A Unitarity Bound for Type-1 Seesaw Models

By: Francis Burk
Advisor: Tao Han

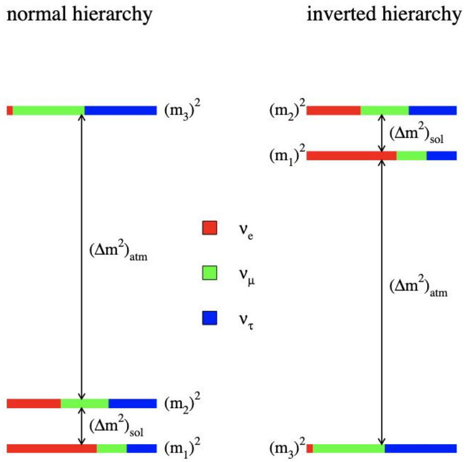
Outline:

1. Neutrino Oscillation and Mass Models overview
2. Simulating New Physics In Madgraph
3. Bolster analysis with Unitarity Bound

Neutrino Masses

- Neutrinos must have distinct masses to oscillate between lepton flavor eigenstates
- Mass eigenstates different from Flavor eigenstates
- Mass states evolve over distances
- Transform between mass and flavor bases with Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix
- Current data constrains “mixing angles”, yielding two possible “mass orderings”
- Still unknown: Origin of Neutrino Masses

$$|\nu_i(t, \vec{x})\rangle = e^{-ip_i x}|\nu_i(0, \vec{0})\rangle$$



Theory: Type 1 Seesaw Models

- Lagrangian: $$\mathcal{L}_N = -\bar{L}\, Y_\nu^D\, \tilde{H}\, N_R - \frac{1}{2}\overline{(N^c)}_L\, M_R\, N_R + \text{H.c.}$$
- Mass Matrix: $$\mathbb{N}^\dagger \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \mathbb{N}^* = \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix}$$
- The diagonalization matrix elements go as: $m_\nu \approx -m_D M_R^{-1} m_D^T \quad , \quad M_N \approx M_R$
- Elements of unitary diagonalization matrix yield mixing between SM and Majorana states

Type 1 Coupling to Standard Model

- In the lepton flavor basis, the coupling to the standard model takes the form:

$$\mathcal{L} \supset \frac{gU_\ell}{\sqrt{2}} \left(W_\mu \bar{l}_L \gamma^\mu N + \text{h.c.} \right) - \frac{gU_\ell}{2\cos\theta_w} Z_\mu \left(\bar{\nu}_L \gamma^\mu N + \bar{N} \gamma^\mu \bar{\nu}_L \right)$$

Features of Type 1:

- Mixing between Majorana Neutrinos and leptons must be small (empirically)
- Majorana Neutrinos can undergo lepton number-violating processes like neutrinoless double beta decay
- Large Majorana mass might be required to reproduce observed neutrino masses

Inverse Seesaw Extension

- Modification of Type 1 Seesaw
- Lagrangian:
  - X field is a Majorana singlet
  - μ_X is small
  - Smallness of μ_X allows for smaller mediator masses
- New mass matrix:
  - M_R is a diagonal 3 x3 matrix with masses separated by:
  - Diagonalization yields mass eigenstates for each generation:

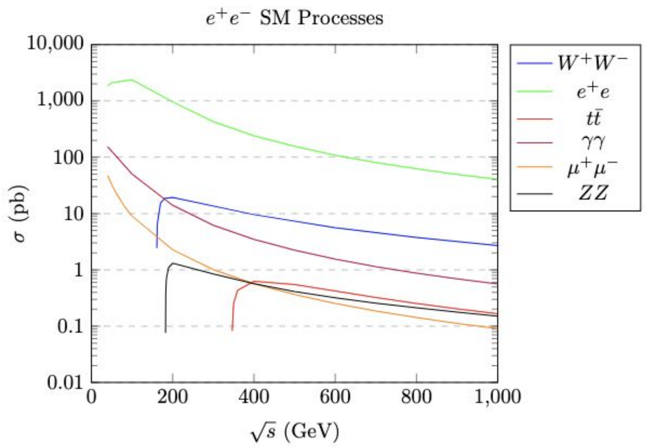
$$\mathcal{L}_{\text{ISS}} = -Y_\nu^{ij}\overline{L_i}\widetilde{\Phi}\nu_{Rj} - M_R^{ij}\overline{\nu_{Ri}^C}X_j - \frac{1}{2}\mu_X^{ij}\overline{X_i^C}X_j + h.c.$$

$$\begin{pmatrix} 0_{3\times3} & m_{D_{3\times3}} & 0_{3\times3} \\ m^T_{D_{3\times3}} & 0_{3\times3} & M_R \\ 0_{3\times3} & M_R^T & \mu_X \end{pmatrix}$$

$$m_\nu \simeq \frac{m_D^2}{m_D^2 + M_R^2}\mu_X \,, \quad m_{N_1,N_2} \simeq \sqrt{M_R^2 + m_D^2} \mp \frac{M_R^2\mu_X}{2(m_D^2 + M_R^2)}$$

Process Simulation Method:

- Tools:
  - Feynrules - Mathematica package that will take model parameters and generate Feynman Rules readable by an event generation tool
  - Madgraph - MonteCarlo event generation tool
- Example SM Processes:
- Goal: Use these tools to analyze BSM processes for specific collider conditions (e.g. pp collision at 13 TeV)

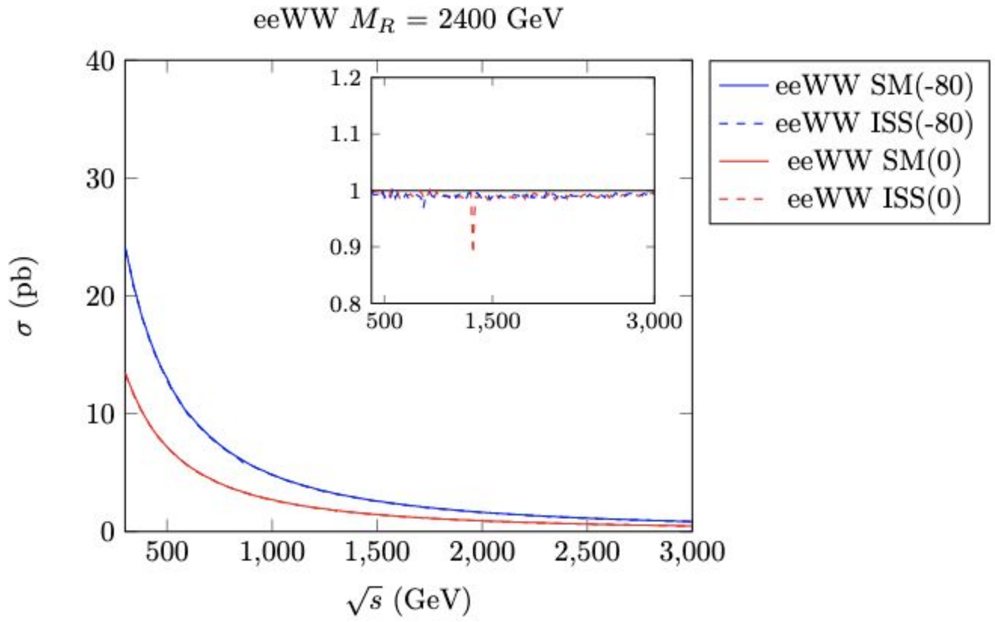


Testing Inverse Seesaw at a lepton collider (1)

- Consider two processes: e+ e- -> W+ W- and e+ e- -> W+ W- H
- Fix M_R = 2400 GeV (all massive neutrinos have the same mass)
- Use numerically generated mass parameters and mixing angles
- Generate cross sections using Madgraph, scan over energy

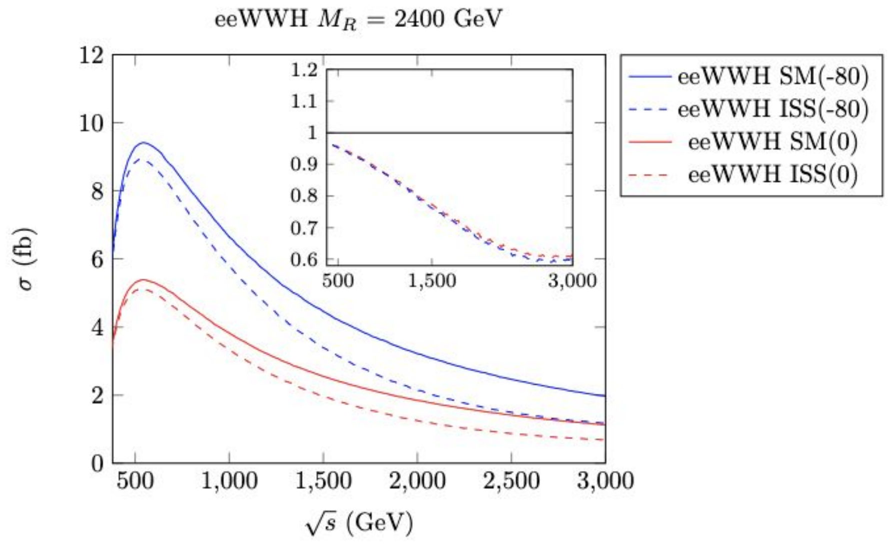
Testing Inverse Seesaw at a lepton collider (2)

- Cross section is larger, but signal is not significant



Testing Inverse Seesaw at a lepton collider (3)

- Discernible signal
  - Drawback: Third-order process suppresses the cross section significantly



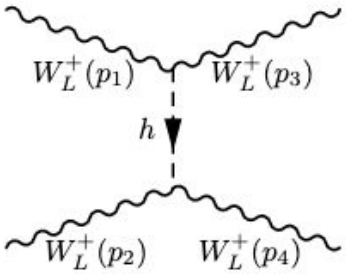
Unitarity and the S-wave constraint

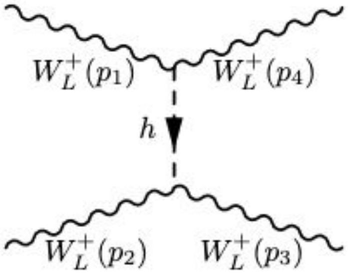
- Unitarity
  - Requirement for the time evolution operator
  - Deviation from unitarity:
    - Inner product no longer preserved
    - Non-physical matrix elements

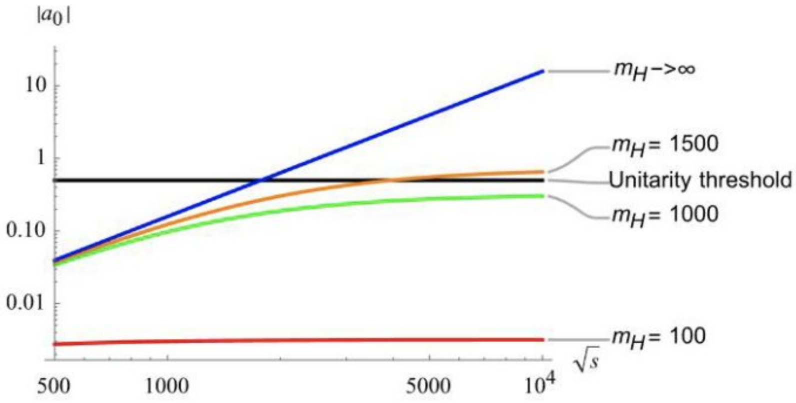
- Angular Momentum Spherical Wave Decomposition: $a_l(s) = \frac{1}{64\pi} \int_{-1}^{1} (i\mathcal{M}(s,t)) P_l(\cos\theta) d\cos\theta$
  - S-wave for l = 0
  - Thus: $a_0 = \frac{1}{64\pi} \int_{-1}^{1} dcos\theta(-i\mathcal{M})$

Lesson from the Past - Bound on Higgs Mass:

- Assume large Higgs mass
- Main contribution from t, u Higgs mediator channels
  - Longitudinal vector bosons have bad high energy behavior
- Take the high-energy limit ($\sqrt{s} \to \infty$)
- Upper bound on Higgs mass is O(1 TeV)

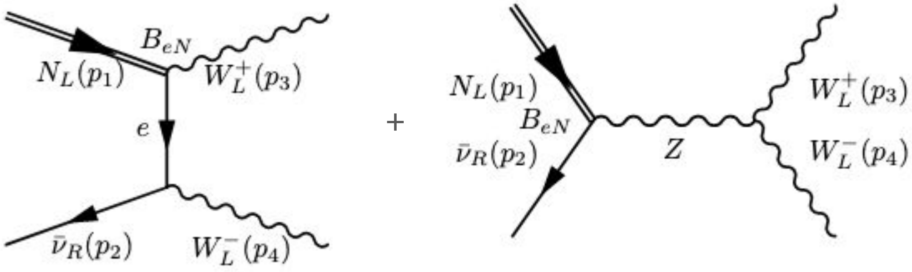


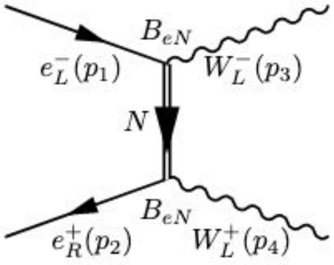




Bound on Neutrino Mixing:

- Assume general Type 1-like model
- Assume general neutrino mixing
- So far, considered processes such as:
    - (among others to be shown)
- Take Longitudinal Vector Bosons





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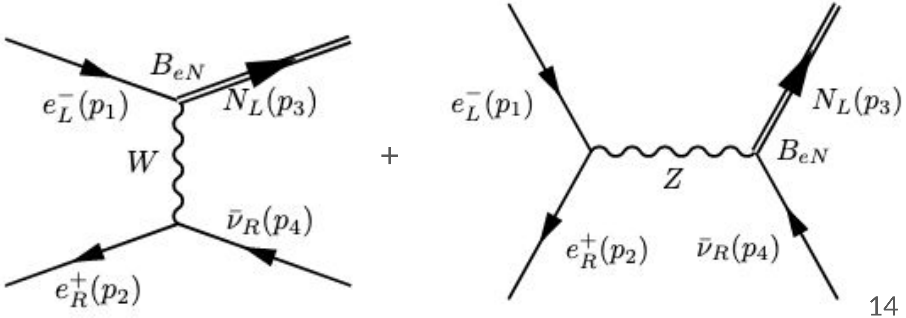
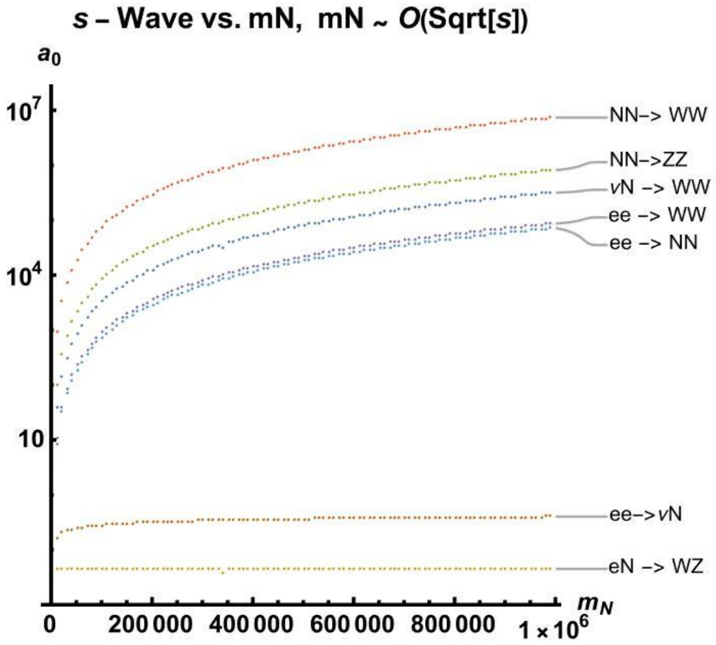
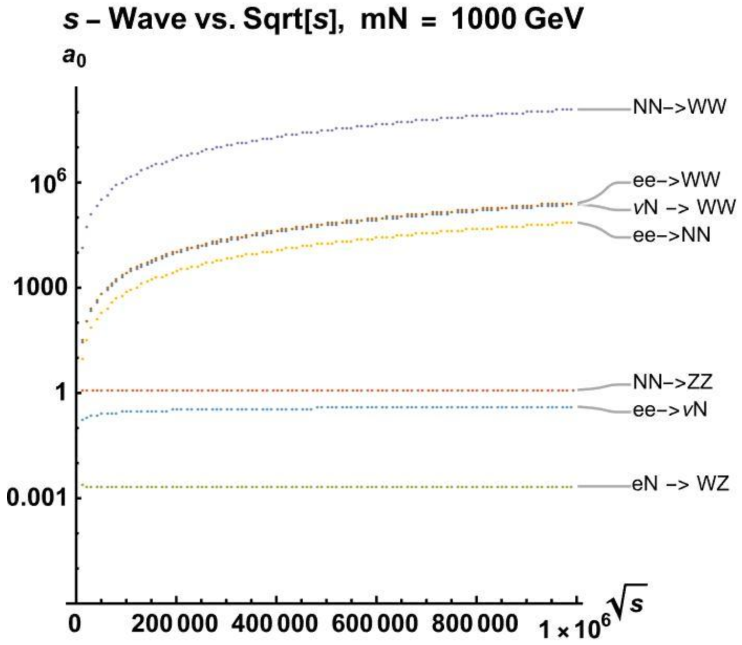


NN -> ZZ (in red) is interesting!

Preliminary Results:





Next Steps:

- Analyze the matrix elements to determine a competitive unitarity bound
- Use the new unitarity bound for neutrino masses to motivate a collider search.
- Use the new bound as input parameters for a madgraph simulation
- Compare this new data against current constrained phase space
- Conduct a detailed analysis looking at various different upcoming collider and neutrino experiments

References:

[1] Baglio, J., Weiland, C., "The Triple Higgs Coupling: A New Probe of Low-Scale Seesaw Models". arXiv:1612.06403

[2] Cai, Y., Han, T., Ruiz, R., "Lepton Number Violation: Seesaw Models and Their Collider Tests". arXiv:1711.02180

[3] Cárcamo Hernández A.E., King, S. F. "Littles Inverse Seesaw Model". arXiv:1903.02565

Backup 1: Type 2 Seesaw

- Lagrangian: $\Delta\mathcal{L}_{II}^{m} = -\overline{L^c}\, Y_\nu\, i\sigma_2\, \Delta_L\, L + \text{ H.c.},$, $\Delta\mathcal{L}_{H\Delta_L} \ni \mu H^T\, i\sigma_2\, \Delta_L^\dagger H + \text{ H.c.}$
- Scalar Triplet Higgs acquires vev through the coupling to the Higgs mechanism
- Consequently generates left-handed Majorana mass
- Model results in seven total Higgses (singly and doubly charged)
- M_υ has direct connections with collider physics, neutrino mass measurements, and neutrino oscillation experiments since Higgses couple to gauge bosons directly

Backup 2: Type 3 Seesaw

- Add SU(2)_L Triplet Leptons transforming as (1,3,0)
- Coupling to the SM Leptons and Higgs Doublet takes form:
- Lagrangian implies mixing between SM and triplet leptons
- Diagonalizing the mass eigenstates yields masses for heavy and light neutrinos and leptons:
- Heavy charged leptons can be searched for in colliders through charged and neutral Drell-Yan scattering

$$\mathcal{L}_Y = -Y_\Sigma \overline{L}\, \Sigma_R^c\, i\sigma^2 H^* + \text{H.c.}$$

$$m_\nu \approx \frac{Y_\Sigma^2 v_0^2}{2M_\Sigma}, \quad M_N \approx M_\Sigma,$$

$$m_l - m_l \frac{Y_\Sigma^2 v_0^2}{2M_\Sigma^2} \approx m_l, \quad M_E \approx M_\Sigma.$$

Backup 3: Type 1 Decay Width Calculations

$$\Gamma^{lW_L} = \frac{g^2}{64\pi M_W^2}|V_{l4}|^2 m_N^3 (1 - \frac{m_W^2}{m_N^2})^2 \qquad = \frac{g^2}{64\pi M_W^2} m_{\nu_l} m_N^2 (1 - \frac{m_W^2}{m_N^2})^2$$

$$\Gamma^{lZ_L} = \frac{g^2}{64\pi^2 M_W^2}|V_{l4}|^2 m_N^3 (1 - \frac{m_Z^2}{m_N^2})^2 \qquad = \frac{g^2}{64\pi^2 M_W^2} m_{\nu_l} m_N^2 (1 - \frac{m_Z^2}{m_N^2})^2$$

$$\Gamma^{lW_T} = \frac{g^2}{32\pi}|V_{l4}|^2 m_N (1 - \frac{m_W^2}{m_N^2})^2 \qquad = \frac{g^2}{32\pi} m_{\nu_l} (1 - \frac{m_W^2}{m_N^2})^2$$

$$\Gamma^{lZ_T} = \frac{g^2}{32\pi \cos^2(\theta_W)}|V_{l4}|^2 m_N (1 - \frac{m_Z^2}{m_N^2})^2 \qquad = \frac{g^2}{32\pi \cos^2(\theta_W)} m_{\nu_l} (1 - \frac{m_Z^2}{m_N^2})^2$$

$$\Gamma^{\nu_l H} = \frac{g^2}{64\pi M_W^2}|V_{l4}|^2 m_N^3 (1 - \frac{m_H^2}{m_N^2})^2 \qquad = \frac{g^2}{64\pi M_W^2} m_{\nu_l} m_N^2 (1 - \frac{m_H^2}{m_N^2})^2$$

Backup 4: Neutrino Unitarity Bound Calculation $e^+e^-\to W^+W^-$

- Matrix element:

$$i\mathcal{M} = \frac{g^2|B_N|^2}{2((l_1 - k_-)^2 - m_{N4}^2)}\bar{v}_L(l_2)\not{\epsilon}^*_L(k_+)(\not{l}_1 - \not{k}_- + m_{N4})\not{\epsilon}^*_L(k_-)u_L(l_1)$$

- Kinematics:

$$\begin{aligned} l_1^\mu &= (E,0,0,E)^T \\ l_2^\mu &= (E,0,0,-E)^T \\ k_-^\mu &= (E,-k\sin\theta,0,-k\cos\theta)^T \\ k_+^\mu &= (E,k\sin\theta,0,k\cos\theta)^T \end{aligned}$$

$$\begin{aligned} \varepsilon^*_{L\mu}(k_-) &= \frac{1}{m_W}(k, E\sin\theta, 0, E\cos\theta)^T \\ \varepsilon^*_{L\mu}(k_+) &= \frac{1}{m_W}(k, -E\sin\theta, 0, -E\cos\theta)^T. \end{aligned}$$

- Evaluating the terms in the Chiral basis yields:

$$i\mathcal{M} = -\frac{|B_N|^2 s^2}{4v^2((\frac{gv}{2})^2 - \frac{s}{2}(1+\beta\cos\theta) - m_N^2)}(\beta^3 + \beta - 2\cos\theta)\sin\theta$$

Backup 4: Neutrino Unitarity Bound Continued

- High-energy limit of amplitude: $$i\mathcal{M} = \frac{|B_N|^2 s}{v^2} \frac{1-\cos\theta}{1+\cos\theta} \sin\theta$$

- Using unitarity constraint: $$|B_N| \leq \frac{8v}{3\sqrt{s}}$$

Backup 5: Neutrino Oscillation Calculation

- To see why neutrino oscillations must result from a mass, consider high energy limit
  - E ~ p + m^2/(2p) ~ E + m^2/(2E)
  - Take t ~ L (length)
  - Obtain:

$$|\nu_j(L)\rangle = e^{-i\left(\frac{m_j^2\, L}{2\, E}\right)}\, |\nu_j(0)\rangle$$

- Moreover,
  - Probability is proportional to squared mass difference
- Neutrino lepton flavor oscillation probabilities are a direct result from the small mass difference

Backup 6: Inverse Seesaw Coupling to Standard Model

- In Feynman-t'Hooft Gauge and mass basis, the SM interactions with the neutrinos are:
- B is proportionate to the mixing between neutrinos and leptons, while C is proportionate to the mixing between neutrinos and neutrinos

$$\mathcal{L}_{\text{int}}^{Z} = -\frac{g_2}{4\cos\theta_W}\bar{n}_i \not{Z}\left[C_{ij}P_L - C_{ij}^* P_R\right] n_j\,,$$

$$\mathcal{L}_{\text{int}}^{H} = -\frac{g_2}{4m_W}H\bar{n}_i\left[(C_{ij}m_{n_i} + C_{ij}^* m_{n_j})P_L + (C_{ij}m_{n_j} + C_{ij}^* m_{n_i})P_R\right] n_j$$

$$\mathcal{L}_{\text{int}}^{G^0} = \frac{\imath g_2}{4m_W}G^0\bar{n}_i\left[-(C_{ij}m_{n_i} + C_{ij}^* m_{n_j})P_L + (C_{ij}m_{n_j} + C_{ij}^* m_{n_i})P_R\right] n_j$$

$$\mathcal{L}_{\text{int}}^{W^\pm} = -\frac{g_2}{\sqrt{2}}\bar{l}_i B_{ij} \not{W}^- P_L n_j + h.c\,,$$

$$\mathcal{L}_{\text{int}}^{G^\pm} = \frac{-g_2}{\sqrt{2}m_W}G^-\left[\bar{l}_i B_{ij}(m_{l_i}P_L - m_{n_j}P_R)n_j\right] + h.c\,,$$

Backup 7: Inverse Seesaw Parameters

- Use these mixings to compute the massive neutrino decay widths.
- Example decay width formulae:

$$\Gamma^{eW_L} = \frac{g^2}{64\pi M_W^2}|B_{eN4}|^2 m_{N4}^3(1-\frac{M_W^2}{m_{N4}^2})^2$$

$$\Gamma^{\mu W_L} = \frac{g^2}{64\pi M_W^2}|B_{\mu N4}|^2 m_{N4}^3(1-\frac{M_W^2}{m_{N4}^2})^2$$

$$\Gamma^{\tau W_L} = \frac{g^2}{64\pi M_W^2}|B_{\tau N4}|^2 m_{N4}^3(1-\frac{M_W^2}{m_{N4}^2})^2$$

$$\Gamma^{eW_T} = \frac{g^2}{32\pi}|B_{eN4}|^2 m_{N4}(1-\frac{M_W^2}{m_{N4}^2})^2$$

$$\Gamma^{\mu W_T} = \frac{g^2}{32\pi}|B_{\mu N4}|^2 m_{N4}(1-\frac{m_W^2}{m_{N4}^2})^2$$

$$\Gamma^{\tau W_T} = \frac{g^2}{32\pi}|B_{\tau N4}|^2 m_{N4}(1-\frac{m_W^2}{m_{N4}^2})^2$$

Backup 8: Numerical Inverse Seesaw Mass Parameters

- Mass Parameters are and mixing angles computed numerically from a single M_R input
  - Fix Yukawa coupling to Dirac mass as 1
  - To enforce unitarity conditions, set:
    - Viz. analysis by Weiland and Baglio, 2017
  - Use Casas-Ibarra Parametrization adapted to the Inverse Seesaw model.
    - Method of diagonalizing the new mass matrix by using the PMNS matrix and measured neutrino masses
  - Back-calculate μ_X with empirical constraints and M_R input
  - End result: Diagonalized masses and mixing angles

$$M_{R_1} = 1.51 M_R, \quad M_{R_2} = 3.59 M_R, \quad M_{R_3} = M_R$$

Backup 9: Analytical Inverse Seesaw Mass parameters

- Diagonalizing the mass matrix to get M_{Ni} and m_{ui} eigenmasses yields:

$$d_1 \approx \frac{M_{R1} m_{\nu_3}}{m_D \mu_1} \qquad e_1 \approx \frac{M_{R1}}{m_D}\sqrt{\frac{m_{\nu_2}}{\mu_5}} \qquad f_1 \approx \frac{M_{R1}}{m_D}\sqrt{\frac{m_{\nu_1}}{\mu_3}}$$

$$d_2 \approx \frac{M_{R2} m_{\nu_3}}{m_D \mu_2} \qquad e_2 \approx \frac{M_{R2}}{m_D}\sqrt{\frac{m_{\nu_2}}{\mu_4}} \qquad f_2 \approx \frac{M_{R2}}{m_D}\sqrt{\frac{m_{\nu_1}}{\mu_5}}$$

$$d_3 \approx \frac{M_3 m_{\nu_3}}{m_D \mu_3} \qquad e_3 \approx \frac{M_{R3}}{m_D}\sqrt{\frac{m_{\nu_2}}{\mu_2}} \qquad f_3 \approx \frac{M_{R3}}{m_D}\sqrt{\frac{m_{\nu_1}}{\mu_6}}$$

- f_i represents the mixing between the electron and the first massive pseudo-dirac Neutrino mass