Quantum

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Feb 10, 2024

Quantum Sensors

Quantum Sensors

- Superconducting Devices

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  - SQUIDs

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  - Transition Edge Sensors (TES)

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- Optomechanical sensors
- Metamaterials

Thermal Noise

$\hbar\omega \gg \kappa_B T$

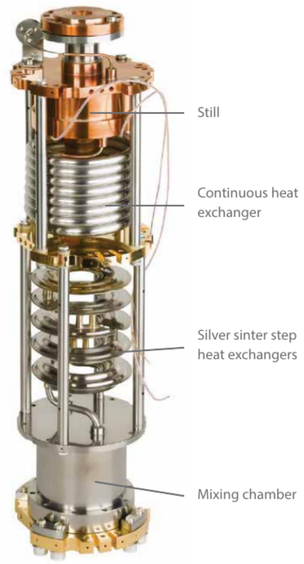
T	Freq
$1\ K$	$20.8\ GHz$
$100\ mK$	$2.08\ GHz$
$50\ mK$	$1.04\ GHz$
$20\ mK$	$416.7\ \mathbf{MHz}$
$10\ mK$	$208.4\ \mathbf{MHz}$

Typical,
Josephson Junction $\sim (4 - 6)\ GHz$,
Resonator $\sim (5 - 9)\ GHz$.
Anisotropy = $\alpha_1 = (E_2 - E_1) - (E_1 - E_0) < 400\ MHz$.

Signal frequency must be 1 decade higher than the thermal noise frequency.

Dilution Refrigerator

Niels Bohr Institute, Copenhagen.

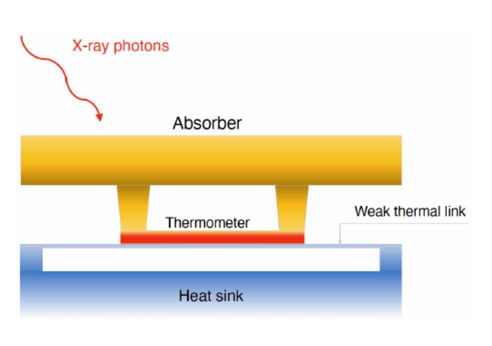


Transition Edge Sensors (TES)
Detects: photon

- SCUBA 2 @ James Clark Maxwell Telescope, Maunakea Hawai'i with 5120 elements at $450\ \mu m$ and $850\ \mu m$.
- Cryogenic Dark Matter Search Experiment (Super-CDMS)
- Detection of Low Energy proton generated in the neutron beta decay.

Wavelength resolution $\Delta\lambda_{FWHM} = \lambda^2 \Delta E_{FWHM}/hc$.
System detection efficiency $\sim 60\%$ at $0.8\ eV$ for Ti/Au TES $8 \times 8\ \mu m^2$.

Transition Edge Sensors (TES)



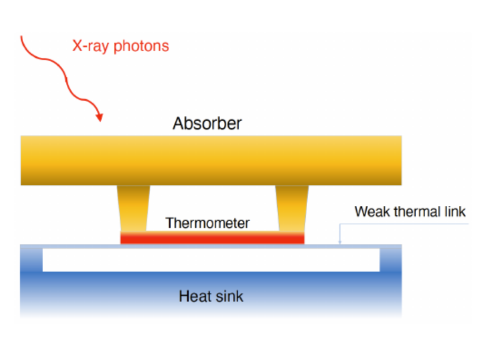
TES : Voltage biased Superconducting thin film based X-ray detector :: thermal equilibrium calorimeter.

A single photon deposits energy E to absorber (heat capacity C)

$$R(T,I) \simeq R_0 + \alpha \frac{R_0}{T_0}\delta T + \beta \frac{R_0}{I_0}\delta I \quad \alpha = \left.\frac{\partial \log R}{\partial \log T}\right|_I \quad \beta = \left.\frac{\partial \log R}{\partial \log I}\right|_T$$

TES Energy resolution $\Delta E \propto \sqrt{4\kappa_B T_0^2 \frac{C}{\alpha}}$

Transition Edge Sensors (TES)



TES : Voltage biased Superconducting thin film based X-ray detector :: thermal equilibrium calorimeter.

Energy gets converted to Heat

$$R(T,I) \simeq R_0 + \alpha \frac{R_0}{T_0}\delta T + \beta \frac{R_0}{I_0}\delta I \quad \alpha = \left.\frac{\partial \log R}{\partial \log T}\right|_I \quad \beta = \left.\frac{\partial \log R}{\partial \log I}\right|_T$$

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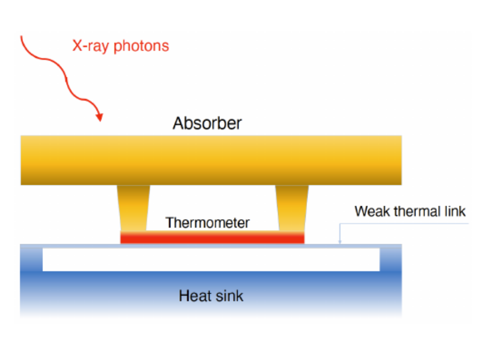

TES : Voltage biased Superconducting thin film based X-ray detector :: thermal equilibrium calorimeter.

Temperature rise is proportional to heat that changes resistance of TES

$$R(T,I) \simeq R_0 + \alpha \frac{R_0}{T_0}\delta T + \beta \frac{R_0}{I_0}\delta I \quad \alpha = \left.\frac{\partial \log R}{\partial \log T}\right|_I \quad \beta = \left.\frac{\partial \log R}{\partial \log I}\right|_T$$

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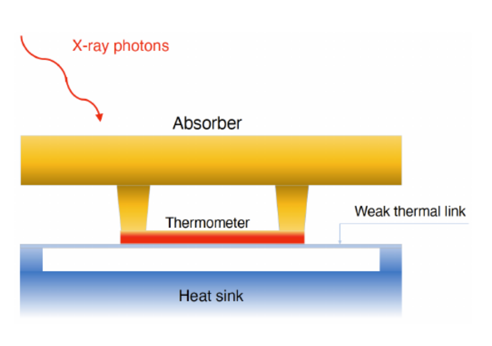

TES : Voltage biased Superconducting thin film based X-ray detector :: thermal equilibrium calorimeter.

Change of resistance changes current through resistor

$\rightarrow$ monitor current through TES using $SQUID$ ammeter.

$$R(T,I) \simeq R_0 + \alpha \frac{R_0}{T_0}\delta T + \beta \frac{R_0}{I_0}\delta I \quad \alpha = \left.\frac{\partial \log R}{\partial \log T}\right|_I \quad \beta = \left.\frac{\partial \log R}{\partial \log I}\right|_T$$

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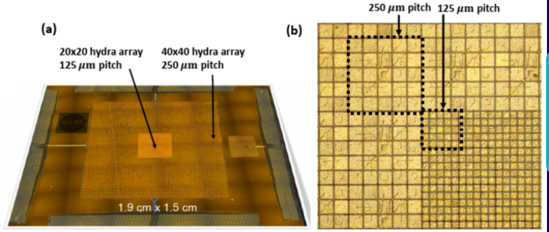
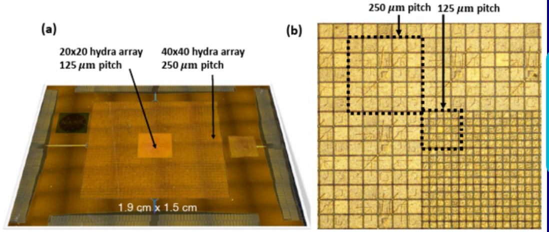


$50K$ pixel LYNX microcalorimeter array at GSFC/NASA.

Readout technology:

Time domain multiplexing (TDM).

Transition Edge Sensors (TES)

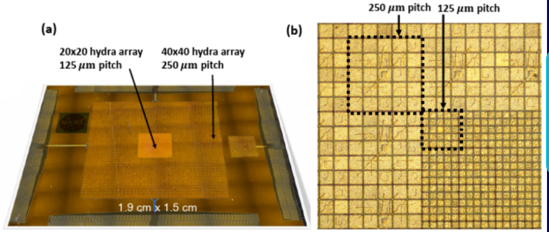


$50K$ pixel LYNX microcalorimeter array at GSFC/NASA.

Readout technology:

Low Frequency domain multiplexing (FDM).

Transition Edge Sensors (TES)

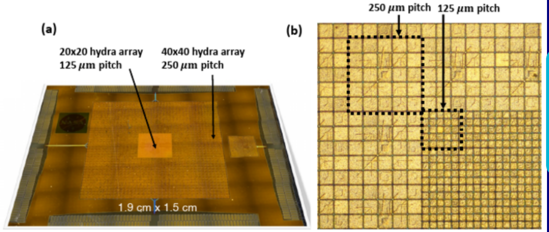


$50K$ pixel LYNX microcalorimeter array at GSFC/NASA.

Readout technology:

Microwave (GHz) multiplexing (μMUX).

Transition Edge Sensors (TES)



$50K$ pixel LYNX microcalorimeter array at GSFC/NASA.

Readout technology:

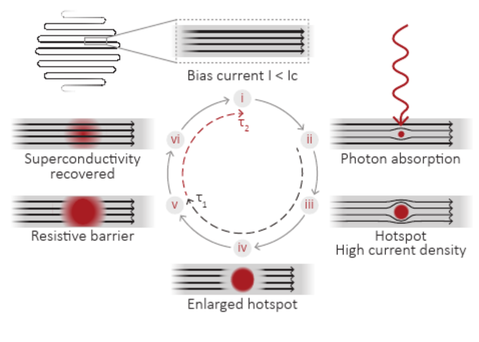
μMUX is most promising that uses RF SQUID with CPW resonator coupled to Traveling Wave Parametric Amplifier (TWPA)

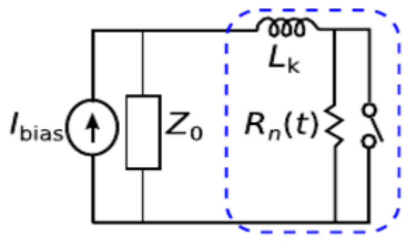
Superconducting Nanowire Single Photon Detector (SNSPD)

Detected: Single Photon, single electron, α, β

Timing: Typ. 15 ps timing zitter

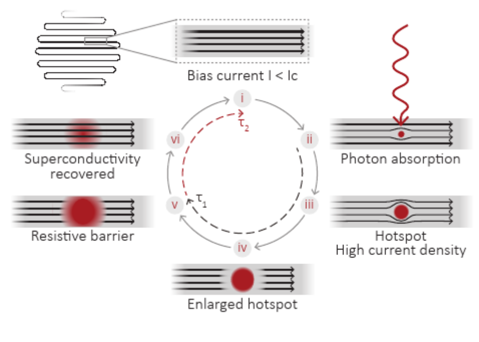
SNSPD

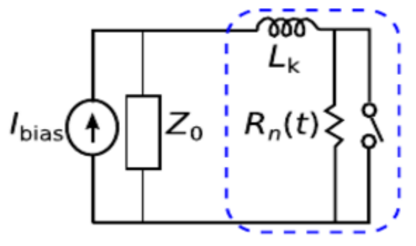




- The superconducting nanowire maintained well below T_c is DC biased just below I_c.

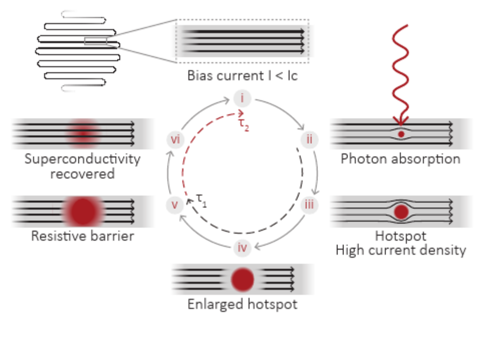
SNSPD

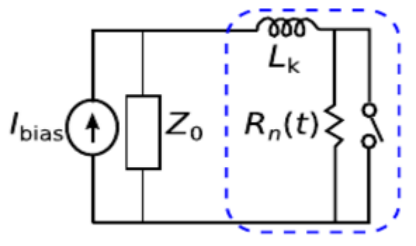




- When a photon is absorbed by the SNSPD a small resistive hotspot is created.

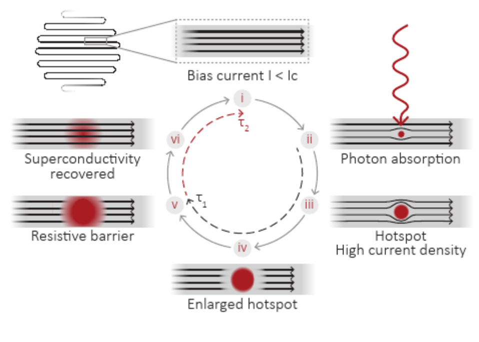
SNSPD

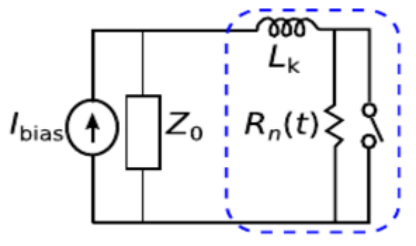




- The supercurrent is forced to flow along the periphery of the hotspot. Since the NbN nanowires are narrow, the local current density around the hotspot increases, exceeding the superconducting critical current density J_c.

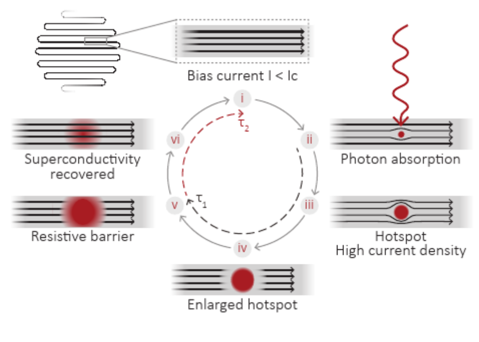
SNSPD

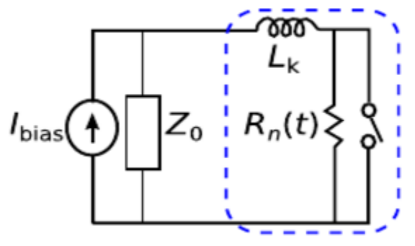




- It leads to the formation of a resistive barrier across the width of the nanowire

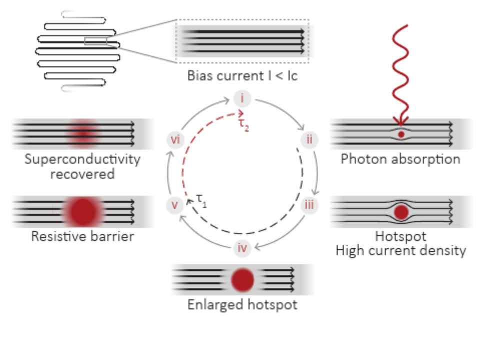
SNSPD

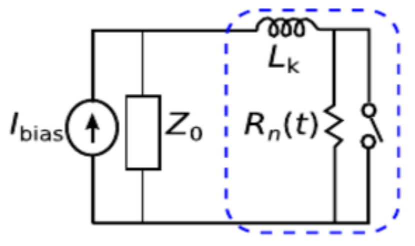




- Joule heating (via the DC bias) aids the growth of resistive region along the axis of the nanowire until the current flow is blocked and the bias current is shunted by the external circuit.

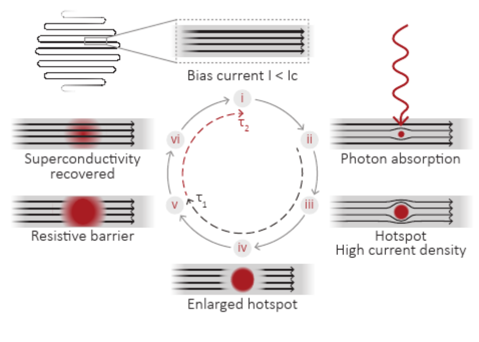
SNSPD

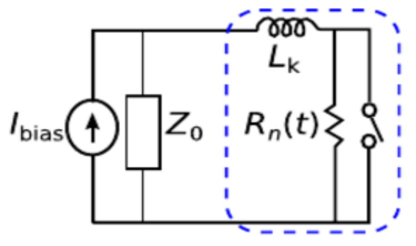




- This allows the resistive region to subside and the wire becomes fully superconducting again. The bias current through the nanowire returns to the original value.

SNSPD





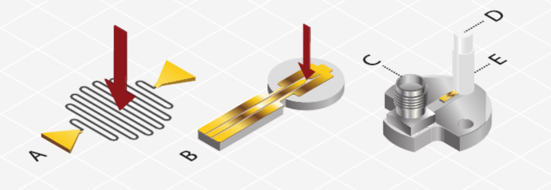
- Model:

$$C_e \frac{dT_e}{dt} = -\frac{C_e}{\tau_{e-p}}\left(T_e - T_p\right) + P(t) \qquad C_p \frac{dT_p}{dt} = -\frac{C_e}{\tau_{e-p}}\left(T_e - T_p\right) - \frac{C_p}{\tau_{es}}\left(T_p - T_0\right)$$

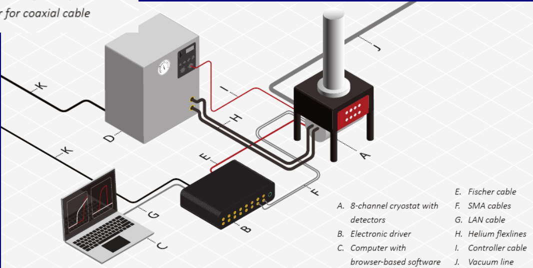
$$R\left(T_e\right) = R_n\left(1 + \exp\left(-4\frac{T_e - T_c}{\Delta T_c}\right)\right)^{-1}$$

$P(t)$: absorbed rad. power, τ_{e-p}: avg. elec-phonon interac. time, τ_{es}: time of phonon escape from superconductor to substrate, R_n: normal state resistance.

SNSPD



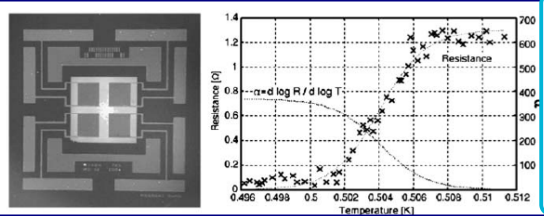
A. The meandering superconducting nanowire
B. Chip containing the superconducting photon detector. The red arrow indicates the light direction
C. Mating sleeve used to align the optical fiber and the detector.
D. Optical fiber termination (ferrule)
E. Electrical connector for coaxial cable



A. 8-channel cryostat with detectors
B. Electronic driver
C. Computer with browser-based software
D. Helium compressor
E. Fischer cable
F. SMA cables
G. LAN cable
H. Helium flexlines
I. Controller cable
J. Vacuum line
K. Power cable

Quantum Sensing Examples

Detection of Low Energy proton generated in the neutron beta decay using Transition Edgeg Sensor (TES).



- 4 pixels array - Ti/Au bilayer - Silicon Nitride membrane - Si wafer
- $\Delta E \sim 30\ eV$ for $500\ eV$
- *(Nuclear Instruments and Methods in Physics Research A,* ***559****, (2006) 573-575)*

Quantum Sensing Examples

High Energy Proton Bombardment having energy ($54\ MeV$ to $280\ MeV$) at Proton Irradiation Facility, PSI on DC SQUIDS.

Relevant physical and electrical properties of the three SQUID sensors.

ATTRIBUTE	SQUID I	SQUID II	SQUID III
Josephson Junction Area (m^2)	$6.1x10^{-12}$	$4x10^{-12}$	$7.1x10^{-12}$
Silicon Substrate Crystal Orientation	<100>	<111>	<100>
Junction Type	Nb/AlO/Nb	Nb/NbOx/Pb InAu	Nb/AlO/Nb
SQUID Loop Area (mm^2)	0.36	1.3	2.1
SQUID Hole Area (mm^2)	$2.5x10^{-3}$	0.14	0.017
Loop Material and Thickness (nm)	PbInAu 550	Nb 300 PbInAu 430	Nb 120
SQUID Critical Current (μA)	18.1	38	10
Carrier Material	Sapphire	Sapphire	Fiberglass
B Field at SQUID (μT)	0.2	40	0.2

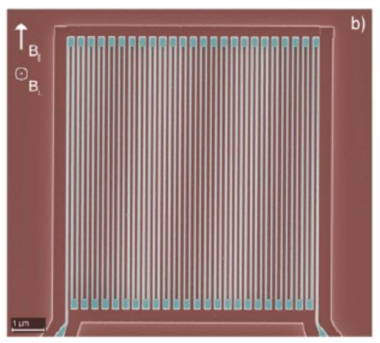
- Proton bombardment with 10^4 to $10^7\ p/cm^2/s$
- Squid voltage output in both open loop AND flux-locked loop.
- *(IEEE Transactions on Applied Superconductivity,* **5**, *Issue: 2, June 1995)*

Quantum Sensing Examples

Single particle detection with superconducting nanowire (SNSPD).

- $500\ \mu m$ long, $100\ nm$ wide, $5\ nm$ thick meandering over $10\ \mu m$ diameter $NbTiN$ ($T_c \sim 12\ K$) nanowire disk.
- Operating temperature $4.2\ K$.
- Saturation count rate $200\ MHz$.
- *(AIP Advances* 2, 032124 (2012))

First detection of $120\ GeV$ protons with $SNSPD$.



- Timing Zitter $\approx 10\ ps$ - pixel size $10\ \mu m$
- SNSPD as charged particle detector with $\hbar\omega \gg 2\Delta \approx 2\ meV$
- Timing difference between plastic scintillators and nano wire.
- Optimal position using $MWPC2$ and $SNSPD$ (+ plastic scintillator)
- *(CPAD Workshop 2023@SLAC reported by S Lee and Whitney Armstrong)*

Josephson Junction

and

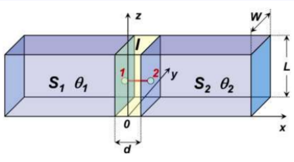
Superconducting QUantum Interference Device (SQUID)

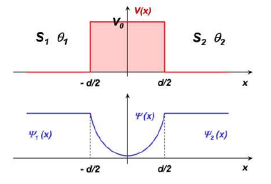
Superconducting sensors

Quantum phenomenon in macroscopic scale

Superconductor

Condensate of electrons bound in pairs (Cooper Pairs)





Superconducting sensors : JJ

Motion of center-of-mass of the electrons (Ginzberg-Landau Order Parameter)

$$\Psi_j = \sqrt{n_j} e^{i\varphi_j(\vec{r},t)} \qquad \text{where, } \phi_j(\vec{r},t) = \phi(\vec{r}) - \frac{2E_F}{\hbar} t$$

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$$i\hbar \frac{\partial \Psi_1}{\partial t} = E_1 \Psi_1 + \kappa \Psi_2 \qquad \text{and} \qquad i\hbar \frac{\partial \Psi_2}{\partial t} = E_1 \Psi_2 + \kappa \Psi_1$$

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Let V be applied voltage.

$$I = 2e \frac{\partial n_1}{\partial t} = -2e \frac{\partial n_2}{\partial t}$$

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Assuming same Cooper pair flows through $n_1 = n_2 = n$

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Let V be applied voltage.

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Assuming same Cooper pair flows through $n_1 = n_2 = n$

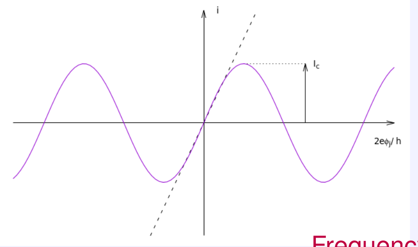
$$\boxed{I} = 2e \frac{\partial n}{\partial t} = 4\kappa \frac{en}{\hbar} \sin\varphi = \boxed{I_c \sin\varphi} : \ \varphi = \varphi_1 - \varphi_2 \text{ and } I_c = \frac{4\kappa en}{\hbar}$$

$$\text{Also } \boxed{V = \frac{\hbar}{2e} \frac{d\varphi}{dt}}$$

JJ

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar}V = \frac{2\pi}{\Phi_0}V$$

$$\varphi(t) = \varphi_0 + \frac{2\pi}{\Phi_0}Vt$$



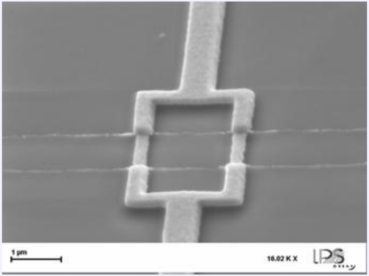
Frequency f

$$I = I_c \sin(\varphi) = I_c \sin\left(\varphi_0 + \frac{2\pi}{\Phi_0}Vt\right) = I_c \sin\left(\varphi_0 + \boxed{\frac{2eV}{\hbar}}t\right)$$

Voltage Controlled Oscillator

Here, $\Phi_0 = h/(2e) = 2.067\times10^{-15}\ Wb = 2.067\ mV.ps$ = Magnetic flux Quantum.
$K_J = \frac{1}{\Phi_0} = (2e)/h = 483.6\ MHz/\mu V$ = Josephson Constant

Superconducting QUantum Interference Device (DC / AC)



DC SQUID Groupe Physique Mesoscopique, LPS, Orsay

$\Phi = \Phi(t) \rightarrow$ SQUID induces:

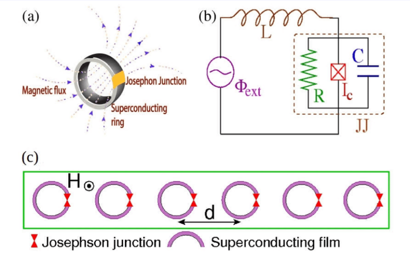
$I_{total} = I_c$ (lossless) + I_{quasi} (Ohmic)

$\Phi_{ext} = \Phi_{dc} + \Phi_{ac}\cos(\omega t)$

$$\Phi = \Phi_{ext} + LI$$

$$I = -C\frac{d^2\Phi}{dt^2} - \frac{1}{R}\frac{d\Phi}{dt} - I_c \sin\left(2\pi\frac{\Phi}{\Phi_0}\right)$$

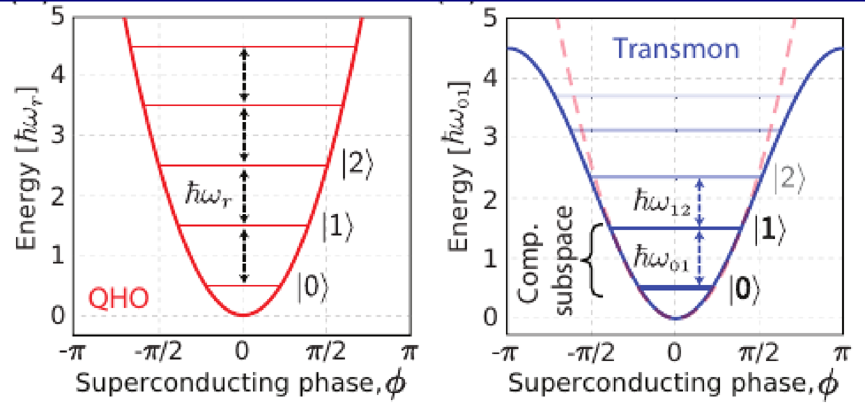
$$C\frac{d^2\Phi}{dt^2} + \frac{1}{R}\frac{d\Phi}{dt} + I_c \sin\left(2\pi\frac{\Phi}{\Phi_0}\right) + \frac{\Phi - \Phi_{ext}}{L} = 0$$



RF SQUID (single JJ) arXiv:1902.02158v1

Nonlinearity

Nonlinearity is essentially required.



Now $\boxed{I} = \boxed{I_c \sin\varphi} \implies \frac{dI}{dt} = I_c \cos\varphi \frac{d\varphi}{dt}$

and $\boxed{V} = \frac{\Phi_0}{2\pi}\frac{d\varphi}{dt} = \boxed{\frac{\Phi_0}{2\pi}\frac{1}{I_c \cos\varphi}\frac{dI}{dt}}$

or, $V = L_j \frac{dI}{dt}$: $\boxed{L_j} = \boxed{\frac{\Phi_0}{2\pi}\frac{1}{I_c \cos\varphi}}$

JJ : cQED

Now $\boxed{I} = \boxed{I_c \sin\varphi} \implies \frac{dI}{dt} = I_c \cos\varphi \frac{d\varphi}{dt}$

and $\boxed{V} = \frac{\Phi_0}{2\pi}\frac{d\varphi}{dt} = \boxed{\frac{\Phi_0}{2\pi}\frac{1}{I_c\cos\varphi}\frac{dI}{dt}}$

or, $V = L_j \frac{dI}{dt}$: $\quad \boxed{L_j} = \boxed{\frac{\Phi_0}{2\pi}\frac{1}{I_c\cos\varphi}}$

Energy stored in JJ =

$$\epsilon_j = \int_0^t IV dt = \int_0^t (I_c \sin\varphi)\left(\frac{\Phi_0}{2\pi}\frac{d\varphi}{dt}\right)dt = \frac{\Phi_0 I_c}{2\pi}\int_0^{\varphi} \sin\varphi d\varphi$$

$$\epsilon_j = \frac{\Phi_0 I_c}{2\pi}(1-\cos\varphi) = E_j(1-\cos\varphi) = E_j\varphi^2/2 - E_j\varphi^4/4! + \ldots$$

Josephson Energy $\boxed{E_j = \frac{\Phi_0 I_c}{2\pi}}$

- order of magnitude:
  - typically: $I_c \sim 1\ mA \implies E_J \simeq 3\times 10^{-19}\ J \sim 2\ eV \sim T \sim 23800\ K$

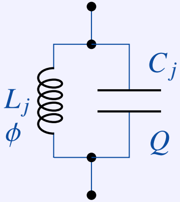
JJ : cQED

$$T_j = \frac{1}{2}C_jV^2 = \frac{Q^2}{2C_j} = \frac{1}{2}C_j\dot{\phi}^2 \quad \text{and} \quad V_j = \frac{1}{2}L_jI^2 = \frac{1}{2}\frac{\phi^2}{L_j}$$

$\because V = -\dot{\phi}$ and $Q = CV = -C\dot{\phi}$

$$\Longrightarrow \hat{\mathcal{L}} = \frac{\hat{Q}^2}{2C_j} - \frac{\hat{\phi}^2}{2L_j} = \frac{1}{2}C_j\dot{\phi}^2 - \frac{1}{2}\frac{\phi^2}{L_j}$$

$$\Longrightarrow \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = C_j\dot{\phi} = Q$$



$$\boxed{\hat{\mathcal{H}}} = \boxed{\frac{\hat{Q}^2}{2C_j} + \frac{\hat{\phi}^2}{2L_j}}$$

$$= \left(\frac{\hat{Q}}{\sqrt{2C_j}} + i\frac{\hat{\phi}}{\sqrt{2L_j}}\right)\left(\frac{\hat{Q}}{\sqrt{2C_j}} - i\frac{\hat{\phi}}{\sqrt{2L_j}}\right) + \frac{i}{2\sqrt{L_jC_j}}\left[\hat{Q},\hat{\phi}\right]$$

$$\boxed{\hat{\mathcal{H}}} = \boxed{\hbar\omega_q\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)} = \boxed{\frac{1}{2}\hbar\omega_q\hat{\sigma}_z \text{ (if 2-level)}}$$

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\boxed{\omega_q = \frac{1}{\sqrt{L_jC_j}} \quad \text{and} \quad Z_q = \sqrt{\frac{L_j}{C_j}}}$$

Ladder operators and ZPF

$$\hat{a} = \frac{1}{\sqrt{\hbar\omega_q}}\left(\frac{\hat{Q}}{\sqrt{2C_j}} - i\frac{\hat{\phi}}{\sqrt{2L_j}}\right) \qquad \hat{a}^\dagger = \frac{1}{\sqrt{\hbar\omega_q}}\left(\frac{\hat{Q}}{\sqrt{2C_j}} + i\frac{\hat{\phi}}{\sqrt{2L_j}}\right)$$

$$\implies \hat{Q} = \sqrt{\frac{\hbar}{2Z_q}}\left(\hat{a} + \hat{a}^\dagger\right) = Q_{ZPF}\left(\hat{a} + \hat{a}^\dagger\right) \qquad Q_{ZPF} = \sqrt{\frac{\hbar}{2Z_q}} = e\sqrt{\frac{R_Q}{2\pi Z_q}}$$

$$\hat{\phi} = i\sqrt{\frac{\hbar Z_q}{2}}\left(\hat{a} + \hat{a}^\dagger\right) = i\phi_{ZPF}\left(\hat{a} - \hat{a}^\dagger\right) \qquad \phi_{ZPF} = \sqrt{\frac{\hbar Z_q}{2}} = \Phi_0\sqrt{\frac{Z_q}{2\pi R_Q}}$$

$$\Phi_0 = \frac{h}{(2e)} \qquad R_Q = \frac{h}{2e^2}$$

Physically, variances in the ground state quantum fluctuation.

$\phi_{ZPF} \equiv \langle 0|\,\hat{\phi}^2\,|0\rangle$ and $Q_{ZPF} \equiv \langle 0|\,\hat{Q}^2\,|0\rangle$

Ground state wave function: cQED

Let ground state wave function be $|\psi_0\rangle$.

$$\hat{a}\,|\psi_0\,(\phi, Q)\rangle = 0.$$

$$\hat{\phi} \equiv i\hbar\frac{\partial}{\partial Q} \qquad \hat{Q} \equiv -i\hbar\frac{\partial}{\partial \phi}$$

$$\hat{a}_\phi = \sqrt{\frac{1}{\hbar\omega_q}}\left(-\frac{i\hbar}{\sqrt{2C_j}}\frac{\partial}{\partial\phi} - \frac{i}{\sqrt{2L_j}}\hat{\phi}\right) \quad \hat{a}_Q = \sqrt{\frac{1}{\hbar\omega_q}}\left(\frac{1}{\sqrt{2C_j}}\hat{Q} + \frac{\hbar}{\sqrt{2L_j}}\frac{\partial}{\partial Q}\right)$$

$$\hat{a}_\phi\left|\phi_{0\phi}\right\rangle = 0,\ \text{or,}\ \left|\phi_{0\phi}\right\rangle = C_\phi e^{-\frac{\phi^2}{4\phi_{ZPF}^2}}$$

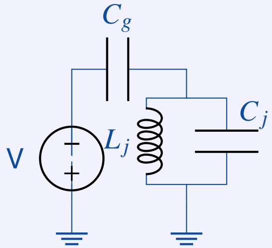
$$\hat{a}_Q\left|\phi_{0Q}\right\rangle = 0,\ \text{or,}\ \left|\phi_{0Q}\right\rangle = C_\phi e^{-\frac{Q^2}{4Q_{ZPF}^2}}$$

Normalised ground state

$$|\phi_0\,(\phi, Q)\rangle = \frac{1}{\sqrt{2\pi\phi_{ZPF}Q_{ZPF}}}e^{-\left(\frac{Q^2}{4Q_{ZPF}^2}+\frac{\phi^2}{4\phi_{ZPF}^2}\right)}$$

Fluctuation is inherent in the state to be measured.

Modeling JJ



$E_j = \frac{\Phi_0 I_c}{2\pi}$ $\quad E_c = \frac{e^2}{2C_\Sigma}$ where, $C_\Sigma = C_g + C_j$

$\hat{\mathcal{H}} = 4E_c\left(\hat{n} - n_g\right)^2 - E_j \cos\hat{\varphi}$ where, $\hat{n} = -i\frac{d}{d\varphi}$

Charge Dispersion for energy level m:

$\epsilon_m = E_m(n_g = 1/2) - E_m(n_g = 0)$

$\hat{\mathcal{H}} = \sqrt{8E_cE_j}\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$ - E_j - $\frac{E_c}{12}\left(\hat{a}^\dagger + \hat{a}\right)^4$

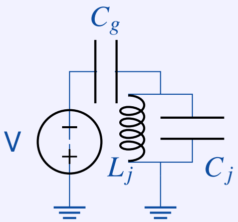
So, $\epsilon_m \simeq -E_j + \sqrt{E_cE_j}\left(m + \frac{1}{2}\right) - \frac{E_c}{12}\left(6m^2 + 6m + 3\right)$

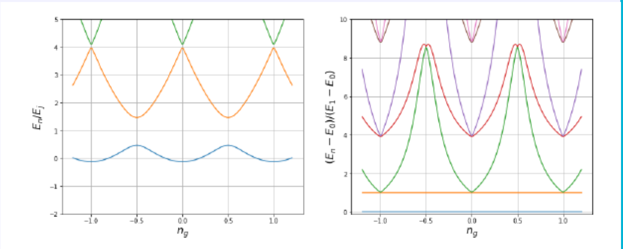
Absolute Anharmonicity

$\alpha_m = E_{m+1,m} - E_{m,m-1} \simeq -E_c$: $E_{mn} = E_m - E_n$

One can check Hamiltonian property numerically

Modeling JJ





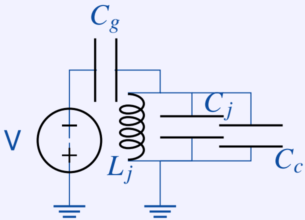
Let E_j/E_c = 1.0. E_{01}/h=5 GHz Let h=1

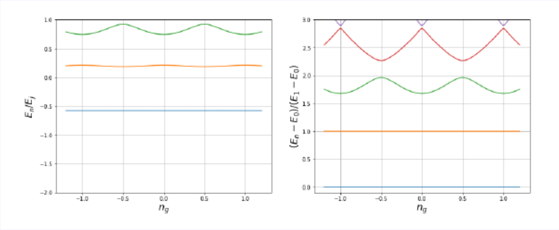
$E_{01} \simeq \sqrt{8E_cE_j} = \sqrt{8 * E_c * E_c} = 5\ GHz$.

$\implies E_c$=1.77 GHz and E_j = 1.77 GHz.

$\alpha_m \sim -E_c = -1.77\ GHz$

Modeling JJ





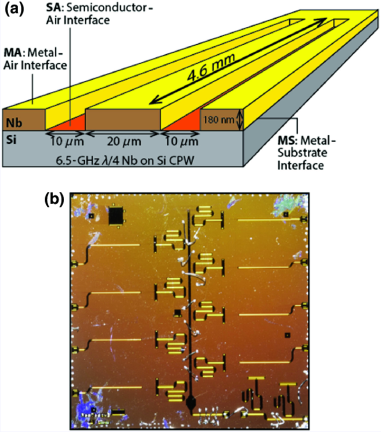
Let $E_j/E_c = 10.0$. E_{01}/h=5 GHz Let h=1

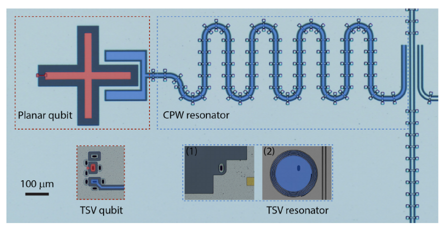
$E_{01} \simeq \sqrt{8E_cE_j} = \sqrt{8 * E_c * 10 * E_c} = 5\ GHz$.

$\implies E_c$=559 MHz and $E_j = 5.59\ GHz$.

$\alpha_m \sim -E_c = -559\ MHz$

Resonators





Appl. Phys. Lett, **123**, 154004, (2023)

TSV: Through superconducting vias

PRX Quantum, **3**, 020312, (2022).

JJ resonator system

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{JJ} + \hat{\mathcal{H}}_{CPW} + \hat{\mathcal{H}}_{int}$$

$H_{int} = -\vec{d}.\vec{E}$ where $\vec{d} \propto (\sigma^+ + \sigma^-)$ and $\vec{E} \propto (\hat{a} + \hat{a}^\dagger)$

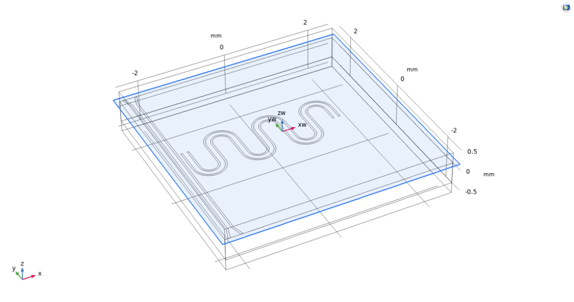
or, $\hat{\mathcal{H}} = \hbar\omega_q\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\omega_r\left(\hat{b}^\dagger\hat{b} + \frac{1}{2}\right) + \hbar g\,(\sigma^+ + \sigma^-)\left(\hat{a} + \hat{a}^\dagger\right)$

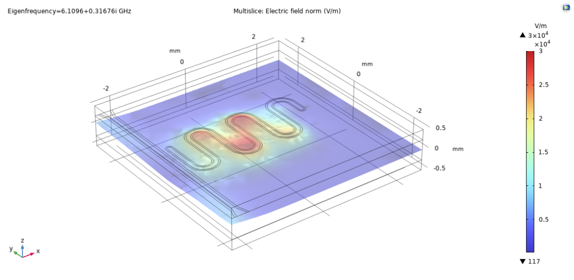
or, $\hat{\mathcal{H}} = \frac{1}{2}\hbar\omega_q\hat{\sigma}_z + \hbar\omega_r\left(\hat{b}^\dagger\hat{b} + \frac{1}{2}\right) + \hbar g\left(\sigma^+\hat{a} + \hat{a}^\dagger\sigma^-\right)$

Important Parameters:

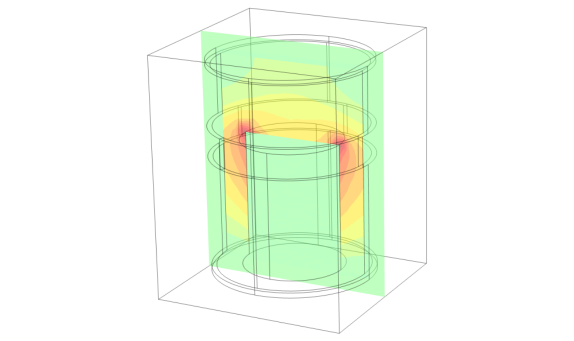
Detuning = Δ = ω_q - ω_r, Dispersive shift = $\chi = g^2/\Delta \propto \left(E_j/E_c\right)^{1/4}$.

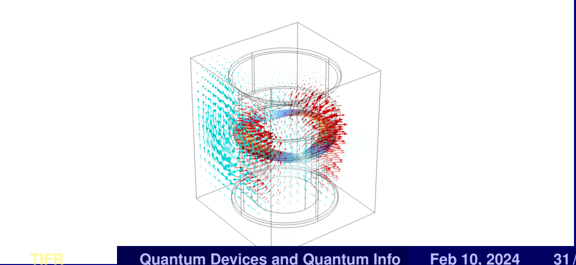
Resonators: 2D:Halfwave



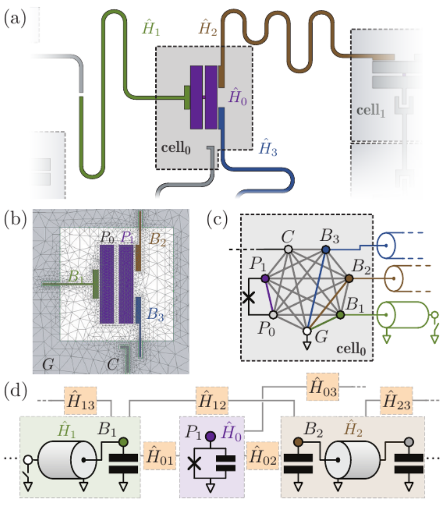


Resonators: 3D : Cylindrical



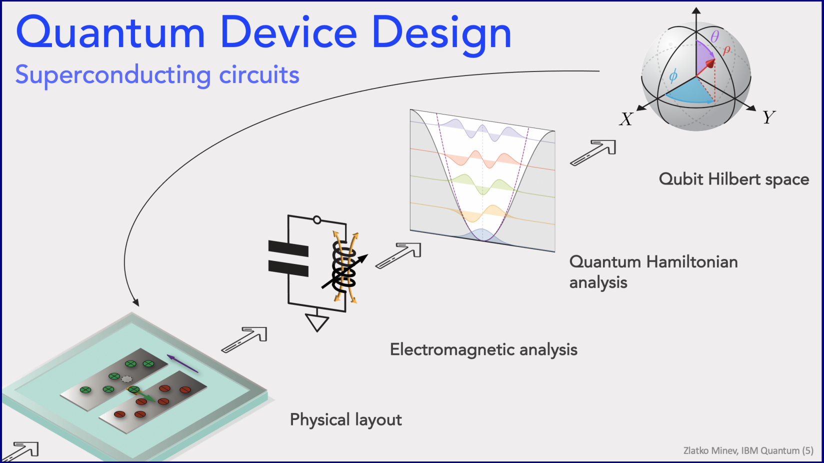


Analysis: Lumped Parameter



Zlatko Minev et al arXiv:2103.10344v1,

quant-ph, 18 Mar 2021


Quantum Device Design
Superconducting circuits
Qubit Hilbert space
Quantum Hamiltonian analysis
Electromagnetic analysis
Physical layout
X
Y
Zlatko Minev, IBM Quantum (5)

Conceptually JJ energy may be decomposed as

$$\epsilon_j(\varphi_j) = E_j\left(1 - \cos\varphi_j\right) = \epsilon_{j,lin}(\varphi_j) + \epsilon_{j,nl}(\varphi_j)$$

.

$$\epsilon_{j,lin} = \frac{1}{2}E_j\left(\varphi_j\right)^2$$

$$\epsilon_{j,nl} = E_j\sum_{p=3}^{\infty} c_{jp}\left(\varphi_j\right)^p$$

$$c_{jp} = \begin{cases} \frac{(-1)^{p/2+1}}{p!} & \text{for even p} \\ 0 & \text{for odd p} \end{cases}$$

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{lin} + \hat{\mathcal{H}}_{nl}$$

$$\hat{\mathcal{H}}_{lin} \equiv \hat{\mathcal{H}}_{EM} + \sum_j \frac{1}{2} E_j \varphi_j^2 = \sum_m \hbar\omega_m \hat{a}_m^\dagger \hat{a}_m \rightarrow \text{ FE simulation for mode } m$$

$\hat{a}_m$ is the m^{th} mode amplitude (annihilation op).

$$\hat{\mathcal{H}}_{nl} \equiv \sum_j \sum_{p=3}^{\infty} E_j c_{jp} \hat{\varphi}_j^p$$

$$\hat{\varphi}_j = \sum_m \phi_{mj} (\hat{a}_m^\dagger + \hat{a}_m)$$

ϕ_m are real-valued ZPF of mode m at junction j

Energy Participation Ratio (EPR)

EPR = Fraction of total inductive energy stored in the junction.

$$p_{mj} = \frac{\text{Inductive energy stored in junction}\, j}{\text{Inductive energy stored in mode}\, m}$$

$$= \frac{\langle n_m| : \frac{1}{2} E_j \hat{\phi}_j^2 \,|n_m\rangle}{\langle n_m| \,\frac{1}{2} \hat{H}_{lin} : |n_m\rangle}$$

$$\phi_{mj}^2 = p_{mj} \frac{\hbar\omega_m}{2E_j} \qquad 0 \leq p_{mj} \leq 1$$

Quantum Information

States: Classical Information

Assume system X : stores INFORMATION

If X is a set of playing cards, possible states are {♠, ♥, ♦, ♣}

Let $|\alpha\rangle$ has 1 in the entry corresponding to α in Φ. Then, $\langle\alpha|$ be the row vector.

If Φ={♠, ♥, ♦, ♣}, then each entity (Basis vector) can be described as

$$|\spadesuit\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad |\heartsuit\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad |\diamondsuit\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad |\clubsuit\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \quad |\heartsuit\clubsuit\rangle = \begin{pmatrix}0\\1\\0\\1\end{pmatrix}$$

Quantum States

- A Quantum state encapsulates all the information of the system being represented by either column or row vector.
- Quantum states may exist in superposition *i*.e. simultaneous combination of different states.
- Quantum states must be normalised so that sum of probabilities of all possible outcome after measuring the state situation is 1.

Key features of Quantum Operators

- Observable (Physical quantity) in QM is a special Operator ($\hat{A}$) which is self-adjoint to yield real eigenvalue.
- $\hat{A}$ is Hermitian. $\hat{A}^{\dagger} = \hat{A}$. $\langle\psi|\hat{A}|\phi\rangle = \langle\phi|\hat{A}|\psi\rangle^{\star} \rightarrow \left\langle\hat{A}\right\rangle = \langle\psi|\hat{A}|\psi\rangle$: REAL.
- In QC, gates are Unitary operators: $\hat{A}^{-1} = \hat{A}^{\dagger}$.
- $\hat{A}|\psi\rangle = \epsilon|\psi\rangle$ Eigenstate & Eigenvalue

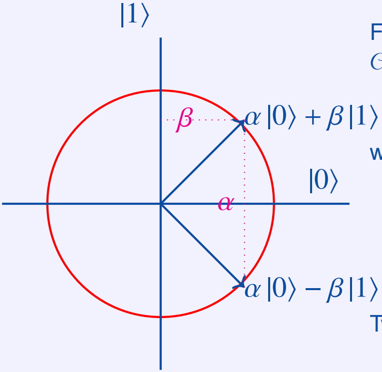
BITS: TWO state system

$$0 \quad \leftrightarrow 1$$

Computation is possible with any system with finite set of discrete and stable states with controlled transitions between them.

BITS: TWO state system

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad \leftrightarrow \quad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$



Formally Qubit state is UNIT VECTOR in $\mathcal{C}^2$ as.

$$|\psi\rangle = \begin{pmatrix}\alpha\\\beta\end{pmatrix},$$

where,

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad \text{and} \quad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

Two Qubits example:

$$c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

Superposition of States.

In general, $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$.

α and β are complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$

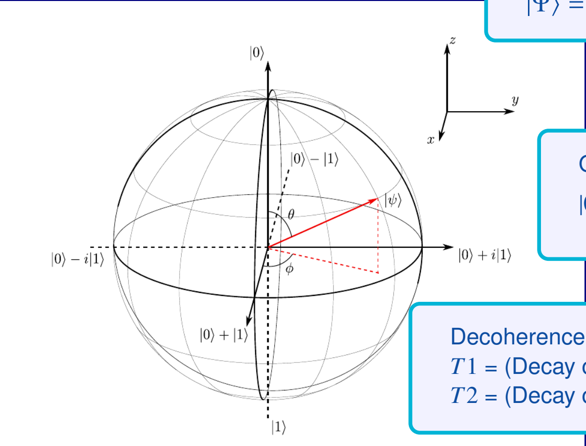
Vector Manipulation

Let us consider $|\psi\rangle = \frac{1+2i}{3}|0\rangle - \frac{2}{3}|1\rangle$.

$\langle 0|\psi\rangle = \frac{1+2i}{3}$ and $\langle 1|\psi\rangle = \frac{2}{3}$.

Quantum Information

Bloch Sphere



$$|\Psi\rangle = \cos(\theta/2)\,|0\rangle + e^{i\phi}\sin(\theta/2)\,|1\rangle$$

Qubit basis vector:

$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \quad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

Decoherence time:

$T1$ = (Decay constant) time for relaxation

$T2$ = (Decay constant) time for dephasing

Superposition :: Mixed State

Superposition of states:

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}$$

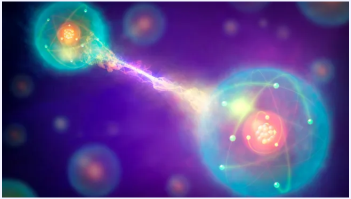
$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$\text{If } |\phi\rangle = \frac{1}{4}|0\rangle + \frac{3}{4}|1\rangle \text{ and } |\psi\rangle = \frac{2}{3}|0\rangle + \frac{1}{3}|1\rangle$$

If ϕ and ψ are independent, then

$$|\pi\rangle = |\phi\rangle \otimes |\psi\rangle = \frac{1}{6}|00\rangle + \frac{1}{12}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{4}|11\rangle$$

Quantum Entanglement

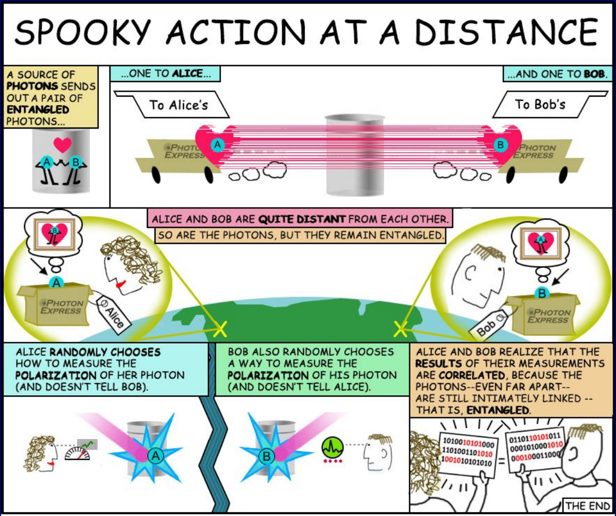


Two subatomic particles gets (non-classically) linked to each other so that their state remains the same even if they are distant away.

Entanglement happens when the system is in superposition of more than one state.

Despite their vast separation, a change induced in one will affect the other.

In 1964, John Bell stated that such changes can be induced and occur instantaneously, even if the particles are very far apart.



Inherent randomness (matching random) of Quantum Mechanics can move faster than light

Superposition :: Entanglement

$$\text{Let, } |\pi\rangle = \frac{1}{2}|00\rangle + \frac{i}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{i}{2}|11\rangle$$
$$= \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle\right)$$

This is an example of Product State.

Can we write $\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right)$ as $(|\phi\rangle \otimes |\psi\rangle)$.

Then $\langle 0|\phi\rangle \langle 1|\psi\rangle = \langle 01| (|\phi\rangle \otimes |\psi\rangle) = 0$

$\implies$ either $\langle 0|\phi\rangle = 0$ and / or $\langle 1|\psi\rangle = 0$

But $\langle 0|\phi\rangle \langle 0|\psi\rangle = \langle 00|\phi \otimes \psi\rangle = \frac{1}{\sqrt{2}} \neq 0$
Similarly, $\langle 1|\phi\rangle \langle 1|\psi\rangle = \langle 11|\phi \otimes \psi\rangle = \frac{1}{\sqrt{2}}$

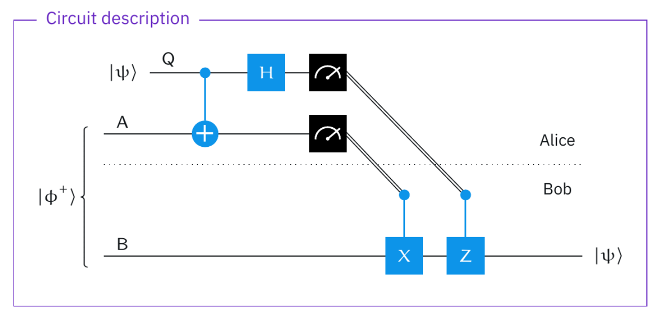
Therefore, Quantum state vector $\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right)$ is NOT a product state but AN ENTANGLED STATE.

Bell States

Bell states are 4 states those can be created from two maximally entangled qubits.

$$\left|\Phi^+\right\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}},$$
$$\left|\Phi^-\right\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}},$$
$$\left|\Psi^+\right\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}},$$
$$\left|\Psi^-\right\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}},$$

Quantum Teleportation



Measurement

- Measurement is not a Gate as it is not reversible.
- Outcome of Measurement of Quantum state is Classical state.

Example

$$\text{Quantum state} = |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Measurement:

$$\text{Probability (Outcome is 0)} = \langle 0|+\rangle^2 = \left|\left(\frac{1}{\sqrt{2}}\right)^2\right| = \frac{1}{2}$$

$$\text{Probability (Outcome is 1)} = \langle 1|+\rangle^2 = \left|\left(\frac{1}{\sqrt{2}}\right)^2\right| = \frac{1}{2}$$

Quantum Gates

- Unitary :: Reversible.

  - Identity Gate
  - Pauli (X, Y, Z) Gates
  - Controlled Gates
  - Phase shift Gate
  - Hadamard Gate
  - Swap Gate

Quantum Computing with QISKIT

- **Install:** Install QISKIT : python -m pip install qiskit[all] (after updating “pip”).
- **Build:** Design and develop quantum circuits with primitives and advanced methods like dynamic circuits and mid-circuit measurements.
- **Transpile:** Compile to optimize your circuits to run efficiently on hardware, with varying degrees of error awareness.
- **Verify:** Validate and evaluate your quantum circuits.
- **Run:** Run on QISKIT hardware with job configuration options such as sessions.