$$\frac{d\vec{p}}{dt} = q\vec{v}\times\vec{B}$$

Using $\vec{P} = \frac{E\vec{v}}{c^2}$ ; E is total energy of particle

$$\frac{d\vec{p}}{dt} = q\frac{c^2}{E}\,\vec{p}\times\vec{B}$$

This gives,
$$\frac{dp_x}{dt} = q\frac{c^2}{E}\,p_x B \qquad -(1)$$

$$\frac{dp_y}{dt} = -q\frac{c^2}{E}\,p_y B \qquad -(2)$$

$$\frac{dp_z}{dt} = 0 \qquad -(3)$$

In x-y plane,

$$\frac{dp_x}{dt} = q\frac{c^2}{E}\,p_x B$$

$$\frac{dp_y}{dt} = -q\frac{c^2}{E}\,p_y B$$

Now,
$$\frac{dp_x}{dt} = \frac{dp_x}{ds}\frac{ds}{dt}$$

$$= \frac{dp_x}{ds}\,|v|$$

$$= \frac{dp_x}{ds}\,\frac{c^2}{E}|p|$$

where $|p|$ is magnitude of ~~po~~ momentum of particle

Similarly,
$$\frac{dp_y}{dt} = \frac{dp_y}{ds}\,\frac{c^2}{E}|p|$$

Thus, eq$^n$ 1 & eq$^n$ 2 becomes

$$\frac{dp_x}{ds} = \frac{qB}{|P|} P_y \quad - 4$$

$$\frac{dp_y}{ds} = -\frac{qB}{|P|} P_x \quad - 5$$

$$\frac{dp_z}{ds} = 0 \quad - 6$$

Since magnetic field do not do work. $|p|$ is constant

Using eq$^n$ 4 & eq$^n$ 5

$$\frac{d^2 p_x}{ds^2} = -\frac{q^2 B^2}{|P|^2} P_x^2$$

$$\frac{d^2 p_y}{ds^2} = -\frac{q^2 B^2}{|P|^2} P_y^2$$

which gives solution

$$P_x = a_1 \cos\left(\frac{qBs}{|P|}\right) + b_1 \sin\left(\frac{qBs}{|P|}\right)$$

$$P_y = a_2 \cos\left(\frac{qBs}{|P|}\right) + b_2 \sin\left(\frac{qBs}{|P|}\right)$$

Let $\rho = \frac{qB}{|P|} = \frac{5.36\times10^{-19}}{1.6\times10^{-19}} \frac{B(T) \quad q(\text{in unit of } e)}{|P| \quad (GeV)}$

$$= 0.3 \frac{qB}{|P|} = \frac{a}{|P|}$$

Thus,

$$P_x = a_1 \cos \rho s + b_1 \sin \rho s \quad \text{— (7)}$$

$$P_y = a_2 \cos \rho s + b_2 \sin \rho s \quad \text{— (8)}$$

Also from eq^n (6)

$$P_z = C \quad \text{— (9)}$$

Using eq^n. (4), we get

$$-a_1 \rho \sin \rho s + b_1 \rho \cos \rho s = \frac{qB}{|P|} a_2 \cos \rho s + \frac{qB}{|P|} b_2 \sin \rho s$$

On comparing coefficients,

$$b_1 = a_2$$

$$-a_1 = b_2$$

Thus,

$$P_x = a_1 \cos \rho s + a_2 \sin \rho s$$

$$P_y = a_2 \cos \rho s - a_1 \sin \rho s$$

$$P_z = C$$

Using $s=0$, $P_x = P_{ox}$, $P_y = P_{oy}$, $P_z = P_{oz}$

we get

$$P_x = P_{ox} \cos P_s + P_{oy} \sin P_s \qquad -(10)$$

$$P_y = P_{oy} \cos P_s - P_{ox} \sin P_s \qquad -(11)$$

$$P_z = P_{oz} \qquad -(12)$$

Now, $P_x = \frac{E}{c^2} V_x = \frac{E}{c^2} \frac{dx}{dt} = \frac{E}{c^2} \frac{dx}{ds} \frac{ds}{dt}$

$$= \frac{E}{c^2} |v| \frac{dx}{ds}$$

$$P_x = |P| \frac{dx}{ds}$$

$eq^n$ (10), (11) & (12) becomes

$$\frac{dx}{ds} = \frac{P_{ox}}{|P|} \cos P_s + \frac{P_{oy}}{|P|} \sin P_s$$

$$\frac{dy}{ds} = \frac{P_{oy}}{|P|} \cos P_s - \frac{P_{ox}}{|P|} \sin P_s$$

$$\frac{dz}{ds} = \frac{P_{oz}}{|P|}$$

Solving these eq$^n$s

$$x = \frac{P_{ox}}{|P|\rho} \sin \rho s - \frac{P_{oy}}{|P|\rho} \cos \rho s + C_1$$

$$y = \frac{P_{oy}}{|p|\rho} \sin \rho s + \frac{P_{ox}}{|p|\rho} \cos \rho s + C_2$$

$$z = \frac{P_{oz} S}{|p|} + C_3$$

At $s = 0$, $x = x_0$, $y = y_0$, $z = z_0$
gives

$$C_1 = x_0 + \frac{P_{oy}}{|P|\rho}$$

$$C_2 = y_0 - \frac{P_{ox}}{|P|\rho}$$

$$C_3 = z_0.$$

Thus,

$$X = \frac{P_{0x}}{|p|\rho}\sin\rho s + (1-\cos\rho s)\frac{P_{0y}}{|p|\rho} + X_0$$

$$y = \frac{P_{0y}}{|p|\rho}\cos\rho s + (1-\cos\rho s)\frac{P_{0x}}{|p|\rho} + Y_0$$

$$Z = \frac{P_{0z}}{|p|} s + Z_0$$

Since $\rho = \frac{0.3qB}{|p|} = \frac{a}{|p|}$

$$X = \frac{P_{0x}}{a}\sin\rho s + \frac{P_{0y}}{a}(1-\cos\rho s) + X_0$$

$$y = \frac{P_{0y}}{a}\cos\rho s + (1-\cos\rho s)\frac{P_{0y}}{\cancel{|p|}\,a} + y_0$$

$$Z = \frac{P_{0z}}{|p|} s + Z_0$$