# Generalized Global Symmetries and Nonperturbative Quantum Flavodynamics



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they/them or whatever

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Fractional charges: 2405.X with Adam Martin

Lepton flavor: 2211.07639 with Clay Córdova, Sungwoo Hong, Kantaro Ohmori

Quark flavor: 2402.12453 with Clay & Sungwoo

See other related things on my inspire

# Generalized Global Symmetries

Symmetries are important!

Usually look at Lagrangian data and consider transforming local operators

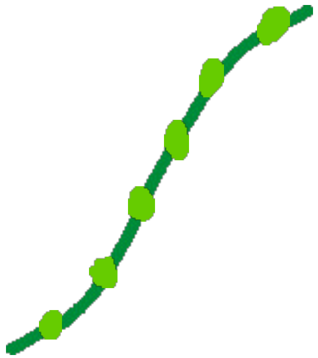
$$\psi^a(x) \to R^a{}_b \psi^b(x)$$

But what about when there are extended operators like domain walls or Wilson/'t Hooft lines?

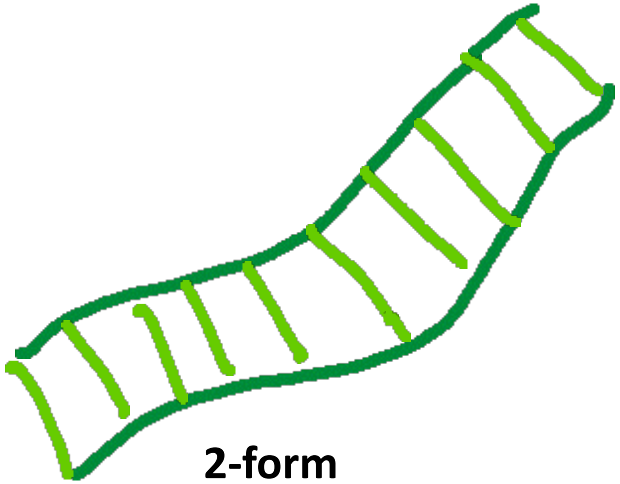
GGS Framework
Gaiotto, Kapustin, Seiberg, Willett
1412.5148

# Higher-form symmetries

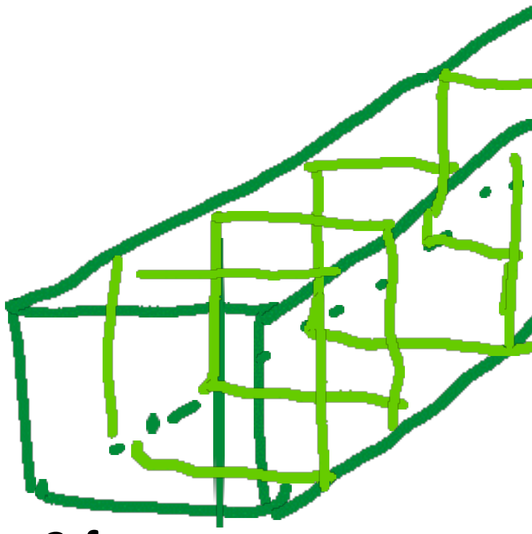
**0-form symmetry**
charged local operators
e.g. particles



**1-form**
line operators
e.g. Wilson line



**2-form**
surface operators
e.g. cosmic string



**3-form**
volume operators
e.g. domain wall

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J^{\mu\nu} = 0$$

Generally $\partial_\mu J^{\mu_1 \mu_2 \cdots \mu_{p+1}} = 0$ antisymmetric

# Higher-form symmetries

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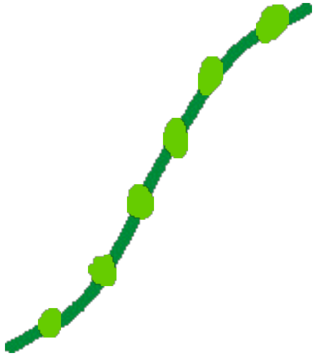

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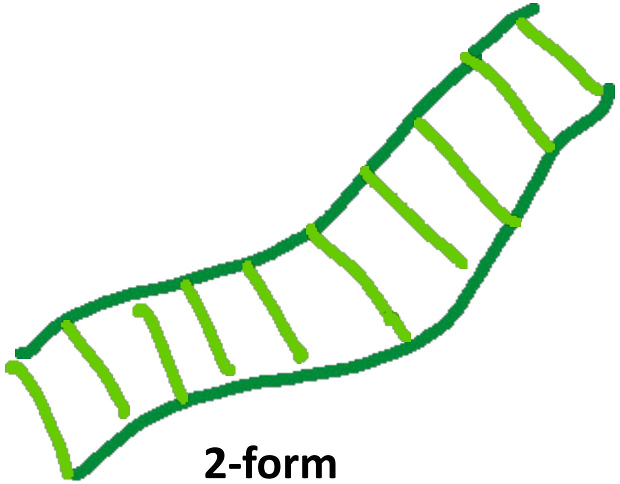

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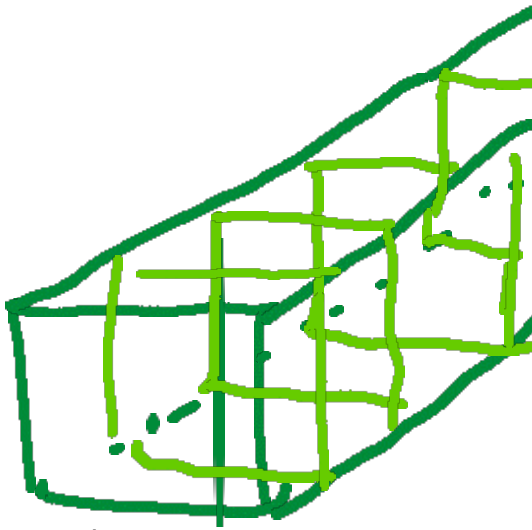

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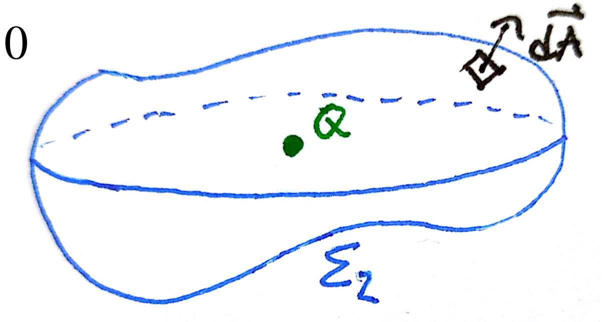
Break by adding charged operator to Lagrangian e.g. $\delta\mathscr{L} = MNN$

Break only with the appearance of new dynamical degrees of freedom!

# Generalized Global Symmetry of Electromagnetism

Recall Gauss' law: The Gaussian surface is topological and so computes an invariant charge.

$t = 0$



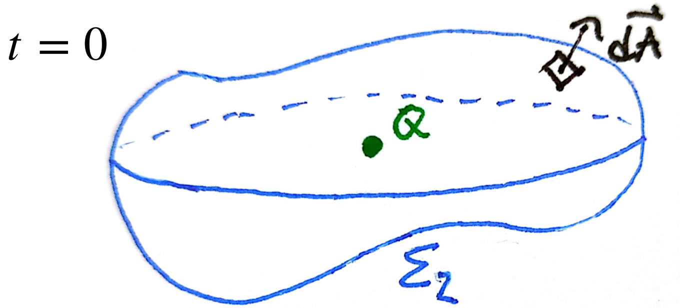

$$Q_{\text{enclosed}} = \int_{\Sigma_2} \vec{E} \cdot d\vec{A}$$

# Generalized Global Symmetry of Electromagnetism

Recall Gauss' law: The Gaussian surface is topological and so computes an invariant charge.

In pure electromagnetism, the photon field strength is conserved $J_E^{\mu\nu} \sim \frac{1}{e^2} F^{\mu\nu}, \quad \partial_\mu J_E^{\mu\nu} = 0$

Gauss' law computes a Noether charge for an electric 1-form symmetry!

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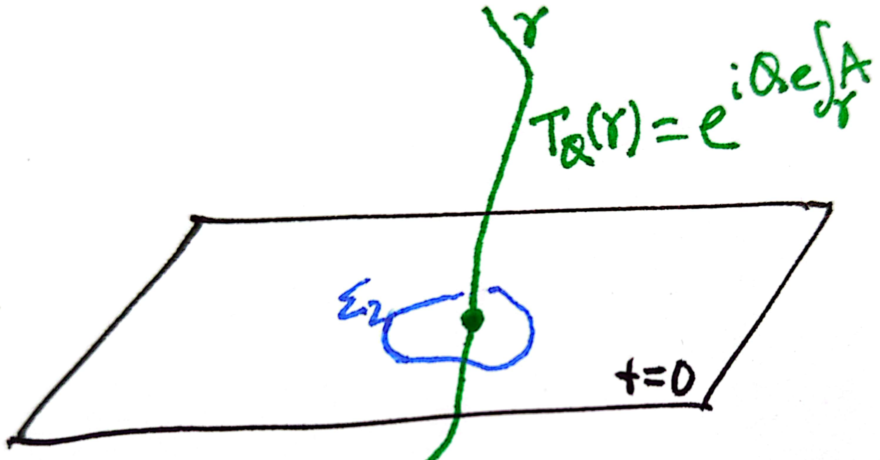
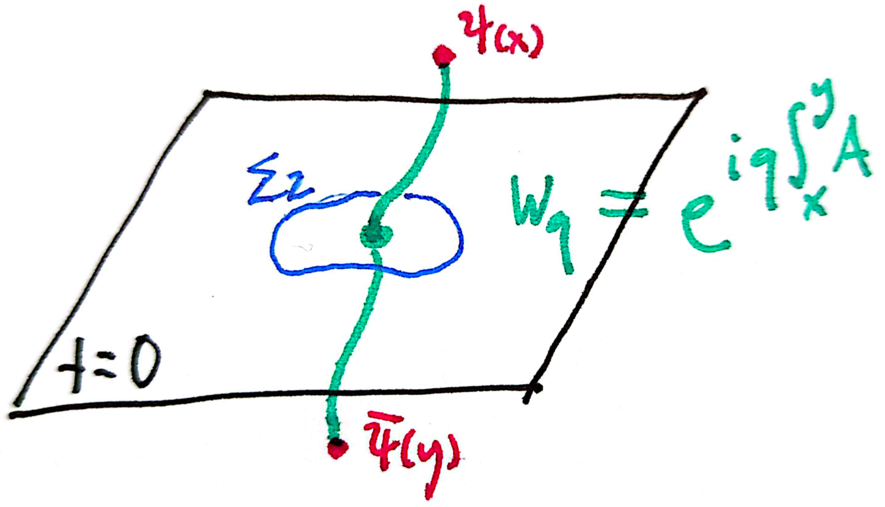



$$Q_{\text{enclosed}} = \int_{\Sigma_2} F^{\mu\nu} dS^{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}$$

# Emergent 1-form symmetry

The 1-form symmetry is emergent in the low-energy, long-distance theory $E \ll m_e$.

Once we see the dynamical electron, then Wilson lines can 'end'.




That is, Gauss' law really breaks for $E > m_e$ because the Gaussian surface is no longer topological.

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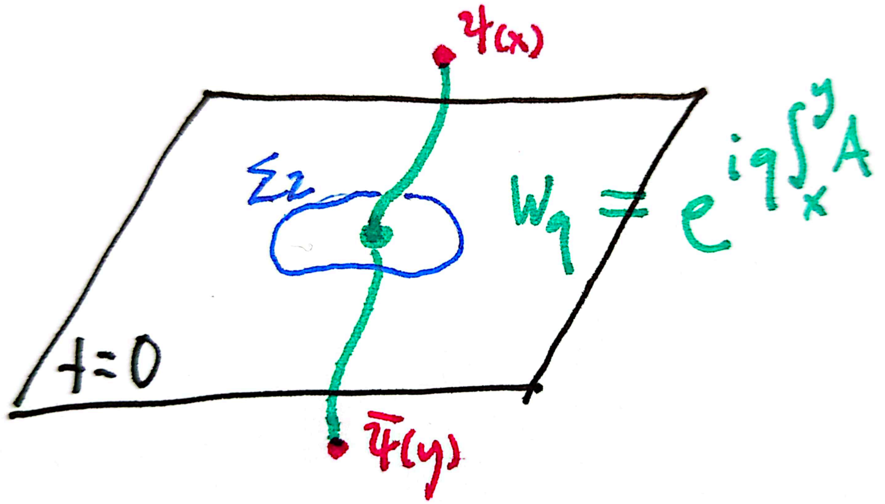



That is, Gauss' law really breaks for $E > m_e$ because the Gaussian surface is no longer topological.

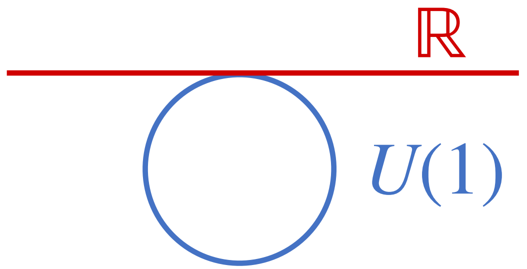
*Mutatis mutandis* a magnetic one-form symmetry for a theory $H$ with 't Hooft lines classified by $\pi_1(H)$

# Fractionally Charged Particles at the Energy Frontier: The SM Gauge Group and One-Form Global Symmetry

SK & Adam Martin, next week

# Nonperturbative aspects of gauge theory

Local operators really only probe the Lie *algebra*, e.g. the Lagrangian cannot tell $\mathbb{R}$ from $U(1) \simeq \mathbb{R}/\mathbb{Z}$



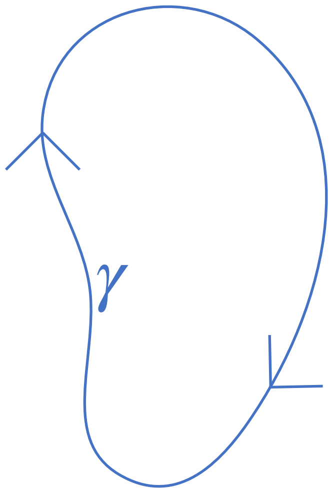

But the Yang-Mills theories *do* differ in important ways, most simply in their representations

$\mathbb{R}$ : any $q \in \mathbb{R}$

$U(1)$ : only $q \in \mathbb{Q}$

So if this information is not in the Lagrangian, where can we find it? In the spectrum of line operators!

$$W_q(\gamma) = e^{iq \int_\gamma A_\mu dx^\mu}$$



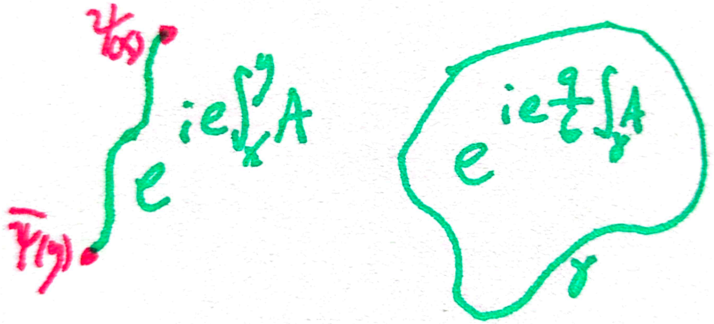
# What's the gauge *group*?

Hucks 1991
Tong 2017

The chiral spectrum of the Standard Model we have observed are consistent with *multiple* Lie groups

$$G_{\mathrm{SM}_q} = \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right)/\mathbb{Z}_q, \quad q \in \{1,2,3,6\}$$

$G_{\mathrm{SM}_q}$ allows particles of fractional electric charge $e \times \left(\frac{q}{6}\right)$!



This means an emergent electric one-form symmetry $\mathbb{Z}^{(1)}_{6/q}$ as the electron does not 'cut' all the Wilson lines

A discrete set of Gaussian surfaces still topological above $m_e$!

# Fractionally-charged particles

$$G_{\mathrm{SM}_q} = \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right)/\mathbb{Z}_q, \quad q \in \{1,2,3,6\}$$

So for $q \neq 6$ perhaps first vector-like particles have fractional charge

There have been some searches, but not enough attention. Bounds on fractionally-charged particle $\psi$ with $e/6$ only $m_\psi \gtrsim 50$ GeV.

Unification demands a quotient! $SU(5), SO(10)$ say $q = 6$, $3^3$ says $q = 2$, PS says $q = 3$.

If we discover $\psi$ we rule out all minimal GUTS!

# Small Instanton Model Building with Noninvertible Symmetries

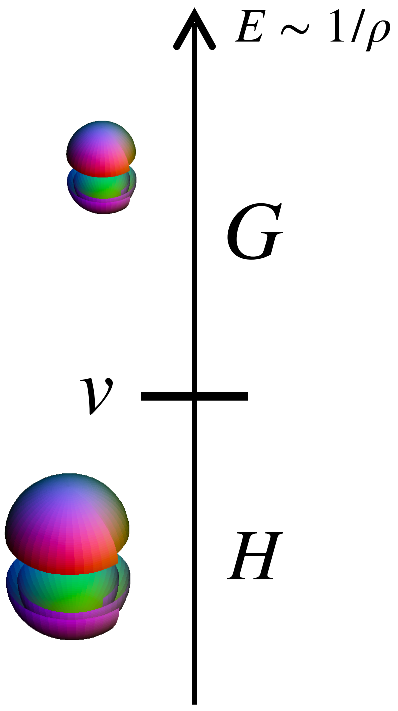
## Nonperturbative Quantum Lepton Flavodynamics

arXiv:2211.07639, Clay Córdova, Sungwoo Hong, SK, Kantaro Ohmori

## Nonperturbative Quantum Quark Flavodynamics

arXiv:2402.12453, Clay Córdova, Sungwoo Hong, SK

# Small instanton model building



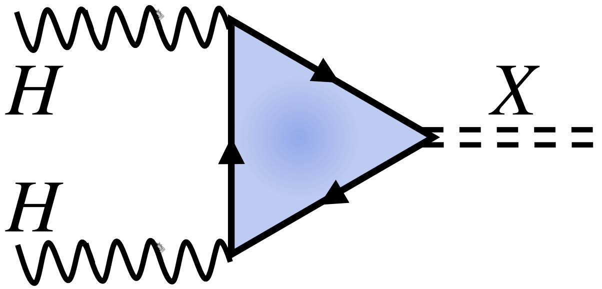

$G$-instanton effects suppressed below Higgsing at $v$, and $H$-instantons (if any) may not have the same effects

What can we tell about small instantons at low energies? Normally, nothing. Need $E \gtrsim v$.

But if $H$ has 't Hooft lines, in fact this information can subtly be preserved in the form of non-invertible symmetries in the $H$ theory

# Non-invertible symmetries

If some $H$ instantons do not appear on $\mathbb{R}^4$, an anomalous global zero-form symmetry $X$ can be converted to a non-invertible symmetry!



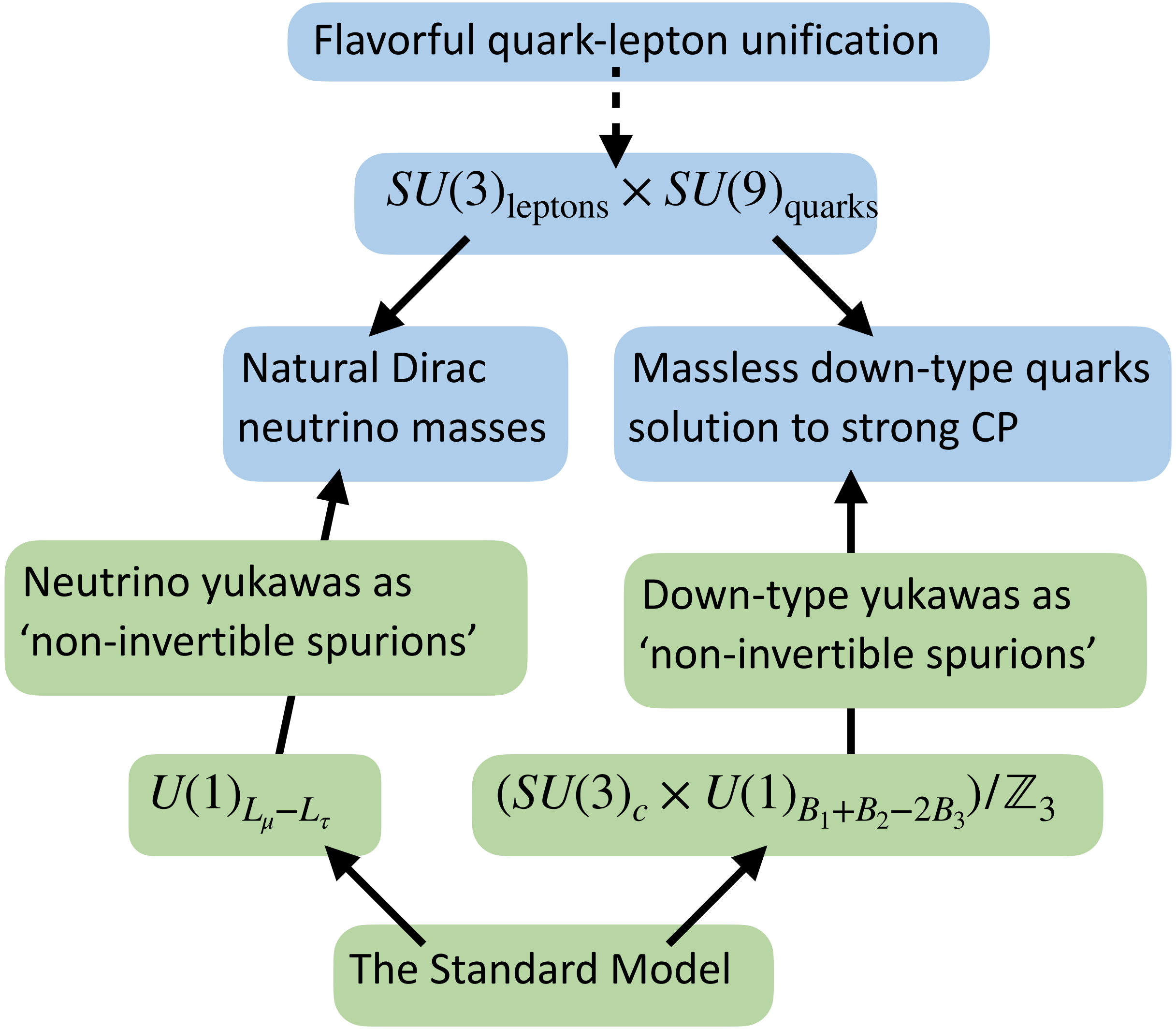

This must act both on local fields and on ’t Hooft lines, roughly as

$$\psi(x) \to \psi(x) e^{i\alpha} \qquad e^{i\oint_\gamma A_m} \to e^{i\oint_\gamma A_m + i\alpha \oint_\gamma A}$$

Strategy: A spurion for a non-invertible symmetry can be generated *solely by instantons* in $G \supset H$ which contains $G/H$ monopoles

# Non-invertible symmetry model building

- Top-down: Theories of quantum flavodynamics have previously-unnoticed nonperturbative effects with super-cool pheno!
- Bottom-up: We uncovered these using powerful new ideas from generalized global symmetries.



# Backup slides

Rants and other things I didn't have time for

# Wrong conclusion

- Incorrect takeaway: “They used these fancy new symmetry ideas but in the end the UV model could be explained in terms of instantons. We’ve known about that stuff since the 80s. So who cares about generalized symmetries?”

- Correct takeaway: “These intriguing instanton effects have been sitting this close to the SM for decades and nobody saw it?! What can generalized symmetries tell me about my favorite BSM model??

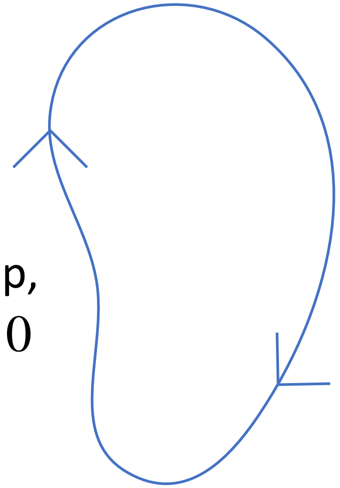
# Violation of non-invertible symmetries

The IR generalized symmetry picture is loops of dynamical monopoles which break magnetic one-form symmetry so violate non-invertible symmetry

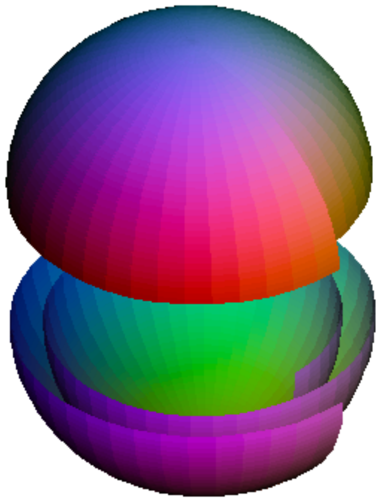
The connection between monopole loops and small instantons is not yet well explored

Fan, Fraser, Reece, Stout 2105.09950

Dyon loop, $q_e, q_m \neq 0$

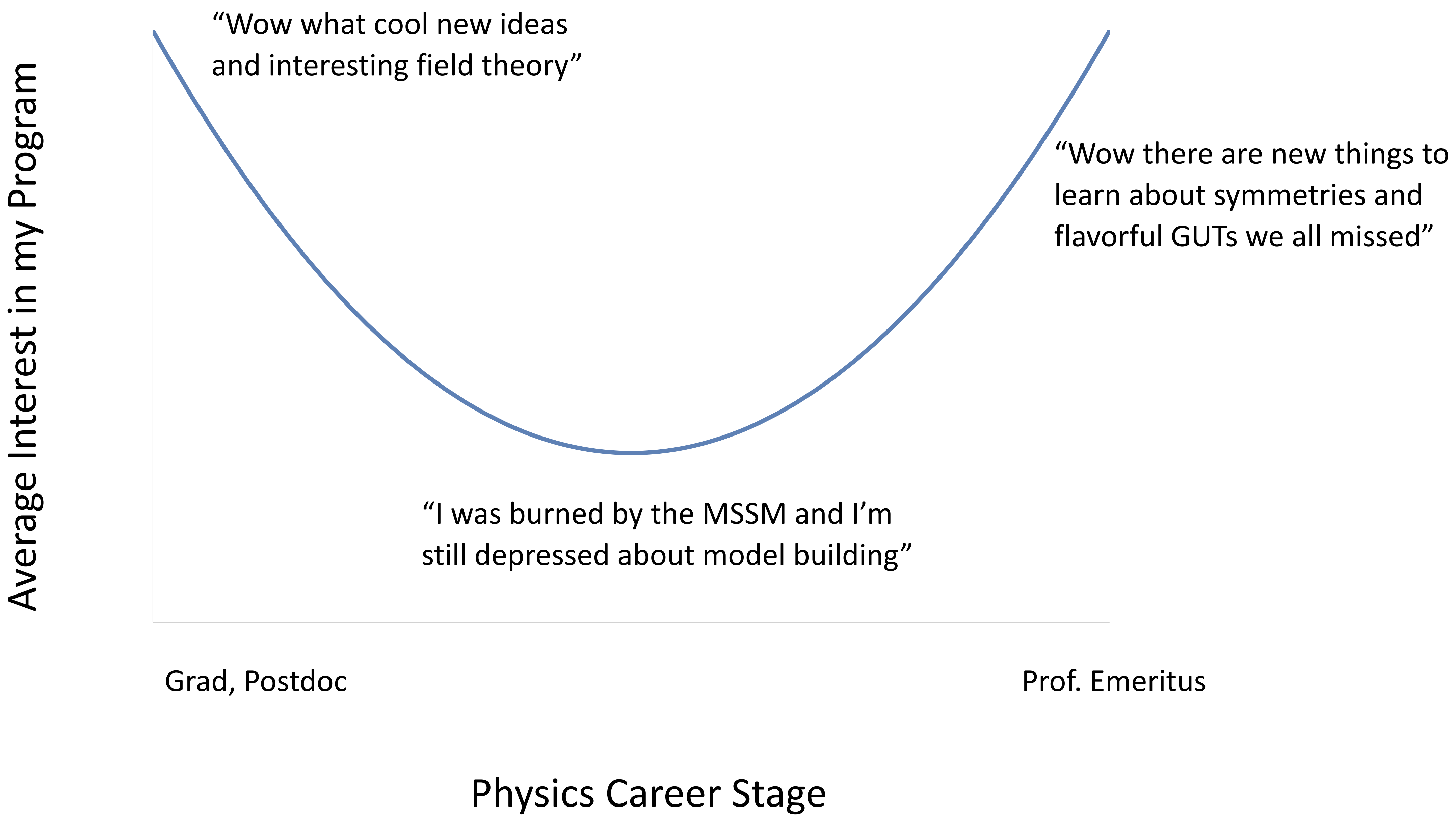


$$\int E \cdot B \neq 0 \neq \int F\tilde{F}$$



$A^{(n-\text{inst})}$

Some deep relation to Callan-Rubakov


“Wow what cool new ideas and interesting field theory”
“Wow there are new things to learn about symmetries and flavorful GUTs we all missed”
“I was burned by the MSSM and I’m still depressed about model building”
Average Interest in my Program
Grad, Postdoc
Prof. Emeritus
Physics Career Stage

# Quark Weak CP and Strong CP Violation

The 'strong CP angle' $\bar{\theta} = \arg e^{-i\theta} \det\left(y_u y_d\right)$ is constrained to $\bar{\theta} \lesssim 10^{-10}$ !

Even worse, we also have the 'weak CP angle' $\tilde{J} = \mathrm{Im}\det\left(\left[y_u^\dagger y_u, y_d^\dagger y_d\right]\right)$

oft parameterized by $m_i, \theta_{ij}$, and the phase $\delta_{\mathrm{CKM}} \sim 1.14$

A small value of $\bar{\theta}$ is not technically natural ⇒ the strong CP problem.

Upon RG evolution, $\delta\bar{\theta} \propto c\,\delta_{\mathrm{CKM}}$

Ellis & Gaillard '79

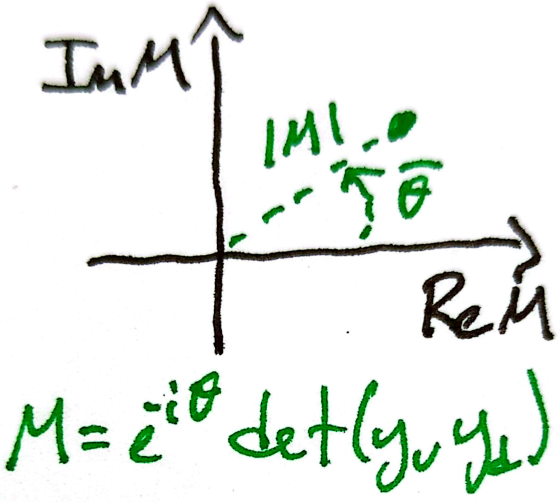
# Peccei-Quinn for Strong CP

Now consider a Peccei-Quinn symmetry protecting the up quark mass

$$U(1)_{\mathrm{PQ}}: \quad \bar{u} \to \bar{u} e^{i\alpha} \quad \Rightarrow \quad \tilde{H} Q \bar{u} \text{ charged so } y_u = 0$$

If the PQ symmetry is good, $y_u \to 0$, and so $\det y_u \to 0$ and there's no strong CP violation

Easier to parameterize in 'Cartesian coordinates' for complex parameter $M \in \mathbb{C}$




Def $M = e^{-i\theta} \det\left(y_u y_d\right)$, so $\bar{\theta} = \arg M$

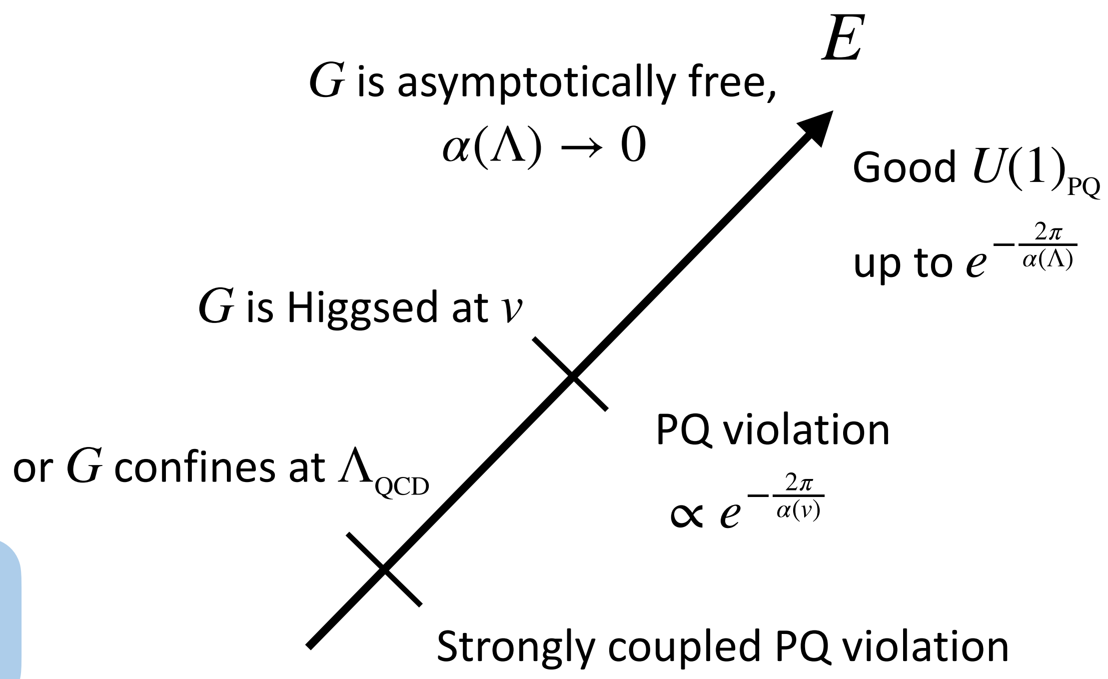
Transforms as $CP$ : $\mathrm{Im}(M) \to -\mathrm{Im}(M)$

# Peccei-Quinn Violation

Massless up quark?! Not in the IR.
A PQ symmetry which begins good is violated by instantons at low energies

UV $y_u = 0$ is then violated by QCD instantons to generate mass, automatically $M \in \mathbb{R}_+$.

Georgi-McArthur '81
Kaplan-Manohar '86
Choi, Kim, Sze '88




Heroic efforts by lattice physicists tell us the SM does not bear out the massless up quark solution

Flavour Lattice Averaging Group 2019

Could there be any UV model where instantons revive this solution?

# Massless quark wins on quality

Both axion and massless quark solutions rely on good quality Peccei-Quinn symmetries, but only the former has a quality ‘problem’ because its required quality is ridiculously unnatural

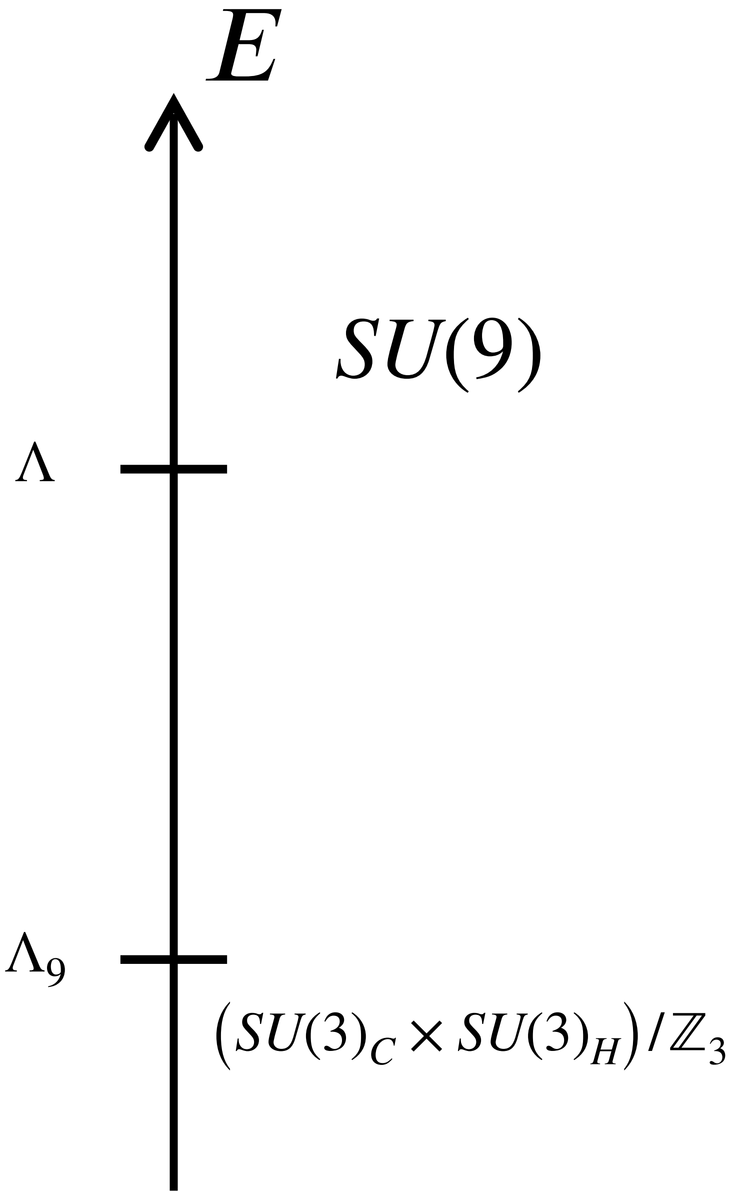
Worse issue for the axion because

- With PQ-charged scalar $\phi$ can have all sorts of PQ-violating ops e.g. $\mathscr{L} \supset c_n M_{\rm Pl}^{4-n} \phi^n$
- We have strong astrophysical bounds on $\langle \phi \rangle = f_a \gtrsim 10^8$ GeV
- The *potential* $V_{\rm grav} \sim f_a^4 \left( f_a / M_{\rm pl} \right)^{n-4}$ cannot overpower $V_{\rm inst} \sim \Lambda_{\rm QCD}^4$

Whereas we can sustain some extra additive contribution to $M$ as long as its magnitude is small

$\mathscr{L} \supset c_\Sigma \tilde{H} Q \Sigma \bar{d} / M_{\rm Pl}$ can have some random phase and $O(1)$ coupling as long as $\langle \Sigma \rangle / M_{\rm pl} \lesssim \bar{\theta}$. Quark flavor physics is not too far away!

# Strong CP in more detail




We begin in the far UV with a good $U(1)_{\mathrm{PQ}}$

$$\mathscr{L}_0 = y_t \tilde{H}\mathbf{Q}\bar{\mathbf{u}} + \text{h.c.} + \frac{i\theta_9}{32\pi^2} F\tilde{F}$$

And so of course $M = e^{-i\theta} \det\left(y_u y_d\right) = 0$

We flow down in energies and begin to generate

$$\mathscr{L}(\Lambda) \sim y_t H\mathbf{Q}\bar{\mathbf{u}} + y_t^{\star} e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9(\Lambda)}} H\mathbf{Q}\bar{\mathbf{d}} + \text{ h.c.} + \frac{i\theta_9}{32\pi^2} F\tilde{F}$$

With exactly the right phase to ensure

$$\bar{\theta} = \arg e^{-i\theta_9} \det y_u y_d = -\theta_9 + \arg |y_t|^2 e^{i\theta_9} = 0$$

Further at the matching scale

$$\mathscr{L}(\Lambda_9) \sim y_t H\mathbf{Q}\bar{\mathbf{u}} + y_t^{\star} e^{i\theta_9} e^{-\frac{2\pi}{3\alpha_s(\Lambda_9)}} H\mathbf{Q}\bar{\mathbf{d}} + \text{ h.c.} + \frac{i3\theta_9}{32\pi^2}\left(G\tilde{G} + K\tilde{K}\right)$$

And the matching accounts for the yukawas now being 3x3 matrices

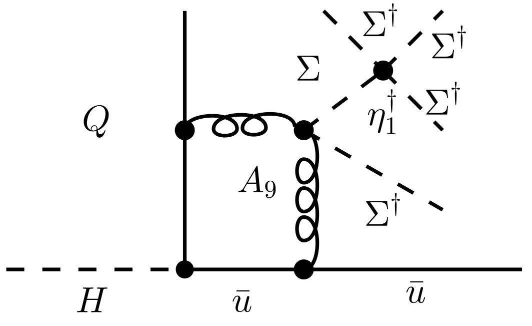
$$\bar{\theta} = -3\theta_9 + \arg\det |y_t|^2 e^{i\theta_9} = 0$$

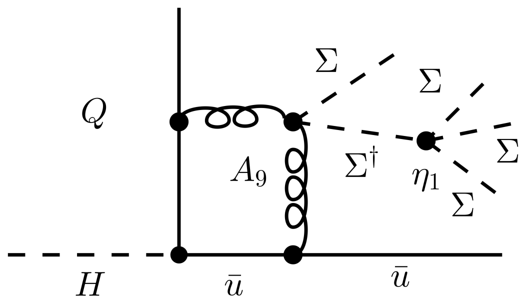
# Generating CKM

Idea: Communicating flavor-breaking $\langle \Sigma^a{}_b \rangle$ through gauged flavor symmetry lets you generate *hermitian* yukawas

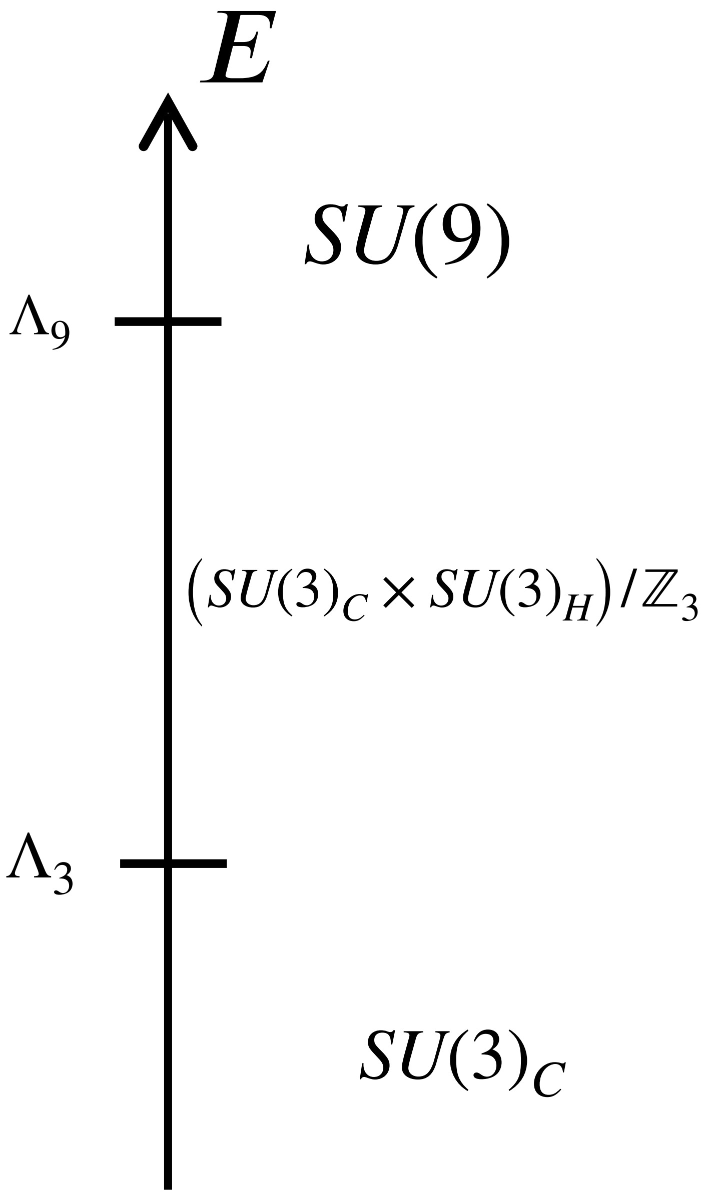
$\bar{\theta} = \arg\det e^{-i\theta} y_u y_d$ automatically zero

$$V_{\mathbb{Z}_4}(\Sigma) = \eta_1 \mathrm{Tr}\left(\Sigma^4\right) + \eta_2 \mathrm{Tr}\left(\Sigma^2\right)^2 + \text{ h.c.}$$




$$(y_u)^a{}_b \sim y_t \left( \mathbb{1}^a{}_b + \frac{\alpha_9}{(4\pi)} \frac{\eta_1^\dagger (\Sigma^{\dagger 4})^a{}_b + \eta_2^\dagger \mathrm{Tr}(\Sigma^{\dagger 2})(\Sigma^{\dagger 2})^a{}_b}{\Lambda_9^4} + \frac{\alpha_9}{(4\pi)} \frac{\eta_1 (\Sigma^4)^a{}_b + \eta_2 \mathrm{Tr}(\Sigma^2)(\Sigma^2)^a{}_b}{\Lambda_9^4} + \dots \right)$$



# Strong CP Technical Naturalness

- Think of it like the hierarchy problem
- The SM strictly by itself does not have a hierarchy problem because the electroweak scale is the largest scale
- But the lack of technically naturalness already in the SM, $\delta m_h^2 \sim c\Lambda_{QCD}^2$ like $\delta\bar{\theta} \sim c\delta_{\mathrm{CKM}}$, to an effective field theorist shows a generic issue one faces in UV theories
- UV theories which often introduce new violations of dilations or of CP introduce severe issues e.g. $\delta m_h^2 \propto M_{GUT}^2$ or $\delta\bar{\theta} \propto \theta_{\mathrm{VLQ}}$

# The Standard Model

| | $Q_i$ | $\overline{u}_i$ | $\overline{d}_i$ | $L_i$ | $\overline{e}_i$ | $H$ |
|---|---|---|---|---|---|---|
| $SU(3)_C$ | $\mathbf{3}$ | $\overline{\mathbf{3}}$ | $\overline{\mathbf{3}}$ | – | – | – |
| $SU(2)_L$ | $\mathbf{2}$ | – | – | $\mathbf{2}$ | – | $\mathbf{2}$ |
| $U(1)_Y$ | $+1$ | $-4$ | $+2$ | $-3$ | $+6$ | $-3$ |

What are its generalized global symmetries?

| | $Q_i$ | $\overline{u}_i$ | $\overline{d}_i$ | $L_i$ | $\overline{e}_i$ | $H$ |
|---|---|---|---|---|---|---|
| $SU(3)_C$ | $\mathbf{3}$ | $\overline{\mathbf{3}}$ | $\overline{\mathbf{3}}$ | $-$ | $-$ | $-$ |
| $SU(2)_L$ | $\mathbf{2}$ | $-$ | $-$ | $\mathbf{2}$ | $-$ | $\mathbf{2}$ |
| $U(1)_Y$ | $+1$ | $-4$ | $+2$ | $-3$ | $+6$ | $-3$ |

# Zero-form symmetries:

Start with large classical flavor symmetry $\left(U(N_g)^{(0)}\right)^5$

$$\mathscr{L} = y_u^{ij}\tilde{H}Q_i\bar{u}_j + y_d^{ij}HQ_i\bar{d}_j + y_e^{ij}HL_i\bar{e}_j$$

Left-over classical $U(1)_B \times U(1)_L \rightarrow U(1)_{B-L} \times \mathbb{Z}_{N_g}^L$ broken by electroweak instantons

This last factor, since we have $N_g > 1$, is responsible for SM proton stability

SK '22; Wang, Wan, You '22

The SM with massless neutrinos has exact $U(1)_{L_\mu - L_\tau} \times U(1)_{L_e - L_\mu}$ but we know from oscillations that these are not symmetries of the real world

| | $Q_i$ | $\overline{u}_i$ | $\overline{d}_i$ | $L_i$ | $\overline{e}_i$ | $H$ |
|---|---|---|---|---|---|---|
| $SU(3)_C$ | $\mathbf{3}$ | $\overline{\mathbf{3}}$ | $\overline{\mathbf{3}}$ | – | – | – |
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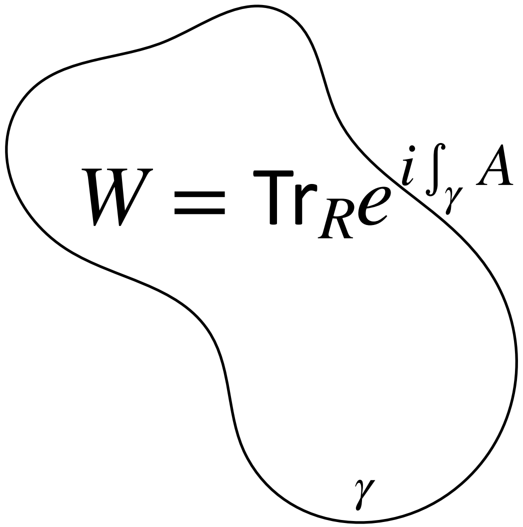
| $U(1)_Y$ | $+1$ | $-4$ | $+2$ | $-3$ | $+6$ | $-3$ |

# An aside on SM one-form symmetries




Hypercharge magnetic one-form symmetry: $U(1)^{(1)}_m$

Electric one-form symmetry? We don't know!

See D. Tong '17

Certain center transformations do not act on any of the SM fields, e.g. consider $\mathbb{Z}_2 \subset SU(2)_L \times U(1)_Y$ under which $\psi \mapsto \psi\left((-1)I_L\right) e^{\pi i Y}$

So the *global structure* of the SM gauge group is

$G_{SM_q} \equiv \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right)/\mathbb{Z}_q$ with q=1,2,3,6

Which has electric one-form symmetry $\mathbb{Z}^{(1)}_{6/q}$

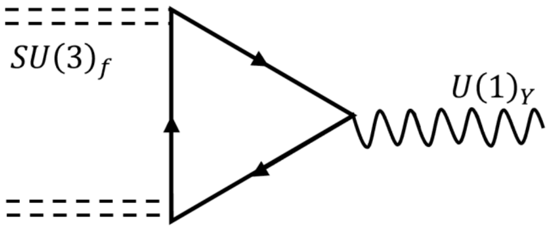
Global structure, fractionally-charged particles, and SMEFT
SK & A. Martin coming

See also recent discussion in axion theories by Reece; Choi, Forslund, Lam, Shao; Cordova, Hong, Wang

# Approximate higher structure:

The non-abelian parts $SU(3)^5$ are intertwined with the magnetic one-form symmetry $U(1)_m^{(1)}$ in the form of a 2-group

Córdova & SK ’22



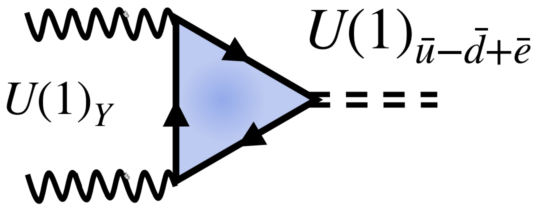

| $\text{Flavor}^2$ | $U(1)_Y$ |
|---|---|
| $SU(3)_Q^2$ | $+1 \cdot 2 \cdot N_c$ |
| $SU(3)_u^2$ | $-4 \cdot N_c$ |
| $SU(3)_d^2$ | $+2 \cdot N_c$ |
| $SU(3)_L^2$ | $-3 \cdot 2$ |
| $SU(3)_e^2$ | $+6$ |

| | | | |
|---|---|---|---|
| One | None | | |
| Two | $\{L, Q\}$ | $\{L, \bar{d}\}$ | $\{L, \bar{e}\}$ |
| Three | $\{\bar{u}, \bar{d}, \bar{e}\}$ | $\{\bar{u}, \bar{e}, Q\}$ | $\{\bar{u}, \bar{d}, Q\}$ |
| Four | None | | |
| Five | $\{Q, \bar{u}, \bar{d}, L, \bar{e}\}$ | | |

Zero-form symmetries so intertwined must be broken by the scale of magnetic one-form symmetry breaking, which (at zero yukawa) tells you the possible unified multiplets

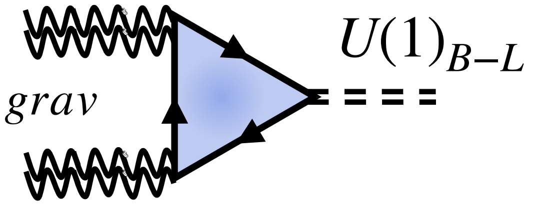
Finite yukawas are ‘spurions’ of 2-group symmetry-breaking and can perturb away from this structure if they control the mass of some vector-like fermions

# Non-invertible symmetries:




Approximate $U(1)_{\bar{u}-\bar{d}+\bar{e}}$ is non-invertible due to a mixed anomaly with hypercharge, $U(1)_Y^2 U(1)_{\bar{u}-\bar{d}+\bar{e}} = 72N_g$

No BSM model-building use yet, but Shao, Lam, Choi '22 use this for a 'symmetry-based' derivation of $\pi^0 \to \gamma\gamma$



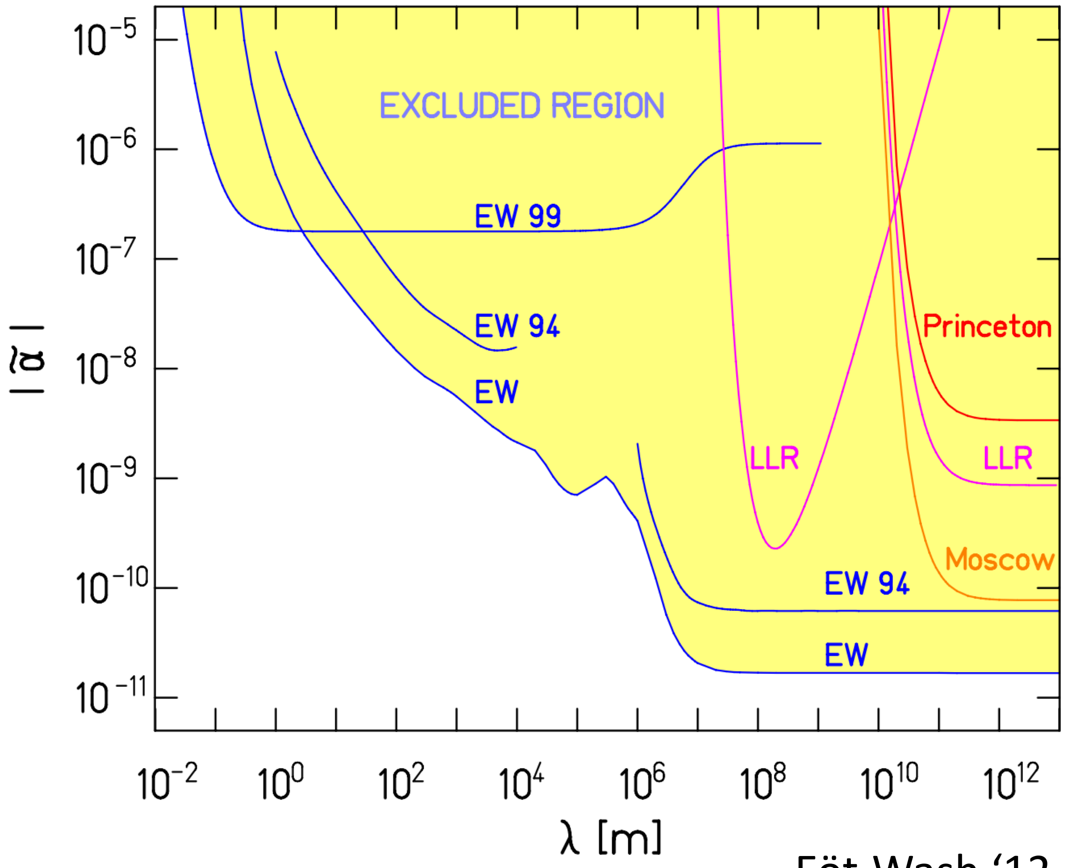

$U(1)_{B-L}$ has a mixed gravitational anomaly without exactly $N_g$ right-handed neutrinos, and Putrov, Wang '23 showed this also leads to a non-invertible symmetry! Could be used for 'gravitational leptogenesis' Alexander, Peskin, Sheikh-Jabbari '06.

# One further IR 'ambiguity'

Given the SM matter content, it's an empirical question whether $U(1)_{B-L}$ is actually a global symmetry or perhaps a weakly coupled gauge symmetry

A $\mathbb{Z}_N$ subgroup may be gauged and unbroken: this "B-L BF theory" is an extension of the SM with 0 new dof

Comes with magnetic two-form symmetry $\mathbb{Z}_N^{(2)}$




Eöt-Wash '12

Remarkably little work on this. I suggested for $N = 2N_g$ these cosmic strings could resolve the cosmological lithium problem.

# Color-flavor embedding

This 'special embedding' $SU(9) \to \left(SU(3)_C \times SU(3)_H\right)/\mathbb{Z}_3$ has a non-trivial 'index of embedding': the fundamental $9$ branches to the $(3,3)$ so the Dynkin index changes non-trivially $k = \mu_{IR}/\mu_{UV} = 3$

See Csaki, Murayama '98 for good discussion

So the $SU(9)$ theory has 'extra' instantons that the IR theory does not: a fermion has $k = 3$ times as many zero-modes in the $SU(3)_C$ instanton background, so we must interpret this as a 3-instanton of the $SU(9)$ theory

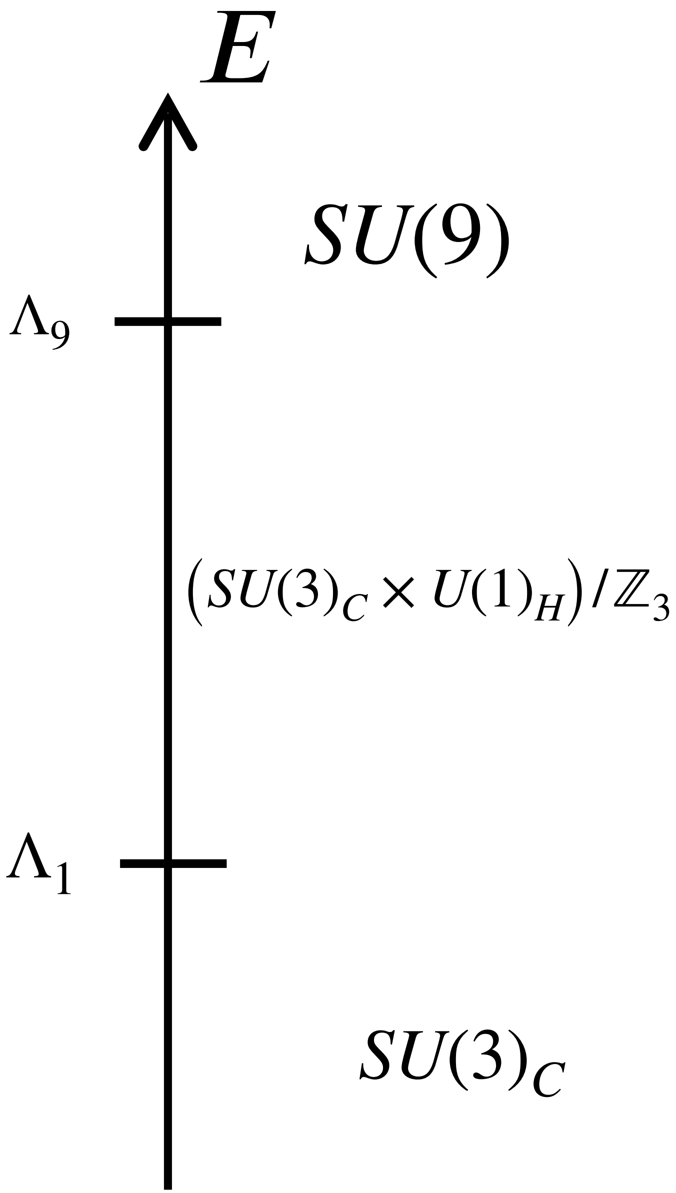
Matching the instanton actions implies a non-trivial matching of the gauge coupling across $\Lambda_9$, as $e^{-\frac{8\pi^2}{\alpha_{IR}}} = e^{-k\frac{8\pi^2}{\alpha_{UV}}}$, so $\alpha_9(\Lambda_9) = 3\alpha_C(\Lambda_9)$

# Abelian Z’ also promising

In the case of $\left(SU(3)_C \times U(1)_H\right)/\mathbb{Z}_3$ the new gauge boson (and new non-invertible symmetry) can observationally appear much sooner

And furthermore maybe don’t need as large Higgs representations to do breaking, so less suppression of instanton density

But need to more clearly understand generating flavor texture in this scheme



# Fractional instanton analysis on $S^2 \times S^2$

See Anber, Hong, Son 2109.03245

Turn on general allowed magnetic fluxes (background 2-form fields for the magnetic 1-form symmetry) and calculate instanton numbers

$$Q_c = \frac{N_c - 1}{N_c} \int_{M_4 = M_2 \times \Sigma_2} \frac{w_2 \wedge w_2}{2} = \frac{N_c - 1}{N_c} \oint_{M_2} w_2 \oint_{\Sigma_2} w_2 = m_1 m_2 \left(1 - \frac{1}{N_c}\right)$$

$$Q_H = \frac{1}{8\pi^2} \int H_2 \wedge H_2 = s_1 s_2$$

Then compute Dirac indices of fermions

$$I_{\psi_i} = n_{\psi_i} T_{\psi_i} Q_c + \mathrm{dim}_{\psi_i} n_{\psi_i} q_{\psi_i}^2 Q_H$$

And find anomaly coefficients for each $U(1)_{\text{global}}[CH]^2$