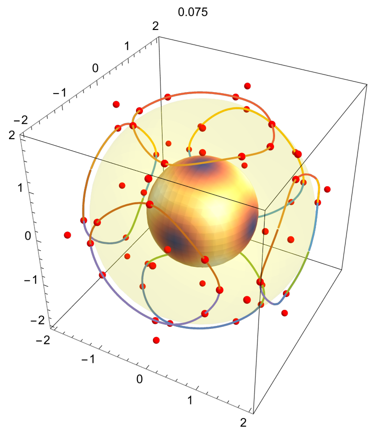
→ use the coupled solution ***for analyticity…*** **[coming up: numeric analysis]**

Outlook

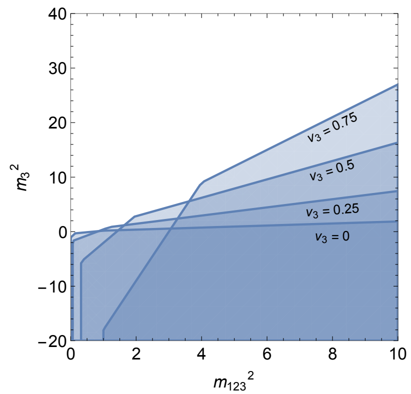
Results & future



Flavon alignments have limited range on the parameter space due to corrections from non-renormalizable terms

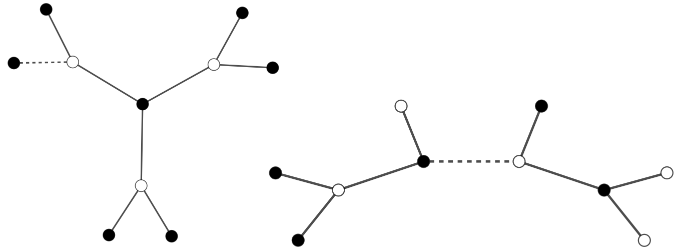


1. Added Δ_{27} to DECO v1.1 which allows enumeration of arbitrary d for effective theory contributions
2. Presented a way to list all order d contributions of Δ_{27}
3. Found conditions of pure and mixed $d = 6$ rescale renormalisable alignment in $\langle\theta_3\rangle = (0,0,1), \langle\theta_{123}\rangle = (1,1,-1)$ and destroy directions depending on k, k_3



Outlook

Results & future



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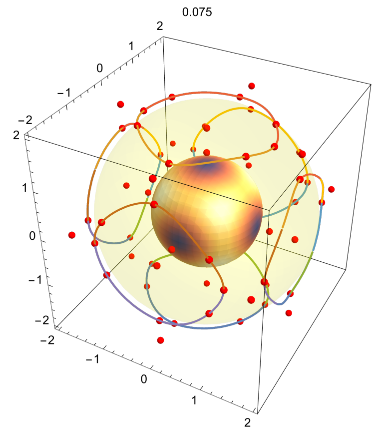

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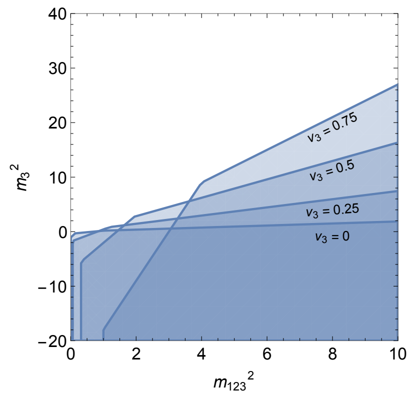
4. **[future] Numeric analysis of N flavon in $d \geq 6$ and CKM & PMNS experimental matches**



Backups

[backup] Stability analysis $N_f = 2$

Non-pure mixed flavon case stability - UTZ perturbation

$$V \supset V_6 = k_3 \left(\theta_3 \theta_3^\dagger\right)_{0,0} \left(\theta_3 \theta_3^\dagger\right)_{0,1} \left(\theta_3 \theta_3^\dagger\right)_{0,2} + k_{123} \left(\theta_{123} \theta_{123}^\dagger\right)_{0,0} \left(\theta_{123} \theta_{123}^\dagger\right)_{0,1} \left(\theta_{123} \theta_{123}^\dagger\right)_{0,2} +$$
$$k \left(\theta_3 \cdot \theta_3^\dagger\right)_{0,0} \left(\theta_{123} \cdot \theta_{123}^\dagger\right)_{0,0}^2$$

Pure $d = 6$ and mixed $d = 6$ for the flavon case

$\rightarrow \langle \theta_{123} \rangle |_{UTZ} = v_{123}/\sqrt{3}\,(1,1,-1)$ is not present ⇒ **<u>DESTROYED</u>**

$\rightarrow \langle \theta_3 \rangle |_{UTZ} = v_3\,(0,0,1)$ is scaled ⇒ **<u>PRESERVED</u>**

$$\langle \theta_3 \rangle \rightarrow (0,0,\sqrt{\frac{\sqrt{\left(\lambda_3 + k_3 v_{123}^2\right)^2 + 3km_3^2} - \lambda_3 - k_3 v_{123}^2}{3k}})$$

There are ~100 invariants, **<u>next step: categorise the combinatorics of them</u>**

<u>IMPLICATION:</u> If we accept a flavon models, the parameter space can be **<u>VERY</u>** narrow

[back up] What are the invariants?

Outlook: graph isomorphism via neural network

What are limitation of this method? (M flavons in N dimension)

Given two flavons θ, θ' that are 3D fundamental representation

The singlets transform as $\mathbf{1}_{r,s} \to \omega^r \mathbf{1}_{r,s}$ or $\omega^s \mathbf{1}_{r,s}$

STEP 1 – Partition of $d = 6 = 2 + 2 + 2 = 2 + 4 = 3 + 3 = 6$

The partition represents the dimension of each singlets that makes up the overall singlet

STEP 2 – Enumerate the triplets it takes to form that singlet

STEP 3 – Enumerate the r, s within each partition

e.g., $6 = 2 + 4$ represents $\mathbf{1}_{r,s} \cdot \mathbf{1}_{r',s'} = \mathbf{1}_{r+r',s+s'}$

Need $r + r' = s + s' = 0 \bmod 3$ ....

Partition is ok for small N

Graph isomorphism :(
Tree isomorphism :)