Higher-order corrections for $t\bar{t}W$ production

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- **Higher-order soft-gluon corrections**
- $t\bar{t}W^+$ **and** $t\bar{t}W^-$ **cross sections**
- **Top-quark** p_T **and rapidity distributions**

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$t\bar{t}W$ production

observation of $t\bar{t}W$ events at 7, 8, 13 TeV collisions at the LHC

measurements are significantly higher than theoretical predictions

QCD corrections at NLO are large, $\sim 47\%$ at 13.6 TeV

electroweak corrections are smaller but significant

the QCD corrections are dominated by soft-gluon emission

further improvement in theoretical accuracy by the inclusion of higher-order soft-gluon corrections

$\rightarrow$ approximate NNLO (aNNLO) and approximate N^{3}LO (aN^{3}LO) predictions

Soft-gluon corrections

They are important for top-quark processes and they approximate known exact results at NLO and NNLO very well

partonic processes $q(p_q) + \bar{q}'(p_{\bar{q}'}) \to t(p_t) + \bar{t}(p_{\bar{t}}) + W(p_W)$

define $s = (p_q + p_{\bar{q}'})^2$**,** $t = (p_q - p_t)^2$**,** $u = (p_{\bar{q}'} - p_t)^2$

we define the threshold variable
$s_4 = (p_{\bar{t}} + p_W + p_g)^2 - (p_{\bar{t}} + p_W)^2 = s + t + u - m_t^2 - (p_{\bar{t}} + p_W)^2$
where extra gluon with p_g **emitted**

At partonic threshold $p_g \to 0$ **and thus** $s_4 \to 0$

Soft corrections $\left[\frac{\ln^k(s_4/m_t^2)}{s_4}\right]_+$ **with** $k \le 2n-1$ **for the order** α_s^n **corrections**

Resum these soft corrections for the double-differential cross section

Finite-order expansions → no prescription needed or used
(this avoids underestimating the size of the corrections)

Approximate NNLO (aNNLO) and approximate N^{3}LO (aN^{3}LO) predictions

for cross sections and differential distributions

Soft-gluon Resummation

$$d\sigma_{pp\to t\bar{t}W} = \sum_{q,\bar{q}'} \int dx_a\, dx_b\, \phi_{q/p}(x_a,\mu_F)\, \phi_{\bar{q}'/p}(x_b,\mu_F)\, d\hat{\sigma}_{q\bar{q}'\to t\bar{t}W}(s_4,\mu_F)$$

take Laplace transforms $d\tilde{\hat{\sigma}}_{q\bar{q}'\to t\bar{t}W}(N) = \int (ds_4/s)\, e^{-Ns_4/s} d\hat{\sigma}_{q\bar{q}'\to t\bar{t}W}(s_4)$

and $\tilde{\phi}(N) = \int_0^1 e^{-N(1-x)}\, \phi(x)\, dx$ **with transform variable** N

Then

$$d\tilde{\sigma}_{q\bar{q}'\to t\bar{t}W}(N) = \tilde{\phi}_{q/q}(N_q,\mu_F)\, \tilde{\phi}_{\bar{q}'/\bar{q}'}(N_{\bar{q}'},\mu_F)\, d\tilde{\hat{\sigma}}_{q\bar{q}'\to t\bar{t}W}(N,\mu_F)$$

Refactorization for the cross section

$$d\sigma_{q\bar{q}'\to t\bar{t}W}(N) = \tilde{\psi}_q(N_q,\mu_F)\, \tilde{\psi}_{\bar{q}'}(N_{\bar{q}'},\mu_F)\, \mathrm{tr}\left\{H_{q\bar{q}'\to t\bar{t}W}\ \tilde{S}_{q\bar{q}'\to t\bar{t}W}\left(\frac{\sqrt{s}}{N\mu_F}\right)\right\}$$

ψ_q, $\psi_{\bar{q}'}$ $\to$ **collinear emission from incoming partons**

$H_{q\bar{q}'\to t\bar{t}W}$ **is hard function** $\to$ **short distance**

$S_{q\bar{q}'\to t\bar{t}W}$ **is soft function** $\to$ **noncollinear soft gluons**

Thus

$$d\tilde{\hat{\sigma}}_{q\bar{q}'\to t\bar{t}W}(N) = \frac{\tilde{\psi}_{q/q}(N_q,\mu_F)\, \tilde{\psi}_{\bar{q}'/\bar{q}'}(N_{\bar{q}'},\mu_F)}{\tilde{\phi}_{q/q}(N_q,\mu_F)\, \tilde{\phi}_{\bar{q}'/\bar{q}'}(N_{\bar{q}'},\mu_F)}\, \mathrm{tr}\left\{H_{q\bar{q}'\to t\bar{t}W}\ \tilde{S}_{q\bar{q}'\to t\bar{t}W}\left(\frac{\sqrt{s}}{N\mu_F}\right)\right\}$$

$S_{q\bar{q}' \to t\bar{t}W}$ **satisfies the renormalization group equation**

$$\left(\mu_R \frac{\partial}{\partial \mu_R} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{q\bar{q}' \to t\bar{t}W} = -\Gamma^{\dagger}_{S\, q\bar{q}' \to t\bar{t}W}\, S_{q\bar{q}' \to t\bar{t}W} - S_{q\bar{q}' \to t\bar{t}W}\, \Gamma_{S\, q\bar{q}' \to t\bar{t}W}$$

Soft anomalous dimension $\Gamma_{S\, q\bar{q}' \to t\bar{t}W}$ **controls the evolution of the soft function which gives the exponentiation of logarithms of** N

Renormalization group evolution → resummation

$$d\tilde{\hat{\sigma}}^{\rm resum}_{q\bar{q}' \to t\bar{t}W}(N) = \exp\left[\sum_{i=q,\bar{q}'} E_i(N_i)\right] \exp\left[\sum_{i=q,\bar{q}'} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\gamma_{i/i}(N_i)\right]$$
$$\times {\rm tr}\left\{H_{q\bar{q}' \to t\bar{t}W}\left(\alpha_s(\sqrt{s})\right) \bar{P} \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\, \Gamma^{\dagger}_{S\, q\bar{q}' \to t\bar{t}W}(\alpha_s(\mu))\right]\right.$$
$$\left. \times \tilde{S}_{q\bar{q}' \to t\bar{t}W}\left(\alpha_s\left(\frac{\sqrt{s}}{N}\right)\right) P \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\, \Gamma_{S\, q\bar{q}' \to t\bar{t}W}(\alpha_s(\mu))\right]\right\}$$

$H_{q\bar{q}' \to t\bar{t}W}$ **and** $\tilde{S}_{q\bar{q}' \to t\bar{t}W}$ **and** $\Gamma_{S\, q\bar{q}' \to t\bar{t}W}$ **are** 2×2 **matrices**

choose color tensor basis of s-channel singlet and octet exchange
$c_1^{q\bar{q}'\to t\bar{t}W}=\delta_{ab}\delta_{12}$, $c_2^{q\bar{q}'\to t\bar{t}W}=T^c_{ba}\,T^c_{12}$

The four matrix elements of $\Gamma_{S\,q\bar{q}'\to t\bar{t}W}$ are at one loop

$$\Gamma^{(1)}_{11\,q\bar{q}'\to t\bar{t}W}=\Gamma^{(1)}_{\rm cusp}\,, \quad \Gamma^{(1)}_{12\,q\bar{q}'\to t\bar{t}W}=\frac{C_F}{2N_c}\Gamma^{(1)}_{21\,q\bar{q}'\to t\bar{t}W}\,, \quad \Gamma^{(1)}_{21\,q\bar{q}'\to t\bar{t}W}=\ln\left(\frac{(t-m_t^2)(t'-m_t^2)}{(u-m_t^2)(u'-m_t^2)}\right),$$

$$\Gamma^{(1)}_{22\,q\bar{q}'\to t\bar{t}W}=\left(1-\frac{C_A}{2C_F}\right)\left[\Gamma^{(1)}_{\rm cusp}+2C_F\ln\left(\frac{(t-m_t^2)(t'-m_t^2)}{(u-m_t^2)(u'-m_t^2)}\right)\right]+\frac{C_A}{2}\left[\ln\left(\frac{(t-m_t^2)(t'-m_t^2)}{s\,m_t^2}\right)-1\right]$$

where $\Gamma^{(1)}_{\rm cusp}=-C_F\left(L_{\beta_t}+1\right)$ is the one-loop QCD massive cusp anomalous dimension,
with $L_{\beta_t}=(1+\beta_t^2)/(2\beta_t)\ln[(1-\beta_t)/(1+\beta_t)]$ and $\beta_t=\sqrt{1-4m_t^2/s'}$,
$s'=(p_t+p_{\bar{t}})^2$, $t'=(p_{\bar{q}'}-p_{\bar{t}})^2$, $u'=(p_q-p_{\bar{t}})^2$

At two loops

$$\Gamma^{(2)}_{11\,q\bar{q}'\to t\bar{t}W}=\Gamma^{(2)}_{\rm cusp}\,, \quad \Gamma^{(2)}_{12\,q\bar{q}'\to t\bar{t}W}=\left(K_2-C_A\,N_2^{\beta_t}\right)\Gamma^{(1)}_{12\,q\bar{q}'\to t\bar{t}W}\,, \quad \Gamma^{(2)}_{21\,q\bar{q}'\to t\bar{t}W}=\left(K_2+C_A\,N_2^{\beta_t}\right)\Gamma^{(1)}_{21\,q\bar{q}'\to t\bar{t}W}\,,$$

$$\Gamma^{(2)}_{22\,q\bar{q}'\to t\bar{t}W}=K_2\,\Gamma^{(1)}_{22\,q\bar{q}'\to t\bar{t}W}+\left(1-\frac{C_A}{2C_F}\right)\left(\Gamma^{(2)}_{\rm cusp}-K_2\,\Gamma^{(1)}_{\rm cusp}\right)+\frac{1}{4}C_A^2(1-\zeta_3)$$

where $K_2=C_A(67/36-\zeta_2/2)-5n_f/18$ with n_f the number of light-quark flavors,

$$N_2^{\beta_t}=\frac{1}{4}\ln^2\left(\frac{1-\beta_t}{1+\beta_t}\right)+\frac{(1+\beta_t^2)}{8\beta_t}\left[\zeta_2-\ln^2\left(\frac{1-\beta_t}{1+\beta_t}\right)-{\rm Li}_2\left(\frac{4\beta_t}{(1+\beta_t)^2}\right)\right]$$

and

$$
\begin{aligned}
\Gamma_{\rm cusp}^{(2)} \quad &= \quad K_2\,\Gamma_{\rm cusp}^{(1)}+C_F C_A\left\{\frac{1}{2}+\frac{\zeta_2}{2}+\frac{1}{2}\ln^2\left(\frac{1-\beta_t}{1+\beta_t}\right)\right. \\
&+\frac{(1+\beta_t^2)}{4\beta_t}\left[\zeta_2\ln\left(\frac{1-\beta_t}{1+\beta_t}\right)-\ln^2\left(\frac{1-\beta_t}{1+\beta_t}\right)+\frac{1}{3}\ln^3\left(\frac{1-\beta_t}{1+\beta_t}\right)-{\rm Li}_2\left(\frac{4\beta_t}{(1+\beta_t)^2}\right)\right] \\
&+\frac{(1+\beta_t^2)^2}{8\beta_t^2}\left[-\zeta_3-\zeta_2\ln\left(\frac{1-\beta_t}{1+\beta_t}\right)-\frac{1}{3}\ln^3\left(\frac{1-\beta_t}{1+\beta_t}\right)-\ln\left(\frac{1-\beta_t}{1+\beta_t}\right){\rm Li}_2\left(\frac{(1-\beta_t)^2}{(1+\beta_t)^2}\right)\right. \\
&\left.\left.+{\rm Li}_3\left(\frac{(1-\beta_t)^2}{(1+\beta_t)^2}\right)\right]\right\}
\end{aligned}
$$

is the two-loop massive cusp anomalous dimension in QCD

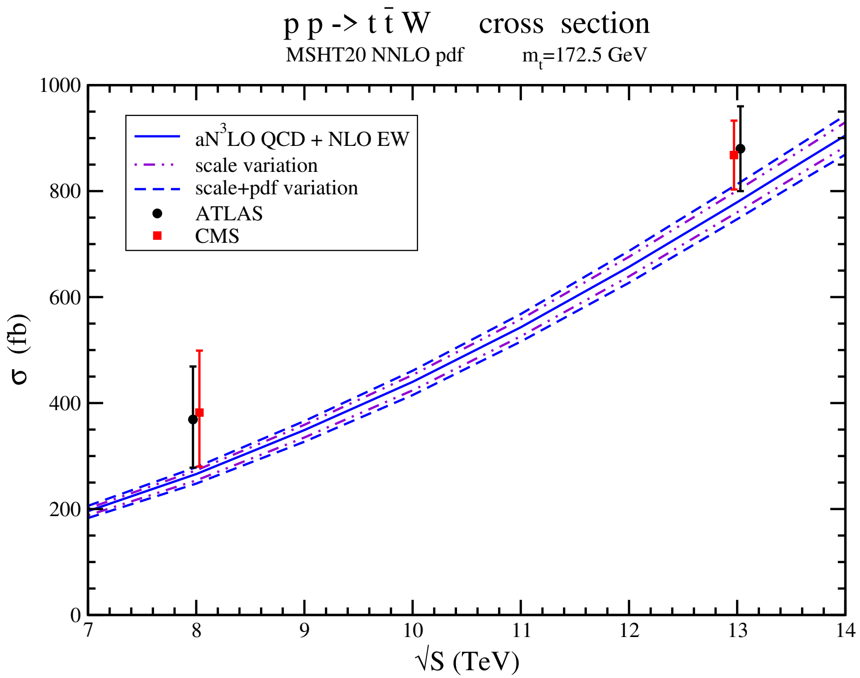
Expansions of the resummed cross section to fixed order

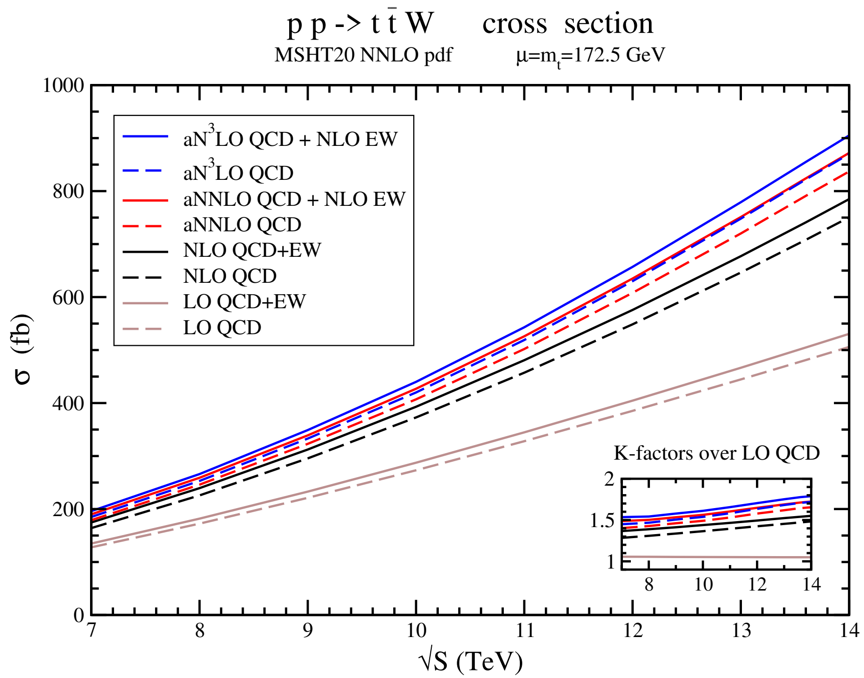
NNLO expansions (aNNLO) are consistent with (almost exact) NNLO results

aN^{3}LO is the state of the art

Electroweak corrections are also included

Cross sections for $t\bar{t}W$ production





large K-factors

improved agreement with data at aN^{3}LO

$t\bar{t}W$ cross sections

$t\bar{t}W$ cross sections (fb) in pp collisions at the LHC					
σ in fb	7 TeV	8 TeV	13 TeV	13.6 TeV	14 TeV
LO QCD	128^{+39}_{-28}	172^{+51}_{-36}	445^{+114}_{-84}	481^{+121}_{-90}	506^{+126}_{-94}
LO QCD+EW	135^{+41}_{-29}	182^{+53}_{-38}	467^{+119}_{-88}	505^{+127}_{-94}	531^{+132}_{-98}
NLO QCD	164^{+13}_{-17}	226^{+20}_{-23}	646^{+83}_{-74}	708^{+94}_{-82}	750^{+101}_{-88}
NLO QCD+EW	175^{+12}_{-17}	239^{+19}_{-23}	677^{+80}_{-74}	741^{+90}_{-82}	785^{+97}_{-88}
aNNLO QCD	179^{+6}_{-10}	246^{+9}_{-15}	720^{+29}_{-43}	791^{+32}_{-47}	837^{+34}_{-50}
aNNLO QCD + NLO EW	190^{+6}_{-10}	259^{+9}_{-15}	751^{+27}_{-43}	824^{+29}_{-47}	872^{+31}_{-50}
aN^{3}LO QCD	185^{+5}_{-8}	253^{+7}_{-12}	748^{+24}_{-19}	822^{+26}_{-20}	870^{+28}_{-21}
aN^{3}LO QCD + NLO EW	196^{+5}_{-8}	266^{+7}_{-12}	779^{+22}_{-19}	855^{+23}_{-20}	905^{+25}_{-21}

At 13.6 TeV

NLO QCD corrections → 47%

aNNLO QCD corrections → 17%

aN^{3}LO QCD corrections → 6%

electroweak NLO corrections → 7%

Total aN^{3}LO QCD+NLO EW cross section is 78% bigger than LO QCD

$t\bar{t}W^+$ and $t\bar{t}W^-$ cross sections

$t\bar{t}W^+$ and $t\bar{t}W^-$ cross sections (fb) in pp collisions at the LHC				
σ in fb	$t\bar{t}W^+$ 13 TeV	$t\bar{t}W^+$ 13.6 TeV	$t\bar{t}W^-$ 13 TeV	$t\bar{t}W^-$ 13.6 TeV
LO QCD	299^{+77}_{-57}	322^{+82}_{-60}	146^{+37}_{-28}	159^{+40}_{-30}
LO QCD+EW	313^{+80}_{-59}	337^{+85}_{-63}	154^{+39}_{-29}	168^{+42}_{-31}
NLO QCD	431^{+54}_{-49}	470^{+61}_{-54}	215^{+29}_{-25}	238^{+33}_{-28}
NLO QCD+EW	450^{+51}_{-48}	490^{+58}_{-53}	227^{+28}_{-25}	251^{+32}_{-28}
aNNLO QCD	480^{+19}_{-28}	525^{+21}_{-31}	240^{+10}_{-15}	266^{+11}_{-16}
aNNLO QCD + NLO EW	499^{+17}_{-28}	545^{+19}_{-31}	252^{+10}_{-15}	279^{+10}_{-16}
aN^{3}LO QCD	498^{+16}_{-12}	545^{+17}_{-13}	250^{+8}_{-7}	277^{+9}_{-7}
aN^{3}LO QCD + NLO EW	517^{+14}_{-12}	565^{+15}_{-13}	262^{+8}_{-7}	290^{+8}_{-7}

the $t\bar{t}W^+$ cross sections are larger than for $t\bar{t}W^-$

but the corrections are slightly bigger for $t\bar{t}W^-$

Comparison with 8 and 13 TeV CMS and ATLAS data

NLO and even aNNLO results are not sufficient

we need aN^{3}LO corrections to describe the data

At 8 TeV, measurements from

CMS: 382^{+117}_{-102} **fb**

and from

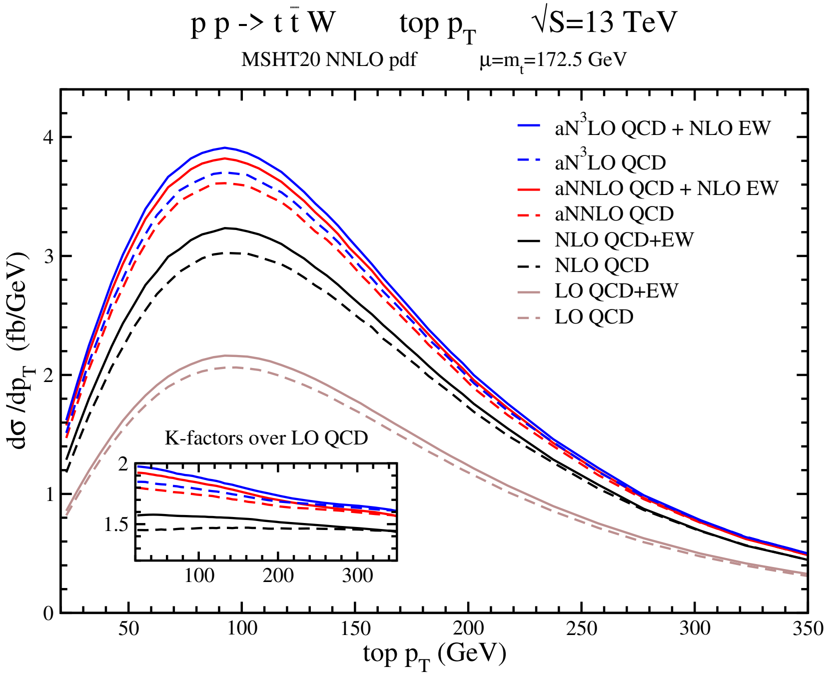
ATLAS: 369^{+100}_{-91} **fb**

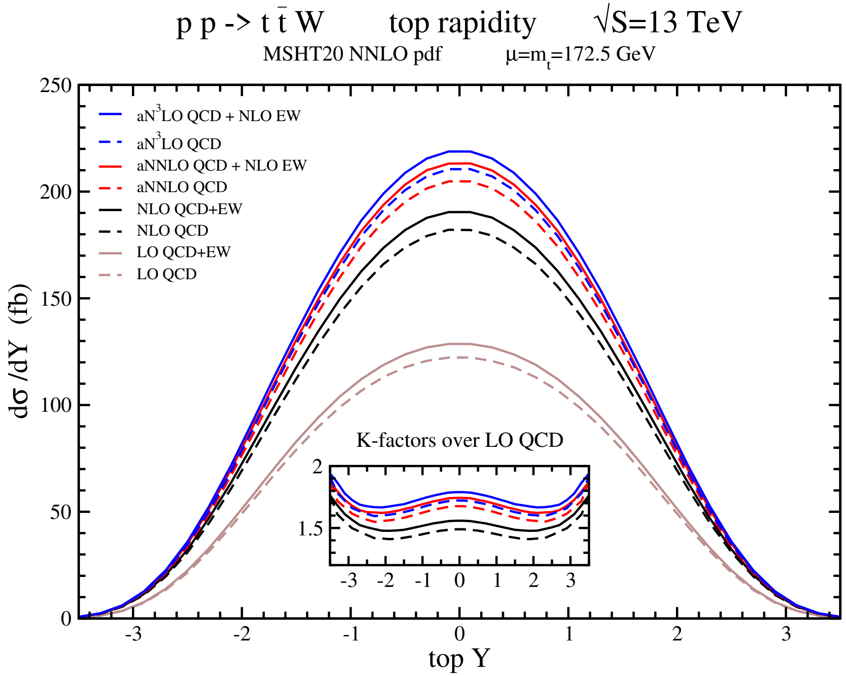
Theoretical prediction is
aN^{3}LO QCD + NLO EW: $266^{+7}_{-12}{}^{+6}_{-6}$ **fb**

At 13 TeV, CMS finds 868 ± 65 **fb with** $t\bar{t}W^+$ 553 ± 42 **fb and** $t\bar{t}W^-$ 343 ± 36 **fb**
while ATLAS finds 880 ± 80 **fb with** $t\bar{t}W^+$ 583 ± 58 **fb and** $t\bar{t}W^-$ 296 ± 40 **fb**

Theoretical prediction is
aN^{3}LO QCD + NLO EW: $779^{+22}_{-19}{}^{+12}_{-13}$ **fb with** $t\bar{t}W^+$ $517^{+14}_{-12}{}^{+8}_{-9}$ **fb and** $t\bar{t}W^-$ $262^{+8}_{-7}{}^{+4}_{-4}$ **fb**

Top-quark p_T and rapidity distributions in $t\bar{t}W$ production at 13 TeV

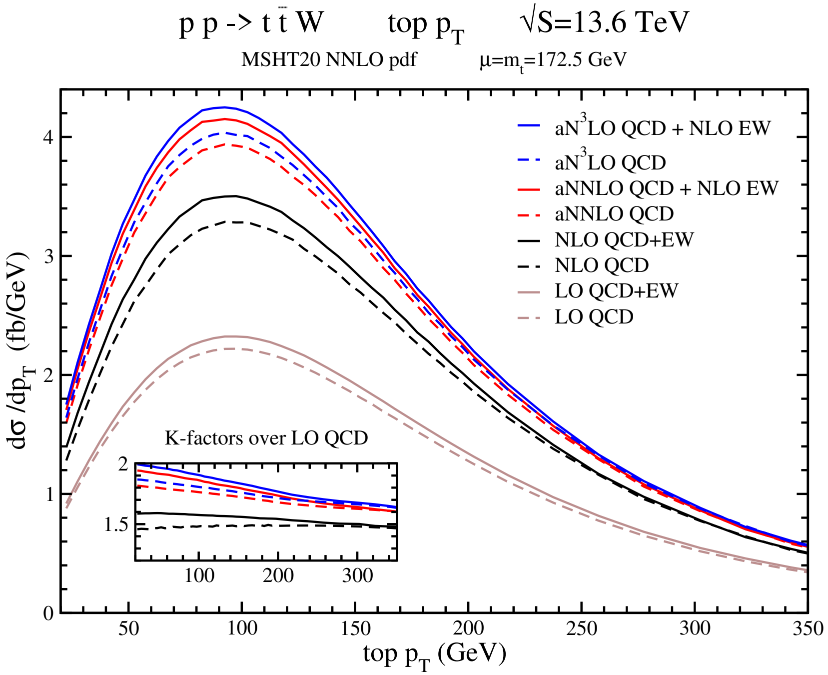


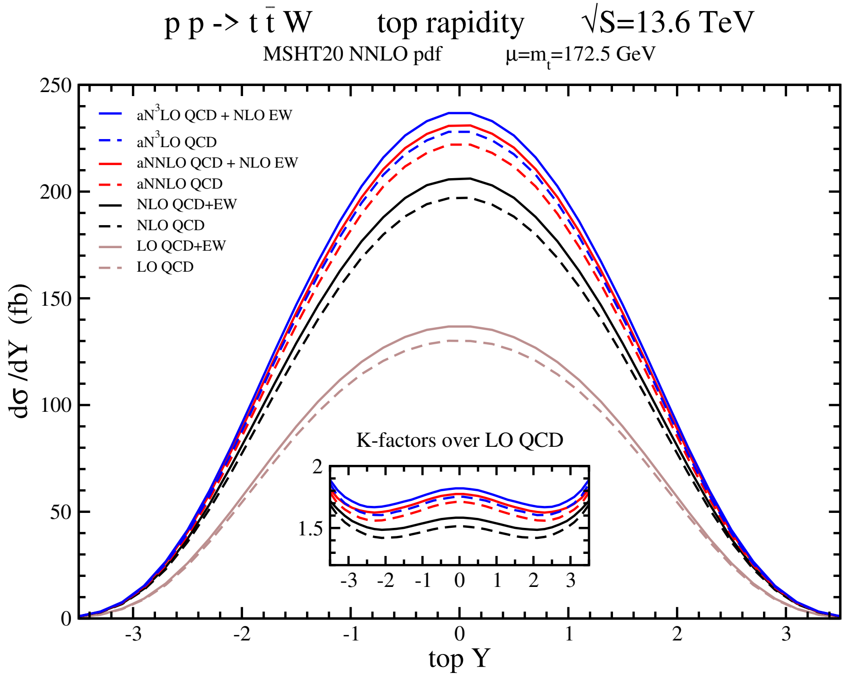


K-factors decrease at larger top p_T

K-factors increase at larger rapidities

Top-quark p_T and rapidity distributions in $t\bar{t}W$ at 13.6 TeV





K-factors decrease at larger top p_T

K-factors increase at larger rapidities

Summary

- $t\bar{t}W$ **production**
- **soft-gluon corrections through aN^{3}LO**
- **results for total cross sections and differential distributions**
- **higher-order corrections further enhance and improve the theoretical predictions**
- **agreement with LHC data within uncertainties**