Predictions from scoto-seesaw with A_4 modular symmetry

Priya Mishra

In collaboration with:
Ranjeet Kumar, Mitesh Behera, Dr. Rahul Srivastava & Prof. Rukmani Mohanta

University of Hyderabad

Outline

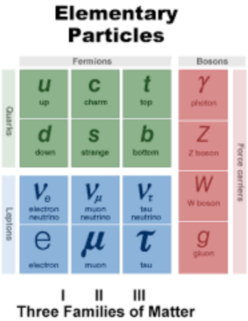
Flavor structure: Hint for new physics

Modular symmetry as a predictive framework

Mass hierarchy of neutrinos shaped by the scoto-seesaw framework

- In Standard Model (SM) there are **three** families of **five** fermions:

$$Q_{Li}, u^c_{Ri}, d^c_{Ri}, l_{Li}, e_{Ri}$$



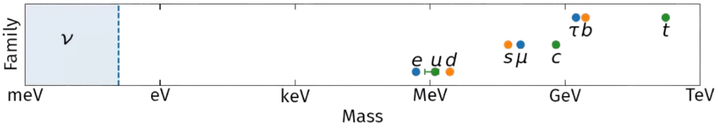
- Flavor problem occurs when three generations have to live together.

- This problem arises from Mass terms:

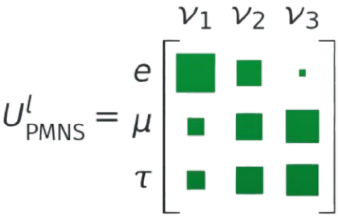
$$e^c_{Ri} M^e_{ij} l_{Lj} \quad \text{: Charged leptons} \tag{1}$$

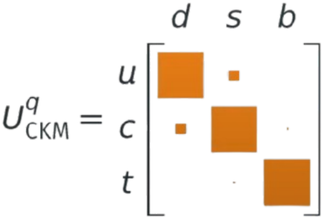
$$l_i M^\nu_{ij} l_j \quad \text{: Neutrinos} \tag{2}$$

$$U_{PMNS} = V_e^\dagger V_\nu \tag{3}$$

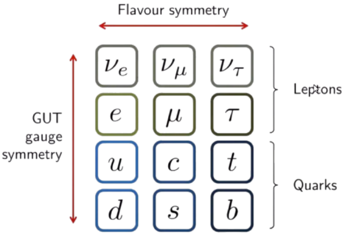


Flavor Puzzle





- This peculiar pattern consists flavor puzzle.
- Leads to new physics : Symmetry approach.



Flavor symmetry approach

There are various ways of doing this:

- **Froggatt Nielsen Mechanism:** Solves hierarchy problem but no explanation for large mixing angles in lepton sector.
- **Discrete Symmetry:** Explains mixing pattern but symmetry breaking mechanism is quite complicated.

In Modular symmetry approach flavor symmetry is realised in a non-linear way hence making it special.

Mass hierarchy of neutrinos shaped by the scoto-seesaw framework

Model Framework

- We extend the SM symmetry by including A_4 modular symmetry.
- $\Delta m^2_{atm} = 2.5 \times 10^{-3}\ \mathrm{eV}^2$ and $\Delta m^2_{sol} = 7.41 \times 10^{-5}\ \mathrm{eV}^2$.
- origins of these two scales may stem from separate mechanisms : Scoto-seesaw.

	Fermions					Scalars				Yukawa couplings					
Fields	L_ℓ	ℓ^c_R	N_{R_1}	N_{R_2}	f	$H_{u,d}$	η	η'	χ	$Y_1^{(4)}$	$Y_{1'}^{(4)}$	$Y_1^{(8)}$	$Y_{1'}^{(8)}$	$Y_{1''}^{(8)}$	$Y_1^{(10)}$
$SU(2)_L$	2	1	1	1	1	2	2	2	1	–	–	–	–	–	–
$U(1)_Y$	$-1/2$	1	0	0	0	$\pm 1/2$	$1/2$	$-1/2$	0	–	–	–	–	–	–
A_4	$1, 1', 1''$	$1, 1'', 1'$	1	$1'$	1	1	1	1	1	1	$1'$	1	$1'$	$1''$	1
k_I	0	0	4	4	5	0	3	3	5	4	4	8	8	8	10

Table: Particle content and modular Yukawa couplings of the model and their charges under $SU(2)_L \times U(1)_Y \times A_4$, where k_I is modular weight.

Atmospheric Neutrino Mass Scale

- Generating the atmospheric scale through type-I seesaw.

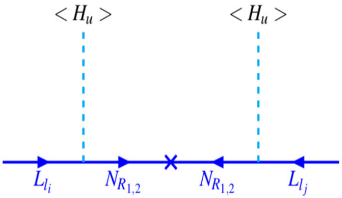


Figure: Feynman diagram at tree level

- **The Superpotential at tree level :**

$$\mathcal{W}_\nu^{\mathcal{T}} = \alpha_T \left(Y_1^{(4)} L_e H_u N_{R_1} + Y_{1'}^{(4)} L_\tau H_u N_{R_1} + Y_{1'}^{(4)} L_\mu H_u N_{R_2} + Y_1^{(4)} L_\tau H_u N_{R_2} \right)$$
$$+ \kappa_1 Y_1^{(8)} N_{R_1} N_{R_1} + \kappa_2 Y_{1''}^{(8)} N_{R_2} N_{R_2}.$$

- **Neutrino mass matrix**:

$$M_D = \begin{pmatrix} Y_1^{(4)} & 0 \\ 0 & Y_{1'}^{(4)} \\ Y_{1'}^{(4)} & Y_1^{(4)} \end{pmatrix} \alpha_T v_u, \qquad M_R = \begin{pmatrix} \kappa_1 Y_1^{(8)} & 0 \\ 0 & \kappa_2 Y_{1''}^{(8)} \end{pmatrix} \tag{4}$$

$$(M_\nu)_{\text{tree}} = -M_D M_R^{-1} M_D^T \tag{5}$$

The Superpotential at loop level:

$$\mathcal{W}_\nu^{\mathcal{L}} \;=\; \beta_L \left(Y_1^{(8)} L_e \eta f + Y_{1''}^{(8)} L_\mu \eta f + Y_{1'}^{(8)} L_\tau \eta f \right) + \kappa_S Y_1^{(10)} ff + \lambda_1 Y_1^{(8)} H_d \eta \chi \, .$$

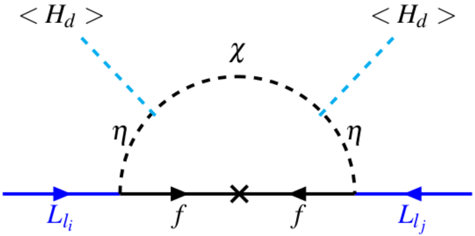


Figure: Radiative neutrino mass generation

Neutrino mass matrices at loop level:

$$(M_\nu)_{\rm loop} = \beta_L^2 M_f \begin{pmatrix} \left(Y_1^{(8)}\right)^2 & \left(Y_1^{(8)} Y_{1''}^{(8)}\right) & \left(Y_1^{(8)} Y_{1'}^{(8)}\right) \\ * & \left(Y_{1''}^{(8)}\right)^2 & \left(Y_{1''}^{(8)} Y_{1'}^{(8)}\right) \\ * & * & \left(Y_{1'}^{(8)}\right)^2 \end{pmatrix} \mathcal{F}\left(m_{\eta_R}, m_{\eta_I}, M_f\right)$$

Total neutrino mass:

$$M_\nu = (M_\nu)_{\rm tree} + (M_\nu)_{\rm loop} \quad (6)$$

Parameter space

Input Parameters	Range
$\mathrm{Re}[\tau]$	$\pm[0.0, 0.5]$
$\mathrm{Im}[\tau]$	$[0.8, 1.5]$
α_T	$[10^{-3}, 10^{-2}]$
β_L	$[10^{-1}, 2]$
M_1 (GeV)	$[1, 10]\times10^{11}$
M_2 (GeV)	$[1, 10]\times10^{11}$
M_f (GeV)	$[1, 10^4]$
m_{η_R} (GeV)	$[1, 400]$
m_{η_I} (GeV)	$[1, 400]$

Model Predictions

Model favors normal ordering of neutrino mass

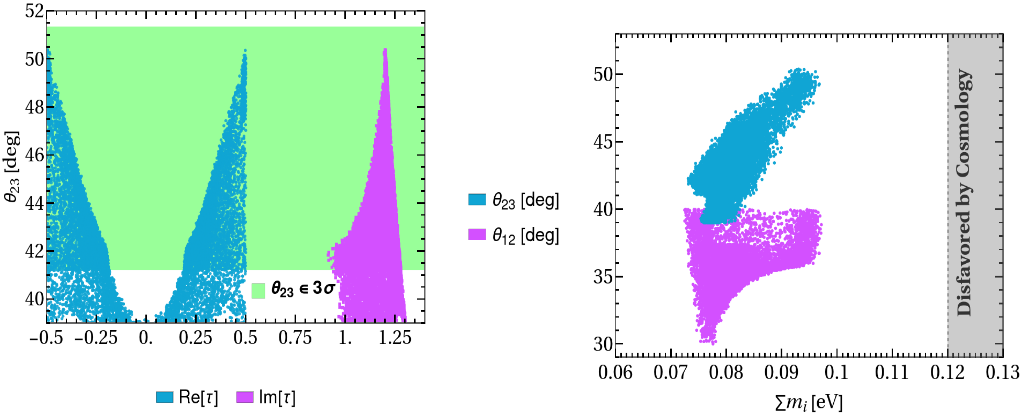


Figure: Predictions of neutrino oscillation parameters from model

Results from Neutrino phenomenology

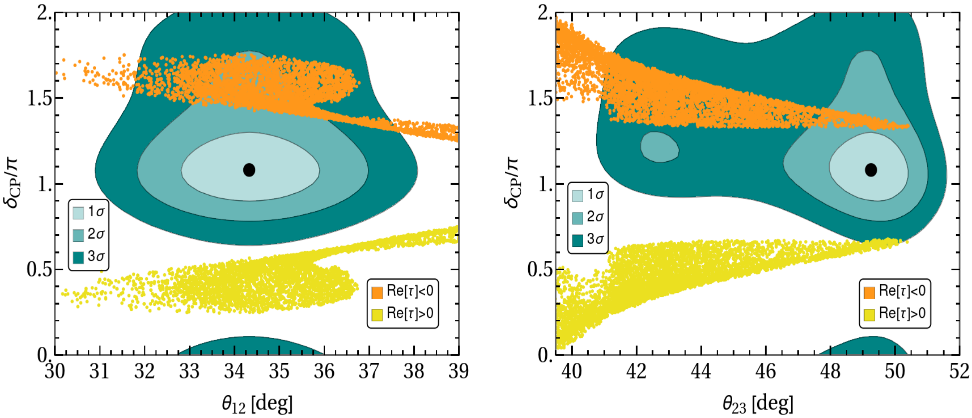
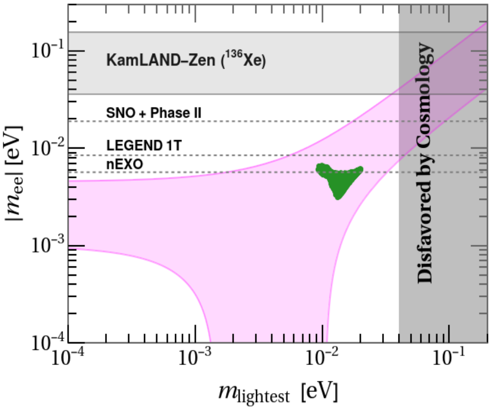


Figure: Predicted range for δ_{CP}

Results from Neutrino phenomenology



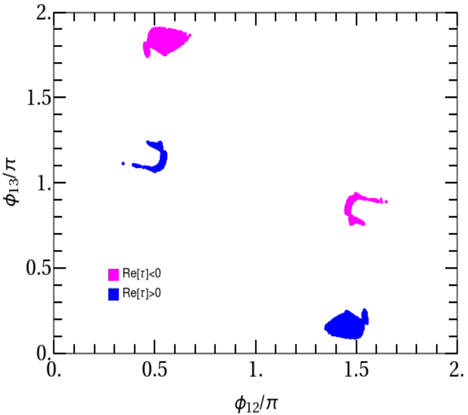
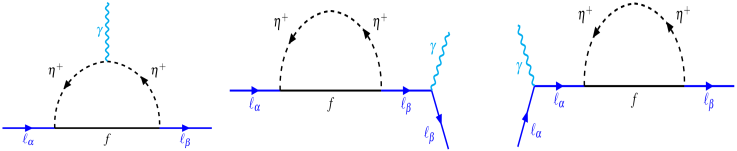


Figure: Neutrinoless double beta decay parameter and Majorana phases



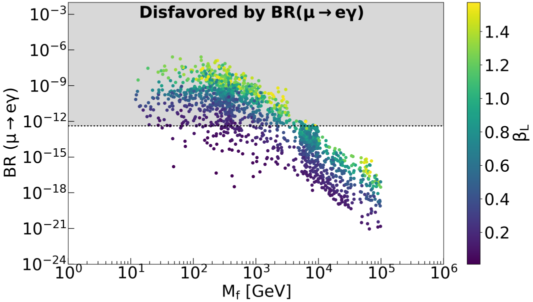
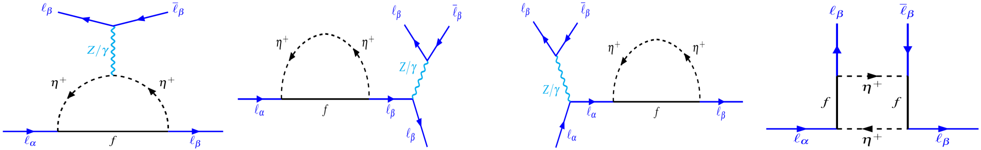


Figure: Expected branching ratio (BR) of $\mu \to e\gamma$ with experimental upper limit as 4.2×10^{-13}

Lepton Flavor Violating Decay mode $\mu \to 3e$



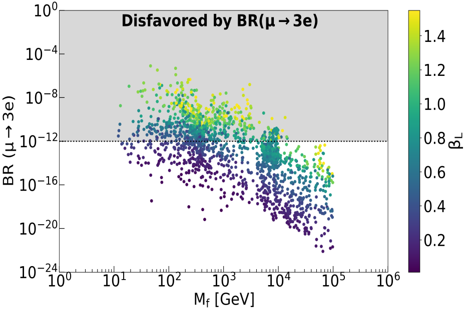


Figure: Expected BR of $\mu \to 3e$ with experimental upper limit as 1×10^{-12}

Conclusion

- Modular symmetry is highly predictive in terms of mixing parameters in lepton sector.
- Neutrino mass is hierarchical by scoto-seesaw.
- Our model predicts normal ordering for neutrino masses and $m_{\text{lightest}} \in (9.2, 20.0) \times 10^{-3}$ eV.
- The neutrinoless double beta decay parameter $|m_{ee}| \in (3.15, 6.66) \times 10^{-3}$ eV, which is within the potential reach of upcoming experiments.
- A_4 modular symmetry within the scoto-seesaw framework leads to a highly predictive model whose predictions can be tested in various experiments.

Thankyou

$$\Gamma^{0\nu}_{\beta\beta} = \frac{1}{T^{0\nu}_{\beta\beta}} = G^{0\nu} \cdot \left|M^{0\nu}\right|^2 \cdot \langle m_{\beta\beta}\rangle^2$$

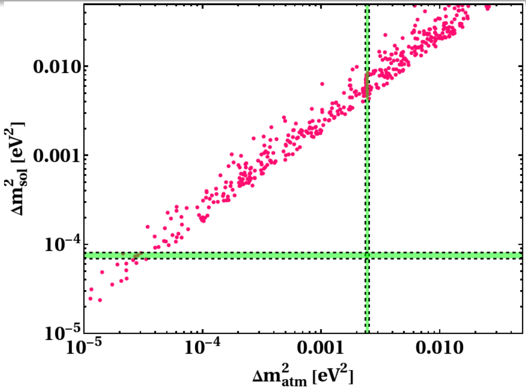
Effective Majorana mass

$$\langle m_{\beta\beta}\rangle = \sum_i U_{ei}^2 m_i ,$$

a larger $< m_{ee} >$ increases the chances of detecting the decay.

$$f(\tau) = (-1)^k f(\tau) \tag{7}$$

Thus for odd weights, modular forms vanishes.


0.010
0.001
10^{-4}
10^{-5}
Δm^2_{sol} [eV^2]
10^{-5}
10^{-4}
0.001
0.010
Δm^2_{atm} [eV^2]

I. q-Series: an example

The generating function, the infinite q-series

$$\sum_{n=0}^{\infty} p(n) q^n = \frac{1}{\prod_{k=1}^{\infty}(1-q^k)}$$
$$= 1 + q + 2\,q^2 + 3\,q^3 + 5\,q^4 + \ldots,$$
$$\in \mathbb{Z}[[q]]$$

Leonhard Euler

is *modular*!

$$q^{-\frac{1}{24}} \sum_{n=0}^{\infty} p(n) q^n = \frac{q^{-\frac{1}{24}}}{\prod_{k=1}^{\infty}(1-q^k)} = \frac{1}{\eta(\tau)}$$

- a shift by $q^{\frac{-1}{24}}$
- $q \to \tau$

I. q-Series: as modular forms

The Dedekind eta function:

$$\eta(\tau) = \frac{q^{\frac{1}{24}}}{\sum_{n=0}^{\infty} p(n) q^n} = q^{\frac{1}{24}} \prod_{k=1}^{\infty} (1 - q^k)$$

is *modular*!

II. Modular Forms: the partition example

$$\eta^{24} : \mathbb{H} \to \mathbb{C}$$

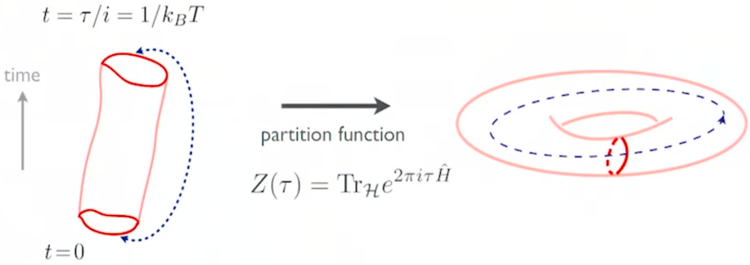
$$\eta^{24}(\tau) = (c\tau + d)^{-12}\, \eta^{24}\left(\frac{a\tau + b}{c\tau + d}\right)$$

$$= q \prod_{k=1}^{\infty} (1 - q^k)^{24}$$

is a modular form of *weight 12*.

II. Modular Forms: as in 2d conformal field theory and string theory

A string moving in time = a cylinder.



The partition functions are computed by identifying the initial and final time. This turns the cylinder into a torus. As a result the string partition functions are modular forms!

Modularity is very helpful in studying these physical theories.

II. Modular Forms: as in 2d conformal field theory and string theory

String theory/CFT is very helpful to understand modularity.

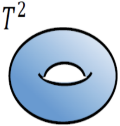
Example 1: free chiral boson/Heisenberg algebra

$$Z(\tau) = q^{-\frac{1}{24}} \sum_{n=0}^{\infty} p(n) q^n = \frac{1}{\eta(\tau)}$$

ground state energy

number of way to increase the energy by n

MODULAR SYMMETRY

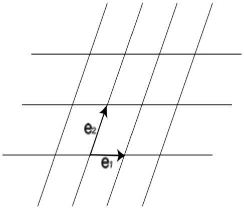


- The modular symmetry is a geometrical symmetry of the two-dimensional torus, T^2.
- The two-dimensional torus is constructed as division of the two-dimensional Euclidean space R^2 by a lattice Λ, $T^2 = \frac{R^2}{\Lambda}$.
- Instead of R^2 , one can use the one-dimensional complex plane.

- The lattice is spanned by two basis vectors, e_1 and e_2 as $m_1 e_1 + m_2 e_2$, where m_1 and m_2 are integer.
- There ratio is

$$\tau = \frac{e_2}{e_1} \tag{16}$$

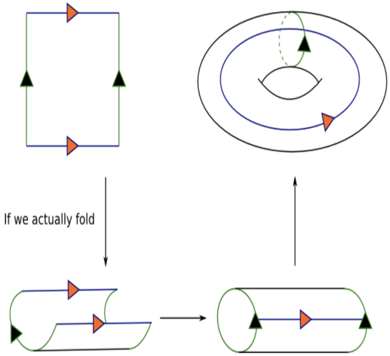
in the complex plane, represents the shape of the Torus T^2 and parameter τ is called the modulus.



- The same lattice can be spanned by other basis vectors such as

$$\begin{pmatrix} e_1' \\ e_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \tag{17}$$

where a, b, c, d are integer satisfying $ad - bc = 1$. That is the $SL(2, Z)$.



- One interesting thing about finite modular groups [7] are: they are isomorphic to discrete symmetry groups like $\Gamma_2 \simeq S_3$, $\Gamma_3 \simeq A_4$, $\Gamma_4 \simeq S_4$, $\Gamma_5 \simeq A_5$, $\Gamma'_3 \simeq A'_4$, $\Gamma'_5 \simeq A'_5$ etc.
- The modular group is defined as a group of 2×2 matrices having integer entries and determinant 1.

$$\boxed{\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,Z); \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (mod\, N) \right\}.}$$

- Therefore, the group $\Gamma(N)$ acts on the complex variable τ, varying in the upper-half $\mathcal{H} = \mathrm{Im}(\tau) > 0$, as linear fractional transformation given by

$$\gamma(\tau) = \frac{a\tau + b}{c\tau + d}, \quad \mathcal{H} = \{\tau \in \mathbb{C}, \mathrm{Im}(\tau) > 0\}, \quad \gamma = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathrm{SL}(2, \mathbb{Z}), \tag{18}$$

[7] Feruglio, Ferruccio, Are neutrino masses modular forms?, From My Vast Repertoire ...

- The generators of modular group being

$$S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \text{ and } T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}. \tag{19}$$

$$S : \tau \to -\frac{1}{\tau} \qquad T : \tau \to \tau + 1 . \tag{20}$$

- A function $\mathcal{F}$ is said to be entire modular of weight k if it satisfies these below conditions:
  1. $\mathcal{F}$ is analytic in the upper plane H, H =$(x + iy \mid y > 0; x, y \in R)$.
  2. $\mathcal{F}\left(\frac{a\tau+b}{c\tau+d}\right) = (c\tau + d)^k \mathcal{F}(\tau)$.
  3. The fourier series of $\mathcal{F}$ is given by the form of (called q expansion form).

$$\mathcal{F}(\tau) = \sum_{n=0}^{\infty} a_n q^n \qquad q = e^{2\pi i\tau} \tag{21}$$

Under the modular transformation, chiral superfields ψ_i (i denotes flavors) with weight -k transform as

$$\psi_i \to (c\tau + d)^{-k_i}\rho(\gamma)ij\psi_j. \tag{8}$$

Weight (k)	d_k	A_4 representations
2	3	**3**
4	5	**3**+**1**+**1**$'$
6	7	**3**+**3**+**1**
8	9	**3**+**3**+**1**+**1**$'$+**1**$''$
10	11	**3**+**3**+**3**+**1** +**1**$'$

Table: A_4 representations for different weight k.

UV completion of SUSY scotogenic loop

$$\mathcal{W}_S = \mu H_u H_d + \mu_\eta Y_1^{(6)} \eta\eta' + \frac{1}{2}\mu_\chi Y_1^{(10)} \chi\chi + \lambda_1 Y_1^{(8)} H_d \eta\chi + \lambda_2 Y_1^{(8)} H_u \eta' \chi.$$

$$(M_\nu^{ij})_{\rm loop} = \sum_{l=1}^{3} \mathcal{F}_{1l} M_f \mathbf{h^i h^j} + \sum_{l=1}^{3} \mathcal{F}_{2l} m_{\tilde{\eta}_l} \mathbf{h^i h^j}\ , \tag{9}$$

where $\mathcal{F}_{1l}$ and $\mathcal{F}_{2l}$ are the loop function given by:

$$\mathcal{F}_{1l} = \frac{1}{32\pi^2}\left[[U_R(2,l)]^2 \frac{m^2_{\eta_{Rl}}}{M_f^2 - m^2_{\eta_{Rl}}} \ln\left(\frac{M_f^2}{m^2_{\eta_{Rl}}}\right) - [U_I(2,l)]^2 \frac{m^2_{\eta_{Il}}}{M_f^2 - m^2_{\eta_{Il}}} \ln\left(\frac{M_f^2}{m^2_{\eta_{Il}}}\right)\right], \tag{10}$$

$$\mathcal{F}_{2l} = [U_\eta(2,l)]^2 \frac{1}{32\pi^2}\left[\frac{m^2_{fR}}{m^2_{\tilde{\eta}_l} - m^2_{fR}} \ln\left(\frac{m^2_{\tilde{\eta}_l}}{m^2_{fR}}\right) - \frac{m^2_{fI}}{m^2_{\tilde{\eta}_l} - m^2_{fI}} \ln\left(\frac{m^2_{\tilde{\eta}_l}}{m^2_{fI}}\right)\right]. \tag{11}$$

In the limit $m_{\tilde{\eta}_l} >> m_{fR,I}$

$$\mathcal{F}_{1I} \rightarrow \mathcal{F} = \frac{1}{32\pi^2}\left[\frac{m^2_{\eta_R}}{M^2_f - m^2_{\eta_R}} \ln\left(\frac{M^2_f}{m^2_{\eta_R}}\right) - \frac{m^2_{\eta_I}}{M^2_f - m^2_{\eta_I}} \ln\left(\frac{M^2_f}{m^2_{\eta_I}}\right)\right]. \quad (12)$$