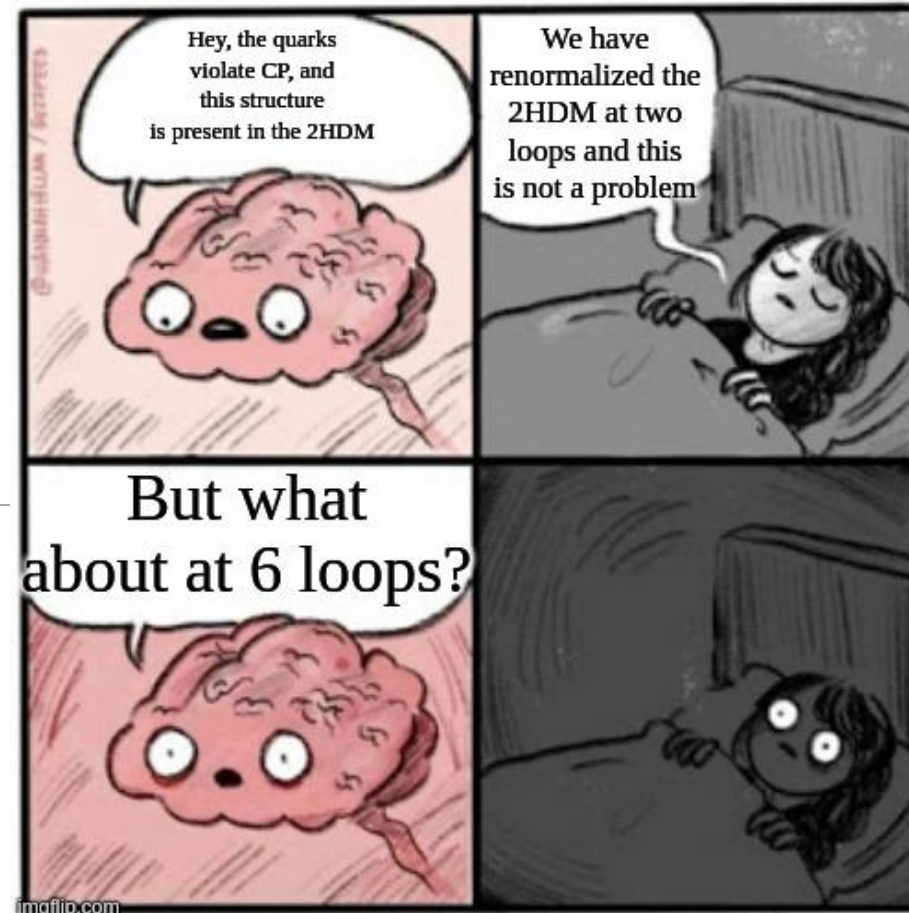


# Can CP be conserved in the two-Higgs doublet model?

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2403.17052

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# Introduction

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- CP violation discovered 1964
  - $K_S \rightarrow \pi\pi$  (CP even),  $K_L \rightarrow \pi\pi\pi$  (CP odd) but also  $K_L \rightarrow \pi\pi$  0.3% of the time!
- CPV from the SM is completely described by the quark mixing matrix (Jarlskog invariant).
- 2HDM extends the SM scalar sector with another doublet which can have additional CPV, literature explore the complex 2HDM and real 2HDM. Most recent phenomenological studies focus on the real 2HDM because of EDM constraints.

Is the real 2HDM a theoretically consistent model since we know that the SM has CPV?

2103.05002 / 2403.17052

# The problem

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- We know there is **CP violation** in the CKM matrix.
- CKM CPV can be transmitted to other operators via loop diagrams
- CKM phase is **hard-breaking** of CP, so no apparent reason why those generated imaginary parts shouldn't be divergent!
- The **real 2HDM** would not have the parameters to absorb this divergence: **not theoretically consistent!**

Is the **real 2HDM** a **theoretically consistent** model since we know that the SM has CPV?

2103.05002 / 2403.17052

# Softly broken $Z_2$ 2HDM

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- Softly broken  $Z_2$  2HDM have one physical CP phase  $\text{Im}((m_{12}^{2*})^2 \lambda_5)$ :

$$\begin{aligned} V_{2HDM} = & m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - m_{12}^2 \phi_1^\dagger \phi_2 \\ & + \frac{1}{2} \lambda_1 (\phi_1^\dagger \phi_1)^2 + \frac{1}{2} \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) \\ & + \frac{1}{2} \lambda_5 (\phi_1^\dagger \phi_2)^2 + \text{h.c.} \end{aligned}$$

- Real 2HDM has both  $\lambda_5$  and  $m_{12}^2$  real on the same basis. The quark sector have CP violation from complex Yukawas:

$$\mathcal{L}_{Yuk} = -Y_d^{(1)} \bar{Q}_L \phi_1 d_R - Y_u^{(2)} \bar{Q}_L \tilde{\phi}_2 d_R + \text{h.c.} \quad (\text{type II})$$

# The primitive diagram

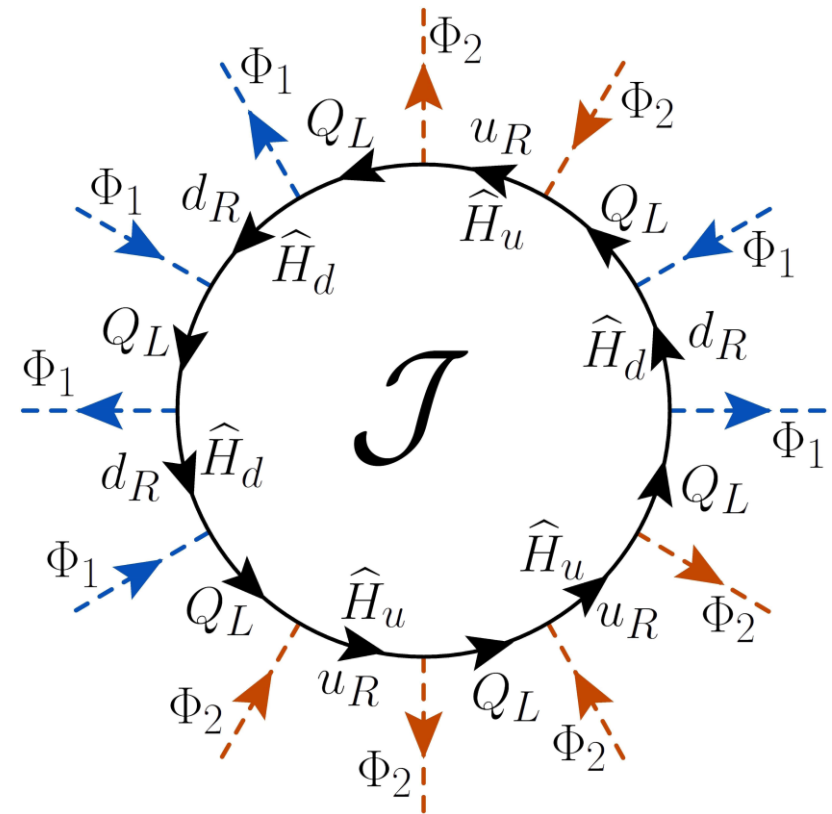
- Jarlskog invariant is proportional to 12 Yukawas

$$\hat{H}_u = Y_u Y_u^\dagger \quad \hat{H}_d = Y_d Y_d^\dagger \quad \mathcal{J} = \text{Tr}(\hat{H}_u \hat{H}_d \hat{H}_u^2 \hat{H}_d^2)$$

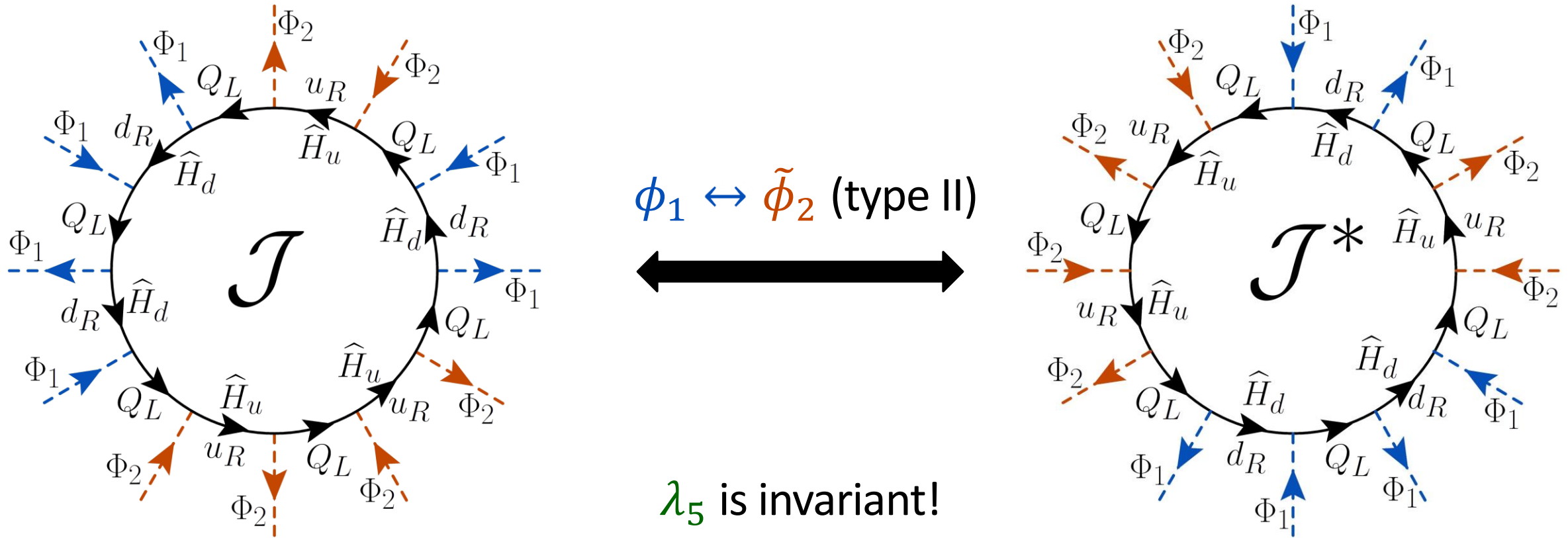
- Any diagram that contribute to  $\text{Im}(\lambda_5)$  comes from the primitive  $\mathcal{J}$  and  $\mathcal{J}^*$  diagrams.

- Can we find relations between the diagrams that are preserved once we start closing legs?

**YES!**



# Symmetries of the primitive diagram



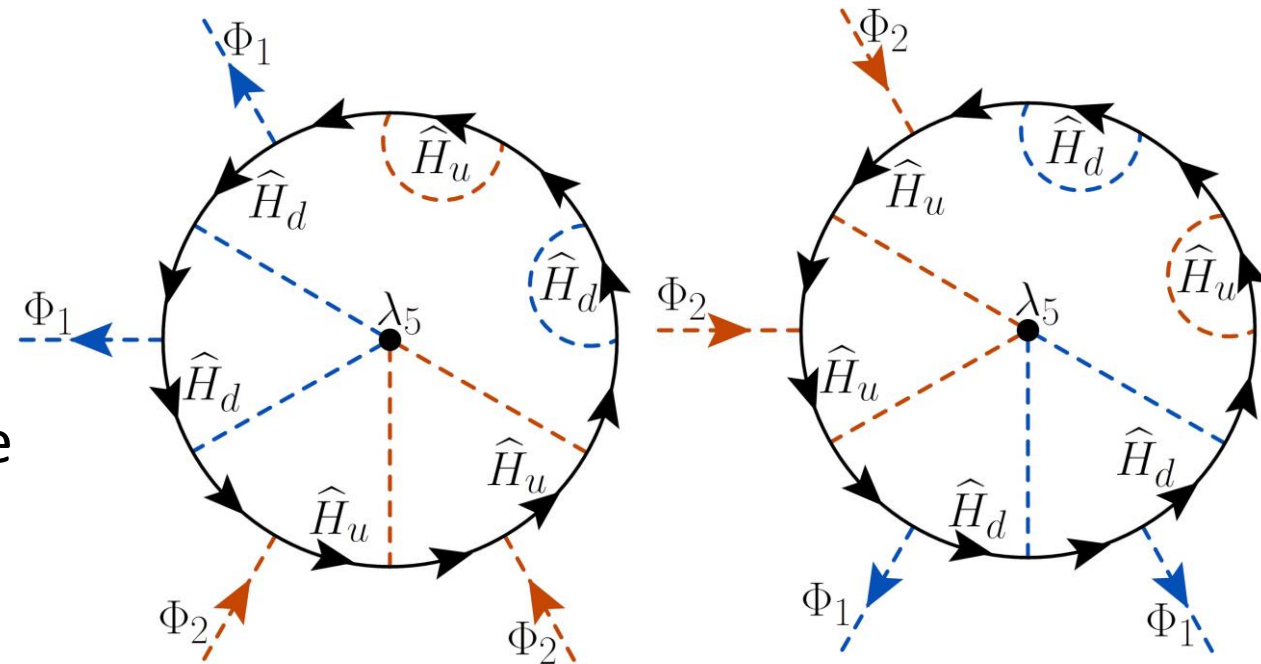
SU(2) structure preserved!

$\lambda_5$  at 6 loops

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# Type II – 6 loops

- For each diagram, there exists a second diagram with identical topology and momentum structure but in which  $u_R \leftrightarrow d_R$
- 3k diagrams proportional to  $\mathcal{J}$
- Always pairwise diagrams as  $\lambda_5$  is invariant under this symmetry! (Note that  $\lambda_1$  and  $\lambda_2$  are not, important later!)
- This means that we have the divergent contribution for  $\lambda_5$  to be REAL!



No divergent leaks at 6 loops



# No leaks of CP in the 2HDM at 6 loops

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- The contribution for both types is **real**, but for **different reasons!**
- Type I – Also cancel, but from the topology of the diagrams, see more in 2403.17052
- Type II – **Generalized CP symmetry** protects the CP violation at 6 loop. The rest of the Lagrangian does not respect this symmetry, do we have **leaks** at 7 loop?
- Complete the sequence:  
 $0, 0, 0, 0, 0, 0, 0, ?$

$\lambda_5$  at 7 loops

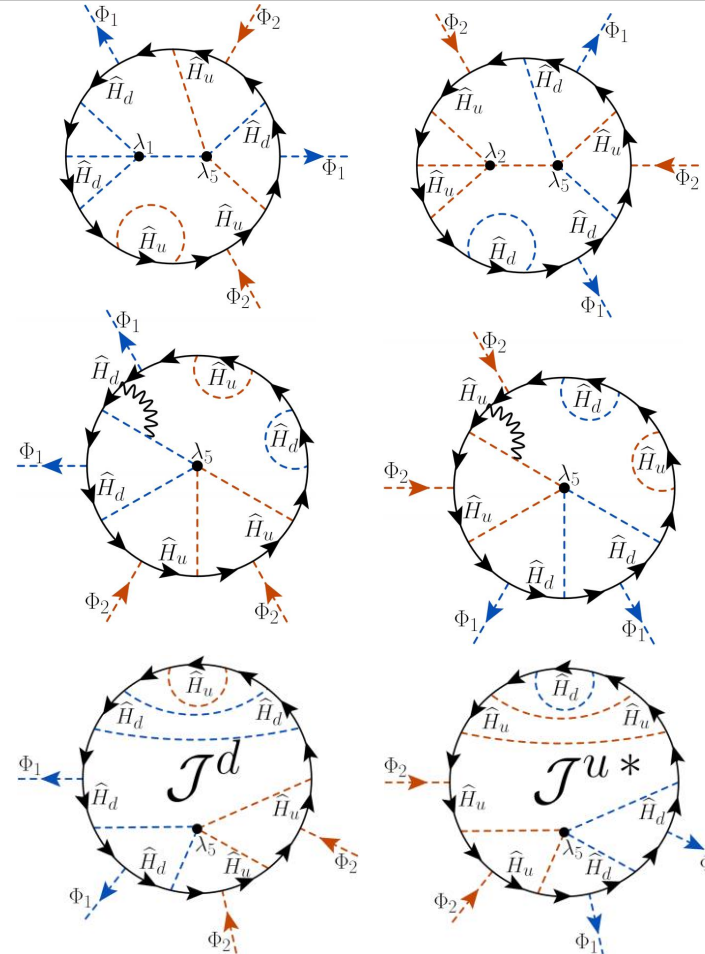


# Type II – Call the plumber! CP is leaking

GCP transformation we have  $\lambda_1 \leftrightarrow \lambda_2$ . This breaks the cancelation of imaginary part if  $\lambda_1 \neq \lambda_2$

The Hypercharge interaction differentiate u and d. This breaks the cancelation of imaginary proportional to the difference of Hypercharges.

Inclusion of one additional Yukawa breaks the symmetry between u and d. The relation now relates different objects and no cancelation of imaginary divergent piece!



# Is the Real 2HDM inconsistent?

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- If the RG evolution is indeed nonzero, even if extremely small, the **real 2HDM would not be a theoretically consistent model!** One should start to question if it makes sense to restrict the CP violation to zero when the model does not respect this decision.
- **There could be additional symmetries which we did not spot canceling the 7-loop diagrams.** Only brute force checking, or another formalism can tackle this.

$$\frac{d \operatorname{Im}(\lambda_5)}{d \ln \mu} = \frac{1}{(16\pi^2)^7} \begin{cases} [a^\lambda(\lambda_1 - \lambda_2) + a^{g'} g'^2 + a^y(y_t^2 - y_b^2 + \dots)] \lambda_5 \operatorname{Im}(\mathcal{J}) & \text{(type II)} \\ [b^{\lambda_3} \lambda_3 + b^{\lambda_4} \lambda_4 + \dots] \lambda_5 \operatorname{Im}(\mathcal{J}) & \text{(type I)} \end{cases}$$

# Extra material

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# General 2HDM

- Add a second Higgs doublet to the SM:
  - Immediately screw up **flavor** and **CP**! Need model building to avoid experimentally-excluded levels of flavor and CPV

$$\begin{aligned}
 V_{2HDM} = & m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - m_{12}^2 \phi_1^\dagger \phi_2 \\
 & + \frac{1}{2} \lambda_1 (\phi_1^\dagger \phi_1)^2 + \frac{1}{2} \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) \\
 & + \frac{1}{2} \lambda_5 (\phi_1^\dagger \phi_2)^2 + \lambda_6 (\phi_1^\dagger \phi_1) (\phi_1^\dagger \phi_2) + \lambda_7 (\phi_2^\dagger \phi_2) (\phi_1^\dagger \phi_2) + \text{h.c.}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}_{Yuk} = & -Y_d^{(1)} \bar{Q}_L \phi_1 d_R - Y_u^{(1)} \bar{Q}_L \tilde{\phi}_1 d_R \\
 & -Y_d^{(2)} \bar{Q}_L \phi_2 d_R - Y_u^{(2)} \bar{Q}_L \tilde{\phi}_2 d_R + \text{h.c.}
 \end{aligned}$$



# Natural flavor conservation

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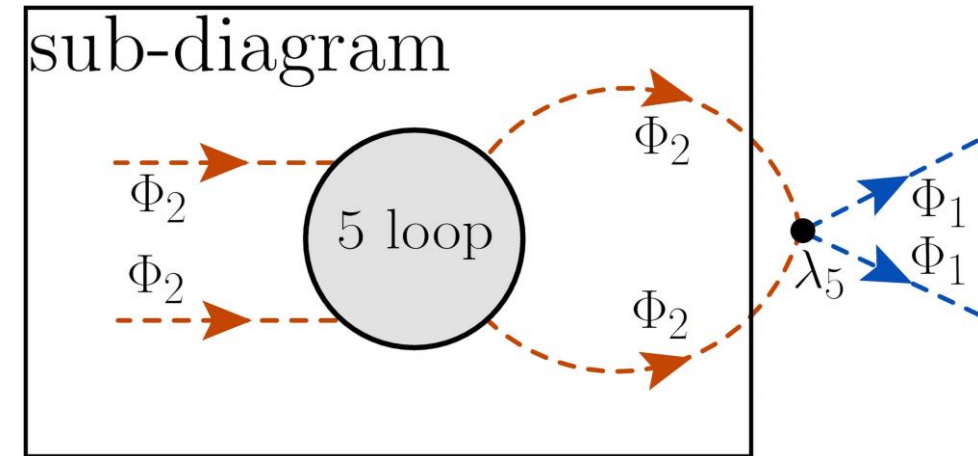
- Sidestep the FCNC problem by imposing Natural Flavor Conservation
- Easy to impose using a  $Z_2$  symmetry:

$$\begin{array}{l} \phi_1 \rightarrow -\phi_1 \quad \phi_2 \rightarrow \phi_2 \\ u_R \rightarrow u_R \quad d_R \rightarrow d_R \quad (\text{type I}) \\ u_R \rightarrow u_R \quad d_R \rightarrow -d_R \quad (\text{type II}) \end{array}$$

- $Z_2$  forces  $\lambda_6 = 0$ ,  $\lambda_7 = 0$  and  $m_{12}^2 = 0$
- $Z_2$  2HDM has no decoupling limit  $\rightarrow$  needs soft breaking ( $m_{12}^2 \neq 0$ )

# Type I – 6 loops

- Type I is special, the only way to create 2  $\phi_1$  are with the  $\lambda_5$  insertion.
- Cut the  $\lambda_5$  internal propagators  $\rightarrow$  separate the diagram into a 5 loop sub-diagram
- The constant contribution of the 5 loop diagram is Hermitian! The divergent contribution of the 6 loop diagram is then also Hermitian!
- The matching is harder to spot
- 10k diagrams proportional to  $\mathcal{J}$



**No divergent leaks at 6 loops**

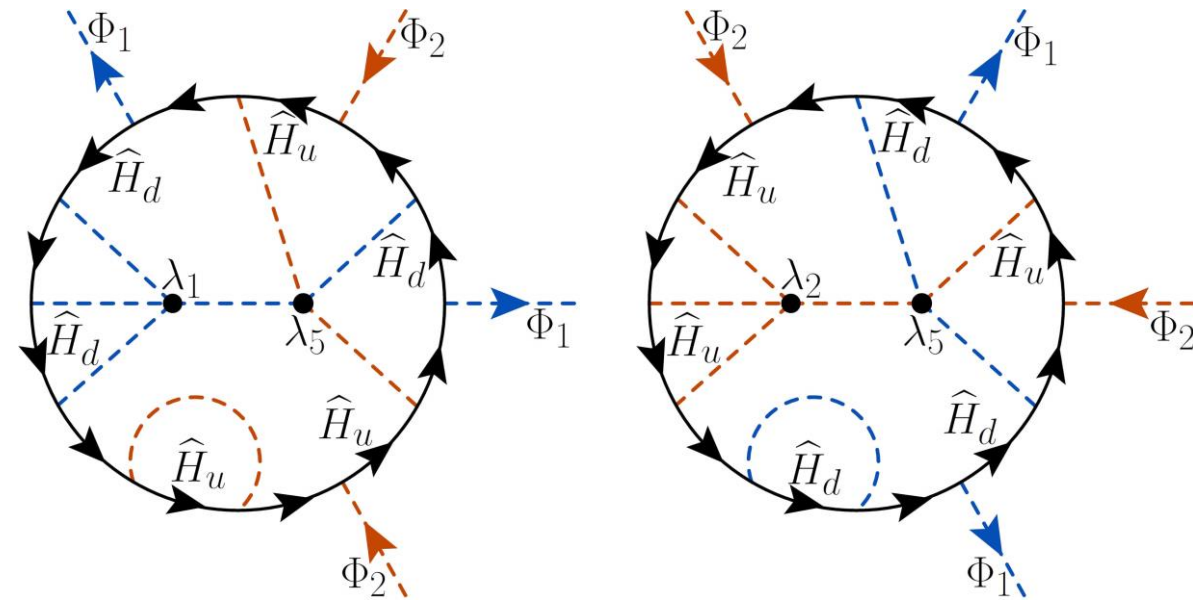




# Type II – Call the plumber! CP is leaking

- Under the GCP transformation we have  $\lambda_1 \leftrightarrow \lambda_2$ . This breaks the cancelation of imaginary part if  $\lambda_1 \neq \lambda_2$ !
- We expect an **imaginary divergent contribution** to  $\lambda_5$  at 7 loops proportional to

$$\lambda_5(\lambda_1 - \lambda_2) \text{Im}(\mathcal{J})$$

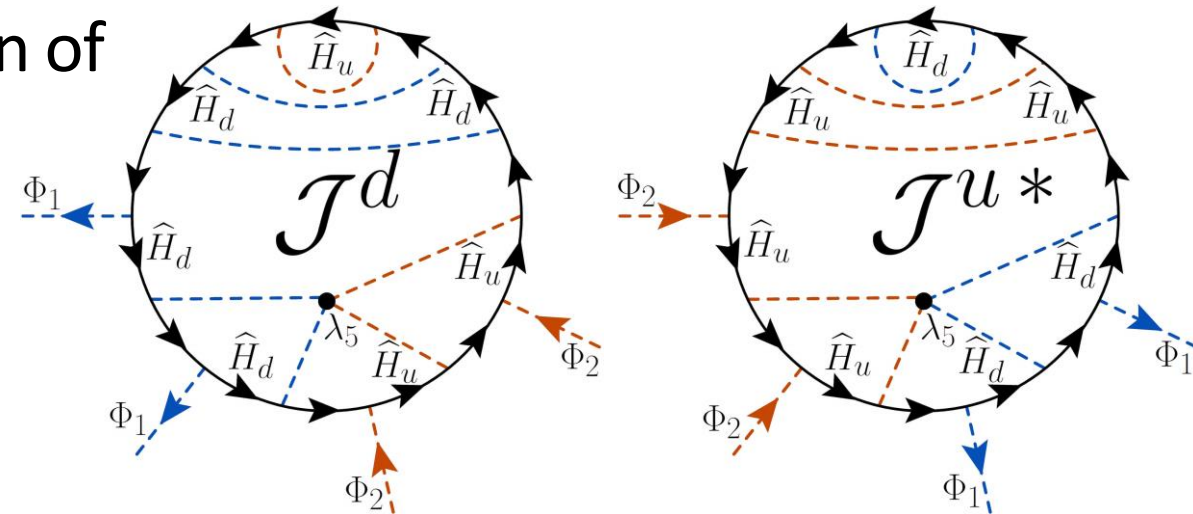


# Type II – Call the plumber! CP is leaking

- Inclusion of one additional Yukawa breaks the symmetry between u and d. The relation now relates different objects and no cancelation of imaginary divergent piece!

- We expect then **an imaginary divergent contribution** proportional to

$$\lambda_5 \text{Im}(\mathcal{J}^u - \mathcal{J}^d) = (y_t^2 - y_b^2 + \dots) \lambda_5 \text{Im}(\mathcal{J})$$

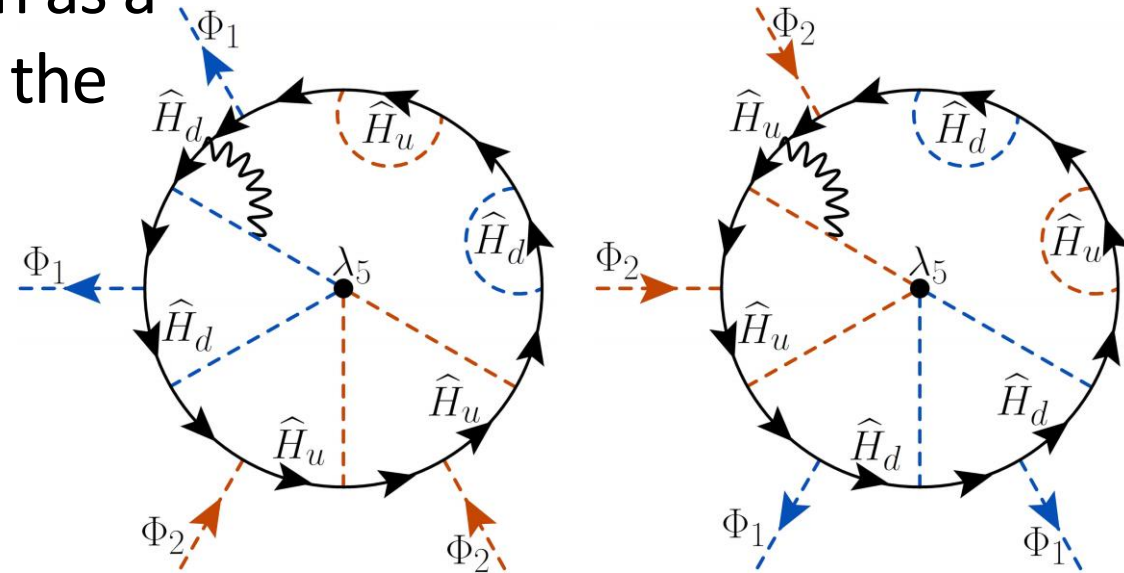


# Type II – Call the plumber! CP is leaking

- The Hypercharge interaction differentiate between u and d. This breaks the cancelation as a function of the difference of Hypercharge of the quarks!

- We expect then **an imaginary divergent contribution** proportional to

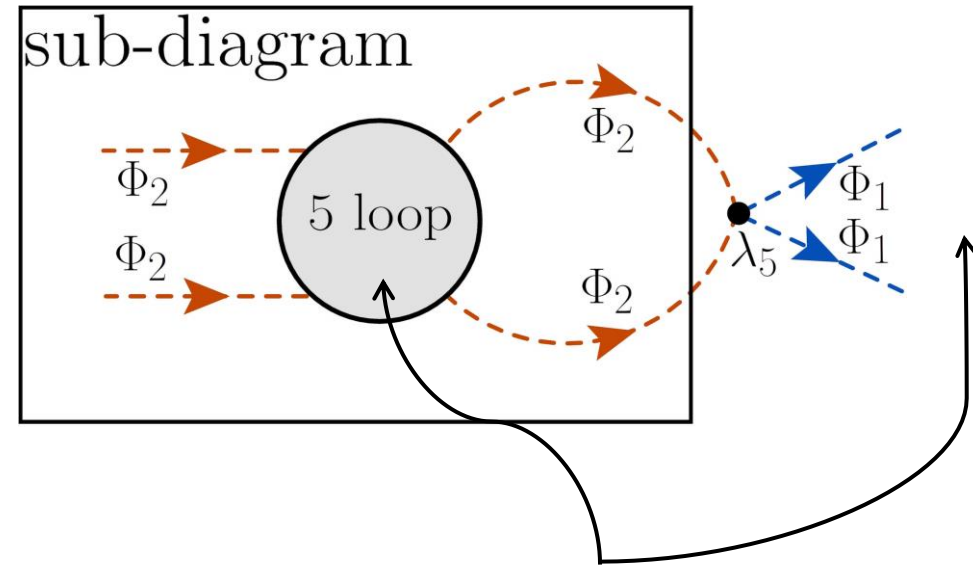
$$\lambda_5 g'^2 \text{Im}(\mathcal{J})$$



# Type I – Call the plumber! CP is leaking

- Diagrams which renormalize  $\lambda_2$  or that renormalize the external line cannot generate an imaginary divergent contribution

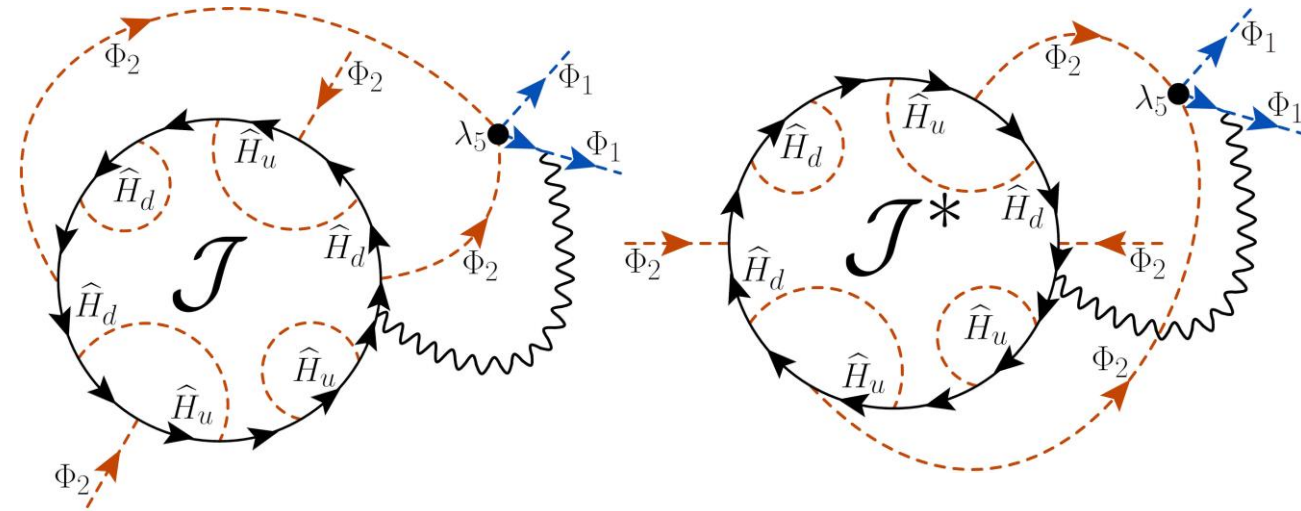
- However, not all diagrams have this topology!



Loops on the external legs or inside preserve the sub diagram structure

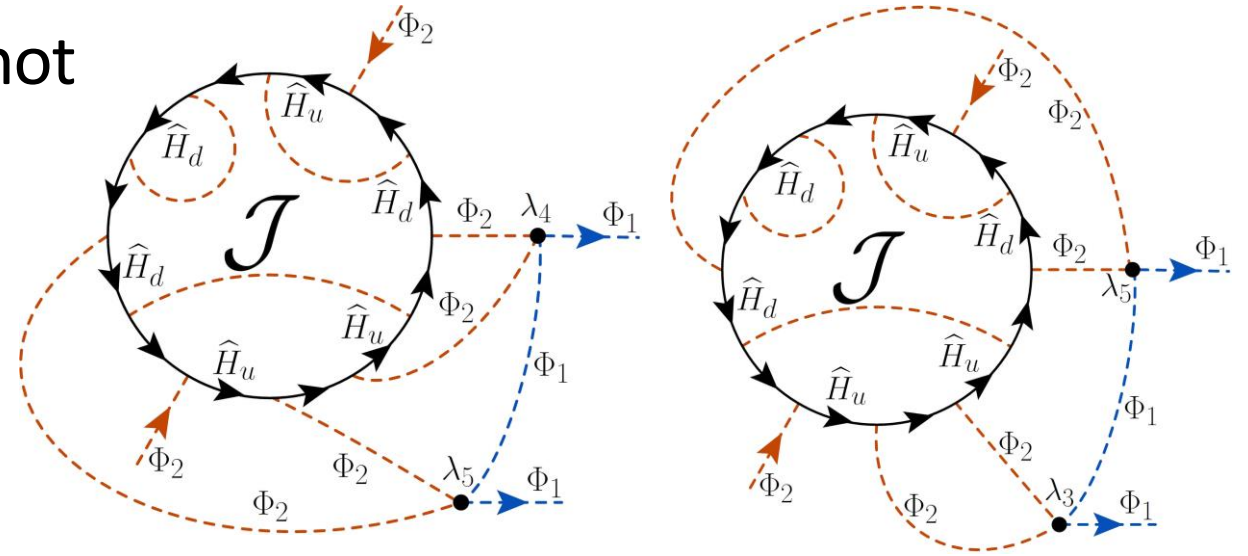
# Type I – Call the plumber! CP is leaking

- Gauge interactions can destroy the sub-diagram structure. Becomes harder to analyze!
- In the fermion line, u and d are indistinguishable. The gauge interactions should not be able to tell the primitive diagrams apart. However, no clear path to proof this



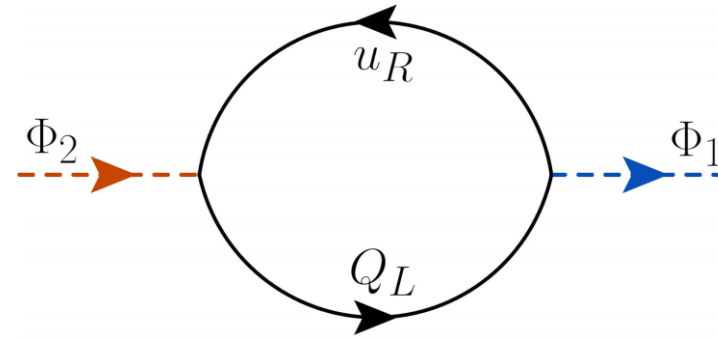
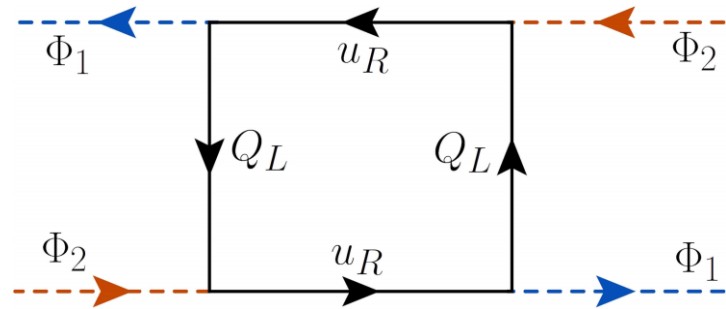
# Type I – Call the plumber! CP is leaking

- Diagrams containing  $\lambda_3$  or  $\lambda_4$  also do not cancel, this structure is impossible to construct from  $\mathcal{J}^*$ !
- We expect an **imaginary divergent contribution** proportional to  $\lambda_3$  and  $\lambda_4$
- No clear relation between the coefficients of these terms!



# Extra material

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# Extra material

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$$\mathcal{L}_{Yuk} = -Y_d^{(1)} \bar{Q}_L \phi_2 d_R - Y_u^{(2)} \bar{Q}_L \tilde{\phi}_2 d_R + \text{h.c.} \quad (\text{type I})$$

$$\frac{d \text{Im}(\lambda_5)}{d \ln \mu} \sim 10^{-24}$$

# Extra material

