EXTENDED BOSE-EINSTEIN CONDENSATE DARK MATTER IN $f(Q)$ GRAVITY

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Topic Outlines

Introduction

- As is widely recognized, the universe is comprised of both visible and dark components. The dark matter stands out as an undetectable entity within the electromagnetic radiation spectrum.
- The dark matter encompasses both baryonic and non-baryonic forms.
- In the baryonic form, dark matter manifests as astronomical entities like massive and compact haloes, primarily made up of ordinary baryonic matter yet emitting negligible electromagnetic radiation.
- The non-baryonic dark matter is characterized by hypothetical and actual particles such as the Weakly Interacting Massive Particles and axions.
- The state of matter known as Bose-Einstein condensate (BEC) arises in the non-baryonic realm, formed when particles called bosons undergo cooling to near absolute zero [1].

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[1] Y. Mambrini, S. Profumo, and F. S. Queiroz, *Phys. Lett. B* **760**, 807-815 (2016).

INTRODUCTION

- The assumption is made that dark matter exists in the form of a bosonic gas below a critical temperature, leading to the formation of Bose-Einstein condensate (BEC). [2].
- It is noteworthy that the EoS of conventional dark matter is derived as that of a barotropic fluid.
- Through the consideration of the dark matter halo existing in a quantum ground state, the equation of state (EoS) was derived[3] as $p \propto \rho^2$.
- Note that, the equation of state (EoS) $p = 0$, $p = \alpha\rho$, and $p = \beta\rho^2$ characterize the cold dark matter, normal dark matter, and dark matter halo respectively.
- These observations motivate to consider the Extended Bose-Einstein Condensate (EBEC) model, a comprehensive model combining normal dark matter and the quantum ground state.

[2] C. C. Bradley, C. A. Sackett, J. J. Tollett and R. G. Hulet, *Phys. Rev. Lett.* **75**, 1687 (1995).
[3] T. Harko, *Phys. Rev. D* **83**, 123515 (2011).

The Mathematical Formulation

The standard general relativity uses spacetime curvature to determine gravity, by incorporating a torsion-free and metric-compatible Levi-Civita connection. In $f(Q)$ gravity theory, we utilize symmetric teleparallel connection that is obtained via imposing following constraints,

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} = 0 \tag{1}$$

and

$$T^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\beta\alpha} - \Gamma^{\lambda}_{\alpha\beta} = 0 \implies \Gamma^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\beta\alpha} \tag{2}$$

The curvature free constraint ensures that the parallel transport is independent of the path taken, preserving the concept of parallelism over long distances. This characteristic is the basis for the term teleparallel. Moreover, the vanishing torsion makes the connection Γ symmetric in its last two indices. Thus, the connection Γ is known as the symmetric teleparallel connection and the corresponding gravity formulation is called symmetric teleparallel gravity. Moreover, we define the following non-metricity tensor arises due to the metric incompatibility of the symmetric teleparallel connection Γ,

$$Q_{\lambda\mu\nu} \equiv \nabla_{\lambda}g_{\mu\nu} = g_{\mu\nu,\lambda} - \Gamma^{\beta}{}_{\lambda\mu}g_{\beta\nu} - \Gamma^{\beta}{}_{\lambda\nu}g_{\mu\beta} \neq 0 \tag{3}$$

The difference between the associated connection $\Gamma^{\lambda}{}_{\mu\nu}$ and the Levi-Civita connection $\mathring{\Gamma}^{\lambda}{}_{\mu\nu}$ is known as the disformation tensor

$$L^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} - \mathring{\Gamma}^{\lambda}{}_{\mu\nu}\,, \tag{4}$$

The Mathematical Formulation

In addition, we define the non-metricity scalar

$$Q = -Q_{\lambda\mu\nu}P^{\lambda\mu\nu} \tag{5}$$

where

$$P^{\lambda}{}_{\mu\nu} := \frac{1}{4}\left(-2L^{\lambda}{}_{\mu\nu} + Q^{\lambda}g_{\mu\nu} - \tilde{Q}^{\lambda}g_{\mu\nu} - \frac{1}{2}\delta^{\lambda}_{\mu}Q_{\nu} - \frac{1}{2}\delta^{\lambda}_{\nu}Q_{\mu}\right), \tag{6}$$

is the superpotential tensor.
The $f(Q)$ gravity action reads as[4]

$$S = \frac{1}{2}\int f(Q)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \tag{7}$$

where, $f(Q)$ is the function of scalar term Q, L_m is the Lagrangian density, and $g = det(g_{\mu\nu})$. The variation of the action term (7) with respect to the metric, corresponds the following metric field equation,

$$\frac{2}{\sqrt{-g}}\nabla_{\lambda}(\sqrt{-g}f_Q P^{\lambda}{}_{\mu\nu}) + \frac{1}{2}g_{\mu\nu}f + f_Q(P_{\mu\lambda\beta}Q_{\nu}{}^{\lambda\beta} - 2Q_{\lambda\beta\mu}P^{\lambda\beta}{}_{\nu}) = -\mathcal{T}_{\mu\nu} \tag{8}$$

Here, $\mathcal{T}$ is the stress-energy tensor given by,

$$\mathcal{T}_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}} \tag{9}$$

[4] J.B. Jiménez, L. Heisenberg, and T. S. Koivisto, *JCAP* **08**, 039 (2018).

The Cosmological $f(Q)$ Model

We begin with following homogeneous and isotropic flat FLRW line element,

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2] \tag{10}$$

The non-metricity scalar Q for the metric (10) is given as

$$Q = 6H^2 \tag{11}$$

Then the corresponding energy momentum tensor becomes

$$\mathcal{T}_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \tag{12}$$

Here ρ denotes matter-energy density and p is the pressure component and $u_\mu = (1, 0, 0, 0)$ is the four velocity vector. Now, the Friedmann equations for an arbitrary $f(Q)$ function is given as,

$$3H^2 = \frac{1}{2f_Q}\left(-\rho + \frac{f}{2}\right) \tag{13}$$

and

$$\dot{H} + 3H^2 + \frac{\dot{f_Q}}{f_Q}H = \frac{1}{2f_Q}\left(p + \frac{f}{2}\right) \tag{14}$$

THE COSMOLOGICAL $f(Q)$ MODEL

We can rewrite equations (13) and (14) as follows,

$$3H^2 = \rho + \rho_{de} \tag{15}$$

$$\dot{H} = -\frac{1}{2}\left[\rho + \rho_{de} + p + p_{de}\right] \tag{16}$$

where ρ_{de} and p_{de} are energy density and pressure of the dark energy fluid part arising due to non-metricity component, and can be expressed as follows,

$$\rho_{de} = \frac{1}{2}(Q - f) + Qf_Q \tag{17}$$

and

$$p_{de} = -\rho_{de} - 2\dot{H}(1 + f_Q + 2Qf_{QQ}) \tag{18}$$

Further, we write the continuity equation for both matter and dark energy component as,

$$\dot{\rho} + 3H(\rho + p) = \mathcal{Q} \tag{19}$$

and

$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = -\mathcal{Q} \tag{20}$$

where $\mathcal{Q}$ is defined as an interaction term arising due to the energy transfer between dark components of the universe. It is evident that the parameter $\mathcal{Q}$ must possess a positive value, indicating the occurrence of energy transfer from dark energy to dark matter. In this context, considering $\mathcal{Q}$ as the product of the energy density and the Hubble parameter is a natural choice, given that it represents the inverse of cosmic time. Therefore, we adopt the specific expression $\mathcal{Q} = 3b^2 H\rho$, where b is the intensity of energy transfer.

We assume an extended form of the equation of state for dark matter known as the Extended Bose-Einstein Condensation (EBEC) for dark matter EoS as [5]

$$p = \alpha\rho + \beta\rho^2 \tag{21}$$

Here, α represent the single-body interaction arising from conventional dark matter, while β is introduced to signify the two-body interaction originating from the dark matter halo. In particular, $\alpha = \beta = 0$ reduces to the cold dark matter case, whereas the case $\beta = 0$ reduces to the normal matter scenario. Further, the case $\alpha = 0$ represents dark matter halo, while $\alpha \neq 0$ and $\beta \neq 0$ represents the contribution from both dark matter halo and the normal matter.

[5] E. Mahichi1 and A. Amani, *Phys. Dark Univ.* **39**, 101167 (2023).

We consider the following power-law $f(Q)$ function[6],

$$f(Q) = \gamma\left(\frac{Q}{Q_0}\right)^n \tag{22}$$

where $Q_0 = 6H_0^2$ and γ and n are free parameters. Then by using equation (22) in the equation (15), we obtained

$$\rho = \frac{(1-2n)}{2}\gamma\left(\frac{H}{H_0}\right)^{2n} \tag{23}$$

On evaluating the equation (23) at present redshift $z = 0$, we have

$$\rho_0 = \frac{(1-2n)}{2}\gamma \tag{24}$$

and therefore, we have

$$\rho = \rho_0\left(\frac{H}{H_0}\right)^{2n} \tag{25}$$

Now, on integrating the continuity equation (19) for the matter component, we acquired

$$\rho = \rho_0\left(\frac{c\eta - \beta}{c\eta(1+z)^{3\eta} - \beta}\right) \tag{26}$$

Here c is the constant of integration and $\eta = \alpha + 1 - b^2$.

[6] H. Shabani, A. De, and Tee-How Loo, *Eur. Phys. J. C.* **83**, 535 (2023).

We obtained the expression of the Hubble parameter, by utilizing equations (25) and (26), as follows,

$$H(z) = H_0 \left(\frac{c\eta - \beta}{c\eta(1+z)^{-3\eta} - \beta} \right)^{\frac{1}{2n}} \tag{27}$$

We estimate the median value of parameters of our $f(Q)$ model. We utilize the Markov Chain Monte Carlo (MCMC) sampling technique and optimization method in Python package emcee [7].

A. Hubble Data:

The Hubble datasets incorporates the 31 H(z) measurements of passively evolving massive galaxies covering the redshift range $0.07 \leq z \leq 2.41$ [8].The χ^2 function corresponding to H(z) data points reads as

$$\chi^2_H = \sum_{k=1}^{31} \frac{[H_{th}(z_k, \theta) - H_{obs}(z_k)]^2}{\sigma^2_{H(z_k)}}. \tag{28}$$

B. Pantheon+SH0ES Samples:

In the last two decades, several compilations of Type Ia supernova data have been introduced, such as Union, Union2, Union2.1, JLA, Pantheon, and the most recent addition, Pantheon+SH0ES. The corresponding χ^2 function is expressed as,

$$\chi^2_{SN} = D^T C^{-1}_{SN} D \tag{29}$$

[7] **D. F. Mackey et al.; Publ. Astron. Soc. Pac. 125; 306(2013).**

[8] **H. Yu; B. Ratra; and F.-Y. Wang; Astrophys. J. 856; 3 (2018).**

Here, C_{SN}[9] represents the covariance matrix associated with the Pantheon+SH0ES samples, encompassing both statistical and systematic uncertainties. Moreover, the vector D is defined as

$$D = \begin{cases} m_{Bi} - M - \mu_i^{Ceph} & i \in \text{Cepheid hosts} \\ m_{Bi} - M - \mu^{th}(z_i) & \text{otherwise} \end{cases} \tag{30}$$

where m_{Bi} and M are the apparent magnitude and absolute magnitude, respectively and μ_i^{Ceph} independently estimated using Cepheid calibrators. In addition, the $\mu^{th}(z_i)$ represents the distance modulus of the assumed theoretical model, and it can be expressed as,

$$\mu^{th}(z_i) = 5log_{10}\left[\frac{D_L(z_i)}{1Mpc}\right] + 25, \tag{31}$$

where, $D_L(z)$ is the luminosity distance assumed theoretical model, and it can be expressed as,

$$D_L(z) = c(1+z)\int_0^z \frac{dx}{H(x,\theta)} \tag{32}$$

where, θ is the parameter space of the assumed model.

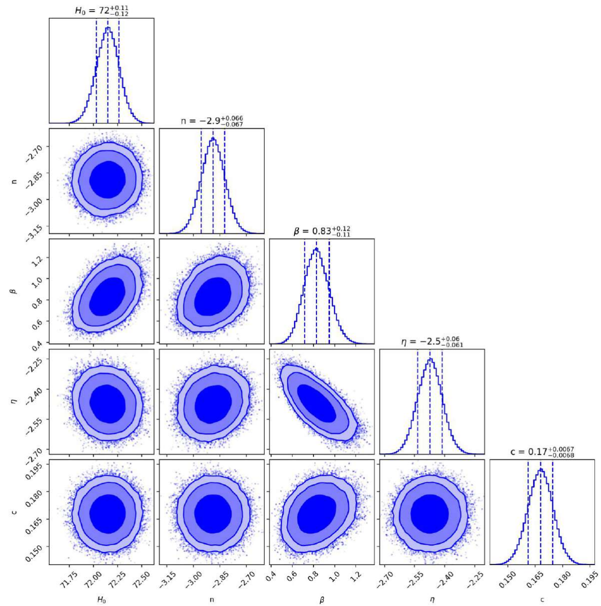
[9] D. M. Scolnic et al., *Astrophys. J.* **938**, 113 (2022).

We obtain the constraints on free parameter space for the combined CC+Pantheon+SH0ES samples utilizing the Gaussian priors as $[50, 100]$ for H_0, $[-5, 0]$ for n, $[0, 5]$ for β, $[-5, 0]$ for η, and $[0, 1]$ for c. In order to obtain the best fit value of parameters, we minimize the total χ^2_{total} function that is defined as follows,

$$\chi^2_{total} = \chi^2_{CC} + \chi^2_{SN} \tag{33}$$

We obtained constraints on the free parameter space with 68% confidence limit as $H_0 = 72^{+0.11}_{-0.12} Km/s/Mpc$, $n = -2.9^{+0.066}_{-0.067}$, $\beta = 0.83^{+0.12}_{-0.11}$, $\eta = -2.5^{+0.06}_{-0.061}$, and $c = 0.17^{+0067}_{-0.0068}$. In addition, we obtained the minimum value of the χ^2_{total} as $\chi^2_{min} = 1642.55$. The corresponding contour plot describing the correlation between different model parameters within the $1\sigma - 3\sigma$ confidence interval is presented in the Figure (1).

The Cosmological $f(Q)$ Model
Model Parameters Estimation



The Cosmological $f(Q)$ Model

Model Comparison

To evaluate the robustness of our MCMC analysis, it is crucial to perform a statistical assessment using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). It can be expressed as follows,

$$AIC = \chi^2_{min} + 2d \tag{34}$$

$$BIC = \chi^2_{min} + dln(N) \tag{35}$$

Here, d represents the number of parameters within the given model and N represents the number of data samples used in the MCMC analysis. A value of ΔAIC less than 2 suggests strong evidence in favor of the assumed theoretical model, while in the range of $4 < \Delta AIC \leq 7$, there is a moderate support. Similarly, ΔBIC is less than 2 suggests strong evidence in favor of the assumed theoretical model, while in the range of $2 < \Delta BIC \leq 6$, there is a moderate support.

We obtained $AIC_{Model} = 1652.55$ and $BIC_{Model} = 1679.8$ and hence we obtained $\Delta AIC = 0.85$ and $\Delta BIC = 15.5$, where ΛCDM value taken to be $AIC_{\Lambda CDM} = 1653.4$ and $BIC_{\Lambda CDM} = 1664.3$. Thus, it is evident from the ΔAIC value that there is strong evidence in favour of the assumed theoretical $f(Q)$ model. However, it is well known that large number of parameters compensate the high ΔBIC value.

The deceleration parameter is an essential tool to quantify the evolutionary phase of expansion of the universe. It is defined as follows,

$$q = -1 - \frac{\dot{H}}{H^2} \tag{36}$$

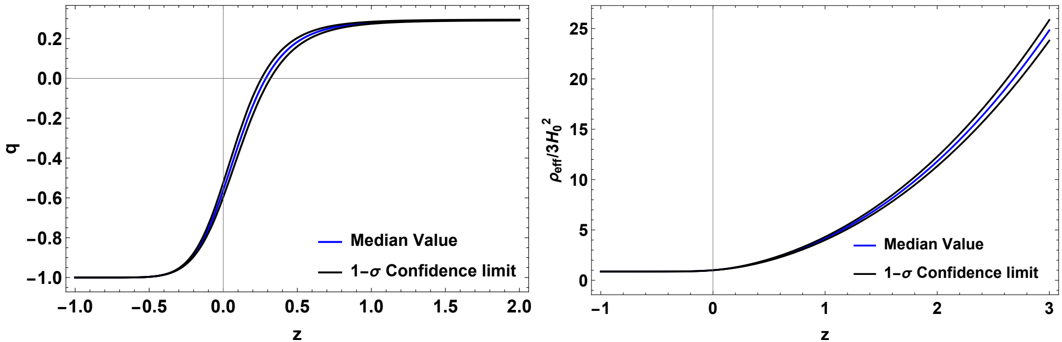


Figure. Profile of the deceleration parameter and effective energy density vs redshift corresponding to obtained parameter constraints with 68% confidence limit.

It is evident that the assumed model shows a transition from decelerated epoch to the de-Sitter type accelerated expansion phase, with the transition redshift $z_t = 0.288^{+0.031}_{-0.029}$. The present value of the deceleration parameter obtained as $q(z=0) = q_0 = -0.56^{+0.04}_{-0.03}$ (68% confidence limit), that is quite consistent with observed ones. Further, we obtained expected positive behavior of the effective energy density.

We investigate the thermodynamical stability of the assumed theoretical model by examining the sound speed parameter. In thermodynamical stability, the variation of pressure in relation to energy density becomes the primary focus, leading us to introduce the sound speed parameter, denoted as c_s^2, in the subsequent expression,

$$c_s^2 = \frac{\partial p}{\partial \rho} = \frac{\partial_z p}{\partial_z \rho} \tag{37}$$

where $\partial_z = \frac{\partial}{\partial z}$. Here, it is noteworthy that the condition $c_s^2 > 0$ indicate stability, while $c_s^2 < 0$ signifies instability.

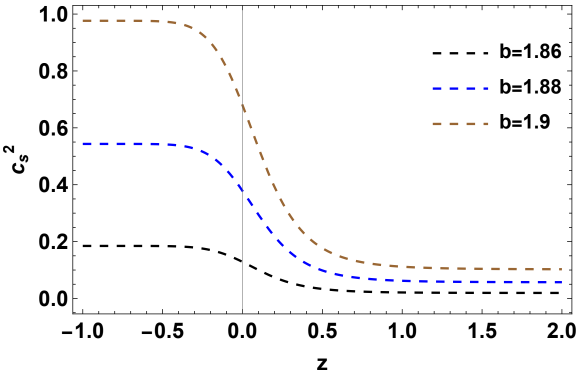


Figure. Profile of the sound speed parameter vs redshift corresponding to the value $b = 1.86$, $b = 1.88$, and $b = 1.9$.

Conclusion

- In this article, we attempted to explore the dark sector of the universe. We considered Extended Bose-Einstein Condensation (EBEC) EoS for dark matter with the modified $f(Q)$ lagrangian.
- We considered the power law $f(Q)$ lagrangian $f(Q) = \gamma \left(\frac{Q}{Q_0}\right)^n$, where γ and n are free parameters. We present the corresponding Friedmann-like equations and the continuity equation for both dark components along with an interacting term that signifies the energy exchange between the dark sector of the universe.
- We obtained the analytical solution of the corresponding equations. Further, we utilize the Bayesian analysis to estimate the posterior probability through the likelihood function and the MCMC sampling technique.
- The obtained constraints on the free parameter space with 68% confidence limit are $H_0 = 72^{+0.11}_{-0.12} Km/s/Mpc$, $n = -2.9^{+0.066}_{-0.067}$, $\beta = 0.83^{+0.12}_{-0.11}$, $\eta = -2.5^{+0.06}_{-0.061}$, and $c = 0.17^{+0067}_{-0.0068}$.
- In addition, to examine the robustness of our MCMC analysis, we estimated the AIC and BIC value. We obtained $\Delta AIC = 0.85$ and $\Delta BIC = 15.5$, and hence it is evident from the ΔAIC value that there is strong evidence in favor of the assumed theoretical $f(Q)$ model. However, it is well known that a large number of parameters compensate for a high ΔBIC value.

Conclusion

- Further, we found that the assumed model shows a transition from the decelerated epoch to the de-Sitter type accelerated expansion phase, with the transition redshift $z_t = 0.288^{+0.031}_{-0.029}$, along with the present value of the deceleration parameter as $q(z=0) = q_0 = -0.56^{+0.04}_{-0.03}$ (68% confidence limit), which is quite consistent with cosmological observations.
- Lastly, we investigated the thermodynamical stability of the assumed theoretical model by examining the sound speed parameter.

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Thank You