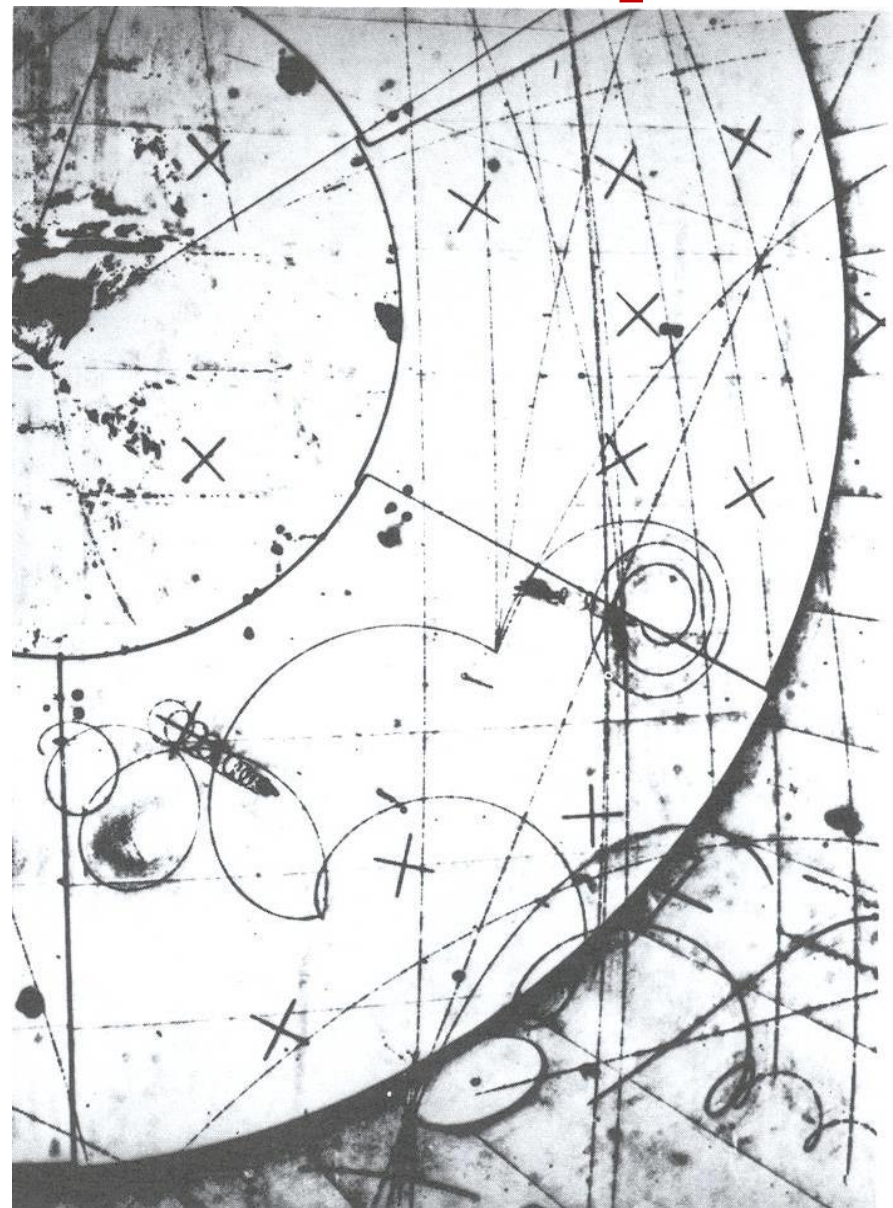
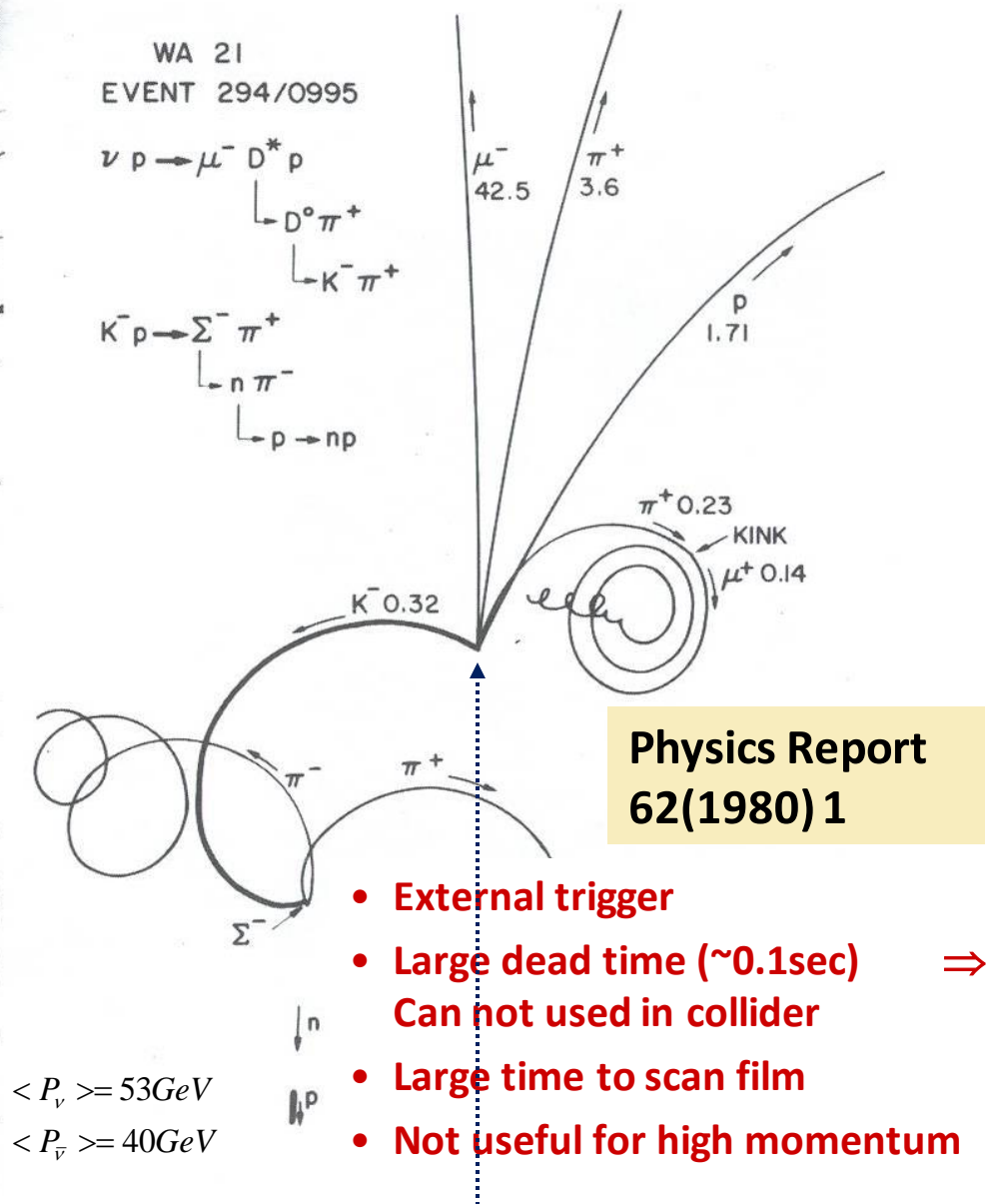


Golden period of particle physics

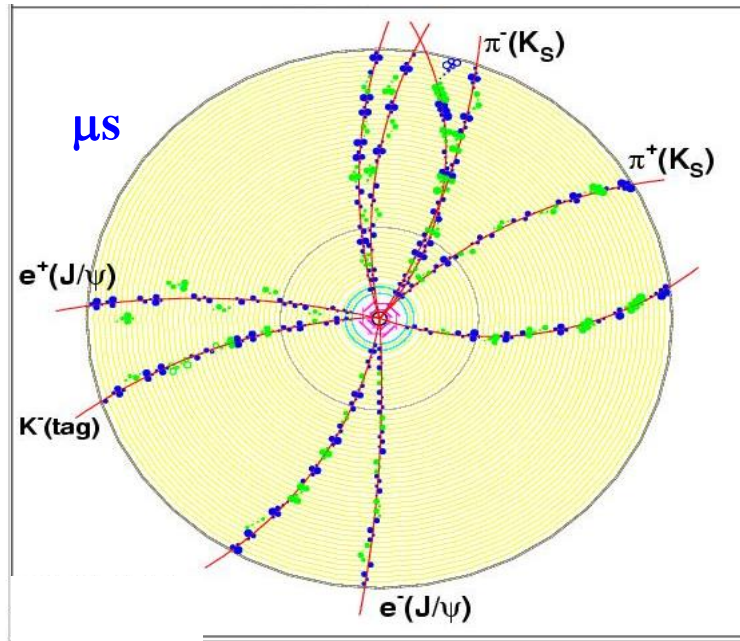


Golden period of particle physics

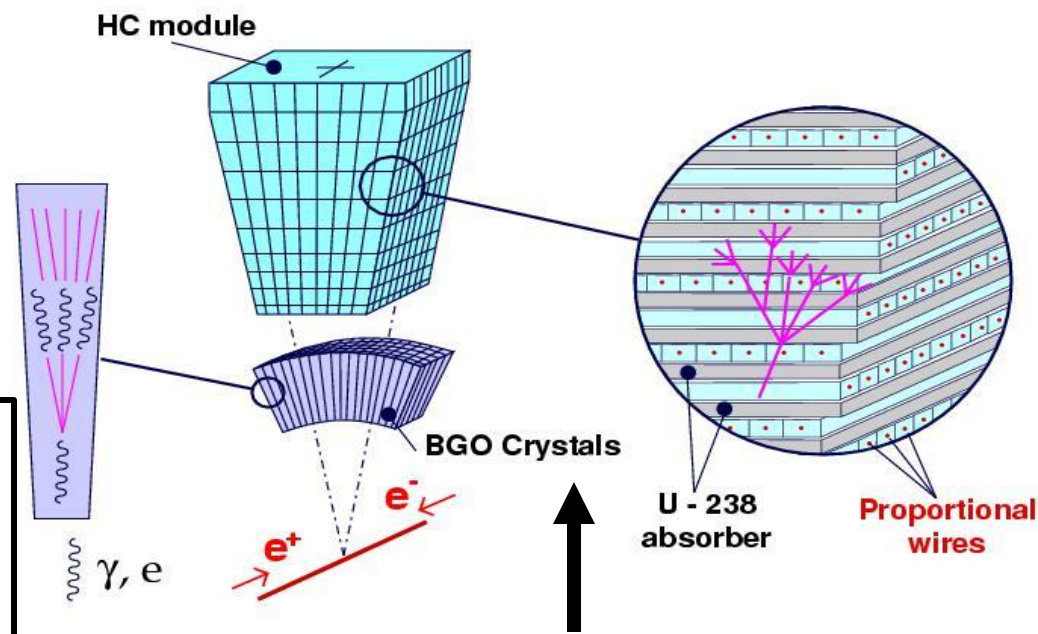
- 4π acceptance
- Trace a charged particle with good efficiency
- Calculate momentum
- Identify particle, $e/\pi/\mu/p/n$
- Separate them out, spatial resolution ($\sim 8\mu\text{m}$) with particle multiplicity up to 150
- Mass identification, identify mother
- Secondary vertex



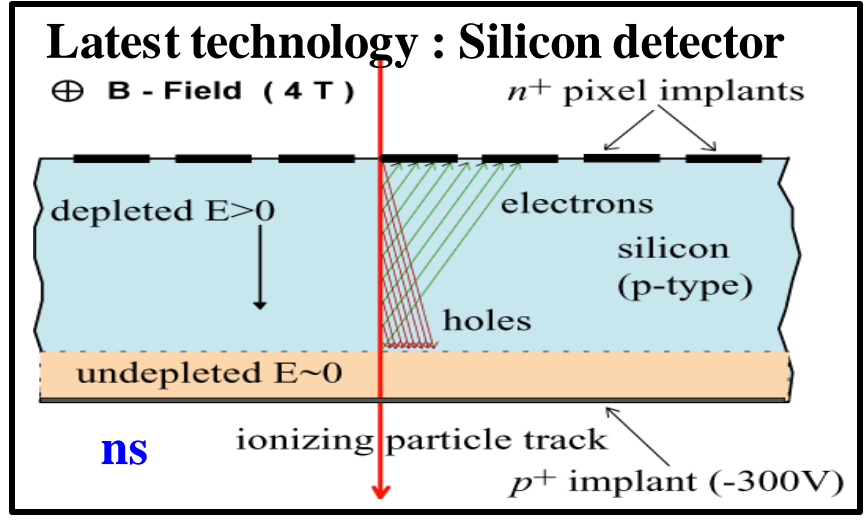
Reconstruction of particles in present day HEP detector



• Bending of charge particle in presence of magnetic field provides the information of momentum of charge particle $\vec{F} = q (\vec{v} \times \vec{B})$



Calorimeter is mainly used for the measurement of energy of neutral particle



Obviously large bias on examples to BELLE/CMS/INO detector

Measurement of track momentum

$$P_T[\text{GeV}/c] = 0.3 B[\text{T}] R[\text{m}]$$

$$\sin \frac{\theta}{2} = \frac{L}{2r}; \text{ for } r \gg L \quad \frac{\theta}{2} \approx \frac{L}{2r} \Rightarrow \theta \approx \frac{0.3BL}{P_T}$$

$$s = r - r \cos\left(\frac{\theta}{2}\right) \approx r \left[1 - \left(1 - \frac{1}{2} \frac{\theta^2}{4} \right) \right] = \frac{r\theta^2}{8} \approx \frac{0.3BL^2}{8P_T} = \frac{L^2}{8R}$$

$$s = x_B - \frac{x_A + x_C}{2}; \therefore ds = dx_B - \frac{dx_A + dx_C}{2}; (ds)^2 = \sigma_s^2 = (3/2)\sigma_x^2$$

$$\frac{dP_T}{P_T} = \frac{\sigma_s}{s} = \frac{\sqrt{3/2}}{s} \sigma_x = \sqrt{\frac{3}{2}} \sigma_x \frac{8P_T}{0.3BL^2}$$

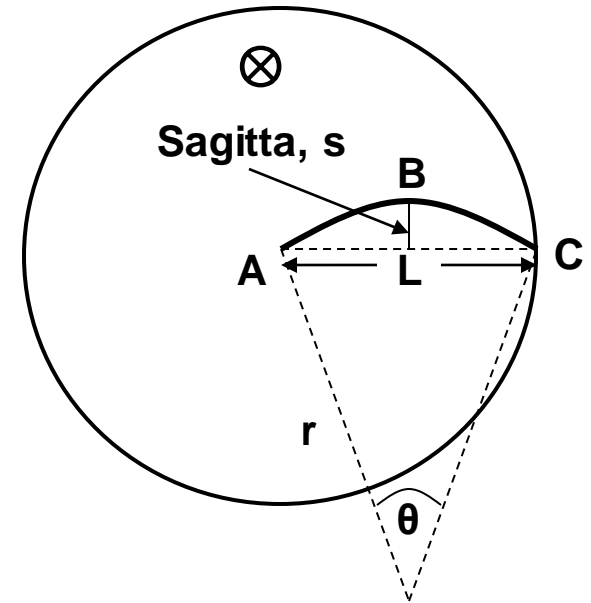
For multiple coulomb scattering, average scattering angle

$$\theta_0 \approx \frac{13.6 \text{ MeV}}{\beta P c} Z \sqrt{\frac{L}{X_0}} \Rightarrow s_{ms} = \frac{L\theta_0}{4\sqrt{3}} \Rightarrow \frac{s_{ms}}{s} = \frac{dP}{P} \Big|_{ms} \approx 0.05 \frac{1}{\beta B \sqrt{LX_0}}$$

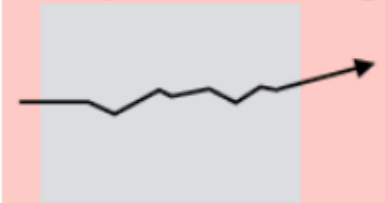

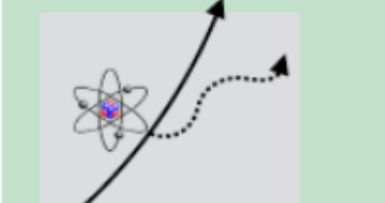
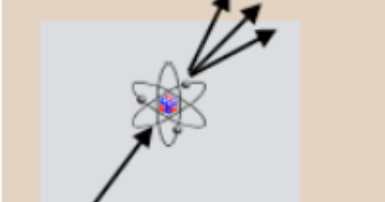
Momentum resolution, σ_P/P varies

- Constant for multiple coulomb scattering (neglect $1/\beta$ term for $p > \sim \text{GeV}$)
- linearly with P_T
- Inversely with BL^2

For higher momentum, detector size increases with \sqrt{L}



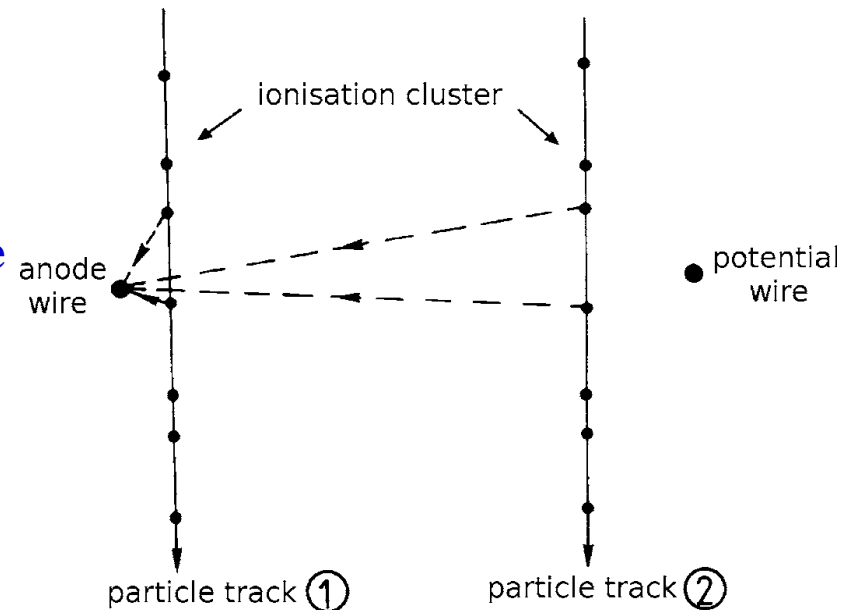
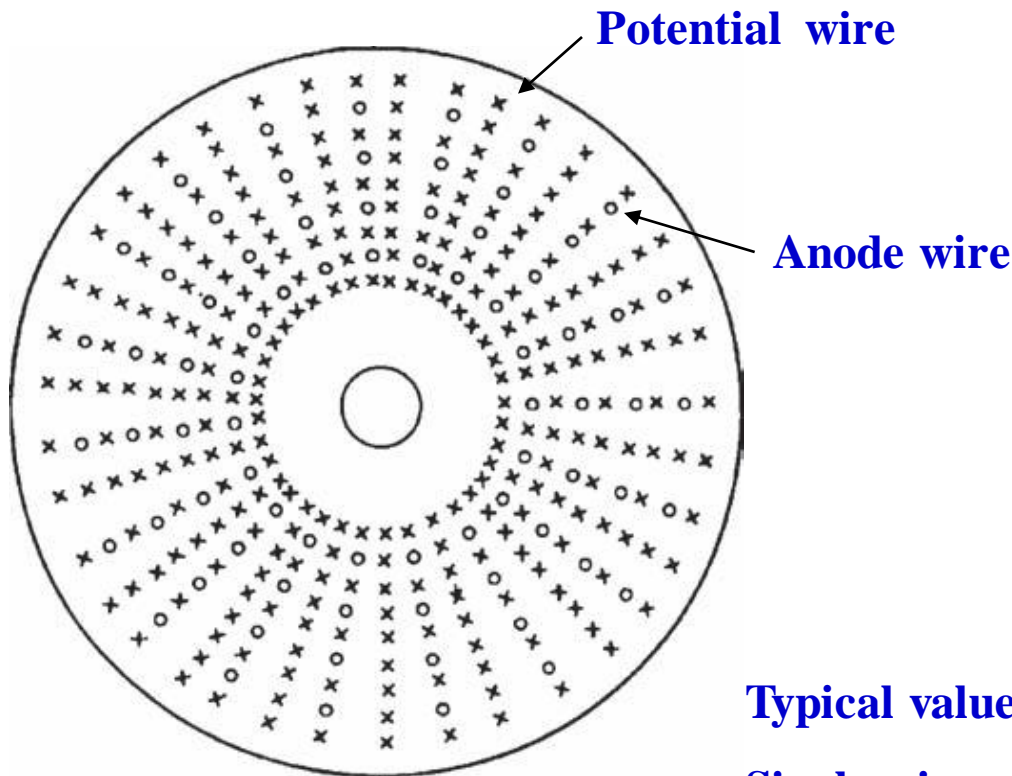
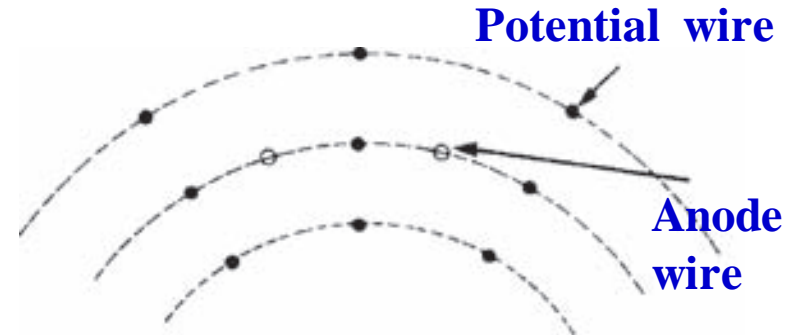
Interactions of charged particle and Track reconstruction

Type	particles	fund. parameter	characteristics	effect
Multiple Scattering 	all charged particle	radiation length X	almost gaussian average effect 0 depends $\sim 1/p$	deflects particles, increases measurement uncertainty
Ionisation loss 	all charged particle	effective density $A/Z * \rho$	small effect in tracker, small dependence on p	increases momentum uncertainty
Bremsstrahlung 	all charged particle, dominant for e	radiation length X	highly non- gaussian, depends	introduces measurement bias
Hadronic Int. 	all hadronic particles	nuclear interaction length Λ	destroys particle, rather constant effect in p	main source of track reconstruction inefficiency

Cylindrical Drift Chamber for collider experiment

- **Characteristics:**

- Cylindrical symmetry
- Open drift cell geometry
- Require: Simple space-time relation
- given by E,B field and drift cell geometry

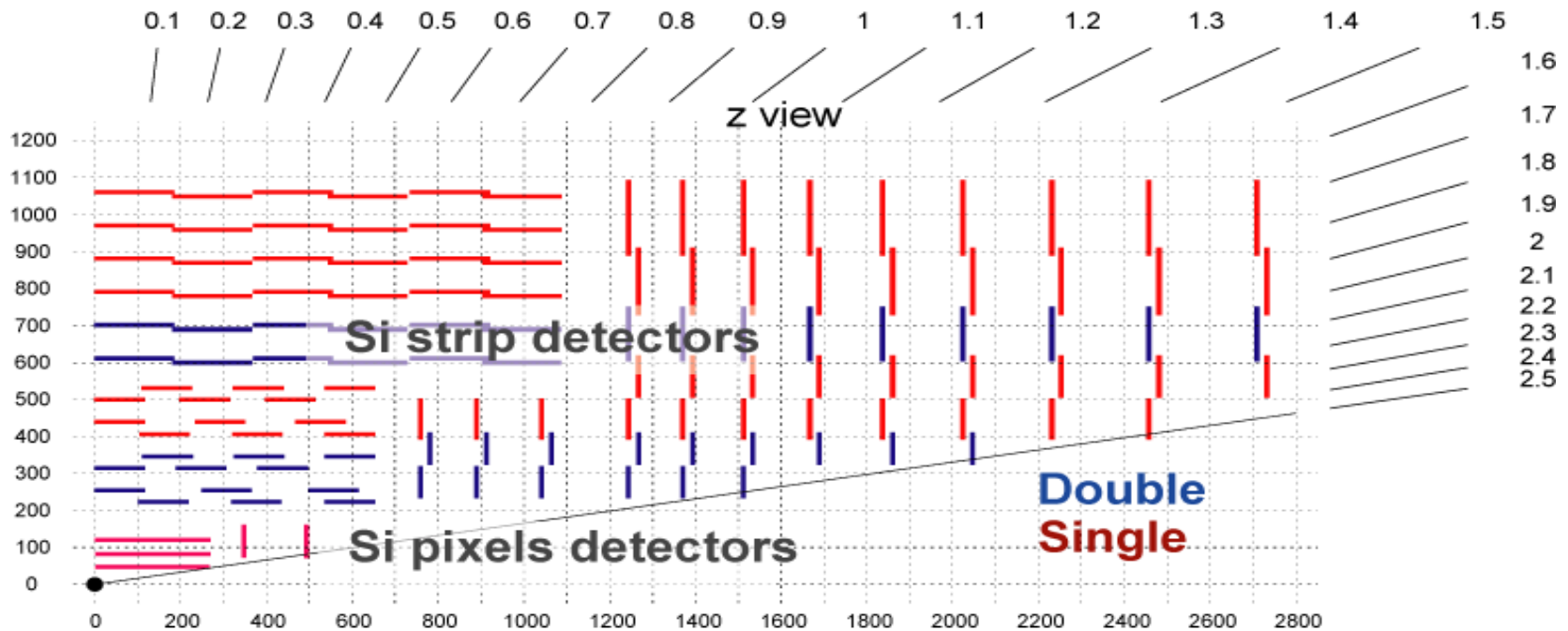


Typical values :

Single wire resolution ~ 50 μm (300 μm)

Double track resolution ~ 450 μm (7mm)





The layout of the CMS inner tracker **(Initial one)**

Pixel : *n-on-n* substrate, **Strip** : *p-on-n* substrate,

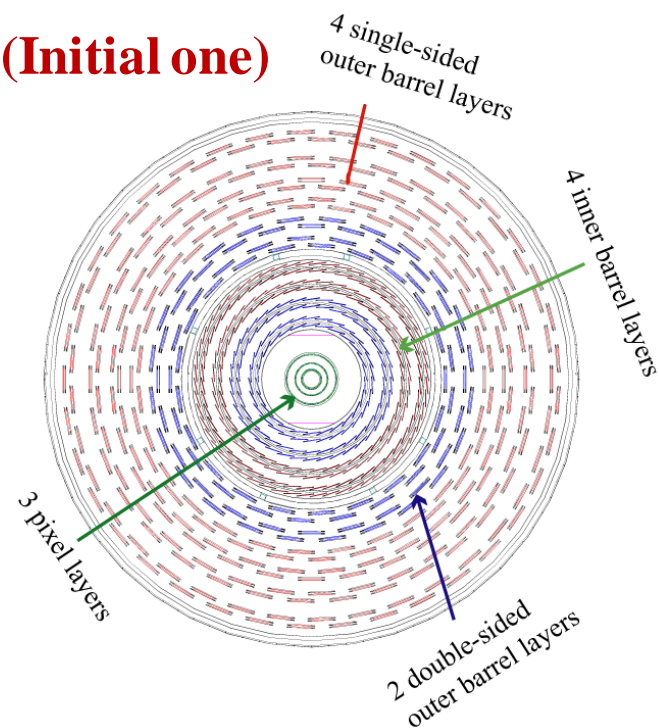
Pixels : $150\ \mu\text{m} \times 150\ \mu\text{m}$

Strip pitch : 80 to 205 μm

Outer region : higher noise due to “long” strip is compensated by larger signal in thick ($500\ \mu\text{m}$) sensor

Total readout channel : 9.6 M+66M **Operating at**
 -10°C

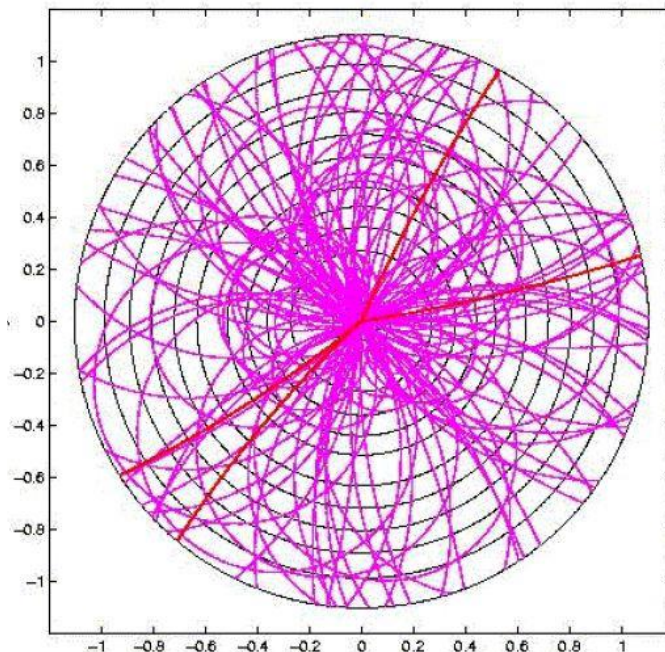
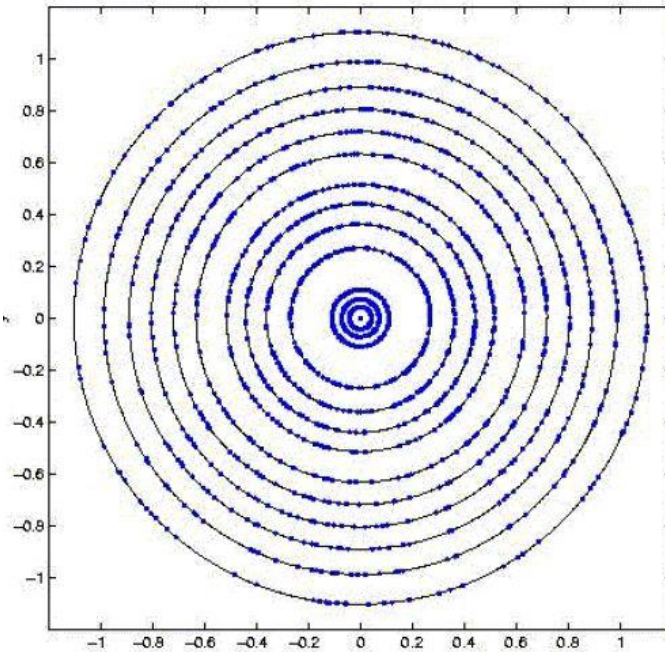
Pixel : $\sigma_{r-\phi/z} = \sim 9/24\ \mu\text{m}$, **Strip** (10-40 μm)



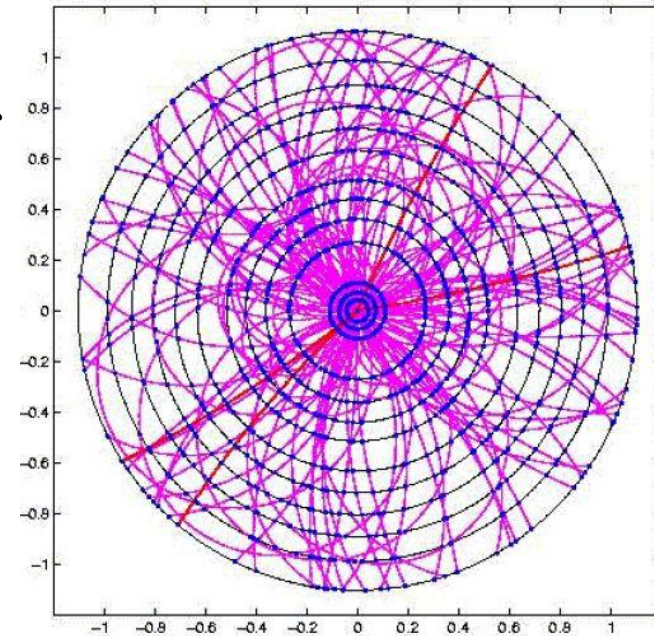
Introduction to track fitting

Tracking detector with cylindrical layers

Input to track finding is all or parts of the measurements in the detector at a given instance



A successful track finder identifies a set of potential tracks as indicated in the figure
Hits along these tracks are given to the track fitter for parameter estimation and final validation of track candidate



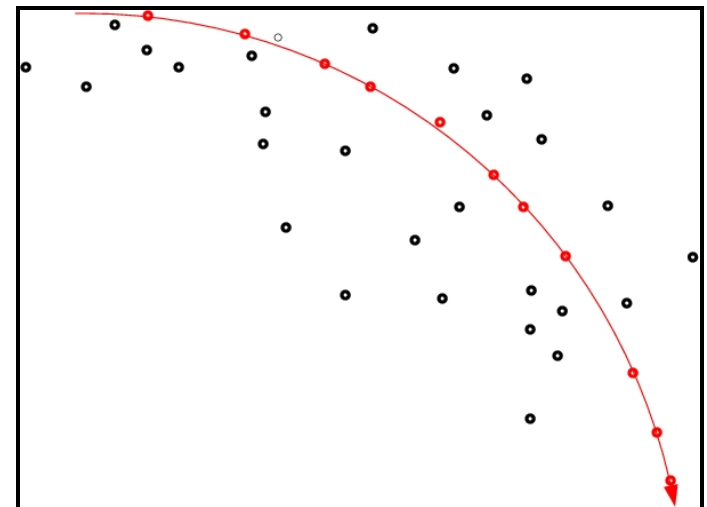
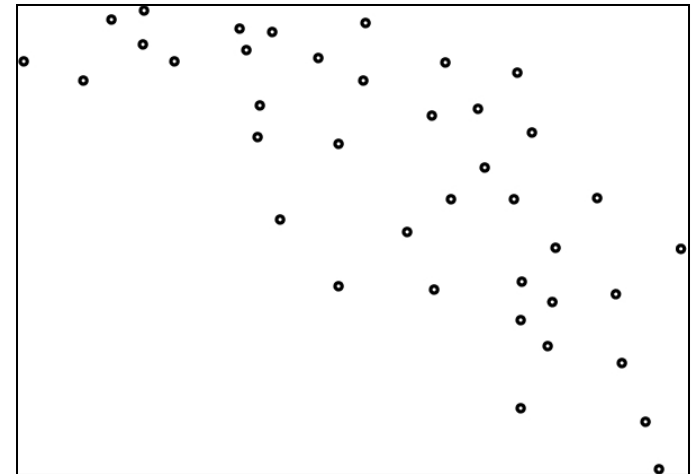
After the track fit one usually forgets about the hits and only cares about a compact representation of the tracks

Track finder

- **Track fitting is involved with inversion of matrices and large dimension**
 - Large computing time to look all combinations of points
 - Most of those combinations might not converge
- **First identify combinations without CPU consuming fitting algorithm, which could be a simple pattern recognition**
 - Track finding algorithm
- **Track fitting algorithm : used hit from finder**

- **Bubble chamber days**
 - scanning team looked at photograph
 - recognition straight forward
- **Electronically read out detectors**
 - less hits per track length
 - environment got more dense (more hits)
 - algorithms needs to replace 'look at'
- **Algorithm (time consuming)**

**Data Analysis Technique for
High Energy Physics
Ed : M. Regler and R.
Frühwirth**



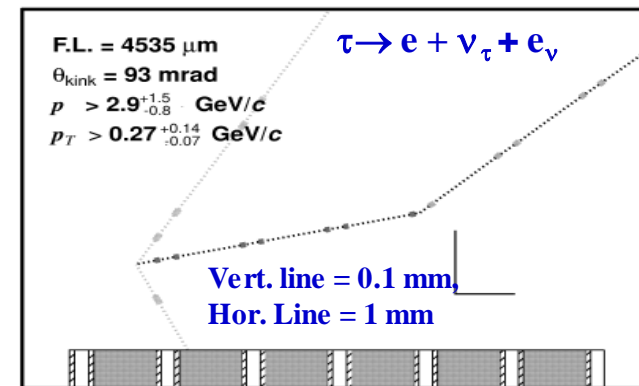
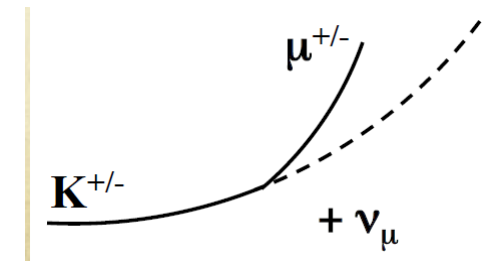
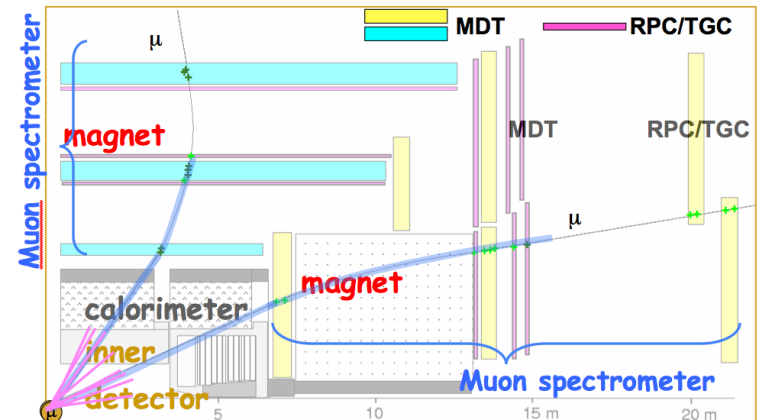
Track finding algorithm

- Prerequisite:

- Detector alignment
- Hit reconstruction (space points with uncertainties)

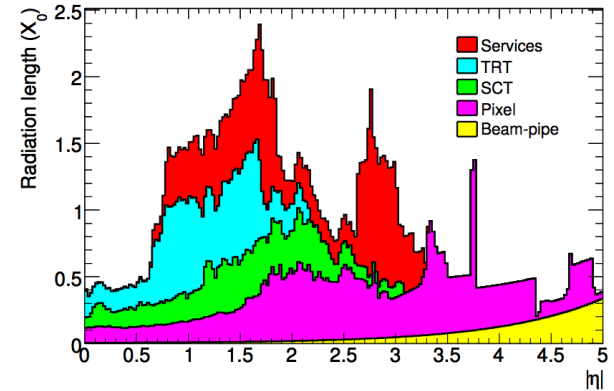
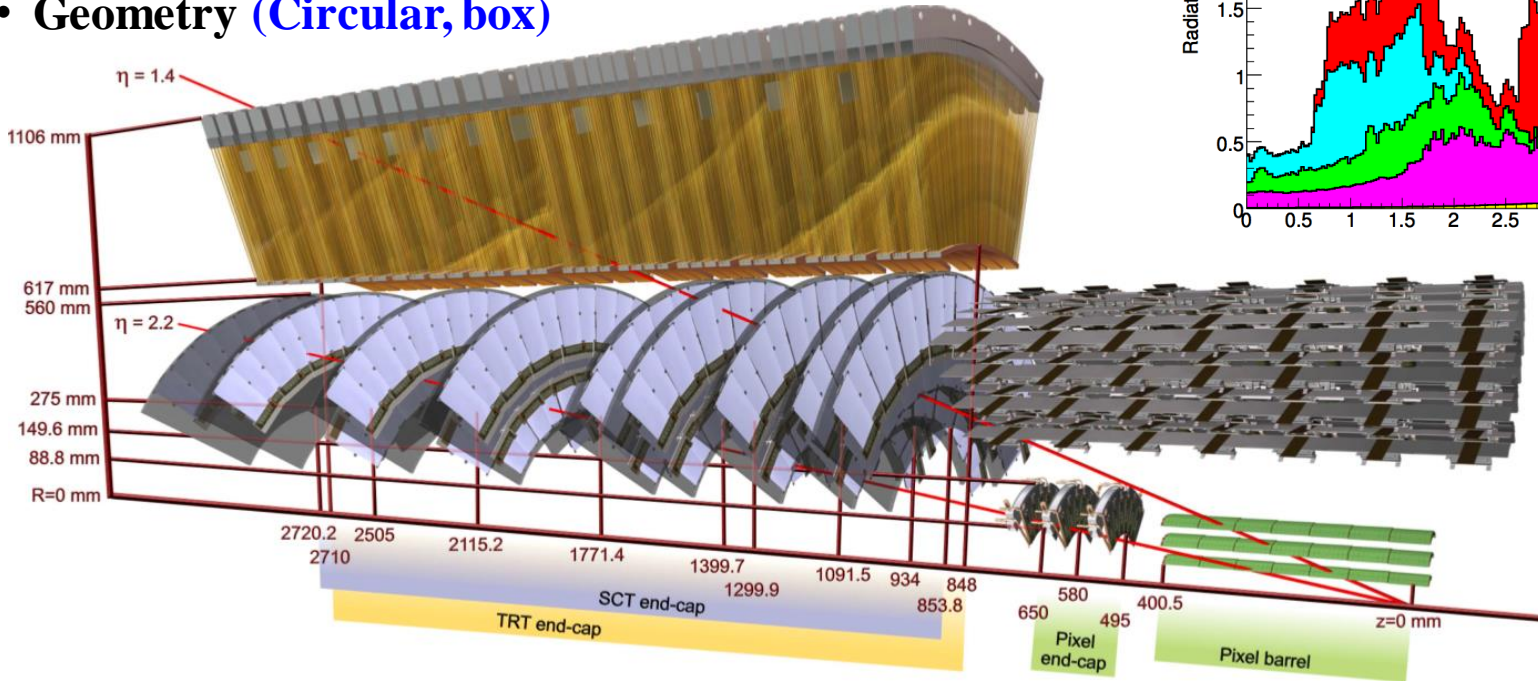
- Depends on :

- Tracker type (detector resolution, dominated in drift chamber, where contribution from electron statistics, diffusion, inhomogeneity of the drift field, electronic jitter, TDC binning, wire sagging....)
- Accuracy in alignment (Shift of different layers)
- Hits it produces (Inefficiency, # of hit in a layer, electronic noise)
- Magnetic field (inhomogeneous, how much can it be linearised)
- Event environment (Beam background, pileup, cosmic)
 - Kink for decay in flight



Some more concern of track finding algorithm

- Complex modular track, where resolution varies from μm to mm , even uses of calorimeter (cm)
- Geometry (Circular, box)

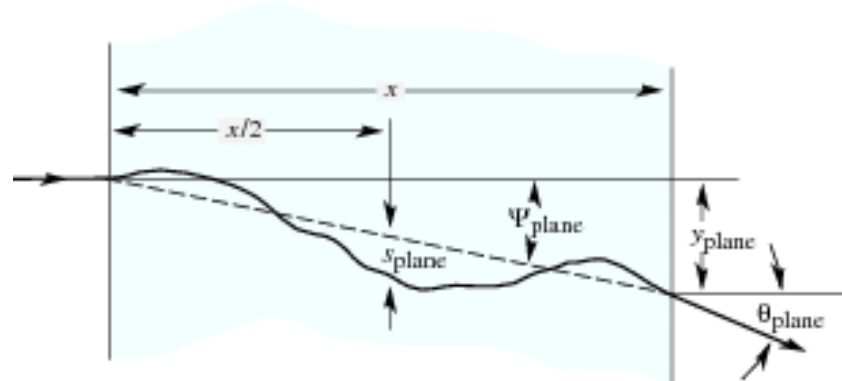


Pixel :
10/115
SCT :
17/580
TRT : 130
Muon :
35/10000μm

- Multiple scattering on sensitive material, support structure, beam pipe

– Not only fluctuation, but also for dE/dx estimation

$$\theta_e = \frac{13.6 \text{ MeV}}{\beta p v} Z_1 \sqrt{\frac{L}{L_R}} \left[1 + 0.125 \log_{10} \left(\frac{L}{L_R} \right) \right]$$



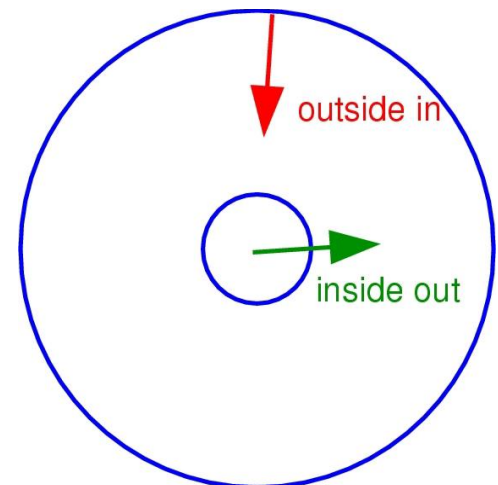
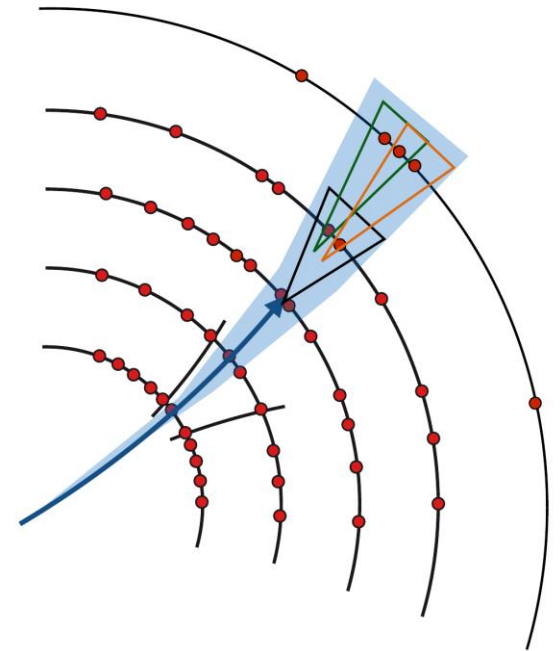
Track finder : Track following

- **Outside in**

- Start with seed at outer end of tracking volume
- Swim in general direction of the beamline
- **Low occupancy outside, easy pattern reco, add hits moving in one knows already where to look**
- For high multiplicity events, look for out-to-in.
- **Certainly it can not find very low p_T tracks**

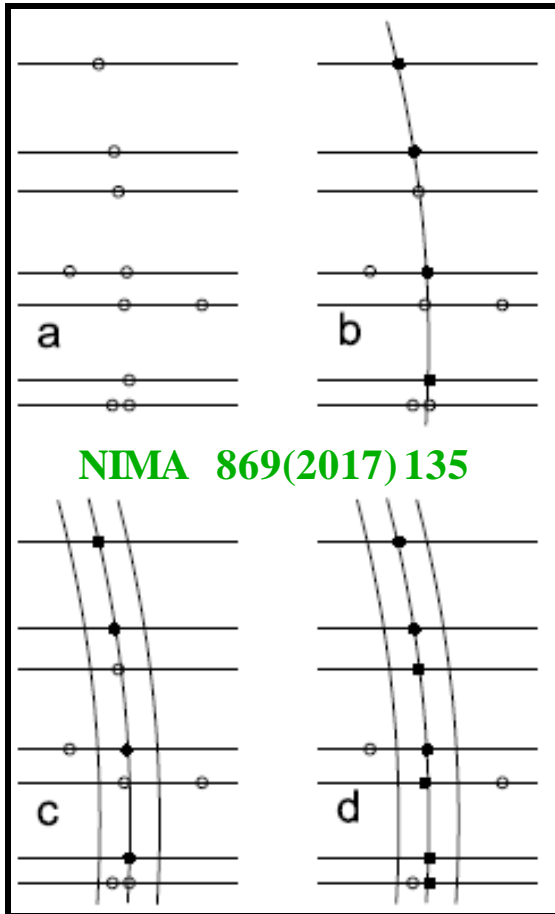
- **Inside out**

- Follow natural particle direction, least MS
- **Detector cutoff in pseudorapidity has minimal effect**
- **Track are built from inside to out because many particles will interact before crossing all layers of the tracker**
- **Seeding difficult because of high occupancy**

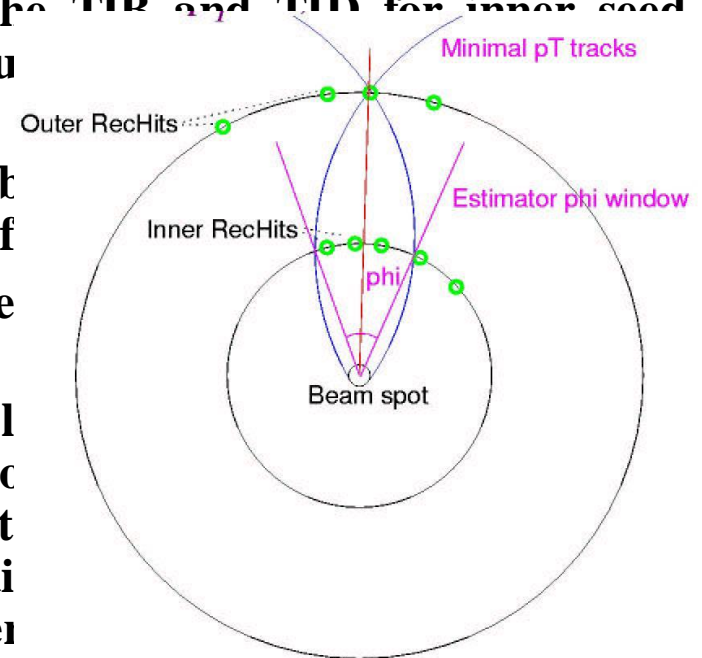


Track finder : Road Search

- First, last and a point at centre and look for other points (or points in inner and outer layers in collider along with interaction vertex as third point (computing time is prop to a factor between n^2 to n^3 , where n is the number of hit points. Slower than following methods, but suitable where redundancy in the co-ordinate measurement is very low.

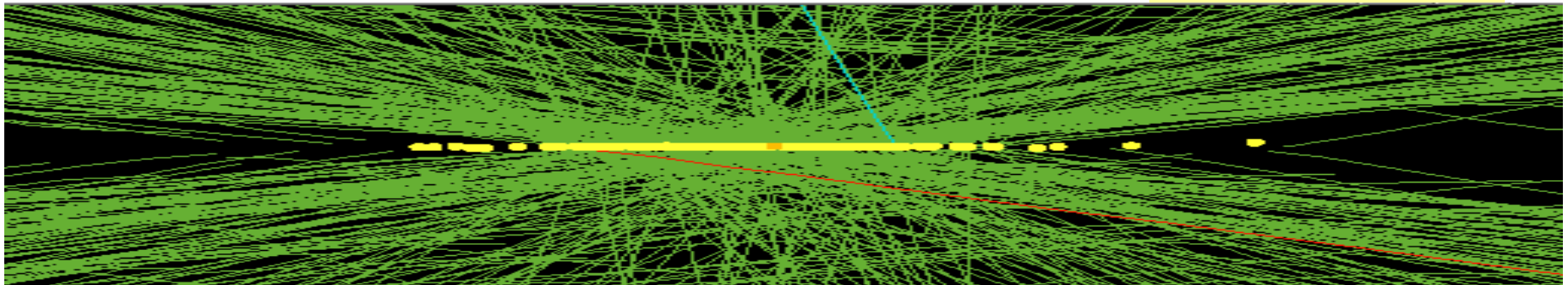
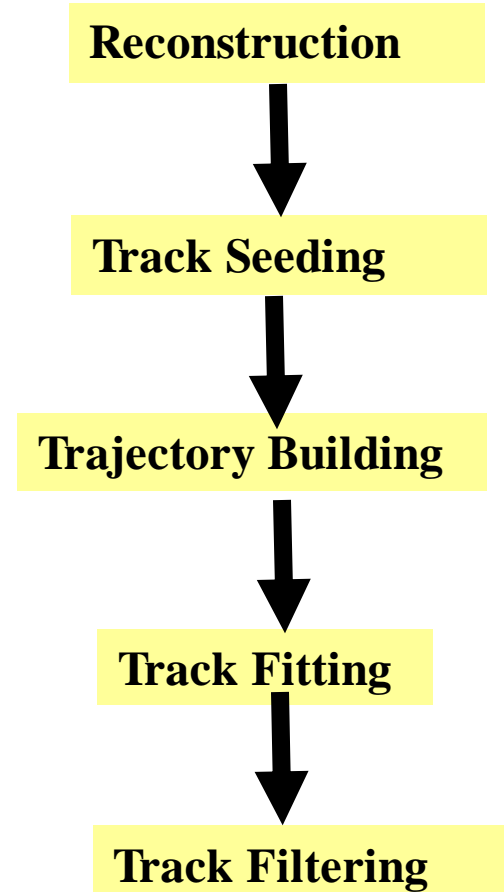


- CMS uses pixel tracklet and outer TOB/TEC hit and build the road with a minimum P_T
 - The standard seeding uses hits from the inner two (matched) layers of the TRP and TID for inner seed hits, and hits in the outer layers for outer seed hits.
 - The trajectory will be built with precision coordinate of the inner seed hits.
 - Initially combine layer by layer.
 - This trajectory is built by combining the beamline and the tracklet again, this time adding higher occupancy layers.



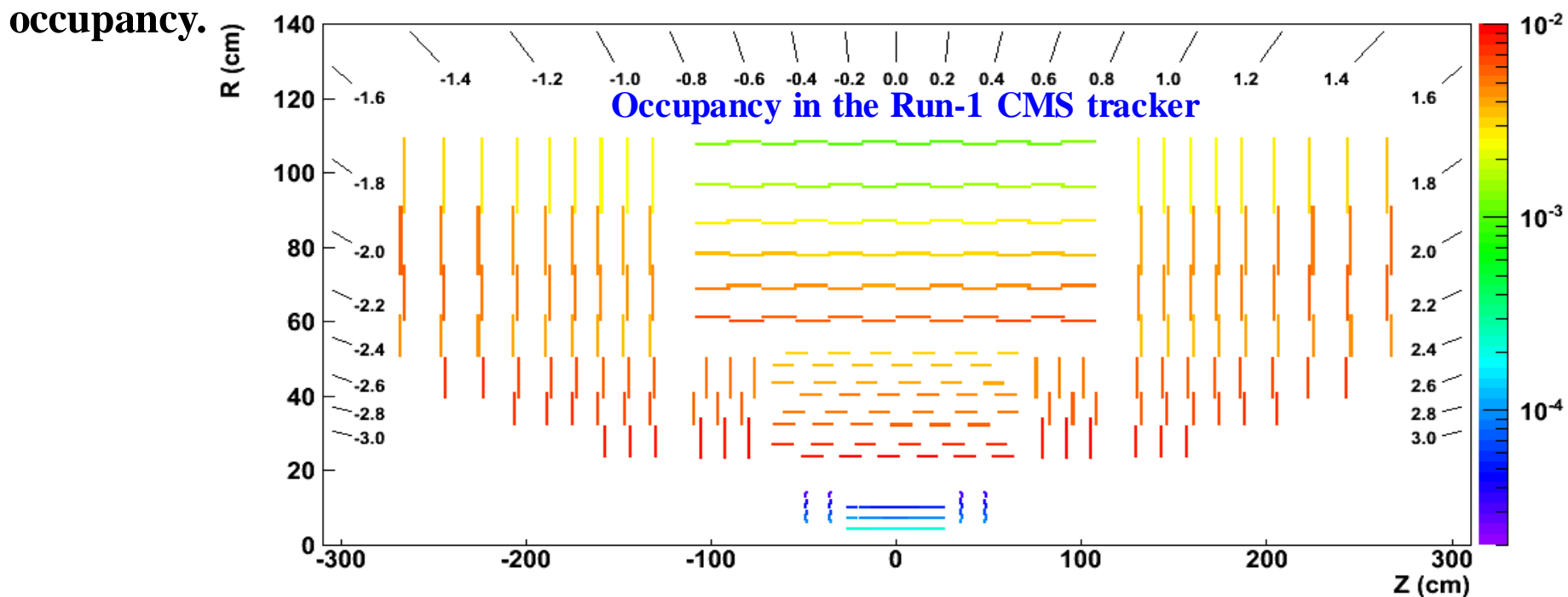
Track Reconstruction in collider (CMS)

- Clustering of silicon strips and pixels to find “hit” positions and errors
- Initial estimate of track parameters using a minimal number of hit
- Collection of the remaining hits associated to the particle trajectory
- Final estimation of the track parameters using the full set of associated hits
- Removal of tracks likely to be fakes

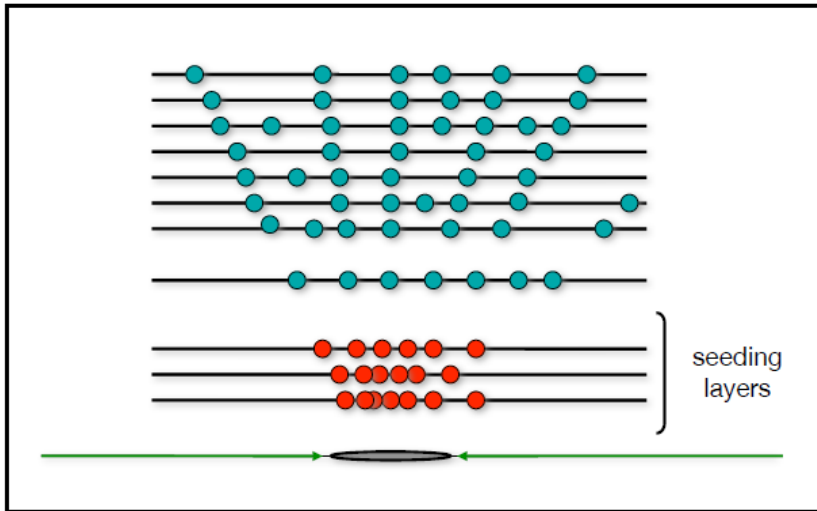


Steps of Track Reconstruction at collider (Track seeding)

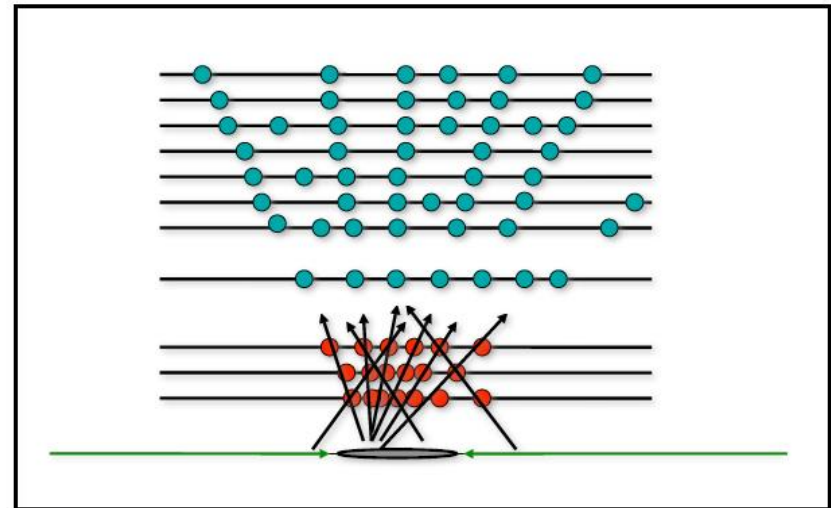
- Default seeding is done in pixels layers
- Tracks are built from inside to out because many particles will interact before crossing all layers of the tracker
- Pixel hits are very precise. So two or three pixel hits can already give good initial estimation of the track parameters
 - single pixel area = $100\mu\text{m} \times 150\mu\text{m} = 1.5 \times 10^{-2} \text{mm}^2$
 - typical strip area = $10\text{cm} \times 100\mu\text{m} = 10\text{mm}^2$
- This plot explains why CMS tracking is heavily pixel driven (seeding) : **Low occupancy.**



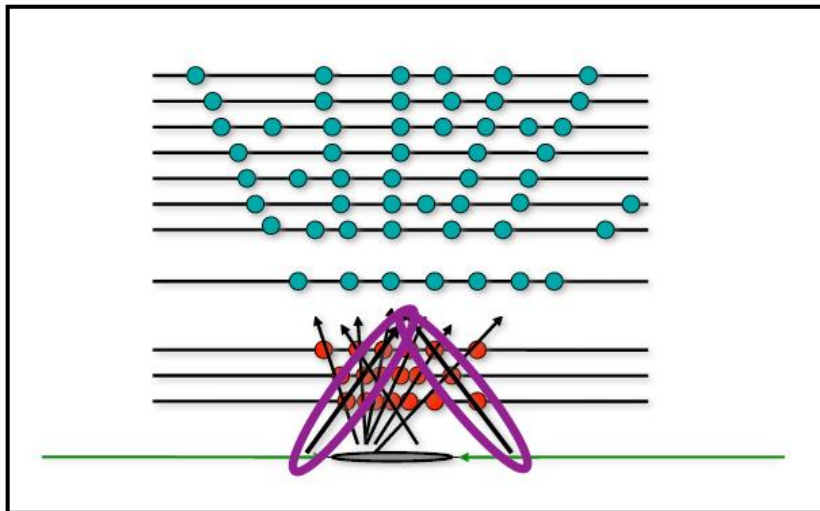
Track follower at collider experiments



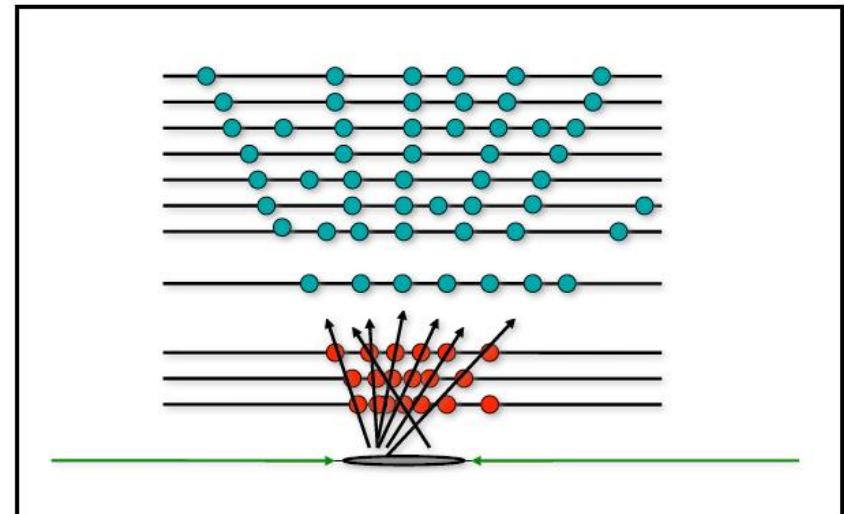
Hits in a subset of tracker layers are used to find trajectory seeds. Default seeds are made from pixels, the innermost layers of the tracker.



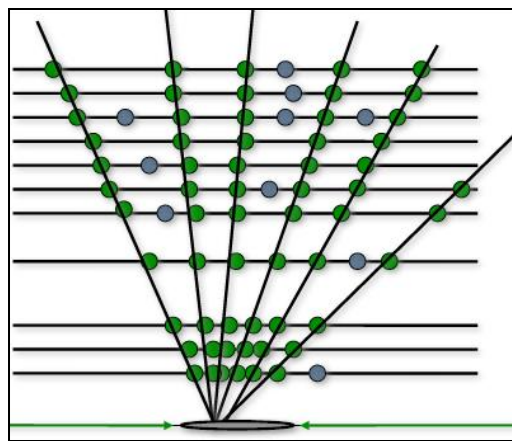
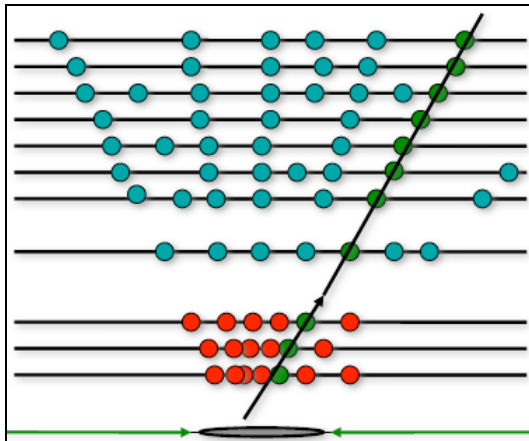
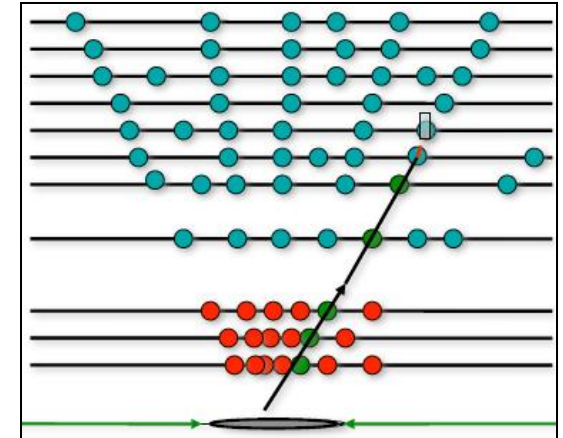
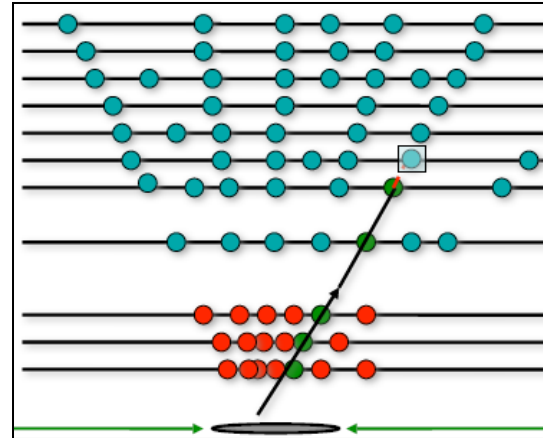
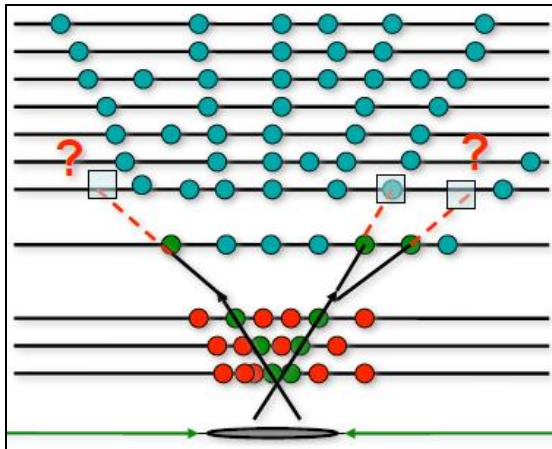
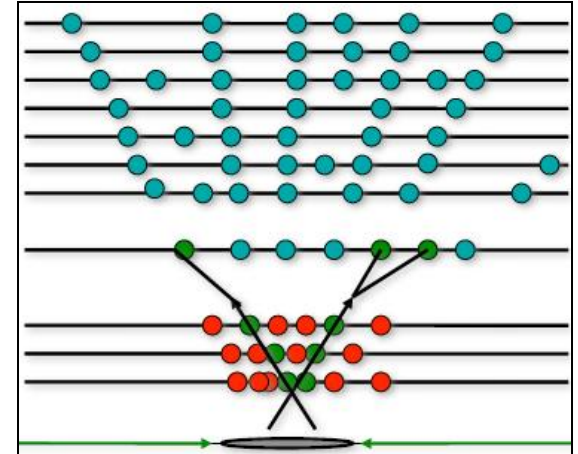
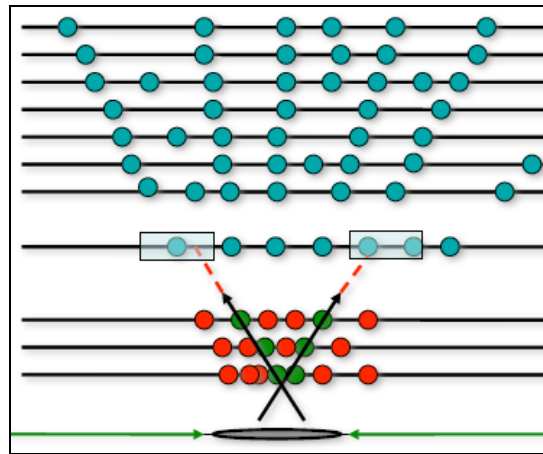
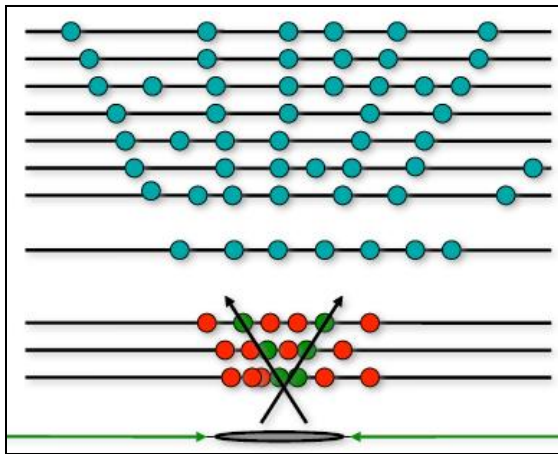
Triplets of hits (or pairs plus the beamspot) are combined to produce trajectory seeds whose directions are **compatible with the beam collision region**.



Seeds that point well outside the collision region are discarded



For the remaining seeds, each one is then propagated outward to collect more hits to find the full trajectory of the charged particle.

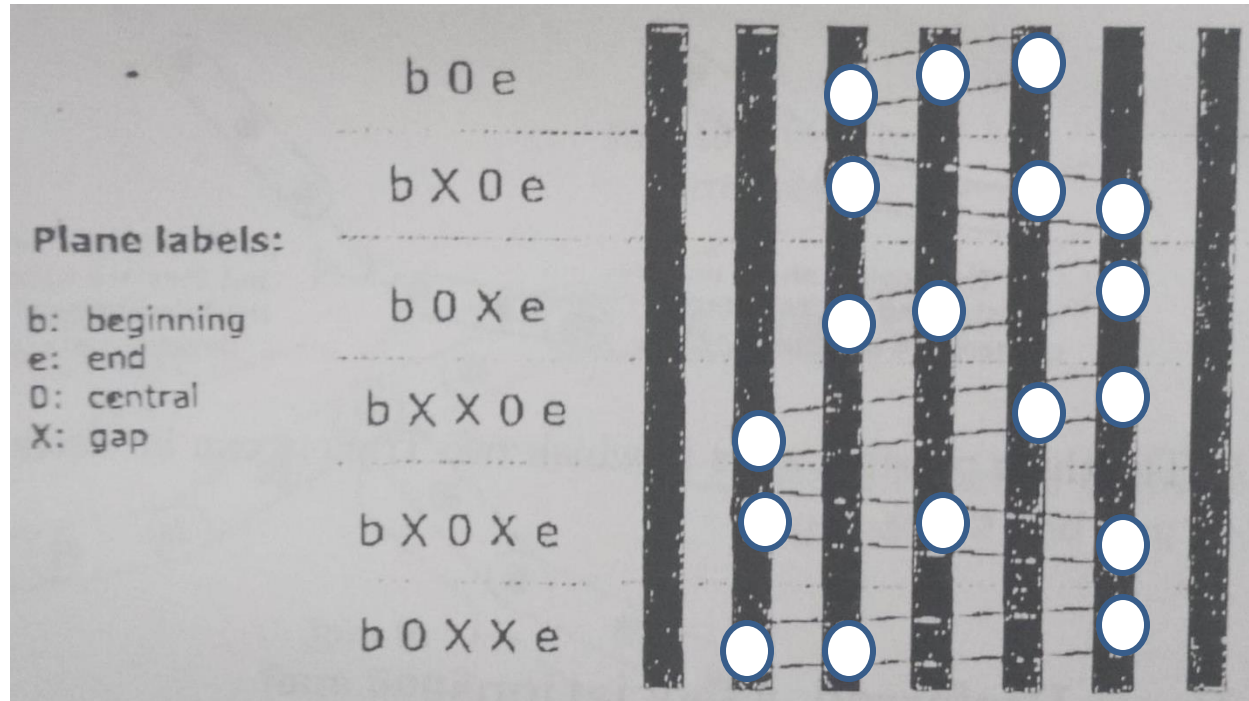


- In the second level remove all hits belongs to these tracks
- Restart with remaining hits with loose criteria

Track finder

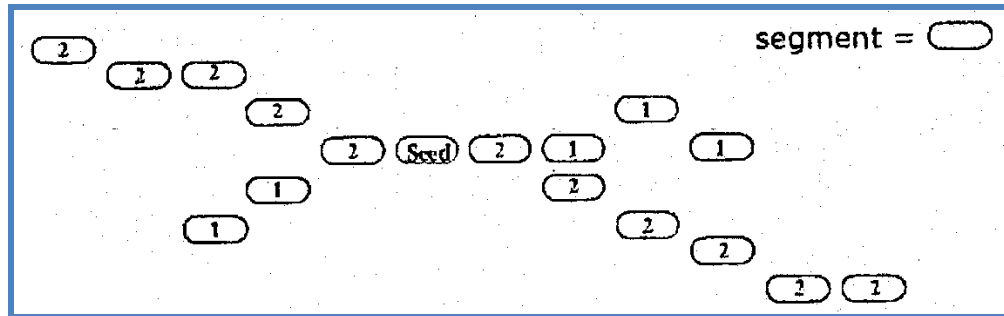
- **Linear approximation, e.g., INO :**

- **Triplet : Formation** (allowed maximum gap of two layers), join them in a chain, sort out the best choice as a track candidate.
- **Need optimisation mainly for a gap**

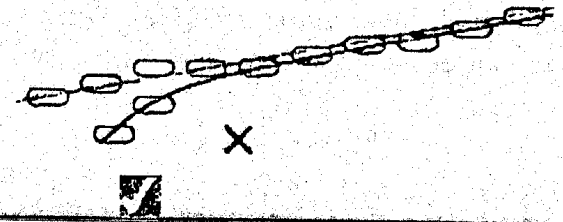


Track finder

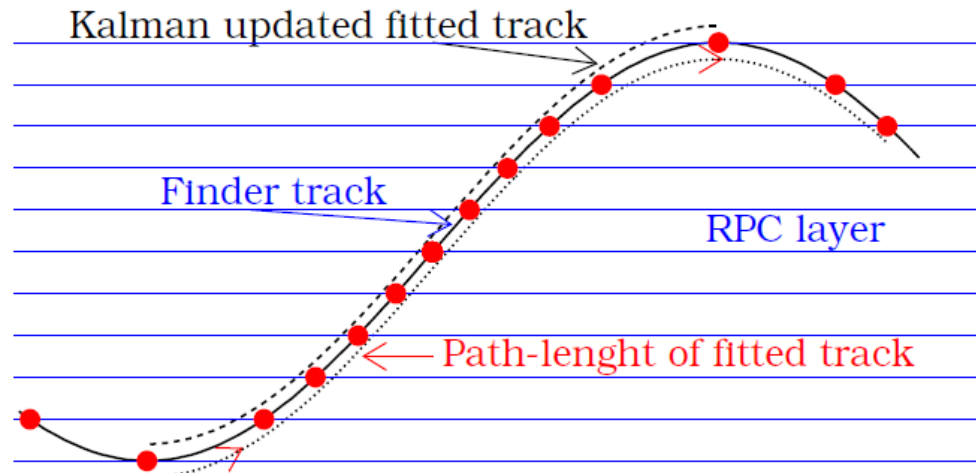
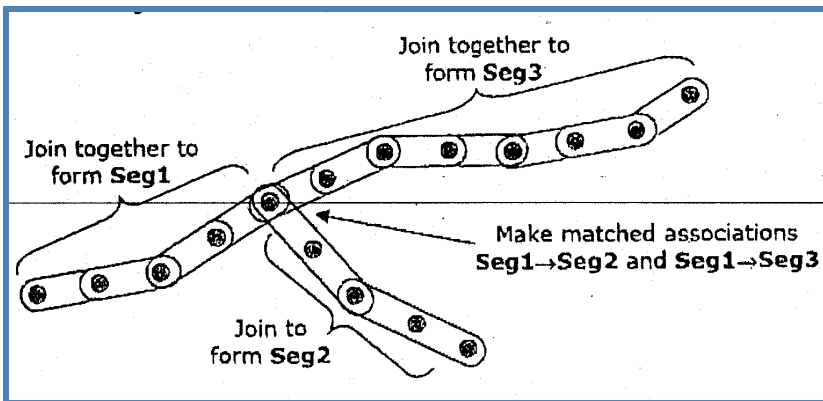
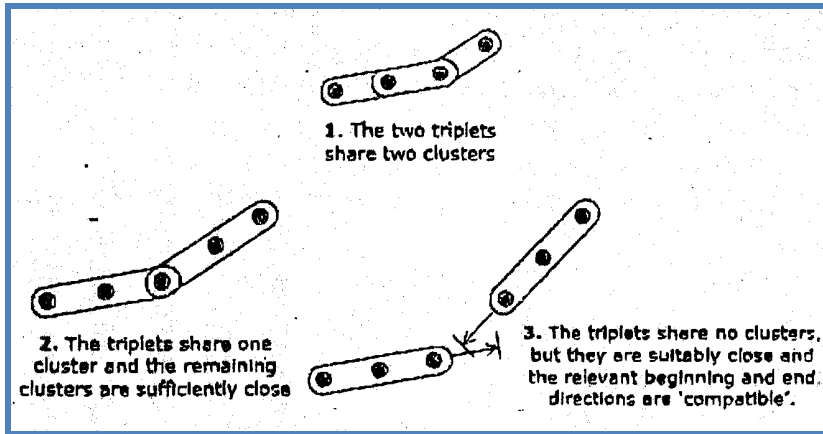
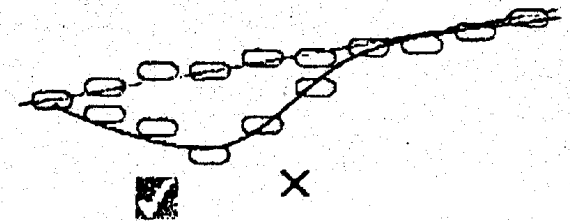
- Linear approximation, e.g., INO :
 - Bending at the end of the track can not be included with this algorithm, but in the fitting time that can be taken care of.



Each possible 2D track is given a score. The first contribution is from the number of clusters in the track.

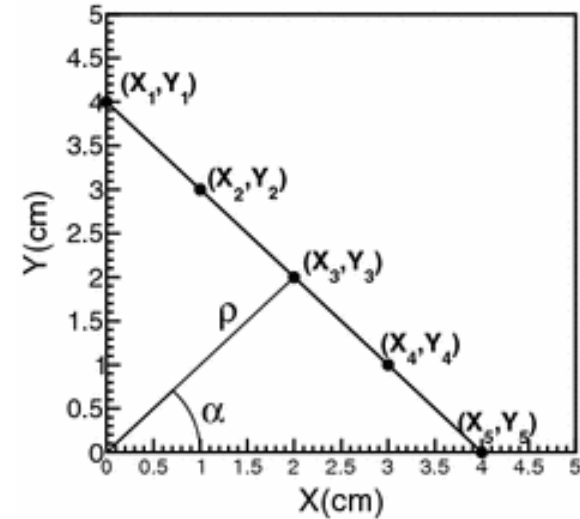
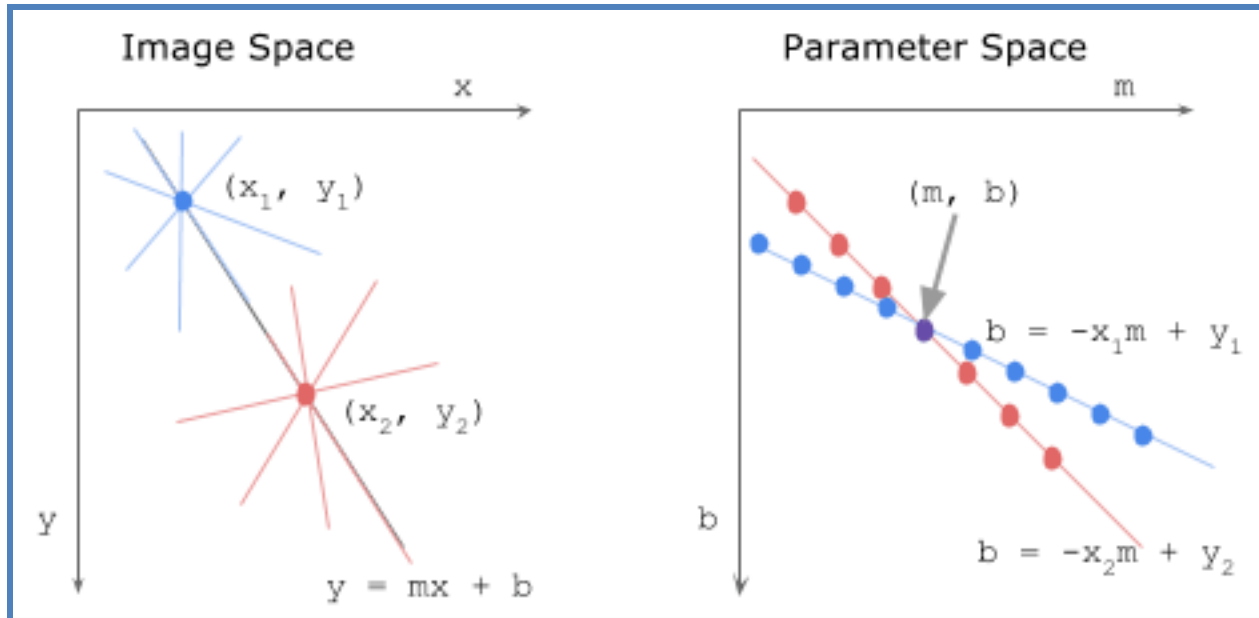


The second contribution is a 'straightness' score. Tracks deviating from local linear fits are penalised.



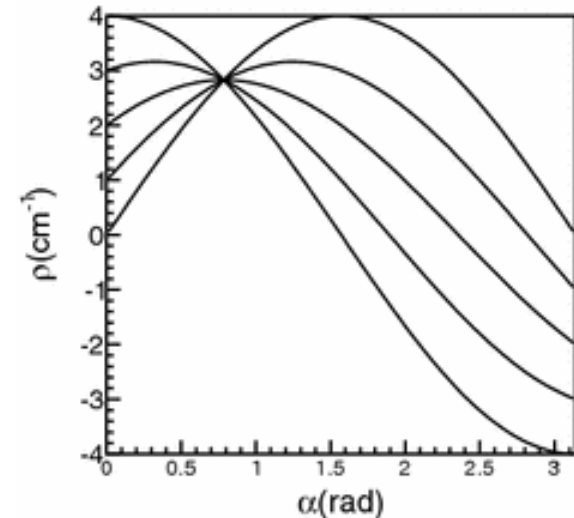
Hough Transformation

- Points in a straight line, $y = mx + b$: transform all points into a line in (m,b) plane by inverting the eqn, $b = -xm + y$



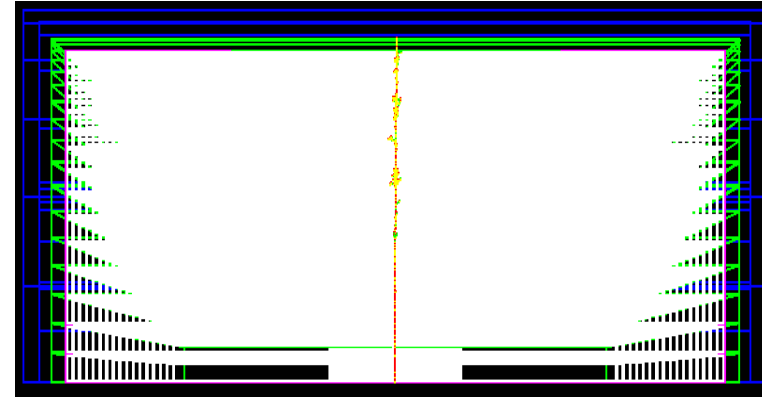
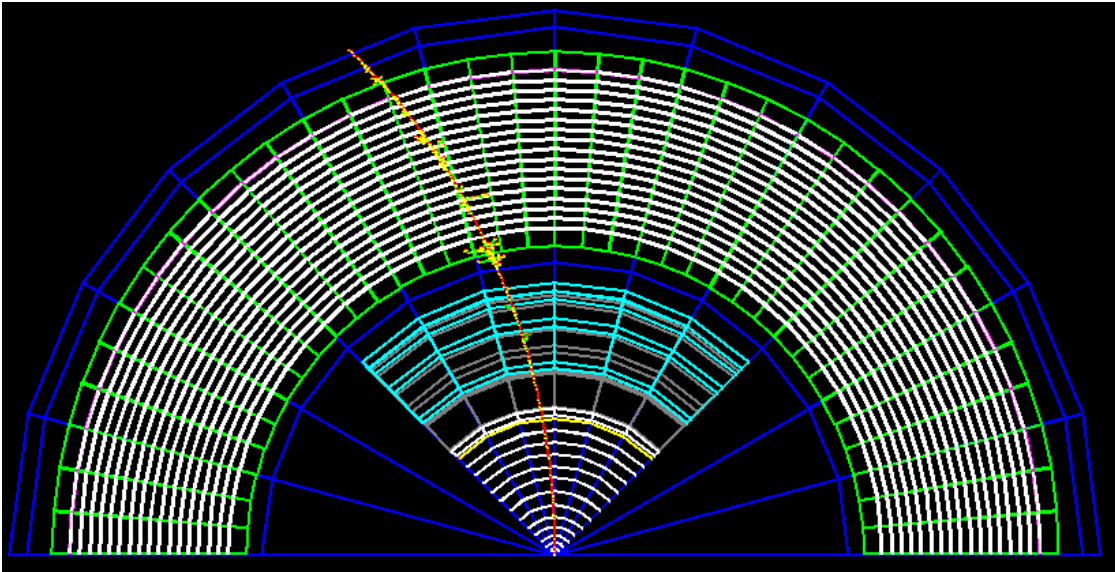
(a) X-Y plane

$$x \cos \alpha + y \sin \alpha = \rho$$



(b) parameter space

Simulated signal in Tracker : (0,0,3.8T) field

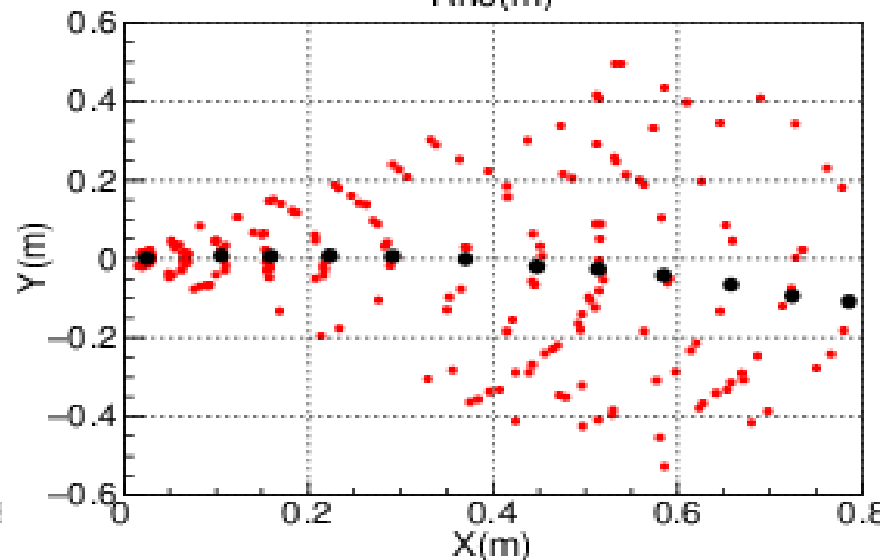
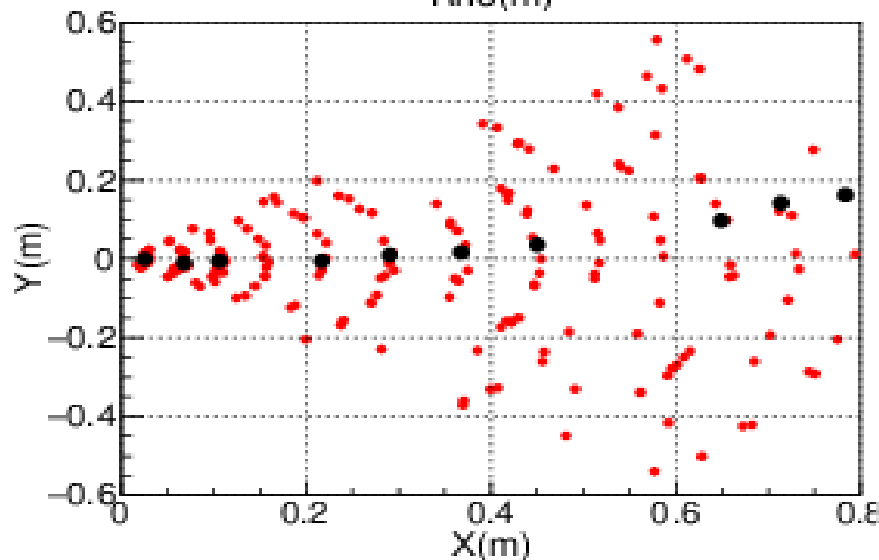
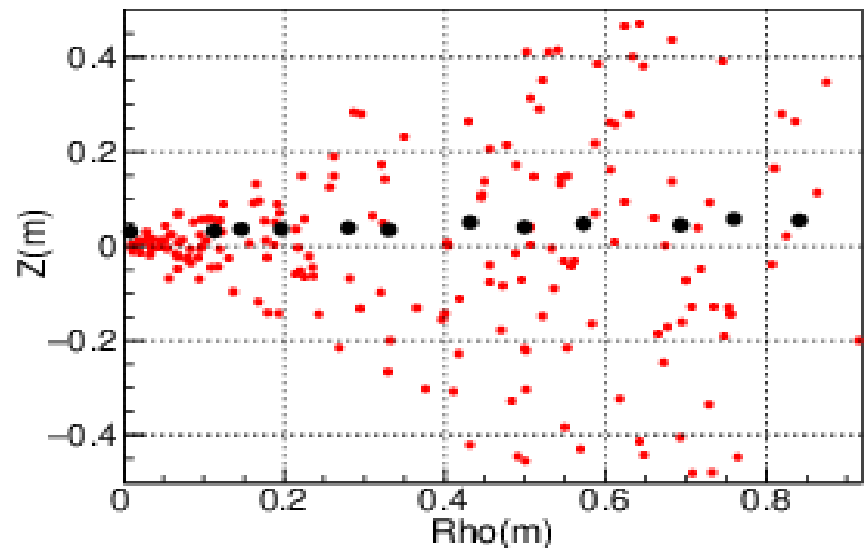
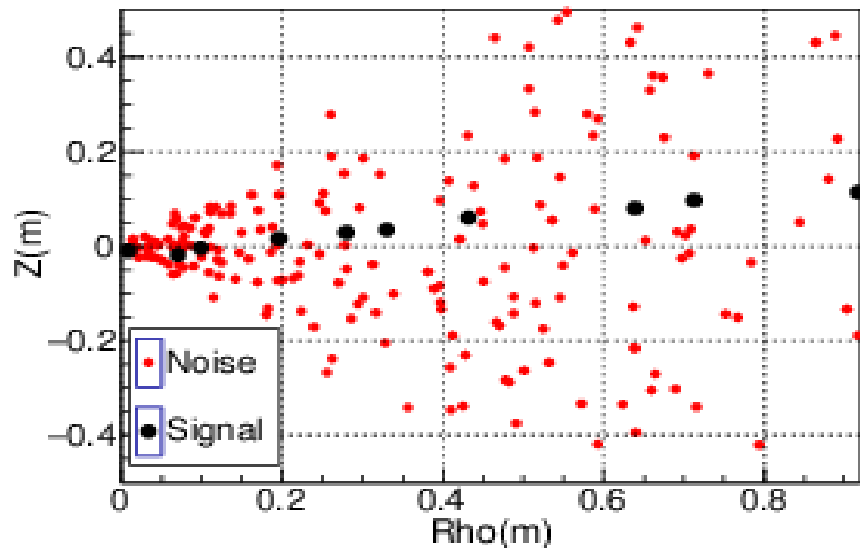


pidin, momin, thein, phiin, posxin, posyin, poszin // pdgId, three momentum vector and position of incident particle

- **Radius[13] = {2.9, 6.8, 10.9, 16.0, 22, 29, 37, 45, 52, 59, 66, 73, 80}; // in cm //4 bit**
- **unsigned int detidTk = theTouchable->GetCopyNumber(0); //Layer number**
- **detidTk<<=13; //For layer number**
- **int nIThe = int((750.0 + localpos.z()) /0.2); //200micon strip length //13bit**
- **detidTk +=nIThe;**
- **detidTk<<=15;**
- **int nIPhi = int((acos(-1.0)/4. + localpos.phi()) *20000); // 50 micon strip width : 15bit**
- **detidTk +=nIPhi;**
- **simenrTk = (*TrkCollection)[ij]->GetEdep(); //in KeV**
- **unsigned int nsimhtTk; //#of simhit in tracker**

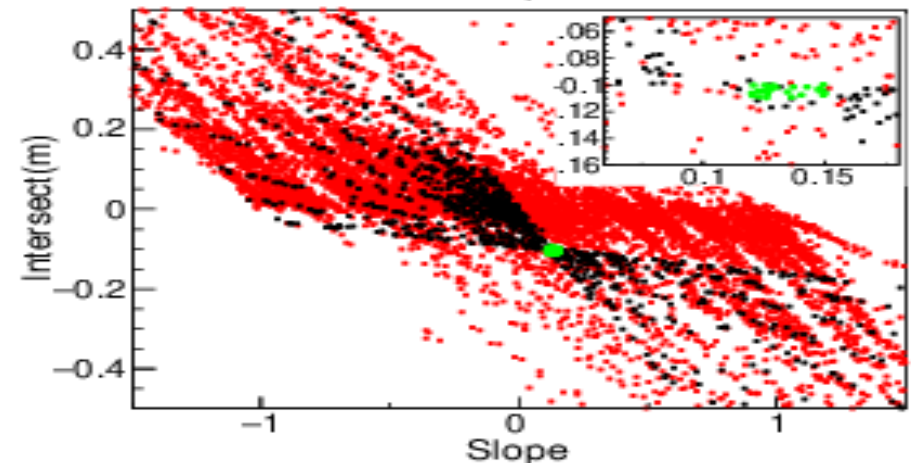
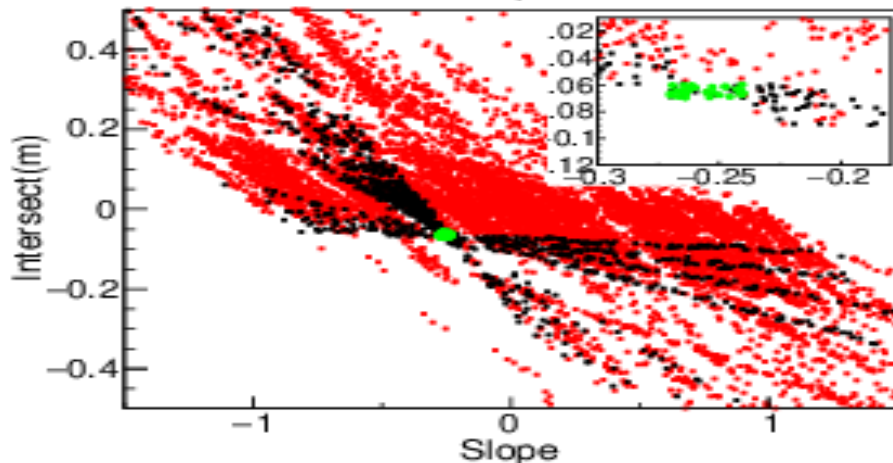
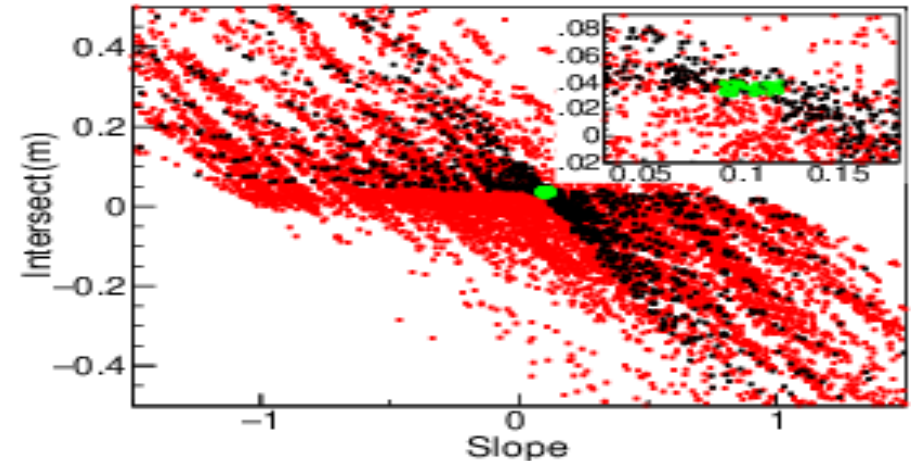
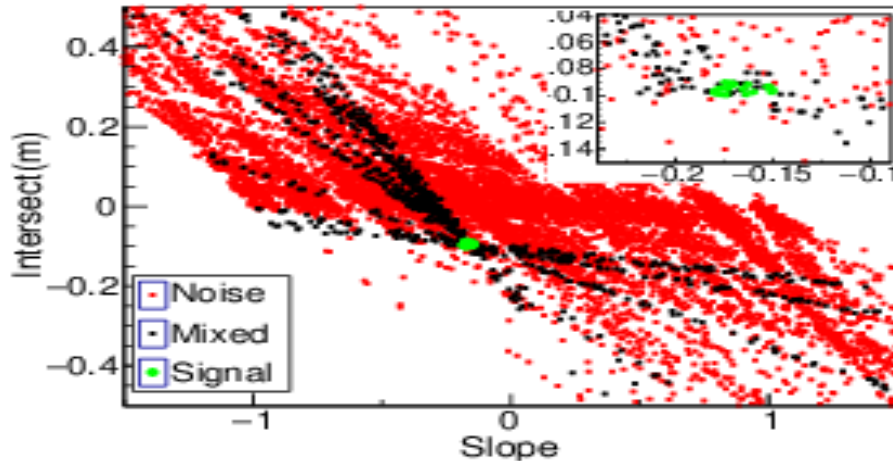
Hough Transformation

- Tracks in uniform magnetic field along Z direction
- Straight line in (ρ, z) plane



Hough Transformation

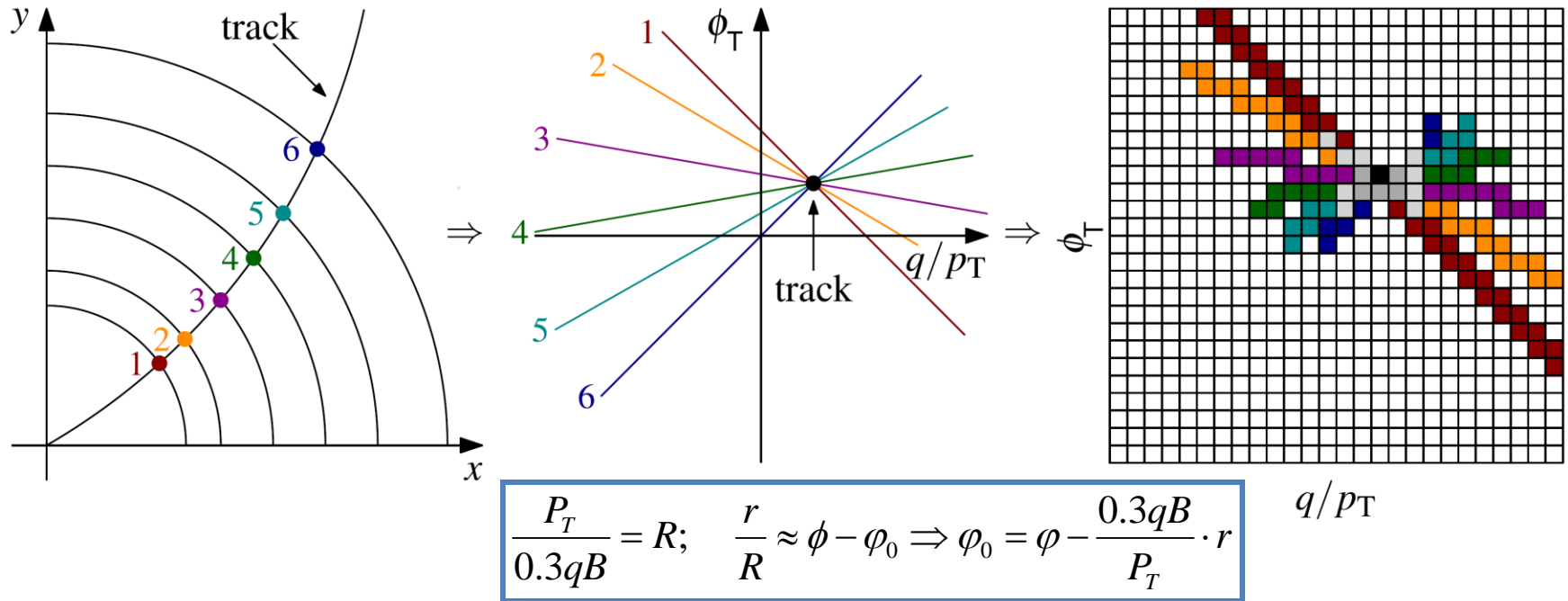
- $y = mx + c$: points in straight line (ρ_i, z_i) in $(\cot\theta, \rho_0)$ plan coincide in one point, whereas noise are not



Divide inverse space into cells and find maxima

- But, useless for collider experiment, mainly because of
 - Much precise measurement of points in $r-\phi$ plane (curvature)
 - No estimation of P_T of track (particularly not in trigger)

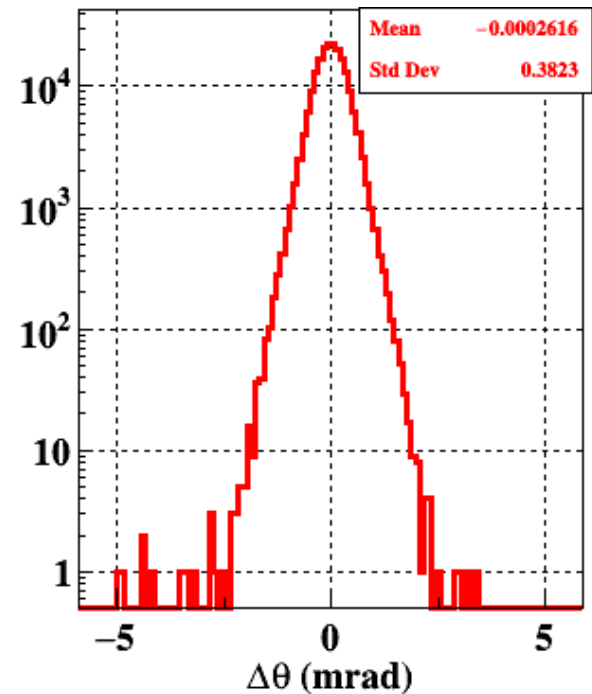
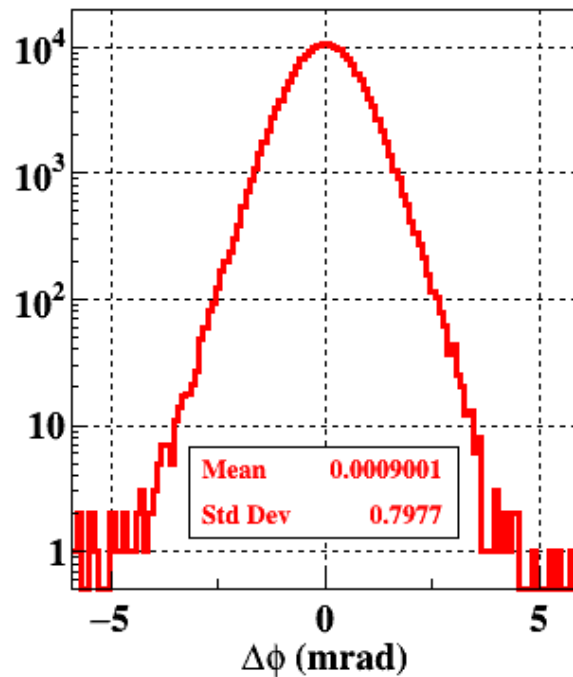
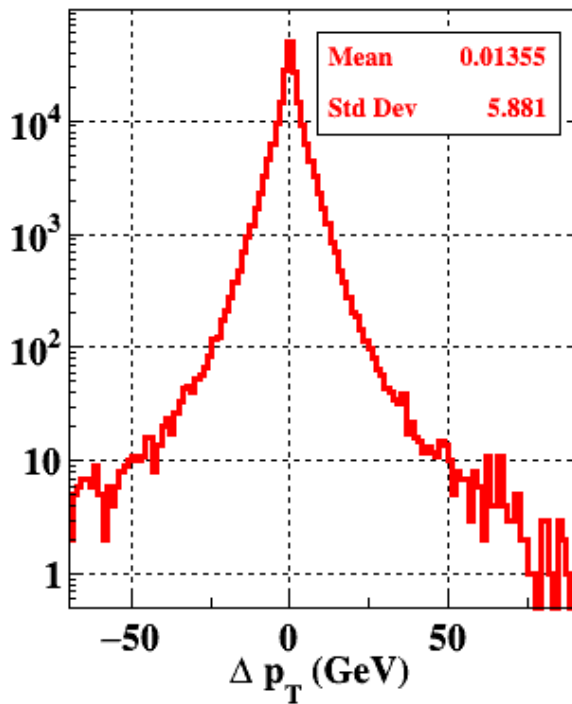
Hough transformation in helical path (P_T criteria)



Single stub with co-ordinate (r, ϕ) maps onto a straight line in track parameter space $(q/P_T, \phi_0)$

The Hough Transform track finding algorithm. Left: Trajectory of a single particle in a quarter of the tracker barrel in the x - y plane. The numbers label the stubs produced by the passage of the particles through the tracking stations. Middle: The produced stubs draw straight lines in the HT space, where the axes correspond to the track parameters $(q/p_T/\phi_T)$. Intersection point identifies a track candidate. Right: The same Hough-space discretised with an array of 32×64 cells. Stubs fill a cell only if their local- p_T estimation is compatible with the bin momentum. [95]

Example of Hough transformation



No firm comments, but

- Looks like Hough transformation is able to reconstruct p_T , ϕ and θ
 - Measurements in non bending plane is much better than bending in plane
- Uncertainties due to
 - Position resolution
 - Energy loss/Multiple scattering

Exercises

- **Hough transformation method**
 - **Using silicon signals in r-z plane**
 - **Make plots of few events in $\cot\theta$ and intersection plane**
 - **Reconstruct the polar angle of muon**
 - **Use the points in 2D grid points as well as fit those data points with straight line to obtain those parameters**
 - **Using silicon signals in r- ϕ plane**
 - **Make plots of few events in ϕ and p_T plane**
 - **reconstruct the azimuthal angle and transverse momentum of muons**
 - **Make the resolution plots of momentum, polar and azimuthal angle of muon**
 - **Compare these results with the inclusion of 1% noise hits in the silicon sensor**
- **Use road search method to find the hits in the muon trajectory and compare that with Hough transformation method**