Imprints of Early Universe Cosmology on Gravitational waves

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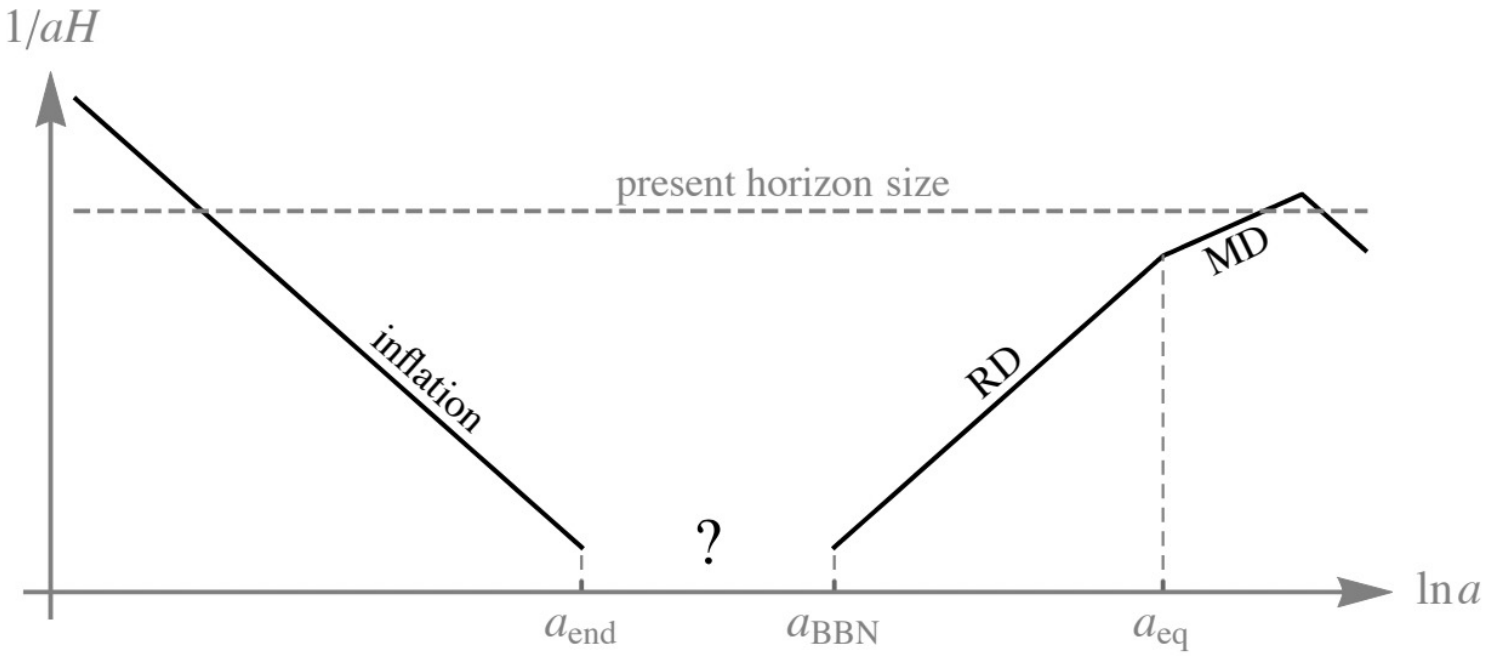
Motivation

- Physics before BBN ($T \sim MeV$) is not well understood due to lack of observational data.
- Gravitational waves can be a natural way to probe this epoch between end of inflation and BBN.



https://arxiv.org/pdf/2006.16182

Introduction

- Energy/entropy injection before BBN has been discussed extensively:
  - Fluctuations generated during inflation and later reentry [Carr & Lidsey, ….]
  - Collapse of domain walls [Cai et al, …]
  - PBH reheating [Bernal et al, …]
  - Bubble collisions during phase transition [Kodama et al, …]
  - Temperature increase during reheating [Co et al, …]

- The rate of energy injection can be either be fast where the field remains stuck as the temperature rises or can be slow where the field tracks its T dependent minima.
- We consider its effects on the GW spectrum for a wide range of FOPT in hidden sectors.

Cosmological setup

- We consider the scenario where the hidden sector is thermally decoupled to the SM.
- We assume that the SM makes up bulk of the energy density of the universe.
- The ratio of hidden sector temperature and that of SM is given by $\xi = \frac{T_h}{T_{SM}} < 1$
- Any small change in the energy density of the universe will thus have more impact on hidden sector as compared to SM and the Hubble will be unaffected.
- Net energy density of the universe is given as, $\rho_R(T) = \frac{\pi^2}{30}\left(g_h^*(T) + \frac{g_{SM}^*(T_{SM})}{\xi^4}\right)T^4$

Model realization

$$V \approx D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda}{4}\phi^4$$

- Initially, at high T, the field is in symmetric phase and there's just 1 minima at $\phi = 0$
- As universe cools, $T < T_1$, there exist a second minima

$$T_1^2 = \frac{T_0^2}{1 - \frac{9E^2}{8\lambda D}}, \quad \phi_1 = \frac{3ET_1}{2\lambda}$$

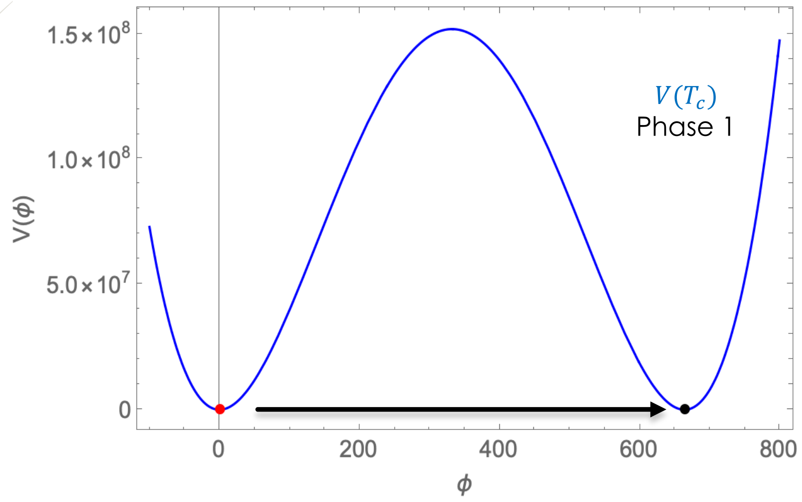
- As it further cools, these two minima become equi-potential and we have an onset of phase transition,

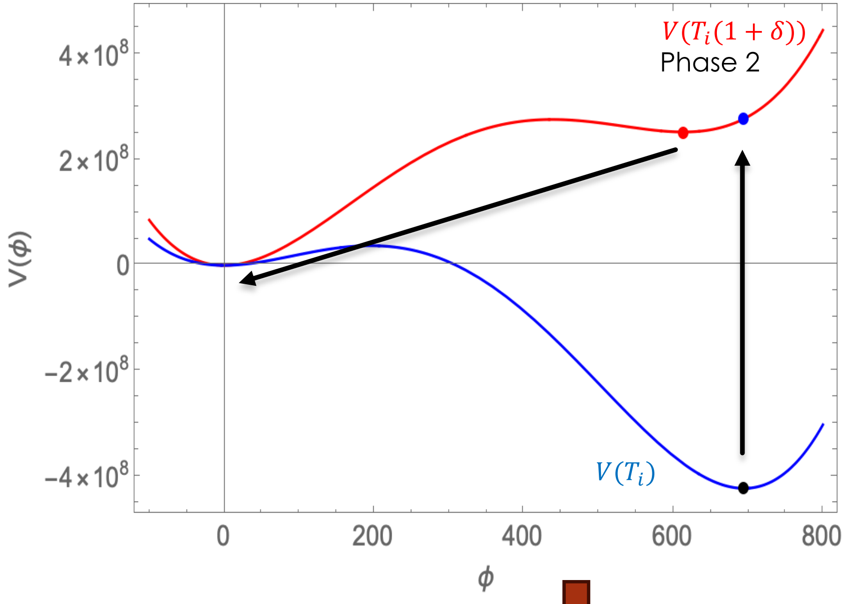
$$V(0, T_c) = V(\phi_c, T_c) \qquad T_c^2 = \frac{T_0^2}{1 - \frac{E^2}{\lambda D}}, \quad \phi_c = \frac{2ET_c}{\lambda}$$

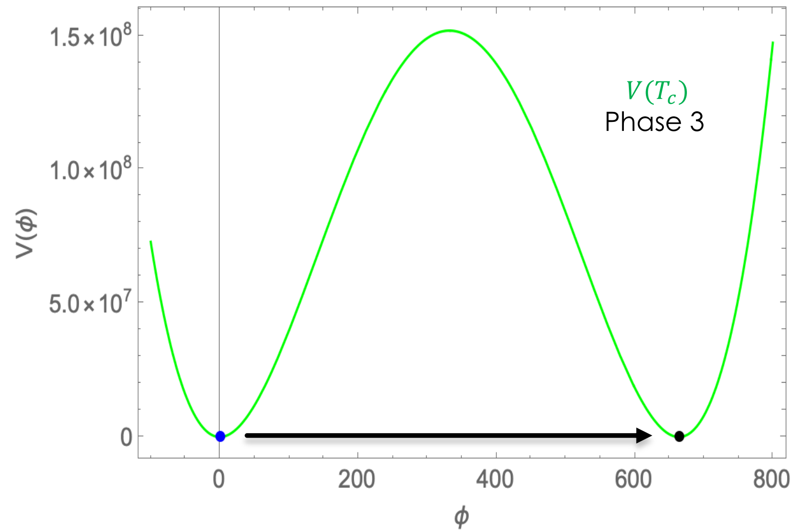
- After $T = T_0$, $\phi = 0$ ceases to be a minima and we are left with,

$$\phi_0 = \frac{3ET_0}{\lambda}$$

Thermal kick effects







- First transition happens at $\mathrm{T} = \mathrm{T_c}$ (Phase 1)
- Due to thermal kick at T_i, $(\mathrm{T_c} > T_i > T_0)$, $T_i \to T_i(1+\delta) > T_c$ whereas the field remains stuck at $\phi_i(T_i)$, leads to PT from $\phi_i \to 0$ (Phase 2)
- As universe cools down, there's another PT from $0 \to \phi_c$, which is like the standard transition but happens at later redshift (Phase 3)

Strength of GW signal

- Amplitude of GW signal is controlled by strength parameter, α given as

$$\alpha = \frac{\Delta(V - \frac{1}{4}\partial_T V)}{\rho_R}\Big|_{T=T_N} \qquad \rho_R(T_N) = \frac{\pi^2}{30}(g_h^* + \frac{g_{SM}^*}{\xi^4})\, T_N^4$$

- For the standard FOPT, we can simplify to get,

$$\alpha\big|_{0\to\phi_c} = \alpha_c \approx \frac{\phi_c^2(-2\, D\, T_0^2)}{4\rho_R(T_c)} = \mu^2\, \frac{-\phi_c^2}{4\rho_R(T_c)}$$

- For the PT due to kick, we have

$$|\frac{\alpha_i}{\alpha_c}| \approx (1+\delta)^2 \left(\frac{\phi_{min}(\tilde{T}_i)\, T_c^2}{\phi_c(T_c)\, \tilde{T}_i^{\,2}}\right)^2 \frac{g_{SM}^*(T_c/\xi)}{g_{SM}^*(T_i/\xi)} > 1$$

Duration of PT

- This parameter gives a measure of the duration of PT
- Defined in terms of the Euclidean bounce action as,

$$\frac{\beta}{H_*} = T\frac{d(S_3/T)}{dT}|_{T_*}$$

- The Euclidean Bounce action is given via,

$$\frac{S_3}{T} = \frac{4.85\, M^3}{E^2\, T^3}\, f(\kappa)$$

$$f(\kappa) = 1 + \frac{\kappa}{4}\left(1 + \frac{2.4}{1-\kappa} + \frac{0.26}{(1-\kappa)^2}\right)$$

$$M^2 = 2\,D\,(T^2 - T_0^2),\ \kappa = \frac{\lambda\, D}{E^2}\left(1 - \frac{T_0^2}{T^2}\right)$$

- For the PT due to kick, the parameters get modified via,

$$\tilde{M}^2(T) = M^2(T) + 3\phi_i(2\,E\,T + \lambda\phi_i)$$

$$\tilde{E}\,T = E\,T + \lambda\phi_i$$

T_N and v_w

- Dedicated numerical simulations are needed to calculate the T_N and v_w
- We will use analytical approximations for T_N:

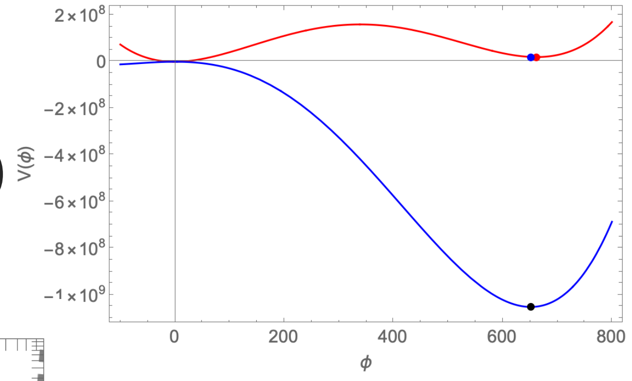
$$\frac{S_3}{T_n} \approx 4 \log \frac{T_n}{H}$$

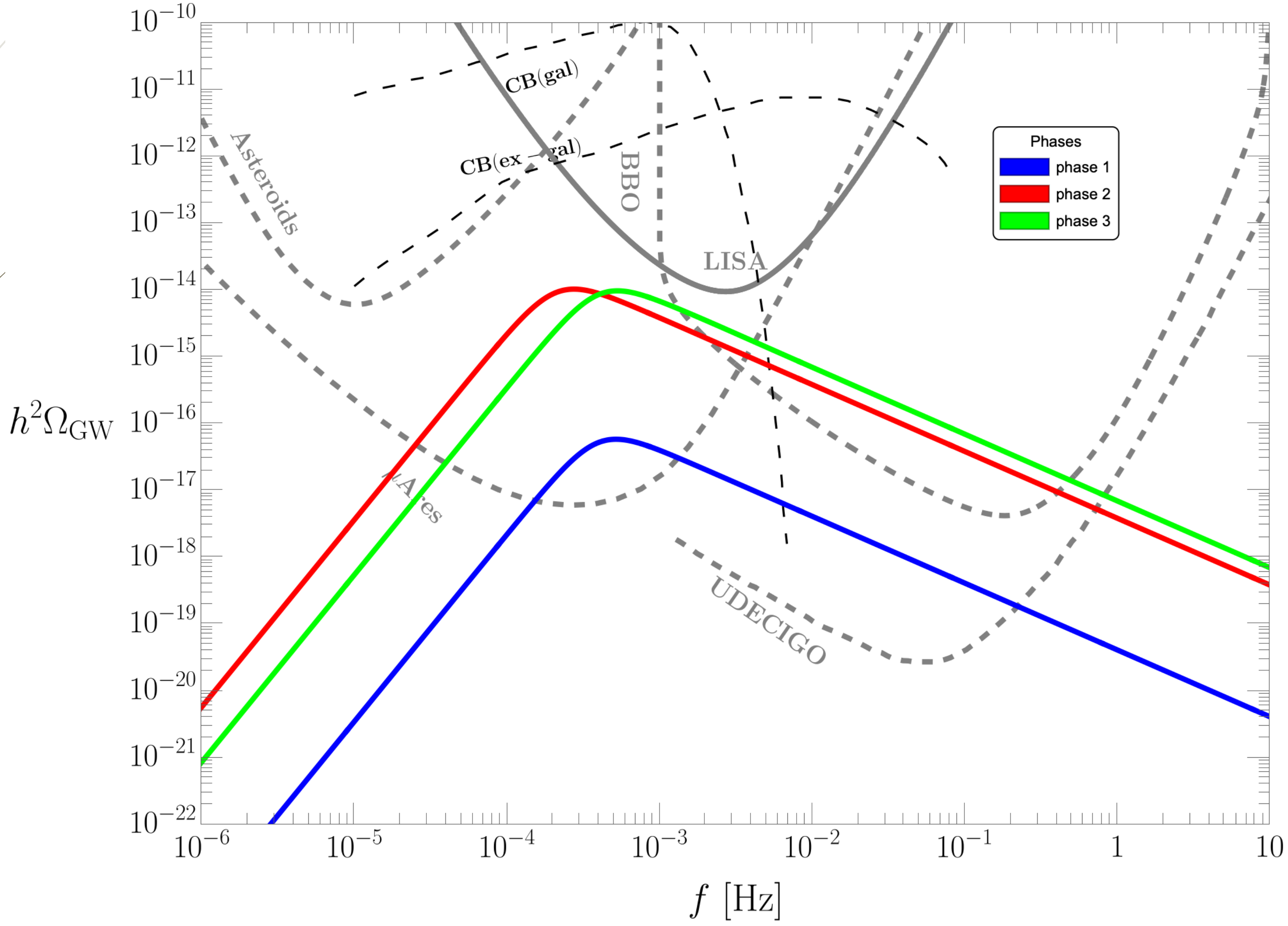
- For wall velocity, a simple demarcation can be made where weaker FOPT attains terminal velocity and for stronger transition, it can overcome friction and wall becomes ultra-relativistic[1]

$$v_w \approx \begin{cases} \frac{1}{\sqrt{3}}, & \alpha \lesssim 10^{-2} \\ 1, & \alpha \gtrsim 10^{-2} \end{cases}$$

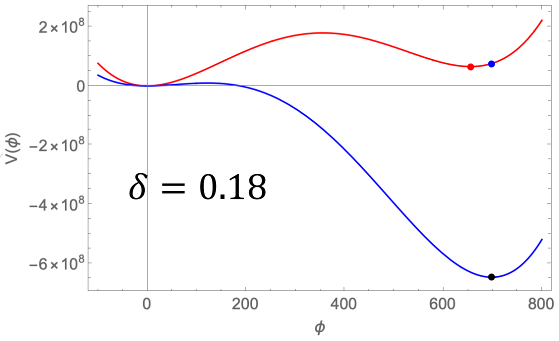
[1] https://arxiv.org/abs/2204.13120

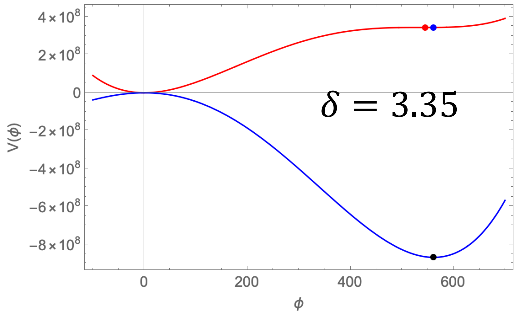
GW spectrum $(\mu = 100\, GeV, \delta = 0.92)$

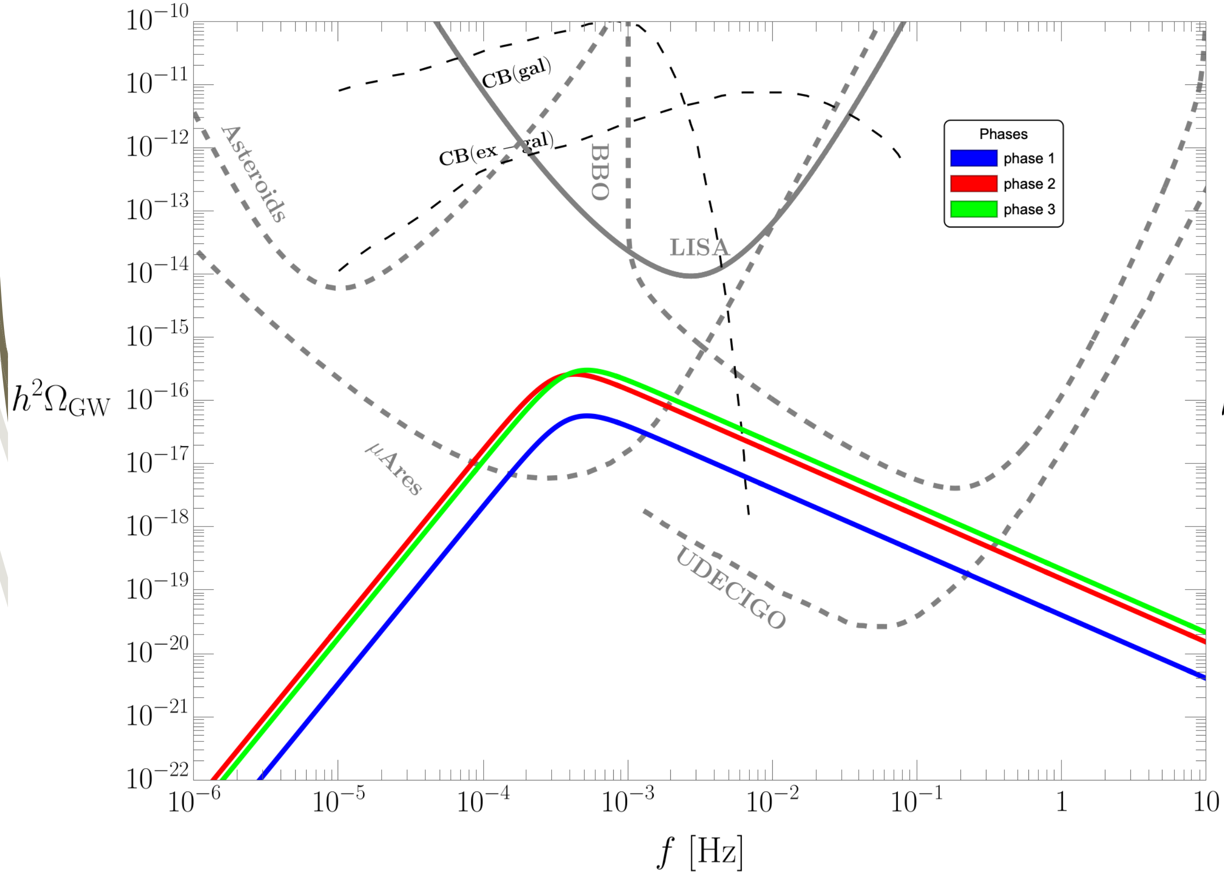


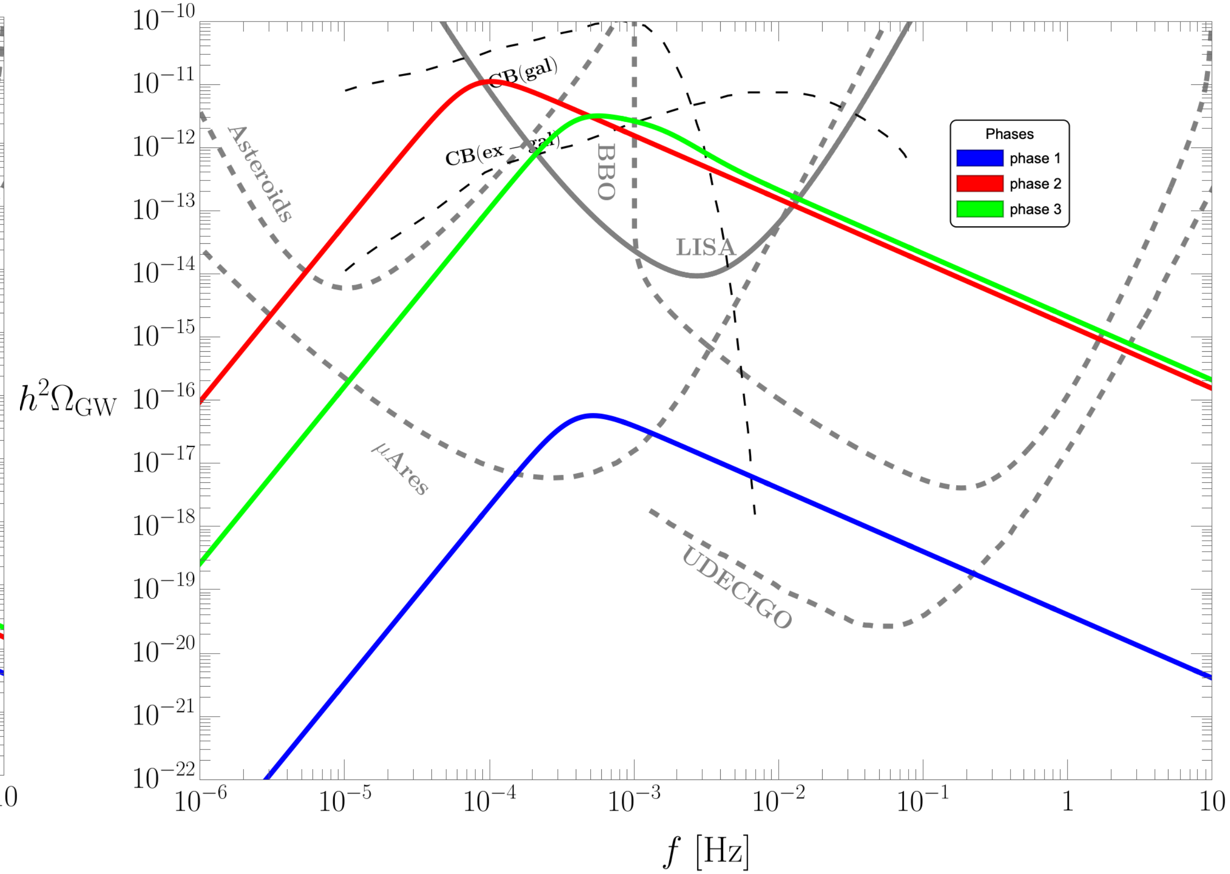


δ dependence









Conclusion

Energy injection (DW, PBH etc) → $T_h \to T_h(1+\delta)$ → 3 GW peaks

- Energy injection leads to more than one peak frequencies for GW from FOPT in hidden sector.
- It is fairly independent w.r.to the mass scale of the hidden sector.
- Even for QCD like transitions, we expect to have multiple peaks due to kick.
- Hidden sectors with GW can probe a variety of new physics scenario in the pre-BBN era.

THANK YOU!

BACKUP Slides

Hidden sector

- For concreteness, we consider a scalar field with U(1) gauge symmetry and a Yukawa like coupling to fermion field,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\phi\, D^\mu\phi + i\,\bar{\psi}\not{D}\psi - \frac{y\phi}{\sqrt{2}}\,\bar{\psi}\psi - V(\phi)$$

- The tree and thermal potential are given as

$$V_0 = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

$$V_{th} = \frac{T^4}{2\pi^2}\left(n_\phi\, J_B\left[\frac{m_\phi^2}{T^2}\right] + n_X\, J_B\left[\frac{m_X^2}{T^2}\right] - n_f\, J_F\left[\frac{m_f^2}{T^2}\right]\right)$$

High T Potential

- At high temperatures, the effective potential is given as,

$$V \approx D(T^2 - T_0^2)\phi^2 - E\,T\phi^3 + \frac{\lambda}{4}\phi^4$$

where,

$$D = \frac{\alpha}{2}, \quad \alpha = \frac{\lambda + g^2}{4} + \frac{y^2}{24}$$

$$E = \frac{1}{12\pi}\left(3g^3 + \left(3\lambda + \frac{\alpha\,T^2 - \mu^2}{\phi^2}\right)^{3/2}\right) \approx \frac{g^3}{4\pi}$$

$$T_0^2 = \frac{\mu^2}{2\,D}$$

Gravitational Waves signal

- Differential GW density parameter characterizes them :

$$\Omega_{GW} = \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\log f}, \quad \rho_c = 3M_{pl}^2 H^2$$

- Semi-analytical parametrizations can be used to describe them,

$$\Omega_{\rm GW}^{\rm em}(f_{\rm em}) = \sum_{I={\rm BW,SW}} N_I\,\Delta_I(v_{\rm w}) \left(\frac{\kappa_I(\alpha_{-1})\,\alpha_{\rm tot}}{1+\alpha_{\rm tot}}\right)^{p_I} \left(\frac{H}{\beta}\right)^{q_I} s_I(f_{\rm em}/f_{{\rm p},I});$$

$$\Rightarrow \quad h^2\,\Omega_{\rm GW}^0(f) = h^2\mathcal{R}\,\Omega_{\rm GW}^{\rm em}\left(\frac{a_0}{a_{\rm perc}}f\right).$$

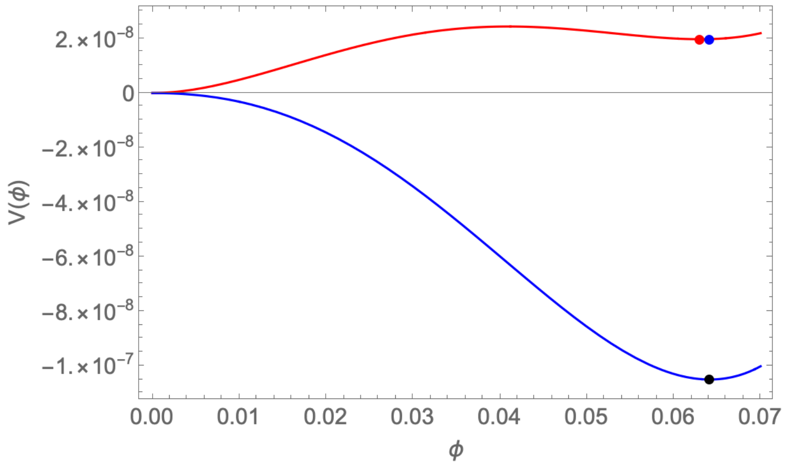
GW parameters for PT Type 2

- The difference is due to phase transition between minimas which are not equipotential.
- Effective potential near transition is given by,

$$V \approx \frac{\tilde{M}^2(T)}{2}\phi^2 - \tilde{E}\,T\,\phi^3 + \frac{\lambda}{4}\phi^4$$

$$\tilde{M}^2(T) = M^2(T) + 3\phi_i(2\,E\,T + \lambda\phi_i)$$

$$\tilde{E}\,T = E\,T + \lambda\phi_i$$



- The bounce action and the transition strength become,

$$\frac{S_3}{T} = \frac{4.85\,\tilde{M}^3(T)}{\tilde{E}^2\,T^3}\,f(\tilde{\kappa})$$

$$\left|\frac{\alpha_i}{\alpha_c}\right| \approx (1+\delta)^2 \left(\frac{\phi_{min}(\tilde{T}_i)\,T_c^2}{\phi_c(T_c)\,\tilde{T}_i^{\,2}}\right)^2 \frac{g^*_{SM}(T_c/\xi)}{g^*_{SM}(T_i/\xi)} > 1$$

GW signal parametrization

- Normalization factors and exponents :

$$(N_{\rm BW}, N_{\rm SW}) = (1, 0.159) \qquad (p_{\rm BW}, p_{\rm SW}) = (2, 2) \qquad (q_{\rm BW}, q_{\rm SW}) = (2, 1)$$

- Potential suppression due to wall velocity :

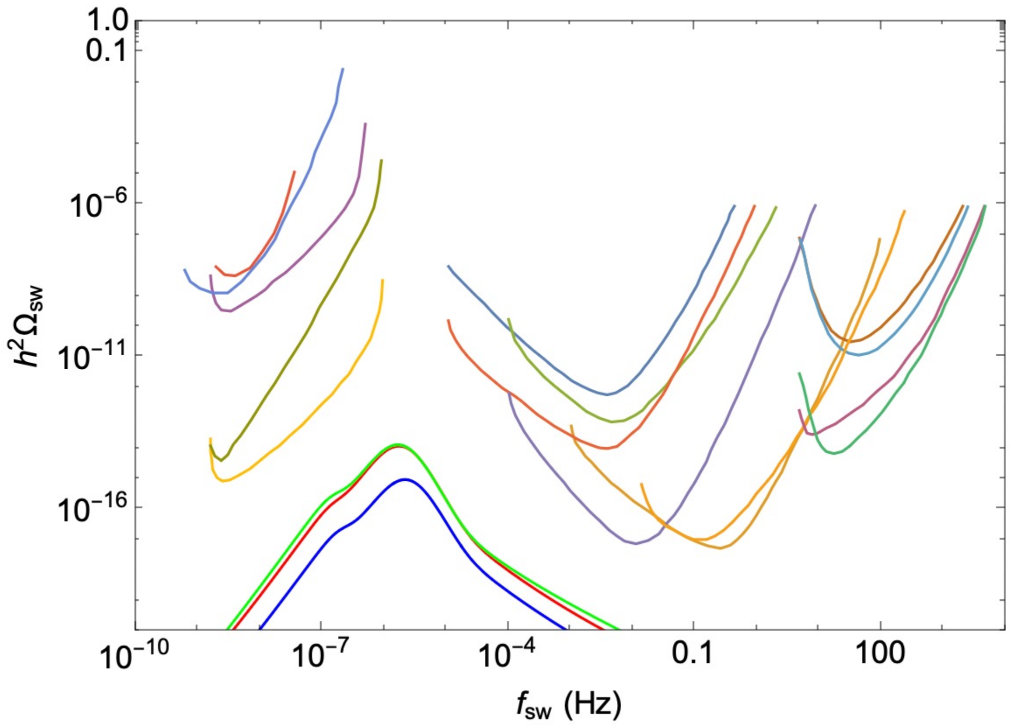
$$(\Delta_{\rm BW}, \Delta_{\rm SW}) = \left(\frac{0.11 v_{\rm w}^3}{0.42 + v_{\rm w}^3}, 1\right)$$

- Spectral shape function and peak frequencies :

$$s_{\rm BW}(x) = \frac{3.8\, x^{2.8}}{1 + 2.8\, x^{3.8}}, \qquad s_{\rm SW}(x) = x^3 \left(\frac{7}{4 + 3\, x^2}\right)^{7/2},$$

$$f_{\rm p,BW} = 0.23\, \beta, \qquad f_{\rm p,SW} = 0.53\, \beta / v_{\rm w}.$$

GW spectrum $\mu = 10\ MeV$

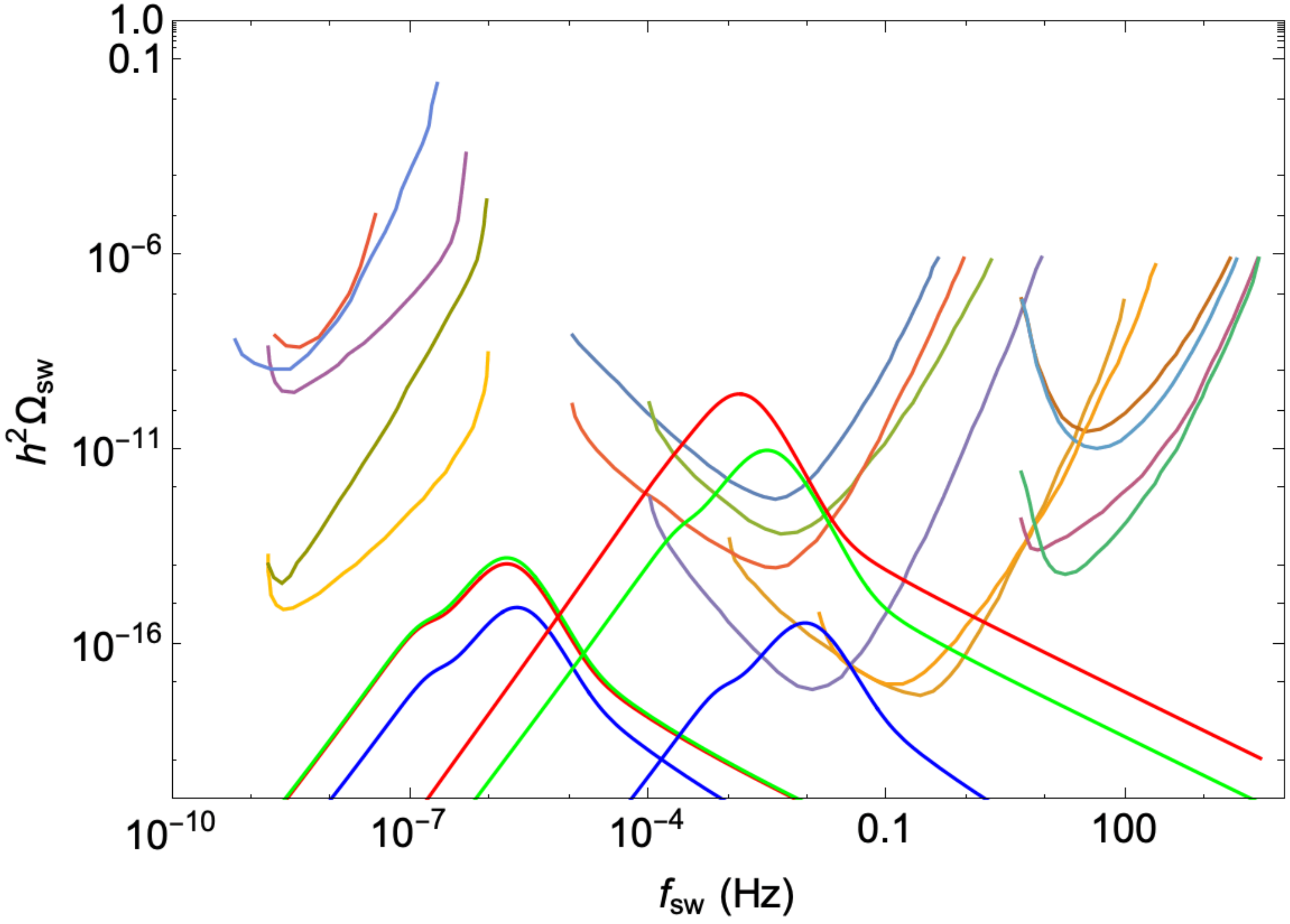


$\{\alpha_{kick},\ \beta_{kick}/H,\ T_{SM,kick}, \delta 0\} = 0.000521944,\ 91.9744,\ 0.108399,\ 0.397682$

$\{\alpha_{late},\ \beta_{late}/H,\ T_{SM,late}\} = 0.000772954,\ 92.2871,\ 0.102398$

$\{\alpha,\ \beta/H, T_{SM,first}\} = 0.000203883,\ 92.2871, 0.14312$

GW spectrum : comparing scales



Slow reheating