College of Natural Sciences
Department of Physics
Fundamental Interactions Theory group

Correlated scalar perturbations and gravitational waves from axion inflation

Sofia Corbà   Lorenzo Sorbo

DPF - PHENO 2024

05/16/2024

Aim:

Study of the **correlation** between the **curvature perturbation** $\zeta(\mathbf{x})$ and the **squared amplitude of the tensor modes** $h_{ij}(\mathbf{x})h_{ij}(\mathbf{x})$, within the framework of **axion inflation**.

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- Gravitational Waves
- Inflation

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- Gravitational Waves
- Inflation
- Axion Inflation

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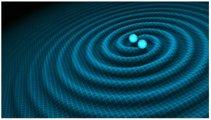
- Gravitational Waves
- Inflation
- Axion Inflation
- Final Results

Gravitational Waves

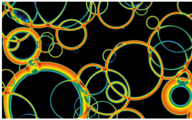
Gravitational Waves



(NASA)

Gravitational Waves

Astrophysical ← → Cosmological



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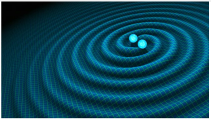

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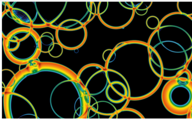

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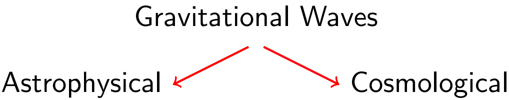
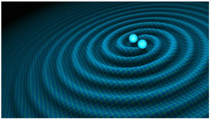


INFLATION

Gravitational Waves



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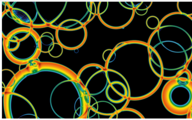

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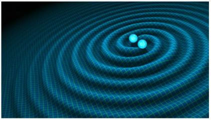
STOCHASTIC GRAVITATIONAL WAVE BACKGROUND



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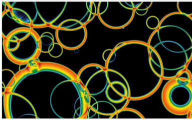

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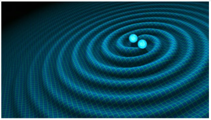
STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

How can we distinguish between astrophysical and cosmological gravitational wave backgrounds?



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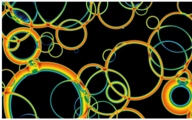

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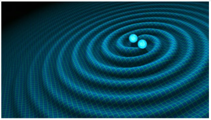

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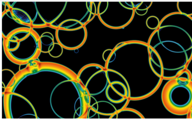

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Cross correlation with CMB anisotropies

$$\mathcal{C}_{h\zeta}(\mathbf{k},\,\tau)=\frac{1}{\mathcal{P}_h\,\sqrt{\mathcal{P}_\zeta}}\frac{k^3}{2\pi^2}\int\frac{d\mathbf{y}}{(2\pi)^{3/2}}e^{-i\mathbf{k}\mathbf{y}}\langle h_{ij}(\mathbf{x}+\mathbf{y},\,\tau)\,h_{ij}(\mathbf{x}+\mathbf{y},\,\tau)\,\zeta(\mathbf{x},\,\tau)\rangle$$

Inflation

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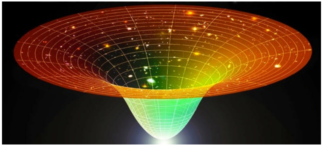
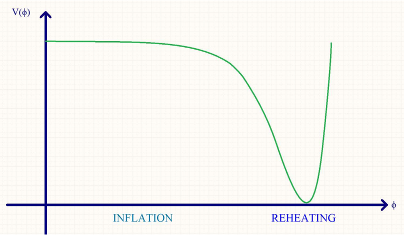


(Siegel)

Initial period of **accelerated expansion** governed by a **scalar** field.

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + V(\phi)\right)$$



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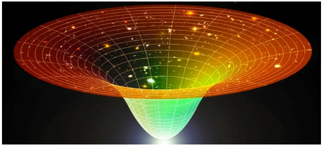

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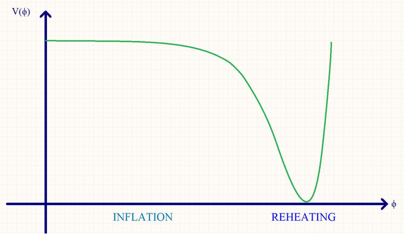

- Horizon problem
- Flatness problem

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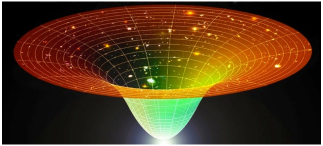

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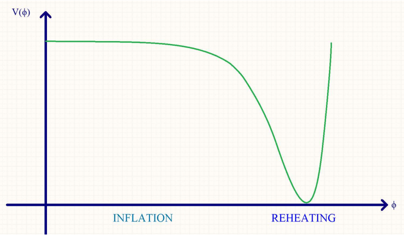

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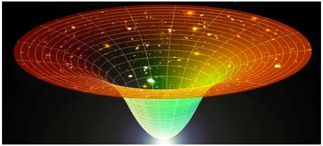

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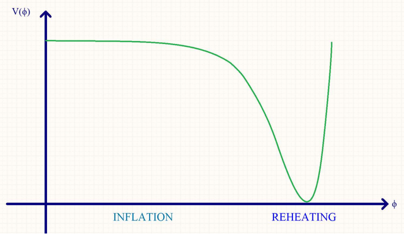

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- Large-scale structure

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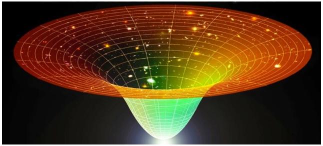

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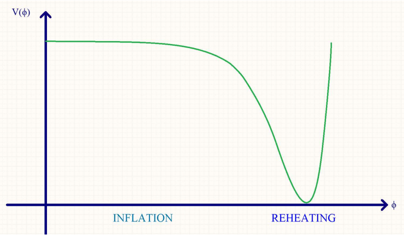

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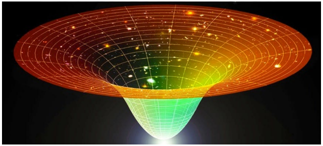

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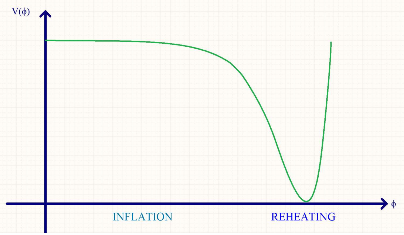

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⟶ Theory of Cosmological Perturbations

Scalar Fluctuations

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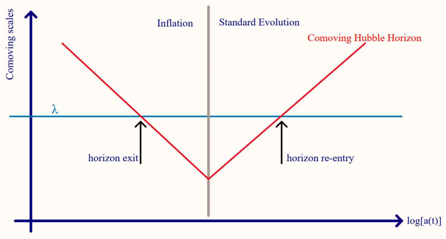

Fourier decomposition:

$$\delta\phi(\tau, \mathbf{x}) = \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} \delta\phi_k(\tau) e^{-i\mathbf{k}\mathbf{x}}$$

$$\delta\phi_k'' + 2\mathcal{H}\delta\phi_k' + k^2\delta\phi_k = 0$$

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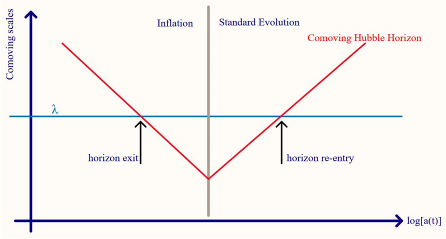

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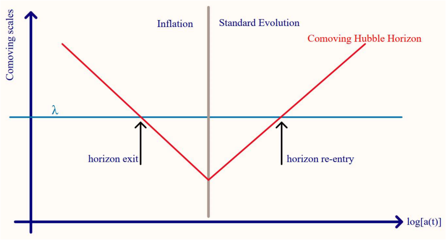

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Wavelengths are stretched $\rightarrow$ Macroscopic modes with constant amplitudes

Scalar Power Spectrum: $$\mathcal{P}_\zeta = \frac{k^3}{2\pi^2}\frac{H^2}{\dot{\phi}_0^2}|\delta\phi_k|^2 \xrightarrow{k\tau\to 0} \left(\frac{H^2}{2\pi\dot{\phi}_0}\right)^2$$

Tensor Fluctuations

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Gravitational Waves

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$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij}(\mathbf{x},\, \tau))\; dx^i\, dx^j\right], \quad h_{ii} = \partial_i h_{ij} = 0$$

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Einstein - Hilbert action 2nd order around FRW

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}R \quad \rightarrow \quad \frac{M_P^2}{8}\int d^4x\,a^2(h'_{ij}h'_{ij} - \partial_k h_{ij}\partial_k h_{ij})$$

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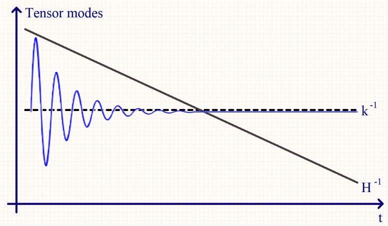

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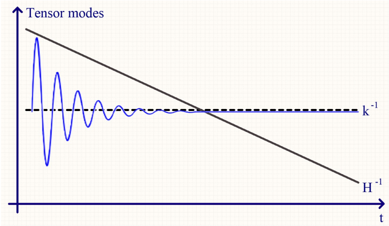

Tensor Power Spectrum:

$$\mathcal{P}_h = 2\left(\frac{2}{M_P}\right)^2\frac{k^3}{2\pi^2}|h_k|^2 \xrightarrow{k\tau\to 0} \frac{2H^2}{\pi^2 M_P^2}$$

Axion Inflation

Axion Inflation

Action:

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} \, F^{\mu\nu} - \frac{\phi}{8\,f} \frac{\epsilon^{\mu\nu\rho\lambda}}{\sqrt{-g}} F_{\mu\nu} \, F_{\rho\lambda} \right]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

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Perturbations:

Inflaton: $\phi(\mathbf{x},\,\tau) = \phi_0(\tau) + \delta\phi(\mathbf{x},\,\tau)$

Metric: $ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij}(\mathbf{x},\,\tau))\;dx^i\,dx^j\right]$

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Canonically Normalized Perturbations:

$$\Phi(\mathbf{x}, \, \tau) = a(\tau) \, \delta\phi(\mathbf{x}, \, \tau) \qquad H_{ij}(\mathbf{x}, \, \tau) \equiv \frac{M_P}{2} \, a(\tau) \, h_{ij}(\mathbf{x}, \, \tau)$$

Axion Inflation

Equations:

$$A_i'' - \nabla^2 A_i - \frac{\phi_0'}{f} \epsilon^{ijk} \, \partial_j A_k = 0$$

$$\Phi'' - \frac{a''}{a} \Phi - \nabla^2 \Phi + \frac{1}{f\,a} \epsilon^{ijk} \, A_i' \, \partial_j A_k = 0$$

$$H_{ij}'' - \frac{a''}{a} H_{ij} - \nabla^2 H_{ij} + \frac{1}{a\,M_P} \left[A_i' \, A_j' - (\partial_i A_k - \partial_k A_i)(\partial_j A_k - \partial_k A_j) \right] = 0$$

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$$\Phi = \underbrace{\Phi_{\rm V}}_{\textbf{Homogeneous}} + \underbrace{\Phi_{\rm S}}_{\textbf{Particular}}$$

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$$H_{ij}'' - \frac{a''}{a} H_{ij} - \nabla^2 H_{ij} + \frac{1}{a\,M_P} \left[A_i' \, A_j' - (\partial_i A_k - \partial_k A_i)(\partial_j A_k - \partial_k A_j) \right] = 0$$

Scalar:

$$\Phi = \underbrace{\Phi_{\mathrm{V}}}_{\text{Homogeneous}} + \underbrace{\Phi_{\mathrm{S}}}_{\textbf{Particular}}$$

Tensor:

$$H_{ij} = \underbrace{H_{ij,\mathrm{V}}}_{\text{Homogeneous}} + \underbrace{H_{ij,\mathrm{S}}}_{\textbf{Particular}}$$

Axion Inflation

$$A_i(\mathbf{x},\,\tau) = \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} \sum_{\lambda=\pm} e_i^{\lambda}(\widehat{\mathbf{k}})\, e^{i\mathbf{k}\mathbf{x}} \left[A_\lambda(k,\,\tau)\,\hat{a}_\lambda(\mathbf{k}) + A_\lambda^*(k,\,\tau)\,\hat{a}_\lambda^\dagger(-\mathbf{k})\right]$$

Axion Inflation

$$A_i(\mathbf{x},\,\tau) = \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} \sum_{\lambda=\pm} e_i^\lambda(\widehat{\mathbf{k}})\, e^{i\mathbf{k}\mathbf{x}} \left[A_\lambda(k,\,\tau)\,\hat{a}_\lambda(\mathbf{k}) + A_\lambda^*(k,\,\tau)\,\hat{a}_\lambda^\dagger(-\mathbf{k})\right]$$

$$\boxed{\frac{d^2\,A_\pm}{d\tau^2} + \left(k^2 \pm 2\,k\,\frac{\xi}{\tau}\right)\,A_\pm = 0} \qquad \xi = \frac{\dot{\phi}_0}{2\,f\,H}$$

Axion Inflation

$$A_i(\mathbf{x},\,\tau) = \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} \sum_{\lambda=\pm} e_i^{\lambda}(\widehat{\mathbf{k}})\, e^{i\mathbf{k}\mathbf{x}} \left[A_\lambda(k,\,\tau)\,\hat{a}_\lambda(\mathbf{k}) + A_\lambda^*(k,\,\tau)\,\hat{a}_\lambda^\dagger(-\mathbf{k})\right]$$

$$\boxed{\frac{d^2\,A_\pm}{d\tau^2} + \left(k^2 \pm 2\,k\,\frac{\xi}{\tau}\right)\,A_\pm = 0} \qquad \xi = \frac{\dot{\phi}_0}{2\,f\,H}$$

$$\downarrow$$

$$\boxed{A_+(k,\,\tau) \simeq \frac{1}{\sqrt{2\,k}}\,\left(-\frac{k\,\tau}{2\,\xi}\right)^{1/4}\,e^{-2\,\sqrt{-2\,\xi\,k\,\tau}+\pi\,\xi}}$$

EXPONENTIALLY AMPLIFIED

[M. M. Anber and L. Sorbo, 2006]

Axion Inflation

$$\Phi_{\rm S}(\mathbf{q},\,\tau) = i\int d\tau'\,G_q(\tau,\,\tau')\frac{H\tau'}{f}\,\epsilon^{ijk}\int\frac{d\mathbf{p}}{(2\pi)^{3/2}}\,A'_i(\mathbf{p},\,\tau')\,(\mathbf{q}-\mathbf{p})_j\,A_k(\mathbf{q}-\mathbf{p},\,\tau')$$

$$H_{ij,\,\rm S}(\mathbf{q},\,\tau) = \int d\tau'\,G_q(\tau,\tau')\,\frac{H\,\tau'}{M_P}\int\frac{d\mathbf{p}}{(2\pi)^{3/2}}\,A'_i(\mathbf{p},\,\tau')\,A'_j(\mathbf{q}-\mathbf{p},\,\tau')$$

$$G_k(\tau,\tau') = \frac{(1+k^2\,\tau\,\tau')\,\sin(k\,(\tau-\tau'))+k\,(\tau'-\tau)\,\cos(k\,(\tau-\tau'))}{k^3\,\tau\,\tau'}\,\Theta(\tau-\tau')$$

Axion Inflation

$$\Phi_{\rm S}(\mathbf{q},\,\tau) = i\,\int d\tau'\,G_q(\tau,\,\tau')\frac{H\tau'}{f}\,\epsilon^{ijk}\int\frac{d\mathbf{p}}{(2\pi)^{3/2}}\,A_i'(\mathbf{p},\,\tau')\,(\mathbf{q}{-}\mathbf{p})_j A_k(\mathbf{q}{-}\mathbf{p},\,\tau')$$

$$H_{ij,\,{\rm S}}(\mathbf{q},\,\tau) = \int d\tau'\,G_q(\tau,\tau')\,\frac{H\,\tau'}{M_P}\int\frac{d\mathbf{p}}{(2\pi)^{3/2}}\,A_i'(\mathbf{p},\,\tau')\,A_j'(\mathbf{q}-\mathbf{p},\,\tau')$$

$$G_k(\tau,\tau') = \frac{(1+k^2\,\tau\,\tau')\,\sin(k\,(\tau-\tau'))+k\,(\tau'-\tau)\,\cos(k\,(\tau-\tau'))}{k^3\,\tau\,\tau'}\,\Theta(\tau{-}\tau')$$

$$\mathcal{P}_\zeta = \mathcal{P}_{\zeta,{\rm V}} + \mathcal{P}_{\zeta,\,{\rm S}} \simeq \frac{H^4}{4\pi^2\dot\phi_0^2} + 4.8\times10^{-8}\,\frac{H^8}{\dot\phi_0^4}\,\frac{e^{4\pi\xi}}{\xi^6}$$

[N. Barnaby and M. Peloso, 2011]

$$\mathcal{P}_h = \mathcal{P}_{h,\,{\rm V}} + \mathcal{P}_{h,\,{\rm S}} \simeq \frac{2\,H^2}{\pi^2\,M_P^2} + 8.7\times10^{-8}\,\frac{H^4}{M_P^4}\,\frac{e^{4\pi\xi}}{\xi^6}$$

[L. Sorbo, 2011]

$\mathcal{C}_{h\zeta}$ correlation

$$\mathcal{C}_{h\zeta}(\mathbf{k},\,\tau) = \frac{1}{\mathcal{P}_h\,\sqrt{\mathcal{P}_\zeta}}\,\frac{k^3}{2\pi^2}\int\frac{d\mathbf{y}}{(2\pi)^{3/2}}e^{-i\mathbf{k}\mathbf{y}}\langle h_{ij}(\mathbf{x}{+}\mathbf{y},\,\tau)\,h_{ij}(\mathbf{x}{+}\mathbf{y},\,\tau)\,\zeta(\mathbf{x},\,\tau)\rangle$$

$\mathcal{C}_{h\zeta}$ correlation

$$\mathcal{C}_{h\zeta}(\mathbf{k},\,\tau) = \frac{1}{\mathcal{P}_h\,\sqrt{\mathcal{P}_\zeta}}\frac{k^3}{2\pi^2}\int\frac{d\mathbf{y}}{(2\pi)^{3/2}}e^{-i\mathbf{k}\mathbf{y}}\langle h_{ij}(\mathbf{x}+\mathbf{y},\,\tau)\,h_{ij}(\mathbf{x}+\mathbf{y},\,\tau)\,\zeta(\mathbf{x},\,\tau)\rangle$$

TWO CONTRIBUTIONS

$\zeta = \zeta_{\mathrm{V}} + \zeta_{\mathrm{S}}$

$(\mathcal{C}_{h\zeta})_{\mathrm{V}}$

Correlation of gravitational waves with the amplified vacuum scalar fluctuations

$$\langle h_{ij,\mathrm{S}}(\mathbf{x}+\mathbf{y},\,\tau)\,h_{ij,\mathrm{S}}(\mathbf{x}+\mathbf{y},\,\tau)\,\zeta_{\mathrm{V}}(\mathbf{x},\,\tau)\rangle$$

$(\mathcal{C}_{h\zeta})_{\mathrm{S}}$

Correlation of gravitational waves with the sourced scalar fluctuations

$$\langle h_{ij,\mathrm{S}}(\mathbf{x}+\mathbf{y},\,\tau)\,h_{ij,\mathrm{S}}(\mathbf{x}+\mathbf{y},\,\tau)\,\zeta_{\mathrm{S}}(\mathbf{x},\,\tau)\rangle$$

$\mathcal{C}_{h\zeta}$ correlation

$$\mathcal{C}_{h\zeta}(\mathbf{k},\,\tau) = \frac{1}{\mathcal{P}_h\,\sqrt{\mathcal{P}_\zeta}}\frac{k^3}{2\pi^2}\int\frac{d\mathbf{y}}{(2\pi)^{3/2}}e^{-i\mathbf{k}\mathbf{y}}\langle h_{ij}(\mathbf{x}+\mathbf{y},\,\tau)\,h_{ij}(\mathbf{x}+\mathbf{y},\,\tau)\,\zeta(\mathbf{x},\,\tau)\rangle$$

TWO CONTRIBUTIONS

$$\zeta = \zeta_{\rm V} + \zeta_{\rm S}$$

$(\mathcal{C}_{h\zeta})_{\rm V}$

Correlation of gravitational waves with the amplified vacuum scalar fluctuations

$$\delta\phi \longrightarrow \delta A \longrightarrow h$$

$(\mathcal{C}_{h\zeta})_{\rm S}$

Correlation of gravitational waves with the sourced scalar fluctuations

$$\phi \longrightarrow A \longrightarrow h,\ \delta\phi$$

$\mathcal{C}_{h\zeta}$ correlation

$$\mathcal{C}_{h\zeta}(\mathbf{k},\,\tau) = \frac{1}{\mathcal{P}_h\,\sqrt{\mathcal{P}_\zeta}}\frac{k^3}{2\pi^2}\int\frac{d\mathbf{y}}{(2\pi)^{3/2}}e^{-i\mathbf{k}\mathbf{y}}\langle h_{ij}(\mathbf{x}+\mathbf{y},\,\tau)\,h_{ij}(\mathbf{x}+\mathbf{y},\,\tau)\,\zeta(\mathbf{x},\,\tau)\rangle$$

TWO CONTRIBUTIONS

$$\zeta = \zeta_{\rm V} + \zeta_{\rm S}$$

$(\mathcal{C}_{h\zeta})_{\rm V}$

Correlation of gravitational waves with the amplified vacuum scalar fluctuations

$$\delta\phi \longrightarrow \delta A \longrightarrow h$$

$(\mathcal{C}_{h\zeta})_{\rm S}$

Correlation of gravitational waves with the sourced scalar fluctuations

$$\phi \longrightarrow A \longrightarrow h,\ \delta\phi$$

Final Results: $(\mathcal{C}_{h\zeta})_{\rm V}$

Final Results: $(\mathcal{C}_{h\zeta})_{\rm V}$

$$\left\langle [h_{ij}h_{ij}]_{\rm S}\,(\mathbf{x}+\mathbf{y})\zeta_{\rm V}(\mathbf{x})\right\rangle \simeq \left\langle [h_{ij}h_{ij}(\phi(\mathbf{x}+\mathbf{y}))]_{\rm S}\,\zeta_{\rm V}(\mathbf{x})\right\rangle$$

$$\simeq \left\langle [h_{ij}h_{ij}(\phi_0)]_{\rm S}\,\zeta_{\rm V}(\mathbf{x})\right\rangle + \left\langle \frac{d\,\left[h_{ij}h_{ij}(\phi_0)\right]_{\rm S}}{d\phi_0}\,\delta\phi_{\rm V}(\mathbf{x}+\mathbf{y})\zeta_{\rm V}(\mathbf{x})\right\rangle$$

Final Results: $(\mathcal{C}_{h\zeta})_{\rm V}$

$$\left\langle [h_{ij}h_{ij}]_{\rm S}\,(\mathbf{x}+\mathbf{y})\zeta_{\rm V}(\mathbf{x})\right\rangle \simeq \left\langle [h_{ij}h_{ij}(\phi(\mathbf{x}+\mathbf{y}))]_{\rm S}\,\zeta_{\rm V}(\mathbf{x})\right\rangle$$

$$\simeq \left\langle [h_{ij}h_{ij}(\phi_0)]_{\rm S}\,\zeta_{\rm V}(\mathbf{x})\right\rangle + \left\langle \frac{d\;[h_{ij}h_{ij}(\phi_0)]_{\rm S}}{d\phi_0}\,\delta\phi_{\rm V}(\mathbf{x}+\mathbf{y})\zeta_{\rm V}(\mathbf{x})\right\rangle$$

$$\frac{d\;[h_{ij}h_{ij}(\phi_0)]_{\rm S}}{d\phi_0} \simeq 4\pi\;[h_{ij}h_{ij}(\phi_0)]_{\rm S}\;\left(\epsilon-\frac{\eta}{2}\right)\frac{1}{f}$$

Final Results: $(\mathcal{C}_{h\zeta})_{\rm V}$

$$\left\langle [h_{ij}h_{ij}]_{\rm S}\,(\mathbf{x}+\mathbf{y})\zeta_{\rm V}(\mathbf{x})\right\rangle \simeq \left\langle [h_{ij}h_{ij}(\phi(\mathbf{x}+\mathbf{y}))]_{\rm S}\,\zeta_{\rm V}(\mathbf{x})\right\rangle$$

$$\simeq \left\langle [h_{ij}h_{ij}(\phi_0)]_{\rm S}\,\zeta_{\rm V}(\mathbf{x})\right\rangle + \left\langle \frac{d\,\left[h_{ij}h_{ij}(\phi_0)\right]_{\rm S}}{d\phi_0}\,\delta\phi_{\rm V}(\mathbf{x}+\mathbf{y})\zeta_{\rm V}(\mathbf{x})\right\rangle$$

$$\frac{d\,\left[h_{ij}h_{ij}(\phi_0)\right]_{\rm S}}{d\phi_0} \simeq 4\pi\,\left[h_{ij}h_{ij}(\phi_0)\right]_{\rm S}\,\left(\epsilon-\frac{\eta}{2}\right)\frac{1}{f}$$

$$(\mathcal{C}_{h\zeta})_{\rm V} \simeq \frac{1}{\mathcal{P}_h\,\sqrt{\mathcal{P}_\zeta}}\frac{k^3}{2\pi^2}\int\frac{d\mathbf{y}}{(2\pi)^{3/2}}\,e^{-i\mathbf{k}\mathbf{y}}\frac{4\pi}{f}\left(\epsilon-\frac{\eta}{2}\right)\,\langle h_{ij}h_{ij}\rangle_{\rm S}\,\langle\delta\phi_{\rm V}(\mathbf{x}+\mathbf{y})\,\zeta_{\rm V}(\mathbf{x})\rangle$$

$$\boxed{(\mathcal{C}_{h\zeta})_{\rm V} = 8\pi\,\xi\frac{\sqrt{\mathcal{P}_{\zeta,\,{\rm V}}}}{(2\pi)^{3/2}}\left(\frac{\eta}{2}-\epsilon\right)}$$

Final Results: $(\mathcal{C}_{h\zeta})_{\rm V}$

$$\left\langle [h_{ij}h_{ij}]_{\rm S}\,(\mathbf{x}+\mathbf{y})\zeta_{\rm V}(\mathbf{x})\right\rangle \simeq \left\langle [h_{ij}h_{ij}(\phi(\mathbf{x}+\mathbf{y}))]_{\rm S}\,\zeta_{\rm V}(\mathbf{x})\right\rangle$$

$$\simeq \left\langle [h_{ij}h_{ij}(\phi_0)]_{\rm S}\,\zeta_{\rm V}(\mathbf{x})\right\rangle + \left\langle \frac{d\,\left[h_{ij}h_{ij}(\phi_0)\right]_{\rm S}}{d\phi_0}\,\delta\phi_{\rm V}(\mathbf{x}+\mathbf{y})\zeta_{\rm V}(\mathbf{x})\right\rangle$$

$$\frac{d\,\left[h_{ij}h_{ij}(\phi_0)\right]_{\rm S}}{d\phi_0} \simeq 4\pi\,\left[h_{ij}h_{ij}(\phi_0)\right]_{\rm S}\,\left(\epsilon-\frac{\eta}{2}\right)\frac{1}{f}$$

$$(\mathcal{C}_{h\zeta})_{\rm V} \simeq \frac{1}{\mathcal{P}_h\,\sqrt{\mathcal{P}_\zeta}}\,\frac{k^3}{2\pi^2}\int\frac{d\mathbf{y}}{(2\pi)^{3/2}}\,e^{-i\mathbf{k}\mathbf{y}}\,\frac{4\pi}{f}\left(\epsilon-\frac{\eta}{2}\right)\,\langle h_{ij}h_{ij}\rangle_{\rm S}\,\langle\delta\phi_{\rm V}(\mathbf{x}+\mathbf{y})\,\zeta_{\rm V}(\mathbf{x})\rangle$$

$$\boxed{(\mathcal{C}_{h\zeta})_{\rm V} = 8\pi\,\xi\frac{\sqrt{\mathcal{P}_{\zeta,\,\rm V}}}{(2\pi)^{3/2}}\left(\frac{\eta}{2}-\epsilon\right)} \quad\longrightarrow\quad \left|(\mathcal{C}_{h\zeta})_{\rm V}\right| \lesssim O(10^{-3})$$

$$\xi \simeq 10,\quad \mathcal{P}_{\zeta,\,\rm V} \simeq 2\times 10^{-9}$$

Final Results: $(\mathcal{C}_{h\zeta})_{\mathrm{S}}$

Final Results: $(\mathcal{C}_{h\zeta})_{\rm S}$

$$(\mathcal{C}_{h\zeta})_{\rm S} = \frac{1}{\mathcal{P}_h\,\sqrt{\mathcal{P}_\zeta}}\,\frac{k^3}{2\pi^2}\int \frac{d\mathbf{y}}{(2\pi)^{3/2}}\,e^{-i\mathbf{k}\mathbf{y}}\,\langle h_{ab,{\rm S}}(\mathbf{x}+\mathbf{y},\,\tau)\,h_{ab,{\rm S}}(\mathbf{x}+\mathbf{y},\,\tau)\,\zeta_{\rm S}(\mathbf{x},\,\tau)\rangle$$

Final Results: $(\mathcal{C}_{h\zeta})_{\rm S}$

$$(\mathcal{C}_{h\zeta})_{\rm S} = \frac{1}{\mathcal{P}_h\,\sqrt{\mathcal{P}_\zeta}}\,\frac{k^3}{2\pi^2}\int\frac{d\mathbf{y}}{(2\pi)^{3/2}}\,e^{-i\mathbf{k}\mathbf{y}}\,\langle h_{ab,{\rm S}}(\mathbf{x}+\mathbf{y},\,\tau)\,h_{ab,{\rm S}}(\mathbf{x}+\mathbf{y},\,\tau)\,\zeta_{\rm S}(\mathbf{x},\,\tau)\rangle$$

$$\langle h_{ab,\,{\rm S}}(\mathbf{k}_1,\tau)\,h_{ab,\,{\rm S}}(\mathbf{k}_2,\tau)\,\zeta_{\rm S}(\mathbf{k}_3,\tau)\rangle = -\frac{4\,H(\tau)}{M_P^2\,\dot\phi_0(\tau)\,a^3(\tau)}\langle H_{ab,\,{\rm S}}(\mathbf{k}_1,\tau)\,H_{ab,\,{\rm S}}(\mathbf{k}_2,\tau)\,\Phi_{\rm S}(\mathbf{k}_3,\tau)\rangle$$

$$= \frac{4\,H(\tau)}{M_P^4\,\dot\phi_0(\tau)\,a^3(\tau)\,f}\int_{-\infty}^{\tau}\frac{d\tau_1}{a(\tau_1)}\,\frac{d\tau_2}{a(\tau_2)}\,\frac{d\tau_3}{a(\tau_3)}\,G_{k_1}(\tau,\tau_1)\,G_{k_2}(\tau,\tau_2)\,G_{k_3}(\tau,\tau_3)$$

$$\times\int\frac{d\mathbf{q}_1\,d\mathbf{q}_2\,d\mathbf{q}_3}{(2\pi)^{9/2}}\,e_a^+(\widehat{\mathbf{q}_1})\,e_b^+(\widehat{\mathbf{k}_1-\mathbf{q}_1})\,e_a^+(\widehat{\mathbf{q}_2})\,e_b^+(\widehat{\mathbf{k}_2-\mathbf{q}_2})\,e_i^+(\widehat{\mathbf{q}_3})\,e_i^+(\widehat{\mathbf{k}_3-\mathbf{q}_3})\,|\mathbf{k}_3-\mathbf{q}_3|$$

$$\times\langle A'_+(q_1,\tau_1)\,A'_+(|\mathbf{k}_1-\mathbf{q}_1|,\tau_1)\,A'_+(q_2,\tau_2)\,A'_+(|\mathbf{k}_2-\mathbf{q}_2|,\tau_2)\,A'_+(q_3,\tau_3)\,A_+(|\mathbf{k}_3-\mathbf{q}_3|,\tau_3)\rangle$$

• • •

$$\boxed{(\mathcal{C}_{h\zeta})_{\rm S} \simeq 2\times 10^{-4}\,\frac{e^{2\pi\,\xi}\,H}{\xi^4\,f}}\qquad \longrightarrow (\mathcal{C}_{h\zeta})_{\rm S} \simeq 1.5\times 10^{-2}\,(f_{\rm NL}^{\rm equil})^{1/3}\quad\lesssim\quad 0.05$$

Thanks for your attention.

DPF-PHENO 2024

Thanks for your attention.

University *of* Massachusetts Amherst

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
University of Massachusetts Amherst

Future work

$$(\mathcal{C}_{h\zeta})_{\mathrm{S}} \simeq 1.5 \times 10^{-2}\,(f_{\mathrm{NL}}^{\mathrm{equil}})^{1/3} \;\;\lesssim\;\; 0.05$$

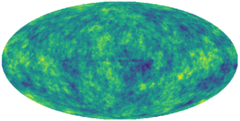
Observability will depend on the amplitude of the anisotropies in the gravitational wave spectra.

$$\Downarrow$$

$$\langle h_{ij}(\mathbf{x})\, h_{ij}(\mathbf{x})\, h_{ab}(\mathbf{y}) h_{ab}(\mathbf{y})\rangle$$

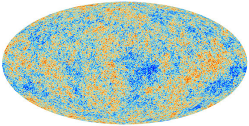
$$\mathcal{C}_{h\zeta}(\mathbf{k},\,\tau) = \frac{1}{\mathcal{P}_h\,\sqrt{\mathcal{P}_\zeta}}\frac{k^3}{2\pi^2}\int\frac{d\mathbf{y}}{(2\pi)^{3/2}}e^{-i\mathbf{k}\mathbf{y}}\langle h_{ij}(\mathbf{x}{+}\mathbf{y},\,\tau)\,h_{ij}(\mathbf{x}{+}\mathbf{y},\,\tau)\,\zeta(\mathbf{x},\,\tau)\rangle$$

GW anisotropies



(Ricciardone, Dall'Armi, Bartolo, Bertacca, Liguori, Matarrese)

CMB anisotropies



(ESA/Planck Collaboration)

$$(\mathcal{C}_{h\zeta})_{\rm S} \gg (\mathcal{C}_{h\zeta})_{\rm V}$$

$$(\mathcal{C}_{h\zeta})_{\rm S} \simeq 1.5\times 10^{-2}\,(f_{\rm NL}^{\rm equil})^{1/3} \quad\lesssim\quad 0.05$$

Pulsar Timing Arrays

NANOGrav: North American Nanohertz Observatory for Gravitational Waves

The Astrophysical Journal Letters, 951:L8 (24pp), 2023 July 1 https://doi.org/10.3847/2041-8213/acdac6

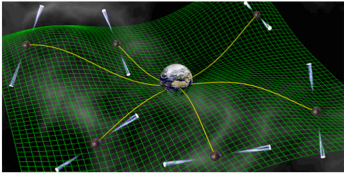
The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background

Gabriella Agazie[1], Akash Anumarlapudi[1], Anne M. Archibald[2], Zaven Arzoumanian[3], Paul T. Baker[4], Bence Bécsy[5], Laura Blecha[6], Adam Brazier[7,8], Paul R. Brook[9], Sarah Burke-Spolaor[10,11], Rand Burnette[5], Robin Case[5], Maria Charisi[12], Shami Chatterjee[7], Katerina Chatziioannou[13], Belinda D. Cheeseboro[10,11], Siyuan Chen[14], Tyler Cohen[15], James M. Cordes[7], Neil J. Cornish[16], Fronefield Crawford[17], H. Thankful Cromartie[7,70], Kathryn Crowter[18], Curt J. Cutler[13,19], Megan E. DeCesar[20], Dallas DeGan[5], Paul B. Demorest[21], Heling Deng[5], Timothy Dolch[22,23], Brendan Drachler[24,25], Justin A. Ellis[26], Elizabeth C. Ferrara[27,28,29], William Fiore[10,11], Emmanuel Fonseca[10,11], Gabriel E. Freedman[1], Nate Garver-Daniels[10,11], Peter A. Gentile[10,11], Kyle A. Gersbach[12], Joseph Glaser[10,11], Deborah C. Good[30,31], Kayhan Gültekin[32], Jeffrey S. Hazboun[5], Sophie Hourihane[13], Kristina Islo[1], Ross J. Jennings[10,11,73], Aaron D. Johnson[1,13], Megan L. Jones[1], Andrew R. Kaiser[10,11], David L. Kaplan[1], Luke Zoltan Kelley[33], Matthew Kerr[34], Joey S. Key[35], Tonia C. Klein[1], Nima Laal[5], Michael T. Lam[24,25], William G. Lamb[12], T. Joseph W. Lazio[19], Natalia Lewandowska[36], Tyson B. Littenberg[37], Tingting Liu[10,11], Andrea Lommen[38], Duncan R. Lorimer[10,11], Jing Luo[39,71], Ryan S. Lynch[40], Chung-Pei Ma[33,41], Dustin R. Madison[42], Margaret A. Mattson[10,11], Alexander McEwen[1], James W. McKee[43,44], Maura A. McLaughlin[10,11], Natasha McMann[12], Bradley W. Meyers[18,45], Patrick M. Meyers[13], Chiara M. F. Mingarelli[30,31,46], Andrea Mitridate[47], Priyamvada Natarajan[48,49], Cherry Ng[50], David J. Nice[51], Stella Koch Ocker[7], Ken D. Olum[52], Timothy T. Pennucci[53], Benetge B. P. Perera[54], Polina Petrov[12], Nihan S. Pol[12], Henri A. Radovan[55], Scott M. Ransom[56], Paul S. Ray[34], Joseph D. Romano[57], Shashwat C. Sardesai[1], Ann Schmiedekamp[58], Carl Schmiedekamp[58], Kai Schmitz[59], Levi Schult[12], Brent J. Shapiro-Albert[10,11,60], Xavier Siemens[1,5], Joseph Simon[61,72], Magdalena S. Siwek[62], Ingrid H. Stairs[18], Daniel R. Stinebring[63], Kevin Stovall[21], Jerry P. Sun[5], Abhimanyu Susobhanan[1], Joseph K. Swiggum[51,72], Jacob Taylor[5], Stephen R. Taylor[12], Jacob E. Turner[10,11], Caner Unal[64,65], Michele Vallisneri[13,19], Rutger van Haasteren[66], Sarah J. Vigeland[1], Haley M. Wahl[10,11], Qiaohong Wang[12], Caitlin A. Witt[67,68], and Olivia Young[24,25]
The NANOGrav Collaboration[69]

Abstract

We report multiple lines of evidence for a stochastic signal that is correlated among 67 pulsars from the 15 yr pulsar timing data set collected by the North American Nanohertz Observatory for Gravitational Waves. The correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background. The presence of such a gravitational-wave background with a power-law spectrum is favored over a model with only independent pulsar noises with a Bayes factor in excess of 10^{14}, and this same model is favored over an uncorrelated common power-law spectrum model with Bayes factors of 200–1000, depending on spectral modeling choices. We have built a statistical background distribution for the latter Bayes factors using a method that removes interpulsar correlations from our data set, finding $p = 10^{-3}$ ($\approx 3\sigma$) for the observed Bayes factors in the null no-correlation scenario. A frequentist test statistic built directly as a weighted sum of interpulsar correlations yields $p = 5 \times 10^{-5}$ to 1.9×10^{-4} ($\approx 3.5\sigma$–4σ). Assuming a fiducial $f^{-2/3}$ characteristic strain spectrum, as appropriate for an ensemble of binary supermassive black hole inspirals, the strain amplitude is $2.4^{+0.7}_{-0.6} \times 10^{-15}$ (median + 90% credible interval) at a reference frequency of 1 yr^{-1}. The inferred gravitational-wave background amplitude and spectrum are consistent with astrophysical expectations for a signal from a population of supermassive black hole binaries, although more exotic cosmological and astrophysical sources cannot be excluded. The observation of Hellings–Downs correlations points to the gravitational-wave origin of this signal.

JUNE 2023



(Max Planck Institute for Radio Astronomy)

Use of a set of pulsars embedded in our Galaxy to probe the passage of gravitational waves that modulate their radio signals.

Axion Inflation

UV sensitivity of inflationary potentials

Solution:

symmetry that protects the potential against large **radiative corrections**

$\Downarrow$

Shift symmetry: $\phi \to \phi + \text{const}$

Axion

First model of axion inflation: Natural inflation (1990)

[K. Freese, J. A. Frieman and A. V. Olinto, 1990]

Natural inflation compatible with phenomenology for $f \gg M_P$ **but** theoretical predictions $f < M_P$.

Axion Inflation

Solutions:

- Spontaneous summetry breaking through the **coupling with 4-forms**.

  [N. Kaloper, L. Sorbo, 2009]

  [N. Kaloper, A. Lawrence, L. Sorbo, 2010]

- **More than one** axion.

  [J. E. Kim, H. P. Nilles, M. Peloso, 2004]

  [M. M. Anber, L. Sorbo, 2006]

  [M. Berg, E. Pajer, S. Sjörs, 2009]

- Additional dynamics through the coupling with **abelian** or **non-abelian gauge fields**.

  [M. M. Anber, L. Sorbo, 2006]

Phenomenology of Axion Inflation

- **Magnetogenesis.**
- **Backreaction** on the inflatonary dynamics.
- Production of **scalar fluctuations**:
  (1). Correction to the power spectrum.
  (2). Non-Gaussianities.
- Production of **tensor fluctuations**.
  (1). Correction to the power spectrum.
  (2). Non-Gaussianities.
- Production of **primordial black holes**.
- Production of **fermions**.

Axion Inflation

$$\mathcal{L} = \underbrace{\left(\frac{1}{2}\Phi'^2 - \frac{1}{2}\partial_k\Phi\,\partial_k\Phi + \frac{a''}{2\,a}\Phi^2\right)}_{\text{free scalar perturbations}} + \underbrace{\left(\frac{1}{2}H'_{ij}\,H'_{ij} - \frac{1}{2}\partial_k H_{ij}\,\partial_k H_{ij} + \frac{a''}{2\,a}H_{ij}\,H_{ij}\right)}_{\text{free tensor perturbations}}$$

$$+ \underbrace{\left(\frac{1}{2}A'_i\,A'_i - \frac{1}{2}\partial_k A_i\,\partial_k A_i - \frac{\phi_0}{f}\epsilon^{ijk}\,A'_i\,\partial_j A_k\right)}_{\text{free gauge field modes}}$$

$$\underbrace{- \frac{H_{ij}}{a\,M_P}\,\left[A'_i\,A'_j - (\partial_i A_k - \partial_k A_i)\,(\partial_j A_k - \partial_k A_j)\right] - \frac{\Phi}{f\,a}\epsilon^{ijk}\,A'_i\,\partial_j A_k}_{\text{interactions}}$$