The Migdal Effect in Semiconductors for the Effective Field Theory of Dark Matter Direct Detection

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Outline

- Background and Motivation for Migdal Effect (in semiconductors)
- Investigate Migdal interactions beyond standard spin-independent scattering
- Results: Direct-detection projections for EFT operators

Migdal Effect (In atoms)

In direct detection experiments, elastic nuclear recoil are only detectable for DM mass O(GeV).
Dark Matter induced electron transitions via the Migdal effect:

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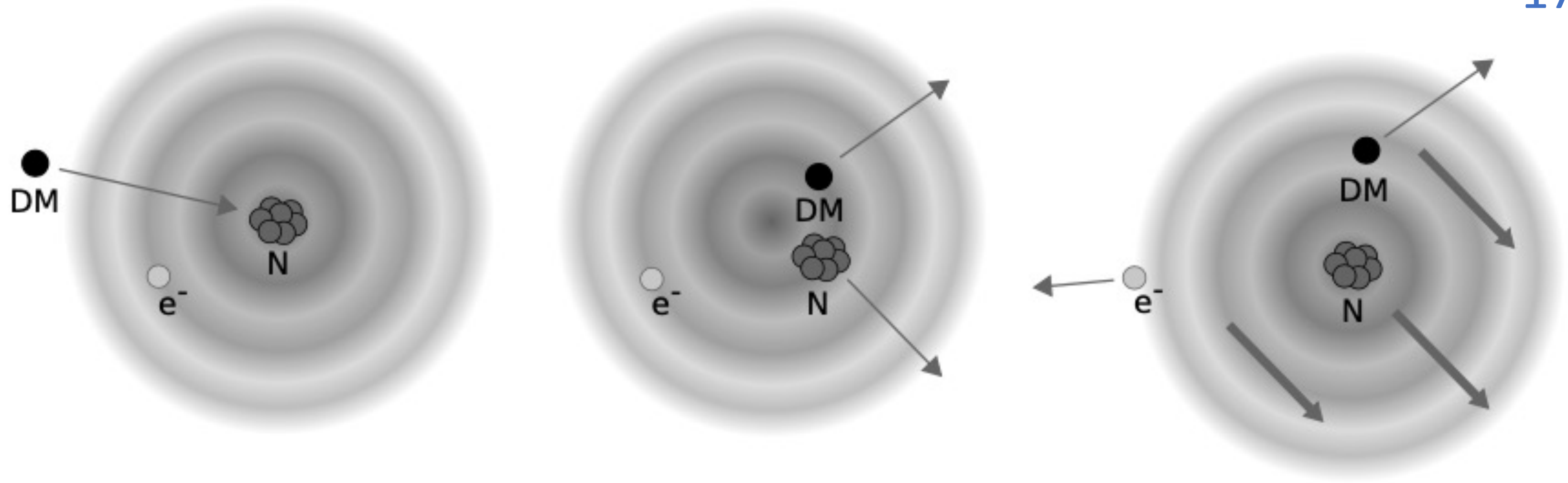
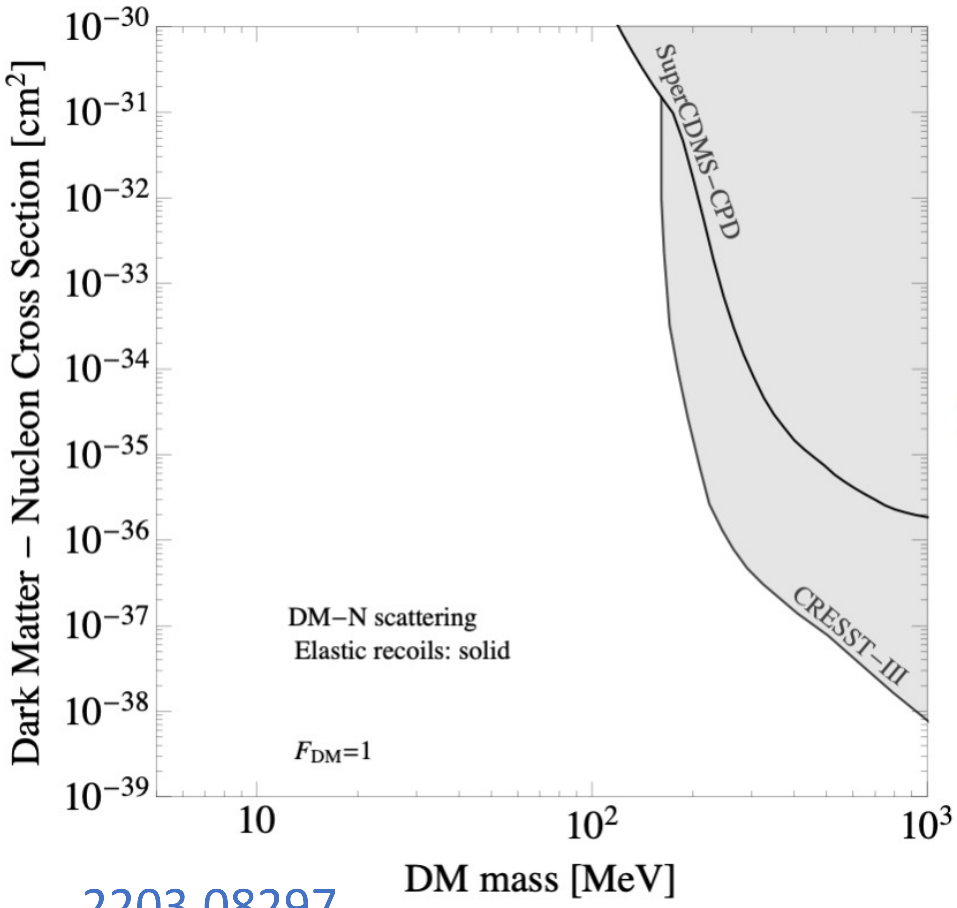


Advantages: Migdal effect allows us to probe sub-GeV DM masses to O(MeV).

Ability to probe lower DM masses

elastic DM-nucleus scattering



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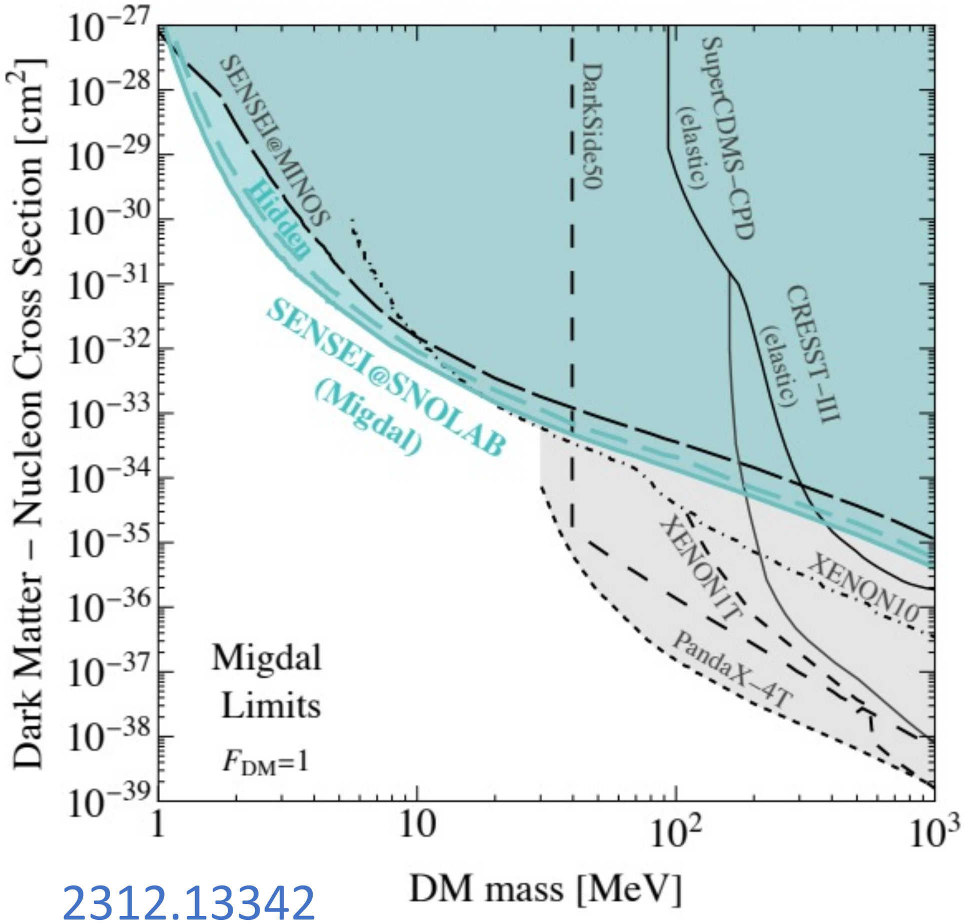
DM-nucleus scattering **w/ Migdal**



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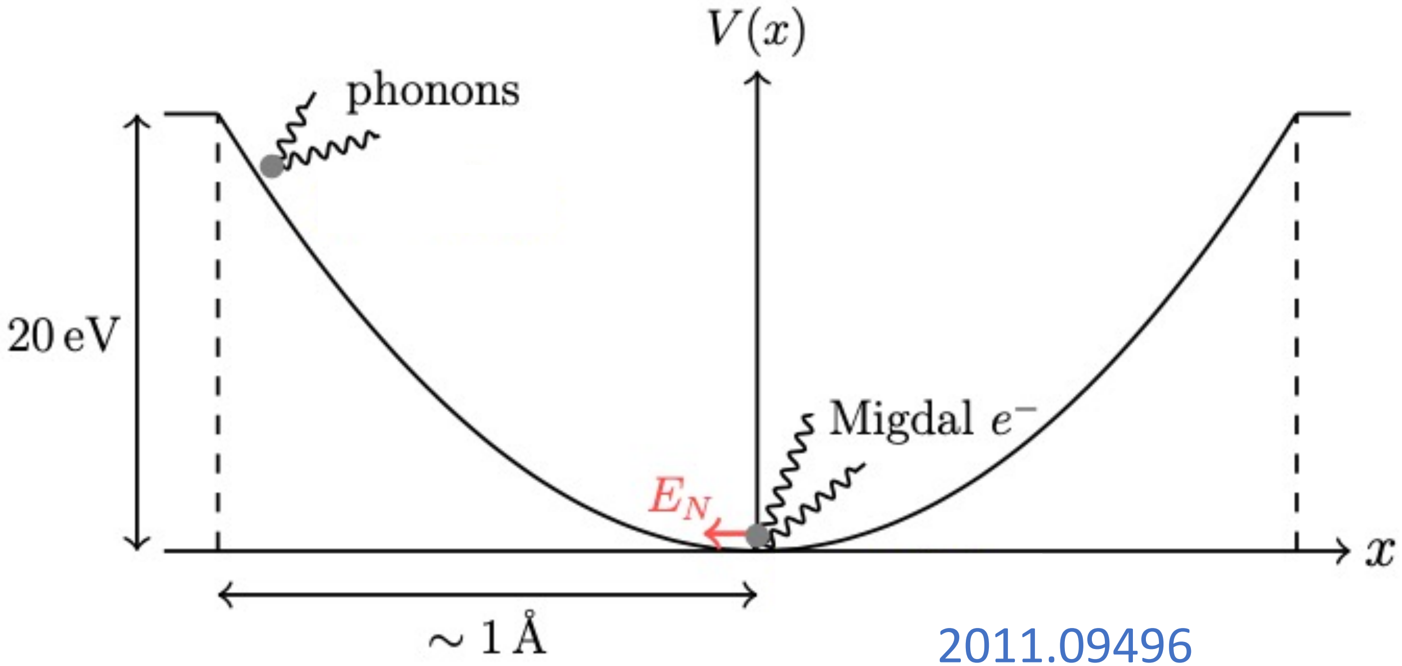
Migdal Effect (In semiconductors)

Advantage: Semiconductors have small energy gaps O(eV) compared to ionization of atoms O(10 eV) → Semiconductor targets have very low thresholds for direct detection experiments.

Complications:

- Lattice potential is introduced
- Phonon production is possible



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Effective DM-Migdal Hamiltonian in semiconductors

2210.06490

$$H_{\text{eff}} = \frac{1}{\omega^2}[H_{\chi\text{L}}, [H_\text{L}, H_{e\text{L}}]] + \mathcal{O}(1/\omega^3)\,,$$

Electron– lattice Hamiltonian is known

$$\longrightarrow \quad H_{\text{eff}} = \frac{1}{m_\text{N}\omega^2}\sum_I \boldsymbol{\nabla}_I H_{\chi\text{L}}^{(I)} \cdot \boldsymbol{\nabla}_I H_{e\text{L}}^{(I)} + \mathcal{O}(1/\omega^3)\,.$$

Take-away: DM-lattice and electron-lattice interactions factorize!

Effective DM-Migdal Hamiltonian in semiconductors

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What interesting DM– lattice interactions can we look at?

Take-away: DM-lattice and electron-lattice interactions factorize!

→ Is this still true for all DM-lattice interactions? → (spoiler!) YES

Non-relativistic operators

Method: Write down every non-relativistic Galilean-invariant operator up to quadratic order in momentum, which can arise from spin-0 or spin-1 exchanges.

1203.3542

$$\mathcal{L}_{\text{int}} = \sum_{N=n,p} \sum_i c_i^{(N)} \mathcal{O}_i \chi^+ \chi^- N^+ N^-,$$

Define, $\vec{v}^\perp \equiv \vec{v} + \frac{\vec{q}}{2\mu_N}.$

Difficult to UV complete

$\mathcal{O}_1 = \mathbf{1}$, ~~$\mathcal{O}_2 = (v^\perp)^2$~~, $\mathcal{O}_3 = i\vec{S}_N \cdot (\vec{q} \times \vec{v}^\perp)$,

Coherent interactions: S_N independent

Rate contains enhancement of $(A-Z)^2$

$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N$, $\mathcal{O}_5 = i\vec{S}_\chi \cdot (\vec{q} \times \vec{v}^\perp)$, $\mathcal{O}_6 = (\vec{S}_\chi \cdot \vec{q})(\vec{S}_N \cdot \vec{q})$,

$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp$, $\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp$, $\mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \vec{q})$ $\mathcal{O}_{10} = i\vec{S}_N \cdot \vec{q}$, $\mathcal{O}_{11} = i\vec{S}_\chi \cdot \vec{q}$.

Non-relativistic operators

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$$\mathcal{L}_{\rm int} = \sum_{N=n,p} \sum_i c_i^{(N)} \mathcal{O}_i \chi^+ \chi^- N^+ N^-,$$

Define, $\vec{v}^\perp \equiv \vec{v}_T^\perp + \vec{v}_N^\perp,$

velocity split into center of mass and intrinsic components (which carry derivatives)

Difficult to UV complete

$\mathcal{O}_1 = \mathbf{1},$ $\mathcal{O}_2 = (v^\perp)^2,$ $\mathcal{O}_3 = i\vec{S}_N \cdot (\vec{q} \times \vec{v}^\perp),$

$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N,$ $\mathcal{O}_5 = i\vec{S}_\chi \cdot (\vec{q} \times \vec{v}^\perp),$ $\mathcal{O}_6 = (\vec{S}_\chi \cdot \vec{q})(\vec{S}_N \cdot \vec{q}),$

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Differential Migdal Rate:

Goal: Calculate the differential Migdal rate in semiconductors using the EFT Hamiltonian for all possible $H_{\chi L}$.

$$\frac{dR}{d\omega}(\omega) = \frac{n_\chi N_T}{m_N M_T} \int_{q_{\min}}^{q_{\max}} dq \int d^3v\, v f_\chi(v) \frac{d\sigma_{\text{spin}}}{dq}(q) \frac{dP}{d\omega}(q,\omega)$$

electron "shake-off" probability, contains electron loss function

$$\frac{d\sigma_{\text{spin}}}{dq}(q) = \frac{\sigma_n q}{\mu_{\chi n}^2 v^2} \sum_{i,j}^{11} \sum_{N,N'}^{n,p} F_{i,j}^{(N,N')}(q,v) \,.$$

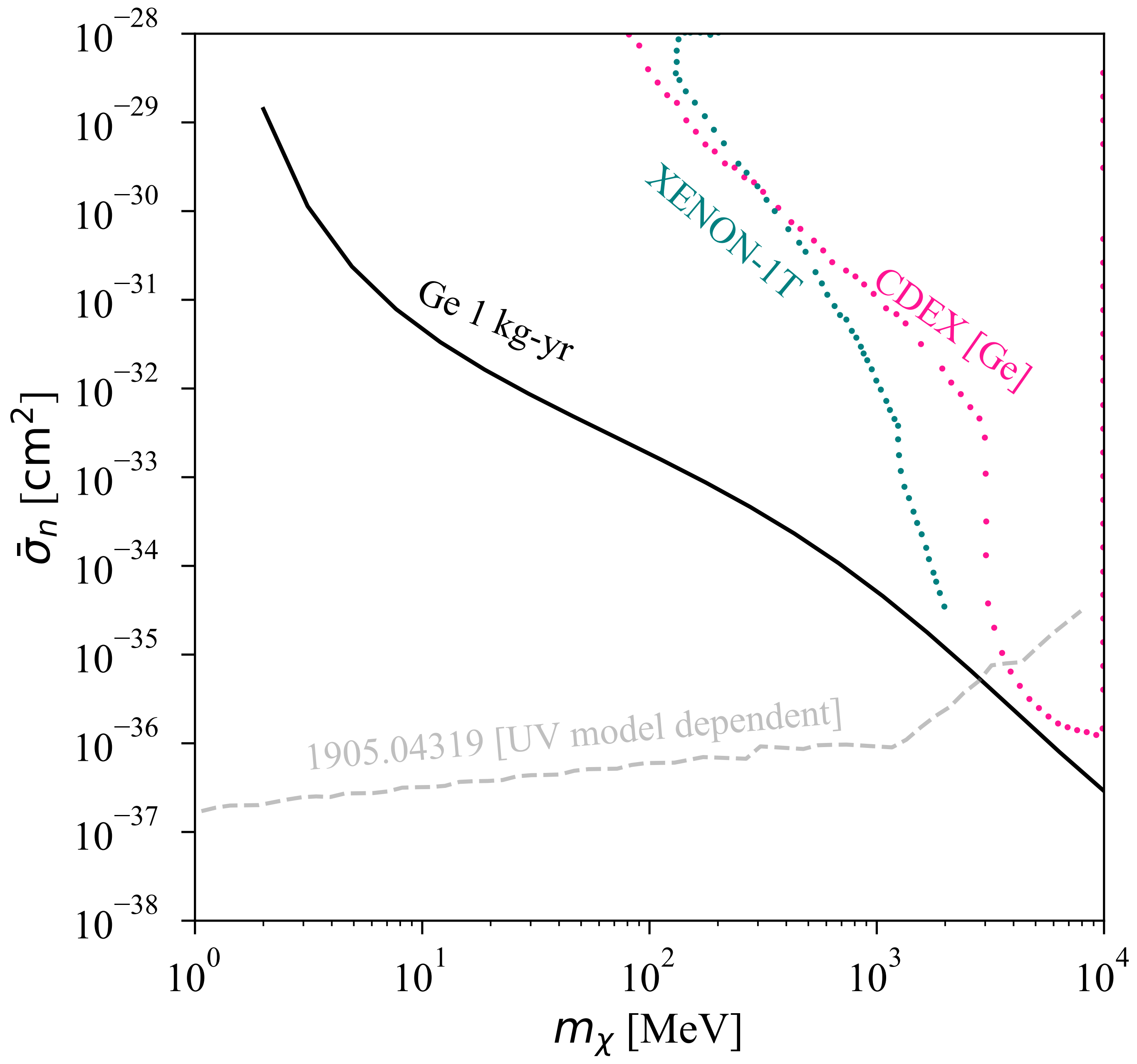
operator form factors, contains nuclear responses

Result: Electron shake-off probability and nuclear responses factorize for all DM – lattice effective operators!

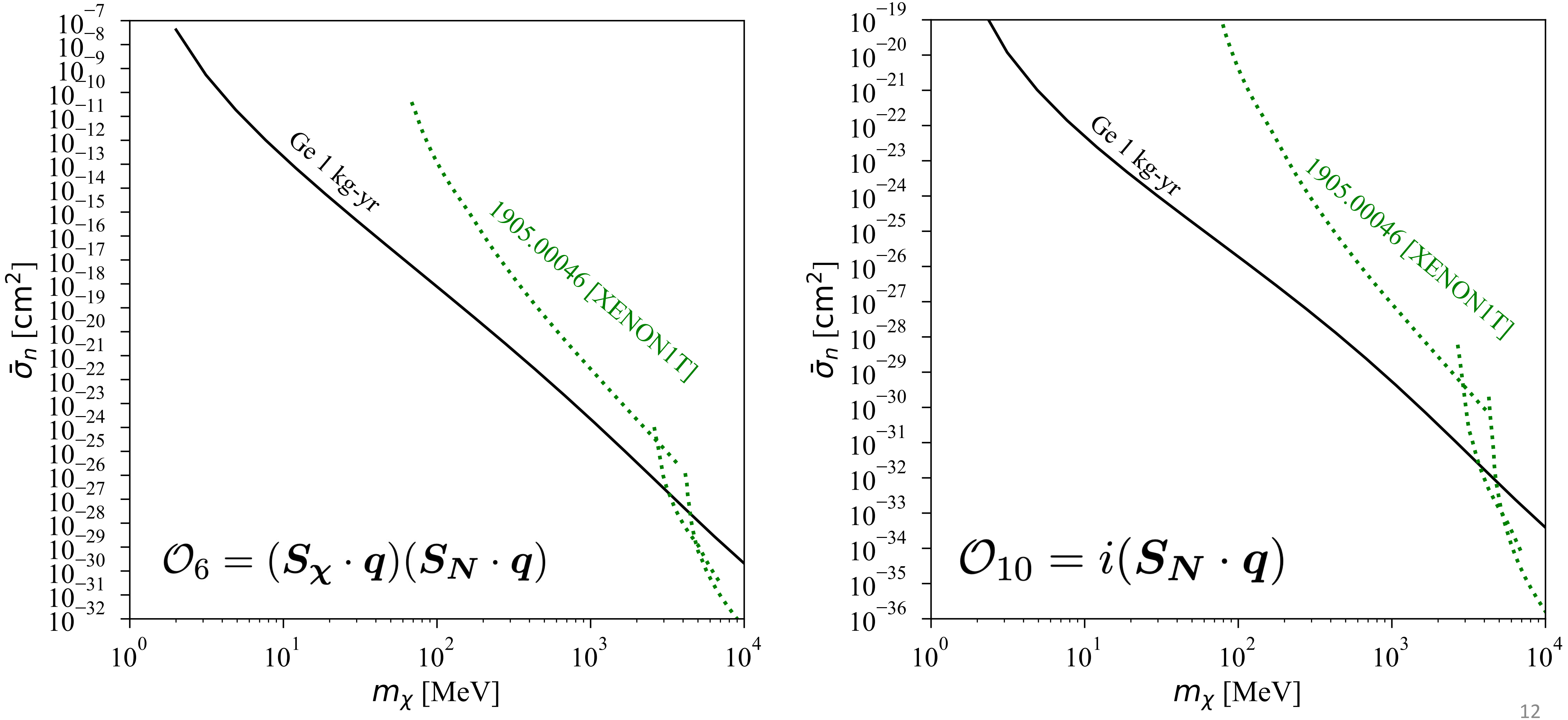
Projections (preliminary)

$$\mathcal{O}_4 = \boldsymbol{S_\chi} \cdot \boldsymbol{S_N}$$

CDEX: 2111.11243
XENON-1T: 1907.12771



Projections (preliminary)



Ge 1 kg-yr Projections (preliminary)

$$\mathcal{O}_3 = i\boldsymbol{S_N} \cdot (\boldsymbol{q} \times \boldsymbol{v}^{\perp})$$

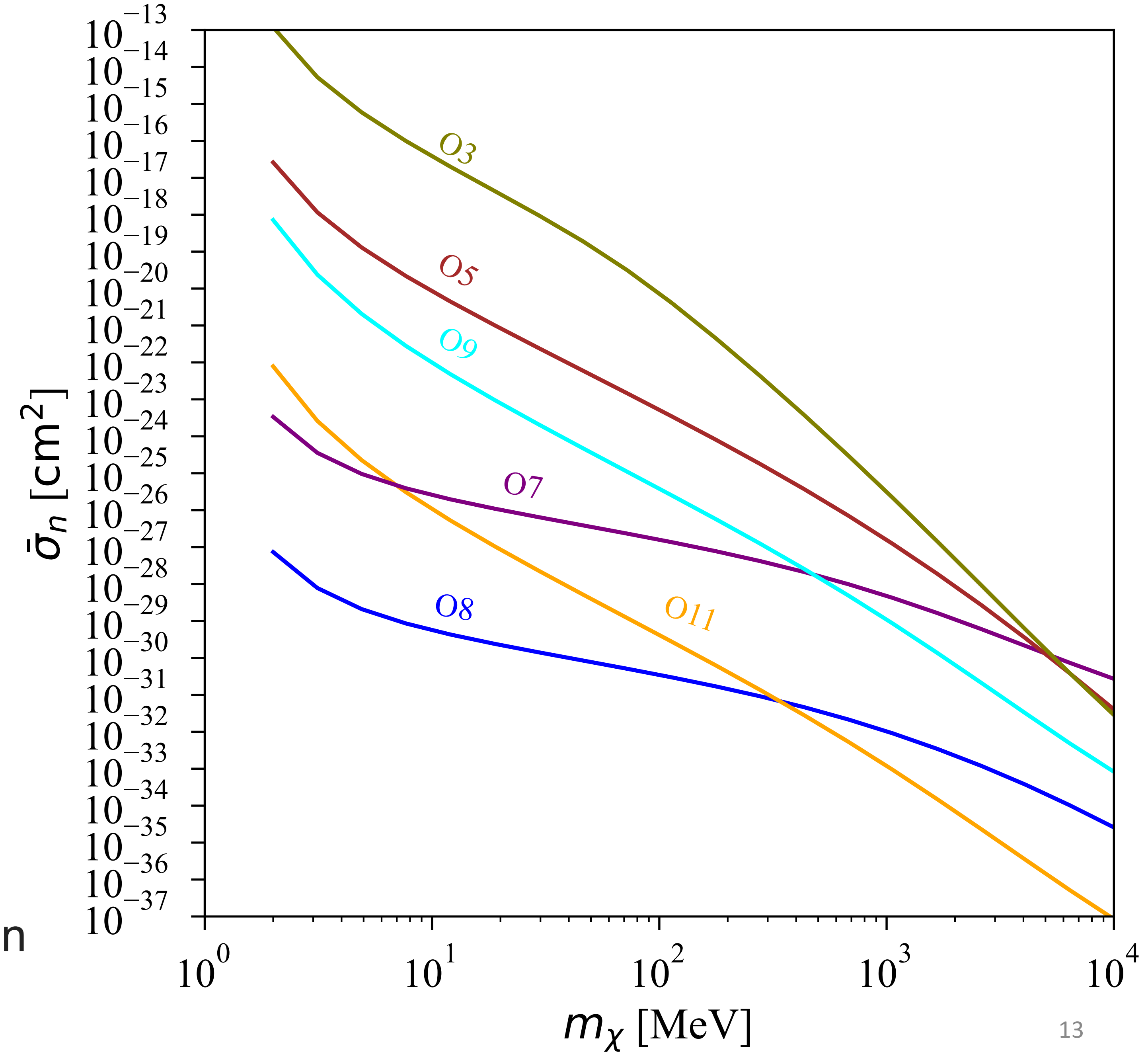
$$\mathcal{O}_5 = i\boldsymbol{S_\chi} \cdot (\boldsymbol{q} \times \boldsymbol{v}^{\perp})$$

$$\mathcal{O}_9 = i\boldsymbol{S_\chi} \cdot (\boldsymbol{S_N} \times \boldsymbol{q})$$

$$\mathcal{O}_7 = i\boldsymbol{S_N} \cdot \boldsymbol{v}^{\perp}$$

$$\mathcal{O}_{11} = i(\boldsymbol{S_\chi} \cdot \boldsymbol{q})$$

$$\mathcal{O}_8 = i\boldsymbol{S_\chi} \cdot \boldsymbol{v}^{\perp}$$



Takeaway: Migdal effects allows us to probe orders of magnitude lower in DM mass for various interactions!

Conclusions

- We have calculated the Migdal effect in semiconductors for many different EFT operators.
- Allows experiments to search for these interactions for DM masses as low as MeV, several orders of magnitude lower than before!

Thank you!

Questions?

DM Migdal EFT Picture

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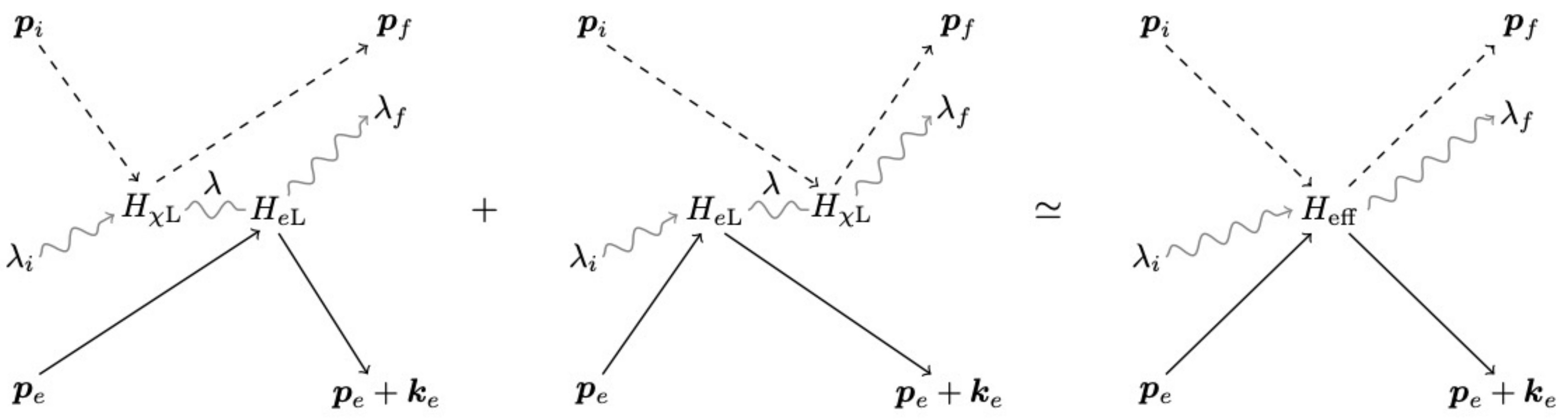

Figure 2: Schematic representation of the EFT procedure applied to the result of old-fashioned perturbation theory. Each line corresponds to a state in the Hilbert spaces of the dark matter (**dashed**), electron (**solid**), and crystal lattice (**wavy**). The intermediate lattice mode has high frequency and, when integrated out, it leads to an effective Hamiltonian that is local in time and independent on the complicated dynamics of the lattice.

$$H_{\mathrm{eL}} = -\frac{4\pi\alpha}{V}\sum_{I}\sum_{\boldsymbol{K},\boldsymbol{K}'}\sum_{\boldsymbol{k}}\frac{\epsilon^{-1}_{\boldsymbol{K}\boldsymbol{K}'}(\boldsymbol{k},\omega)Z(|\boldsymbol{k}+\boldsymbol{K}'|)}{|\boldsymbol{k}+\boldsymbol{K}||\boldsymbol{k}+\boldsymbol{K}'|}e^{i(\boldsymbol{k}+\boldsymbol{K})\cdot\boldsymbol{x}_e}e^{-i(\boldsymbol{k}+\boldsymbol{K}')\cdot\boldsymbol{x}_I}$$

Building blocks

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Complete set of Galilean, Hermitian invariants: $i\vec{q}, \quad \vec{v}^{\perp}, \quad \vec{S}_{\chi}, \quad \vec{S}_N.$

Set of T-conserving operators: $\mathbf{1}, \ \vec{S}_{\chi}\cdot\vec{S}_N, \ v^2, \ i(\vec{S}_{\chi}\times\vec{q})\cdot\vec{v}, \ i\vec{v}\cdot(\vec{S}_N\times\vec{q}), \ (\vec{S}_{\chi}\cdot\vec{q})(\vec{S}_N\cdot\vec{q})$ ← P-conserving

$\vec{v}^{\perp}\cdot\vec{S}_{\chi}, \ \vec{v}^{\perp}\cdot\vec{S}_N, \ i\vec{S}_{\chi}\cdot(\vec{S}_N\times\vec{q}).$ ← P-violating

Set of T-violating operators: $i\vec{S}_N\cdot\vec{q}, \ i\vec{S}_{\chi}\cdot\vec{q},$ ← P-conserving

$(i\vec{S}_N\cdot\vec{q})(\vec{v}^{\perp}\cdot\vec{S}_{\chi}), \ (i\vec{S}_{\chi}\cdot\vec{q})(\vec{v}^{\perp}\cdot\vec{S}_N).$ ← P-violating

Higher spin > 1 mediated operators

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$$\begin{aligned}
\mathcal{O}_{12} &= \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp) \\
\mathcal{O}_{13} &= i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N}) \\
\mathcal{O}_{14} &= i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp) \\
\mathcal{O}_{15} &= -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}) \\
\mathcal{O}_{16} &= -((\vec{S}_\chi \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N}). \quad = \quad \mathcal{O}_{15} + \frac{\vec{q}^{\,2}}{m_N^2}\mathcal{O}_{12},
\end{aligned}$$

Dark Matter – Lattice Hamiltonian

$$H_{\chi L} = \sum_I l_0 \delta(\boldsymbol{x_\chi} - \boldsymbol{x_I}) + \sum_I \frac{l_0^A}{2m_n}[-\frac{1}{i}\overleftarrow{\nabla}_I \cdot \boldsymbol{\sigma}\delta(\boldsymbol{x_\chi} - \boldsymbol{x_I}) + \delta(\boldsymbol{x_\chi} - \boldsymbol{x_I})\boldsymbol{\sigma} \cdot \frac{1}{i}\overrightarrow{\nabla}_I]$$

$$+ \sum_I \boldsymbol{l}_5 \cdot \boldsymbol{\sigma}\delta(\boldsymbol{x_\chi} - \boldsymbol{x_I}) + \sum_I \frac{\boldsymbol{l}_M}{2m_n}[-\frac{1}{i}\overleftarrow{\nabla}_I \delta(\boldsymbol{x_\chi} - \boldsymbol{x_I}) + \delta(\boldsymbol{x_\chi} - \boldsymbol{x_I})\frac{1}{i}\overrightarrow{\nabla}_I]$$

$$+ \sum_I \frac{\boldsymbol{l}_E}{2m_n}[\overleftarrow{\nabla}_I \times \boldsymbol{\sigma}\delta(\boldsymbol{x_\chi} - \boldsymbol{x_I}) + \delta(\boldsymbol{x_\chi} - \boldsymbol{x_I})\boldsymbol{\sigma} \times \overrightarrow{\nabla}_I].$$

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Nucleon spin written in terms of Pauli matrices

Intrinsic velocity dependence from EFT operators rewritten as derivatives terms

Dark Matter – Lattice Hamiltonian

EFT coefficients contain all DM-spin and DM-momentum (q) dependence

$$
\begin{aligned}
H_{\chi L} = & \sum_I l_0 \delta(\boldsymbol{x}_\chi - \boldsymbol{x}_I) + \sum_I \frac{l_0^A}{2m_n}[-\frac{1}{i}\overleftarrow{\nabla}_I \cdot \boldsymbol{\sigma}\delta(\boldsymbol{x}_\chi - \boldsymbol{x}_I) + \delta(\boldsymbol{x}_\chi - \boldsymbol{x}_I)\boldsymbol{\sigma} \cdot \frac{1}{i}\overrightarrow{\nabla}_I] \\
& + \sum_I \boldsymbol{l}_5 \cdot \boldsymbol{\sigma}\delta(\boldsymbol{x}_\chi - \boldsymbol{x}_I) + \sum_I \frac{\boldsymbol{l}_M}{2m_n}[-\frac{1}{i}\overleftarrow{\nabla}_I \delta(\boldsymbol{x}_\chi - \boldsymbol{x}_I) + \delta(\boldsymbol{x}_\chi - \boldsymbol{x}_I)\frac{1}{i}\overrightarrow{\nabla}_I] \\
& + \sum_I \frac{\boldsymbol{l}_E}{2m_n}[\overleftarrow{\nabla}_I \times \boldsymbol{\sigma}\delta(\boldsymbol{x}_\chi - \boldsymbol{x}_I) + \delta(\boldsymbol{x}_\chi - \boldsymbol{x}_I)\boldsymbol{\sigma} \times \overrightarrow{\nabla}_I].
\end{aligned}
$$

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Although it gets complicated when including all operators, we can calculate H_eff!

Operator Form Factors

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$$F_{1,1}^{(N,N')} = F_M^{(N,N')}$$

$$F_{3,3}^{(N,N')} = \frac{q^2}{4m_n^2} F_{\Phi''}^{(N,N')} + \frac{q^2}{8}\left[v^2 - \frac{q^2}{4\mu_N^2}\right] F_{\Sigma'}^{(N,N')}$$

$$F_{4,4}^{(N,N')} = \frac{C(j_\chi)}{16}\left[F_{\Sigma''}^{(N,N')} + F_{\Sigma'}^{(N,N')}\right]$$

$$F_{5,5}^{(N,N')} = \frac{C(j_\chi)}{4}\left[q^2\left[v^2 - \frac{q^2}{4\mu_N^2}\right] F_M^{(N,N')} + \frac{q^4}{m_n^2} F_\Delta^{(N,N')}\right] = q^2 F_{8,8}^{(N,N')}$$

$$F_{6,6}^{(N,N')} = \frac{C(j_\chi)}{16} F_{\Sigma''}^{(N,N')}$$

$$F_{7,7}^{(N,N')} = \frac{1}{8}\left[v^2 - \frac{q^2}{4\mu_N^2}\right] F_{\Sigma'}^{(N,N')}$$

$$F_{8,8}^{(N,N')} = \frac{C(j_\chi)}{4}\left[\left[v^2 - \frac{q^2}{4\mu_N^2}\right] F_M^{(N,N')} + \frac{q^2}{m_n^2} F_\Delta^{(N,N')}\right]$$

$$F_{9,9}^{(N,N')} = C(j_\chi)\frac{q^2}{16} F_{\Sigma'}^{(N,N')}$$

$$F_{10,10}^{(N,N')} = \frac{q^2}{4} F_{\Sigma''}^{(N,N')}$$

$$F_{11,11}^{(N,N')} = C(j_\chi)\frac{q^2}{4} F_M^{(N,N')}$$

$$F_{1,3}^{(N,N')} = \frac{q^2}{2m_n} F_{M,\Phi''}^{(N,N')}$$

$$F_{4,5}^{(N,N')} = -C(j_\chi)\frac{q^2}{8m_n} F_{\Sigma',\Delta}^{(N,N')}$$

$$F_{4,6}^{(N,N')} = C(j_\chi)\frac{q^2}{16} F_{\Sigma''}^{(N,N')}$$

$$F_{8,9}^{(N,N')} = C(j_\chi)\frac{q^2}{8m_n} F_{\Sigma',\Delta}^{(N,N')}$$

Nuclear Responses

X		$\frac{4\pi}{2J+1} W_X^{(p,p)}(0)$
M	spin-independent	Z^2
Σ''	spin-dependent (longitudinal)	$4\,\frac{J+1}{3J}\langle S_p \rangle^2$
Σ'	spin-dependent (transverse)	$8\,\frac{J+1}{3J}\langle S_p \rangle^2$
Δ	angular-momentum-dependent	$\frac{1}{2}\,\frac{J+1}{3J}\langle L_p \rangle^2$
Φ''	angular-momentum-and-spin-dependent	$\sim \langle \vec{S}_p \cdot \vec{L}_p \rangle^2$ [a]

table from 1401.3739

Nuclear Responses

From writing down the matrix element you get an $e^{-i\vec{q}\cdot\vec{x}}$ which can be expanded into the (vector) spherical harmonics depending on term.

$$M_{JM}(q\boldsymbol{x}) \equiv j_J(qx) Y_{JM}(\Omega_x)$$
$$\boldsymbol{M}_{\boldsymbol{JM}}^{\boldsymbol{M}}(q\boldsymbol{x}) \equiv j_J(qx) \boldsymbol{Y}_{\boldsymbol{JLM}}(\Omega_x)$$

Depending on the operator you will have terms which also come with derivatives and/or Pauli matrices

$$\Delta_{JM}(q\boldsymbol{x}) \equiv \boldsymbol{M}_{\boldsymbol{JM}}^{\boldsymbol{M}}(qx) \cdot \frac{1}{q}\overrightarrow{\nabla}$$
$$\Sigma'_{JM}(q\boldsymbol{x}) \equiv -i\left[\frac{1}{q}\overrightarrow{\nabla} \times \boldsymbol{M}_{\boldsymbol{JM}}^{\boldsymbol{M}}(qx)\right] \cdot \boldsymbol{\sigma}$$
$$\Sigma''_{JM}(q\boldsymbol{x}) \equiv \left[\frac{1}{q}\overrightarrow{\nabla} M_{JM}(qx)\right] \cdot \boldsymbol{\sigma}$$
$$\tilde{\Phi}'_{JM}(q\boldsymbol{x}) \equiv \left[\frac{1}{q}\overrightarrow{\nabla} \times \boldsymbol{M}_{\boldsymbol{JM}}^{\boldsymbol{M}}(qx)\right] \cdot \left[\boldsymbol{\sigma} \times \frac{1}{q}\overrightarrow{\nabla}\right] + \frac{1}{2}\boldsymbol{M}_{\boldsymbol{JM}}^{\boldsymbol{M}}(qx) \cdot \boldsymbol{\sigma}$$
$$\Phi''_{JM}(q\boldsymbol{x}) \equiv i\left[\frac{1}{q}\overrightarrow{\nabla} M_{JM}(qx)\right] \cdot \left[\boldsymbol{\sigma} \times \frac{1}{q}\overrightarrow{\nabla}\right]$$

Squaring the Matrix element gives relevant form factors

$$F_{X,Y}^{(N,N')}(q^2) \equiv \frac{4\pi}{2j+1}\sum_{J=0}^{2j+1}\langle j||X_J^{(N)}||j\rangle\langle j||Y_J^{(N')}||j\rangle,$$

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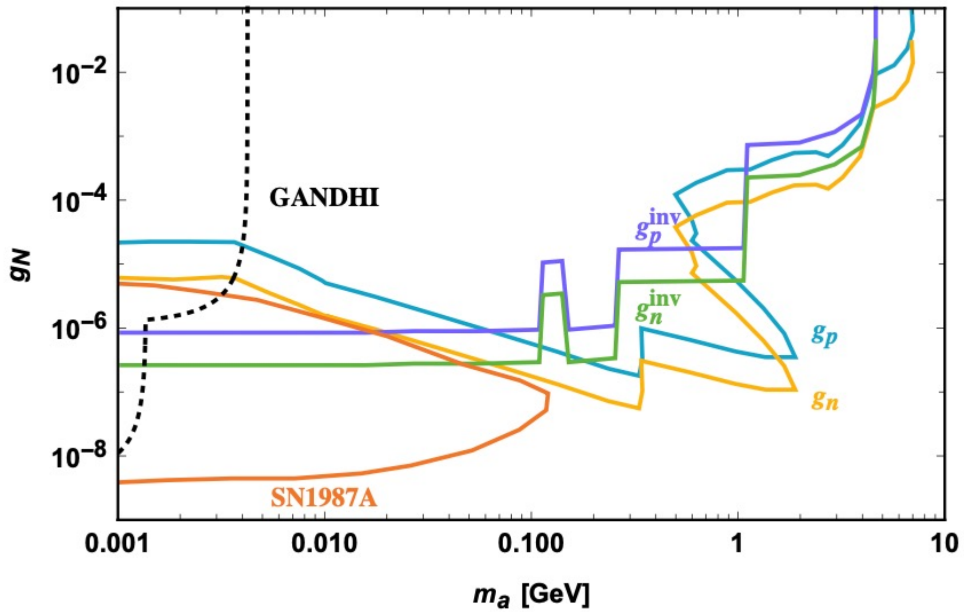
Constraints from UV completions

- PSEUDOSCALAR MEDIATOR
  - meson decays
  - super nova cooling
  - GANDHI experiment from nuclear decays

- AXIAL-VECTOR MEDIATOR
  - Z exchange is widely ruled out, look for BSM exchanges
  - limits from UV completion required to cancel anomalies on axial-vectors

- VECTOR MEDIATOR
  - unexamined.



1905.04319

FIG. 1. Terrestrial and astrophysical limits on the pseudoscalar Yukawa coupling to nucleons are plotted as a function of pseudoscalar mass m_a. Terrestrial limits are adapted from [26] and organized into proton and neutron couplings as well as into visible and invisible decay channels for the pseudoscalar. SN1987A constraints are from [29]. Also shown are projected limits from [10].

UV models

1401.3739

$$\mathcal{L}_{\rm int}^{\rm anapole} = \frac{f_a}{M^2}\bar{\chi}\gamma^\mu\gamma^5\chi\mathcal{J}_\mu^{\rm EM} \quad \to \frac{2f_a}{M^2}\sum_{N=n,p}(Q_N\mathcal{O}_8+\tilde{\mu}_N\mathcal{O}_9)$$

$$\mathcal{L}_{\rm int}^{\rm magnetic\ dipole} = \frac{f_{\rm md}}{M^2}\bar{\chi}\frac{i\sigma^{\mu\nu}q_\nu}{\Lambda}\chi\mathcal{J}_\mu^{\rm EM} \to \frac{2f_{\rm md}}{M^2}\sum_{N=n,p}\left(Q_N\left(\frac{m_N}{\Lambda}\mathcal{O}_5-\frac{\vec{q}^{\,2}}{4m_\chi\Lambda}\mathcal{O}_1\right)+\tilde{\mu}_N\left(\frac{m_N}{\Lambda}\mathcal{O}_6-\frac{\vec{q}^{\,2}}{m_N\Lambda}\mathcal{O}_4\right)\right). \tag{24}$$

$$\mathcal{L}_{\rm int}^{\rm LS} = \frac{f_{\rm LS}}{\Lambda^2}\bar{\chi}\gamma_\mu\chi\sum_{N=n,p}\left(\kappa_1^N\frac{q_\alpha q^\alpha}{m_N^2}\bar{N}\gamma^\mu N+\kappa_2^N\bar{N}\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}N\right)$$

$$\to \frac{f_{\rm LS}}{\Lambda^2}\sum_{N=n,p}\left(\left(\frac{\kappa_2^N}{4}-\kappa_1^N\right)\frac{\vec{q}^{\,2}}{m_N^2}\mathcal{O}_1-\kappa_2^N\mathcal{O}_3+\kappa_2^N\frac{m_N}{m_\chi}\left(\frac{\vec{q}^{\,2}}{m_N^2}\mathcal{O}_4-\mathcal{O}_6\right)\right).$$

UV Models: pseudoscalar mediated

1401.3739

$$\mathcal{L}_{\text{int}}^{\text{pseudoscalar}} = \frac{1}{M^2} \sum_{N=n,p} \left(f_1^N i\bar{\chi}\gamma^5\chi\bar{N}N + f_2^N i\bar{\chi}\chi\bar{N}\gamma^5 N + f_3^N \bar{\chi}\gamma^5\chi\bar{N}\gamma^5 N \right)$$

$$i\bar{\chi}\gamma^5\chi\bar{N}N \to -\frac{m_N}{m_\chi}\mathcal{O}_{11}$$

$$i\bar{\chi}\chi\bar{N}\gamma^5 N \to \mathcal{O}_{10}$$

$$\bar{\chi}\gamma^5\chi\bar{N}\gamma^5 N \to -\frac{m_N}{m_\chi}\mathcal{O}_6,$$