Curvature Perturbations Protected Against One Loop

Keisuke Inomata (Johns Hopkins University)

arXiv: 2403.04682

DPF-PHENO 2024, May. 16, 2024

Main message

Superhorizon curvature perturbations are constant.

Separate Universe picture is valid.

Main message

Superhorizon curvature perturbations are constant.

Separate Universe picture is valid.

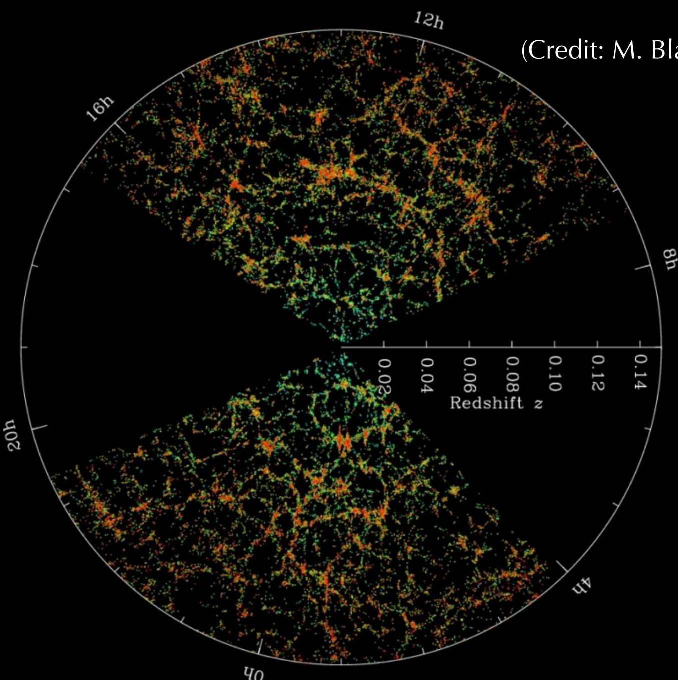
Even at one loop level

Outline

- **Introduction of curvature perturbations**
- Recent claim: Curvature is not conserved?
- Conservation of curvature at one loop
- Summary

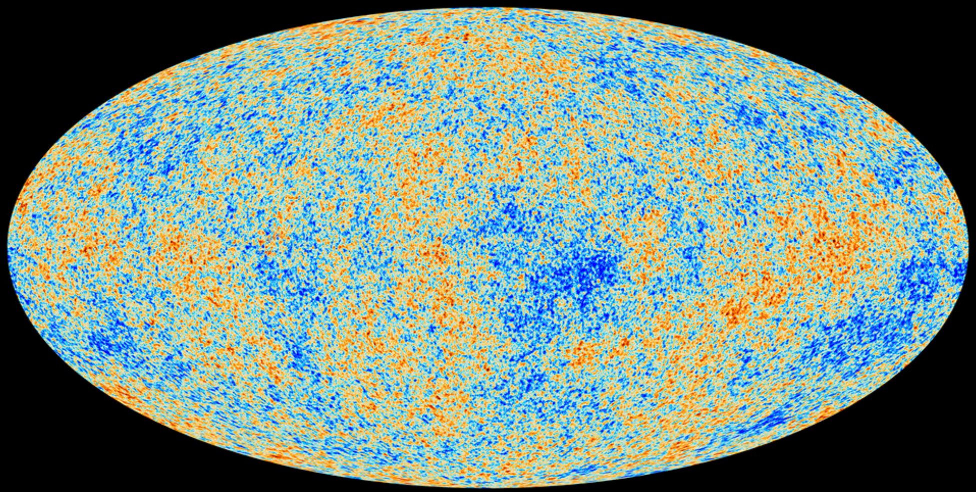
Cosmological perturbations

We have observed cosmological density perturbations in the Universe.



(Credit: M. Blanton and SDSS)

Large Scale Structure
(Galaxies are gathered in the bright regions.)



(Credit: ESA and the Planck Collaboration)

CMB anisotropies
(red: hot = dense, blue: cold = sparse)

$$\mathcal{P}_\zeta = 2.1 \times 10^{-9} \ \text{(Planck 2018)}$$

$$\rightarrow \delta\rho/\bar{\rho} \simeq 10^{-5}$$

ζ: curvature perturbation

What is curvature perturbation?

curvature perturbation

$$\mathrm{d}^2 s = g_{00}\mathrm{d}\eta^2 + 2g_{0i}\mathrm{d}\eta\mathrm{d}x^i + a^2 \mathrm{e}^{-2\psi}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

(In the gauge where $g_{ij}|_{i\neq j} = 0$)

In the isotropic and homogeneous Universe (FLRW metric),

$$a^2\mathrm{e}^{-2\psi}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \to a^2\left(\frac{\mathrm{d}^2 r}{1-Kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\Omega^2)\right)$$

$$R_3 = \frac{6K}{a^2} = 4\nabla^2\psi$$

3-dim. Ricci scalar

What people call curvature perturbation

$$\mathrm{d}^2 s = g_{00}\mathrm{d}\eta^2 + 2g_{0i}\mathrm{d}\eta\mathrm{d}x^i + a^2 \mathrm{e}^{-2\psi}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

ψ itself is gauge dependent (depends on the coordinate choice).

In Cosmology, the following gauge-invariant quantities are often used.

$$\zeta = -\psi + \frac{\delta\rho}{3(\rho + P)}$$

ζ coincides the curvature
in uniform density gauge, $\delta\rho = 0$.

$$\mathcal{R} = -\psi - \frac{H\delta\phi}{\dot{\phi}}$$

$\mathcal{R}$ coincides the curvature
in comoving gauge, $\delta\phi = 0$.

In the superhorizon limit,

$$\zeta = \mathcal{R}$$

People often call ζ and $\mathcal{R}$
curvature perturbations.

Why is curvature perturbation used?

ζ and $\mathcal{R}$ are conserved (constant) on superhorizon scales in single field inflation models.

During inflation

subhorizon perturbation

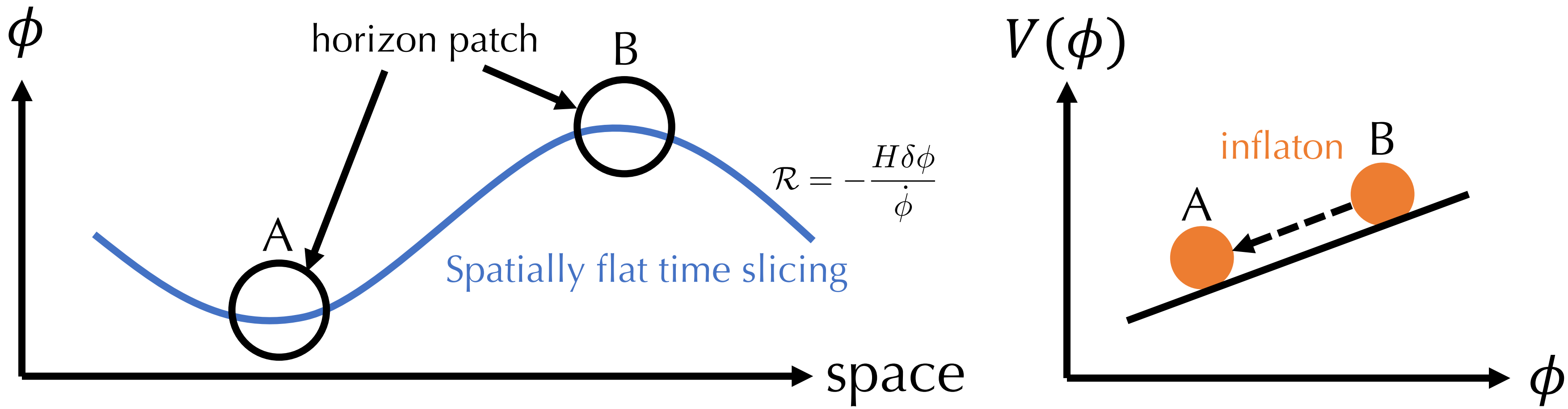
superhorizon perturbation
→ becomes constant!

➡ Useful in characterizing the amplitude of cosmological perturbations.

$$\mathcal{P}_\zeta(k < \mathcal{O}(1)\,\mathrm{Mpc}^{-1}) \simeq 2.1 \times 10^{-9} \;\; (\mathrm{Planck}\ 2018)$$

Separate Universe and curvature conservation

Focus on superhorizon-limit curvature perturbations. (neglect $\mathcal{O}((k/aH)^2)$ contributions)



Separate Universe picture:

For local Universes, superhorizon perturbations can be regarded as the background.

A local observer inside a horizon patch **cannot recognize the existence of the superhorizon-limit curvature perturbations**. To be consistent with this, curvature must be constant.

Q: What if superhorizon curvature is time dependent?

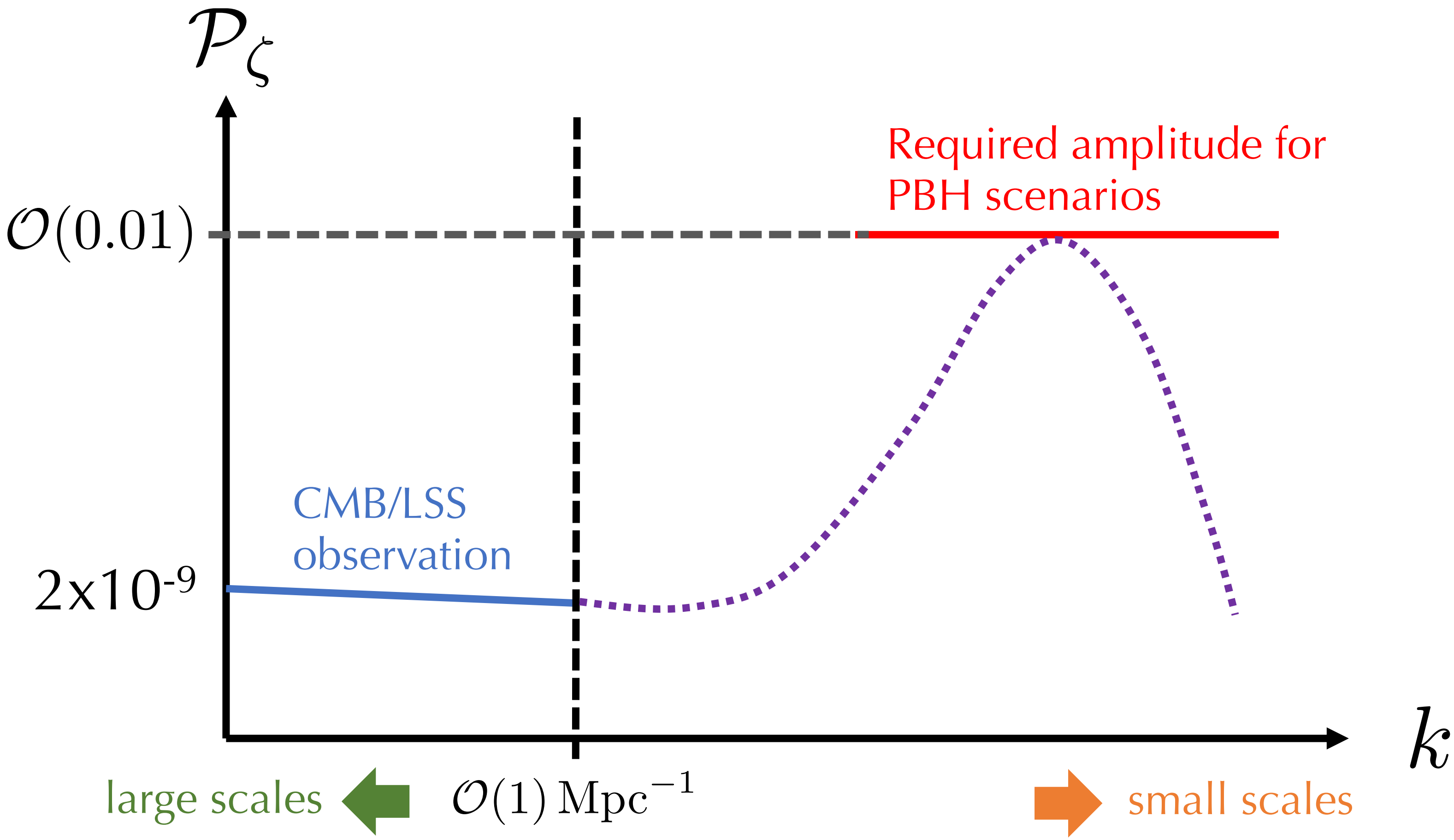
A: The local observer can recognize its existence through $e^{2\zeta(\eta)}a^2dx^2$

Outline

- Introduction of curvature perturbations
- **Recent claim: Curvature is not conserved?**
- Conservation of curvature at one loop
- Summary

Large perturbations for PBH scenarios

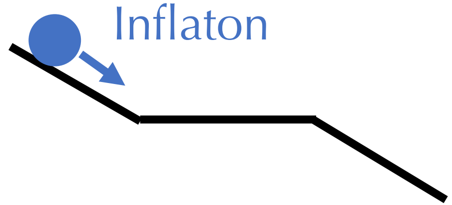
Primordial black holes are candidates of DM and BHs detected by LIGO-Virgo-KAGRA collaborations.



Inflaton potentials for large amplification

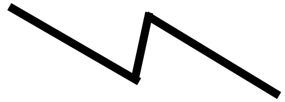
Single field models for large amplification of density perturbations:

flatter region

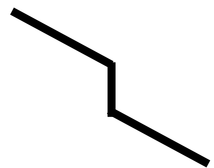


(Starobinsky 1992, Ivanov *et al*. 1994, Inoue and Yokoyama 2001, Kinney 2005)

step feature

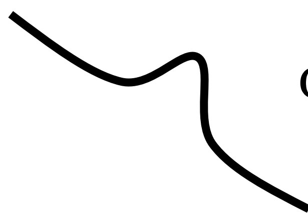


or

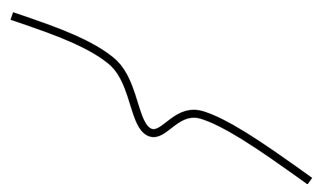


(Kefala et al. 2020, Inomata et al. 2021)

bump/dip feature

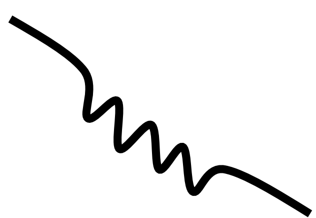


or



(Ozsoy et al. 2018, Mishra and Sahni 2019)

oscillatory feature



(R.G. Cai et al. 2019, Zhou et al. 2020, Peng et al. 2021)

One loop corrections

Lagrangian: $$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$$

E.o.m. for the inflaton fluctuations: (slow-roll-parameter suppressed terms neglected) $\left(V_{(n)} \equiv \partial^n V/\partial\phi^n\right)$

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1}$$

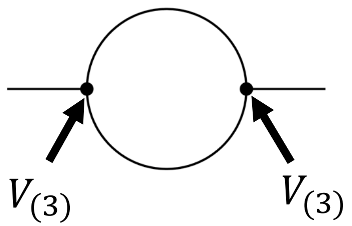
beyond liner order corrections

In-in formalism: (Jordan 1986, Calzetta and Hu 1987, Weinberg 2005)

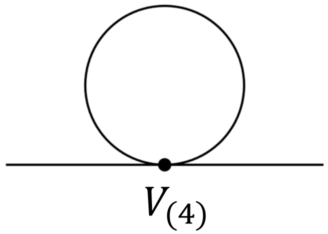
$$\langle\delta\phi_{\mathbf{k}}(\eta)\delta\phi_{\mathbf{k}'}(\eta)\rangle = \langle 0|\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta' H_{\rm int}(\eta')}\right)^\dagger \delta\phi_{\mathbf{k}}(\eta)\delta\phi_{\mathbf{k}'}(\eta)\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta'' H_{\rm int}(\eta'')}\right)|0\rangle$$

$$\left(H_{\mathrm{int},n} \equiv \int \mathrm{d}^3x\, a^4\mathcal{H}_n,\ \mathcal{H}_{n(>2)} = \frac{1}{n!}V^{(n)}(\phi)\delta\phi^n\right)$$

two vertices



one vertex



The lowest order corrections to linear power spectrum appear as one loops.

Superhorizon curvature evolves?

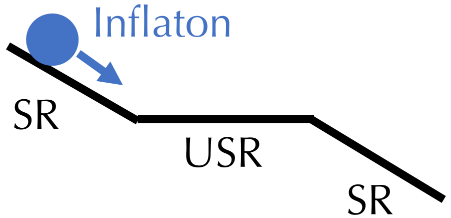
(Note: the conservation of linear ζ is well known.)

Recent claim:
Superhorizon-**limit** curvature perturbations **are not conserved at one-loop level** in the case of slow-roll (SR) → ultra-slow roll (USR) → SR.

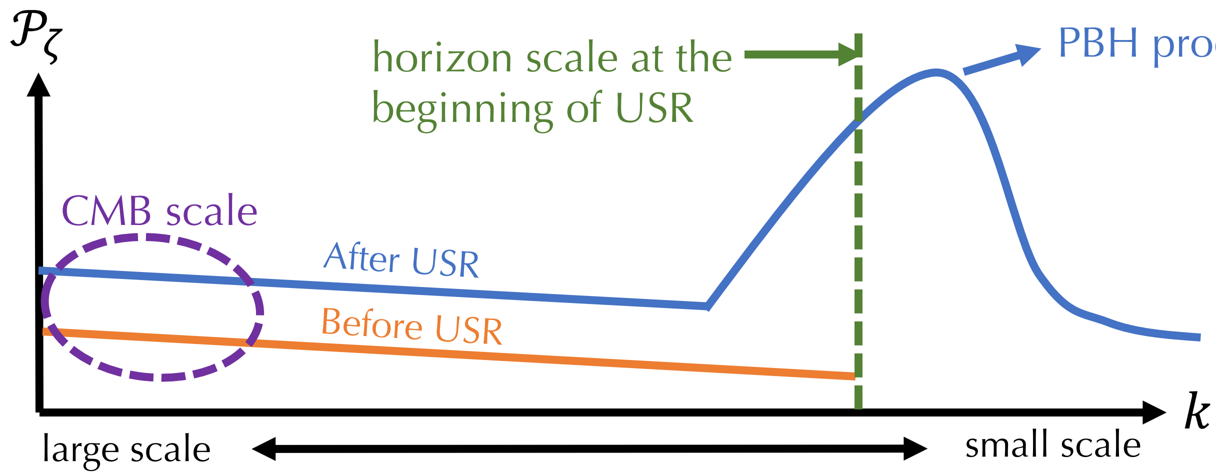
e.o.m.

$$3H\dot{\phi} + V'(\phi) = 0 \text{ (SR)}$$

$$\ddot{\phi} + 3H\dot{\phi} = 0 \text{ (USR)}$$



Kristiano & Yokoyama (2022), followed by
Riotto, Choudhury et al., Firouzhahi, Motohashi & Tada, Franciolini et al., Gianmassimo, Cheng et al., Maity et al., Davies et al. (2023), Saburov & Ketov, Guillermo & Egea (2024)



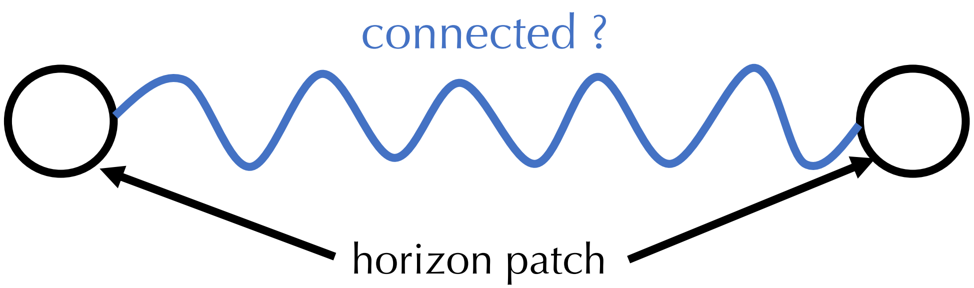
The one loop corrections can be comparable to the tree-level power spectrum in some PBH models. → CMB spectrum changed? → models constrained?

However, this is inconsistent with the separate Universe picture...

Universes connected?

The violation of the separate Universe means the Universes connected through the distance larger than the horizon.

→ The causality is violated?



I am going to show the conservation of superhorizon curvature at one loop level.
(separate universe picture is valid, causality is satisfied)

Outline

- Introduction of curvature perturbations
- Recent claim: Curvature is not conserved?
- **Conservation of curvature at one loop**
- Summary

Ongoing debates

The higher order action is needed for one loop calculation.

Comoving gauge ($\delta\phi$=0) is often taken, where ζ appears as a metric perturbation.

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right]$$

astro-ph/0210603, Maldacena

$$S_3 = \int \frac{1}{4}\frac{\dot\phi^4}{\dot\rho^4}[e^{3\rho}\dot\zeta^2\zeta + e^\rho(\partial\zeta)^2\zeta] - \frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\dot\zeta\partial_i\chi\partial_i\zeta + \\ - \frac{1}{16}\frac{\dot\phi^6}{\dot\rho^6}e^{3\rho}\dot\zeta^2\zeta + \frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\dot\zeta\zeta^2\frac{d}{dt}\left[\frac{1}{2}\frac{\ddot\phi}{\dot\phi\dot\rho} + \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}\right] + \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\partial_i\partial_j\chi\partial_i\partial_j\chi\zeta \\ + f(\zeta)\left.\frac{\delta L}{\delta\zeta}\right|_1$$

arXiv:0709.2708, Jarnhus & Sloth

$$S^{(4)} = \frac{1}{2}\int \mathrm{d}t\mathrm{d}^3x a^3\Big\{ -\frac{1}{3}\zeta^3\partial^2\zeta - 2\alpha^{(1)}(\zeta\partial_i\zeta\partial^i\zeta + \zeta^2\partial^2\zeta) + \dot\phi_c^2\alpha^{(1)2}\left[\frac{9}{2}\zeta^2 - 3\zeta\alpha^{(1)} + \alpha^{(1)2}\right] \\ \left[\frac{1}{2}\zeta^2 + \zeta\alpha^{(1)} + \alpha^{(1)2}\right]\left[\partial_i\partial_j\chi^{(1)}\partial^i\partial^j\chi^{(1)} - \partial^2\chi^{(1)}\partial^2\chi^{(1)}\right] + (6H^2 - \dot\phi^2)\alpha^{(2)2} \\ - 2[\zeta + \alpha^{(1)}]\left[\partial_i\partial_j\chi^{(1)}\partial^i\partial^j\chi^{(2)} - \partial^2\chi^{(1)}\partial^2\chi^{(2)} - 2\partial_i\partial_j\chi^{(1)}\partial^i\chi^{(1)}\partial^j\zeta\right] \\ - 2\left[2\partial_i\partial_j\chi^{(2)}\partial^i\chi^{(1)}\partial^j\zeta + 2\partial_i\partial_j\chi^{(1)}\partial^i\chi^{(2)}\partial^j\zeta - \partial_j\chi^{(1)}\partial_i\zeta\partial^i\chi^{(1)}\partial^j\zeta\right] \\ + \frac{1}{2}\partial_i\beta_j{}^{(2)}\partial^i\beta^{j(2)} - 2\alpha^{(1)}\partial_i\partial_j\chi^{(1)}\partial^i\beta^{j(2)}\Big\}$$

However, these expressions neglect the boundary terms.

e.g. $$\int \mathrm{d}t A\dot B = -\int \mathrm{d}t \dot A B + \int \mathrm{d}t\frac{\mathrm{d}}{\mathrm{d}t}(AB)$$

boundary term

Ongoing debate: the missing boundary terms lead to the curvature conservation?

Fumagalli (2023), Tada et al. (2023), Firouzhahi (2023), Braglia & Pinol (2024), Kawaguchi et al. (2024)

Strategy of this work

In this work, **spatially-flat gauge ($\psi = 0$)** is taken, where $\delta\phi$ is the basic quantity.

The metric perturbations are suppressed by slow-roll parameter ϵ, compared to $V_{(n)}$ terms.

($\epsilon \to 0$ is known as the decoupling limit in effective field theory of inflation.)

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right]$$

$$\downarrow \qquad \left(V_{(n)} \equiv \frac{\partial^n V(\phi)}{\partial\phi^n}\right)$$

$$S_n = -\int \mathrm{d}^4x\, a^4 \frac{V_{(n)}(\bar{\phi})}{n!}\delta\phi^n$$

Simple!

Advantage: The higher order action can be easily obtained.
→ no need to worry about boundary terms!

Strategy: We first calculate the one-loop power spectrum of $\delta\phi$.
Then, we connect it to the one loop-power spectrum of ζ.

One loop calculation

Equation of motion:

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1}$$

$(\delta\phi = \delta\phi^{(1)} + \delta\phi^{(2)} + \delta\phi^{(3)} + \cdots)$

$$\hat{\mathcal{N}}_k\delta\phi^{(1)}_{\boldsymbol{k}} = 0,$$

$$\hat{\mathcal{N}}_k \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + k^2 + a^2V_{(2)}(\bar{\phi})$$

$$\hat{\mathcal{N}}_k\delta\phi^{(2)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}},$$

$$\hat{\mathcal{N}}_k\delta\phi^{(3)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}(\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(2)}_{\boldsymbol{p}} + \delta\phi^{(2)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}}) - \frac{a^2}{6}V_{(4)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\int\frac{\mathrm{d}^3p'}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}'}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{p}'}$$

In-in formalism = equation of motion approach: (Musso (2013), Inomata et al. (2022))

$$\langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k}'}\rangle = \langle 0|\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta' H_{\rm int}(\eta')}\right)^\dagger \delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta'' H_{\rm int}(\eta'')}\right)|0\rangle$$

$$= \dots$$

$$\left(H_{{\rm int},n} \equiv \int \mathrm{d}^3x\, a^4\mathcal{H}_n,\ \mathcal{H}_{n(>2)} = \frac{1}{n!}V^{(n)}(\phi)\delta\phi^n\right)$$

$$= \langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(2)}_{\mathbf{k}}\delta\phi^{(2)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(3)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(3)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle$$

One loop calculation

Equation of motion:

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1}$$

$(\delta\phi = \delta\phi^{(1)} + \delta\phi^{(2)} + \delta\phi^{(3)} + \cdots)$

$$\hat{\mathcal{N}}_k\delta\phi^{(1)}_{\boldsymbol{k}} = 0,$$

$$\hat{\mathcal{N}}_k \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + k^2 + a^2 V_{(2)}(\bar{\phi})$$

$$\hat{\mathcal{N}}_k\delta\phi^{(2)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}},$$

$$\hat{\mathcal{N}}_k\delta\phi^{(3)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}(\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(2)}_{\boldsymbol{p}} + \delta\phi^{(2)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}}) - \frac{a^2}{6}V_{(4)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\int\frac{\mathrm{d}^3p'}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}'}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{p}'}$$

In-in formalism = equation of motion approach: (Musso (2013), Inomata et al. (2022))

$$\langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k}'}\rangle = \langle 0|\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta' H_{\rm int}(\eta')}\right)^\dagger \delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta'' H_{\rm int}(\eta'')}\right)|0\rangle$$

$$\left(H_{{\rm int},n} \equiv \int \mathrm{d}^3x\, a^4\mathcal{H}_n,\ \mathcal{H}_{n(>2)} = \frac{1}{n!}V^{(n)}(\phi)\delta\phi^n\right)$$

$$= \dots$$

$$= \underbrace{\langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle}_{\text{tree}} + \underbrace{\langle 0|\delta\phi^{(2)}_{\mathbf{k}}\delta\phi^{(2)}_{\mathbf{k}'}|0\rangle}_{\text{Poisson fluctuations}} + \underbrace{\langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(3)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(3)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle}_{\text{nonzero even in superhorizon limit}}$$

tree

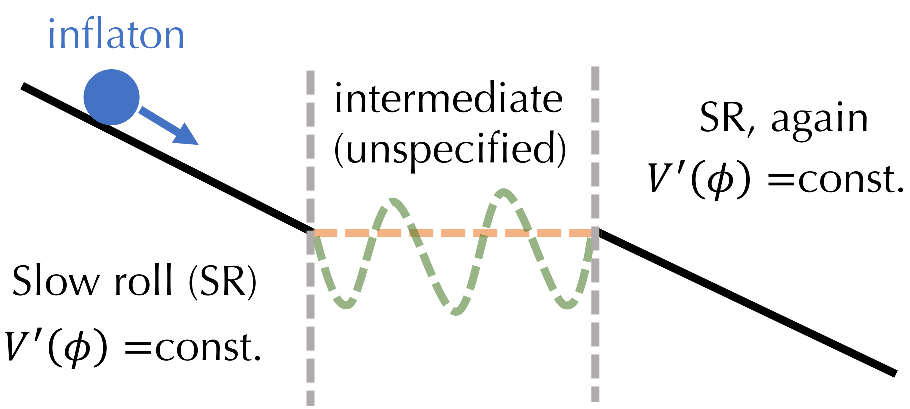
Poisson fluctuations
→ 0 in superhorizon limit

nonzero even in superhorizon limit

Superhorizon curvature perturbations evolve at one loop? → **No. There is a trick!**

Conservation of curvature

The trick lies in the relation between $\delta\phi$ and ζ.
Consider the simplest case:



During the intermediate period, ζ is enhanced.
→ Peak power spectrum

We assume the separate Universe satisfied at least during the SR periods **(do not assume that during the intermediate period)**.

From δN formalism, curvature perturbations during the SR periods are:

$$-\zeta|_{\leq\text{1-loop}} = \left.\frac{H\delta\phi}{\dot{\bar{\phi}}}\right|_{\leq\text{1-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar{\phi}}^{(0)} + \dot{\bar{\phi}}^{(2)}} \quad \text{(SR)}$$

On the other hand, $\frac{H\delta\phi}{\dot{\bar{\phi}}}$ is **always** constant:

$$\left.\frac{H\delta\phi}{\dot{\bar{\phi}}}\right|_{\leq\text{1-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar{\phi}}^{(0)} + \dot{\bar{\phi}}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\bar{\phi}}^{(0)}} = \text{const.}$$

Point: $\dot{\bar{\phi}}$ gets the one-loop backreaction, $\dot{\bar{\phi}}^{(2)}$, which cancels $\delta\phi^{(3)}$.

ζ during the fist and the second SR periods coincide. → ζ is conserved!

One loop backreaction

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + V_{(1)}(\bar{\phi}) = -\frac{1}{2}V_{(3)}(\bar{\phi})\left\langle (\delta\phi^{(1)})^2 \right\rangle$$

⬇ Take time derivative

$$\left(\bar{\Pi} \equiv \dot{\bar{\phi}},\ \hat{\mathcal{N}}_k \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + k^2 + a^2 V_{(2)}(\bar{\phi}) \right)$$

$$\hat{\mathcal{N}}_0 \bar{\Pi}^{(0)} = 0,$$

$$\hat{\mathcal{N}}_0 \bar{\Pi}^{(2)} = -\frac{a^2}{2}\left(V_{(3)} \left\langle \delta\phi^2 \right\rangle^{\cdot} + V_{(4)} \left\langle \delta\phi^2 \right\rangle \bar{\Pi}^{(0)} \right)$$

Recap:

$$\hat{\mathcal{N}}_k \delta\phi^{(1)}_{\boldsymbol{k}} = 0,$$

$$\hat{\mathcal{N}}_k \delta\phi^{(3)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\left(\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(2)}_{\boldsymbol{p}} + \delta\phi^{(2)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}}\right) - \frac{a^2}{6}V_{(4)}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\int \frac{\mathrm{d}^3 p'}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}'}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{p}'}$$

After some (easy) calculation, we find

$$\left.\hat{\mathcal{N}}_q \delta\phi^{(3)}_{\boldsymbol{q}}\right|_{q\to 0} = -\frac{a^2}{2}\left(V_{(3)} \left\langle \delta\phi^2 \right\rangle^{\cdot} + V_{(4)} \left\langle \delta\phi^2 \right\rangle \bar{\Pi}^{(0)} \right)\frac{\delta\phi^{(1)}_{\boldsymbol{q}}}{\bar{\Pi}^{(0)}}$$

We finally obtain

$$\delta\phi^{(3)}_{\boldsymbol{q}} = \frac{\dot{\bar{\phi}}^{(2)}}{\dot{\bar{\phi}}^{(0)}}\delta\phi^{(1)}_{\boldsymbol{q}} \quad \Rightarrow \quad \left.\frac{H\delta\phi}{\dot{\bar{\phi}}}\right|_{\leq 1\text{-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar{\phi}}^{(0)} + \dot{\bar{\phi}}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\bar{\phi}}^{(0)}} = \text{const.}$$

This relation is always satisfied.

Outline

- Introduction of curvature perturbations
- Recent claim: Curvature is not conserved?
- Conservation of curvature at one loop
- **Summary**

Summary

Non-conservation of superhorizon curvature perturbations at one-loop level has recently been claimed.

However, the claim is inconsistent with the separate Universe picture.

I have taken the spatially-flat gauge and focus on $\delta\phi$ evolution at one loop level.

I have finally found that the superhorizon curvature is conserved if we carefully consider the one-loop backreaction.

Main message

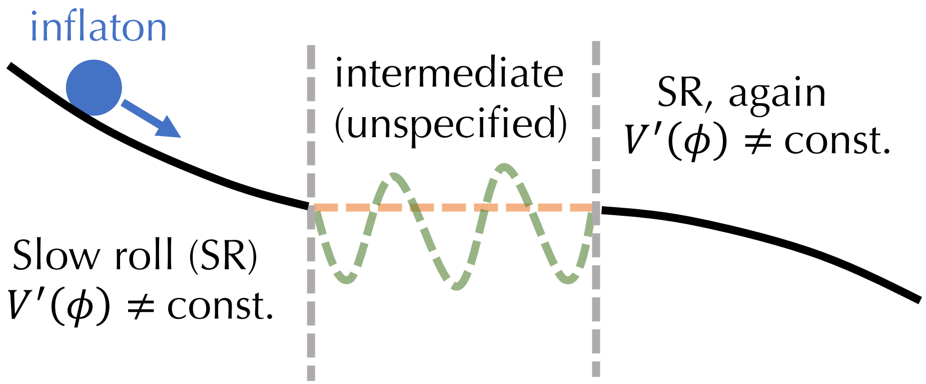
Superhorizon curvature perturbations are constant.

Separate Universe picture is valid.

Even at one loop level

Backup

In general SR potentials



Again, we assume the separate Universe satisfied at least during the SR periods **(do not assume that during the intermediate period)**.

When the separate Universe is satisfied, the curvature perturbations are conserved even at non-perturbative (including one-loop) level. (Lyth, Malik, and Sasaki, 2004)

This means that, if we find one concrete SR potential for the conservation of ζ, the conservation is secured for any types of SR potential.

One concrete example: the SR potentials that have region of $V'(\phi) =$ const.

Renormalization

In general, the loop contributions have divergence, which must be cancelled out by counter terms.

$$\hat{\mathcal{N}}_0 \bar{\Pi}^{(2)} = -\frac{a^2}{2}\left(V_{(3)} \left\langle \delta\phi^2 \right\rangle^{\cdot} + V_{(4)} \left\langle \delta\phi^2 \right\rangle \bar{\Pi}^{(0)} \right)$$

$$\hat{\mathcal{N}}_q \delta\phi^{(3)}_{\boldsymbol{q}} \Big|_{q\to 0} = -\frac{a^2}{2}\left(V_{(3)} \left\langle \delta\phi^2 \right\rangle^{\cdot} + V_{(4)} \left\langle \delta\phi^2 \right\rangle \bar{\Pi}^{(0)} \right) \frac{\delta\phi^{(1)}_{\boldsymbol{q}}}{\bar{\Pi}^{(0)}}$$

Counter terms are introduced in the same way for $\bar{\Pi}^{(2)} (= \dot{\bar{\phi}}^{(2)})$ and $\delta\phi^{(3)}_{\boldsymbol{q}}$ through $\hat{\mathcal{N}}_0$.

$$\hat{\mathcal{N}}_0 \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + a^2 V_{(2)}(\bar{\phi}) + \underline{a^2 m^2_{\text{ct.}}}$$

counter term

The introduction of the counter terms does not break the following relations:

$$\delta\phi^{(3)}_{\boldsymbol{q}} = \frac{\dot{\bar{\phi}}^{(2)}}{\dot{\bar{\phi}}^{(0)}} \delta\phi^{(1)}_{\boldsymbol{q}}$$

$$-\zeta = \frac{H\delta\phi}{\dot{\bar{\phi}}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar{\phi}}^{(0)} + \dot{\bar{\phi}}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\bar{\phi}}^{(0)}} = \text{const.}$$