Minimal Z' for Radiative generation of fermion masses

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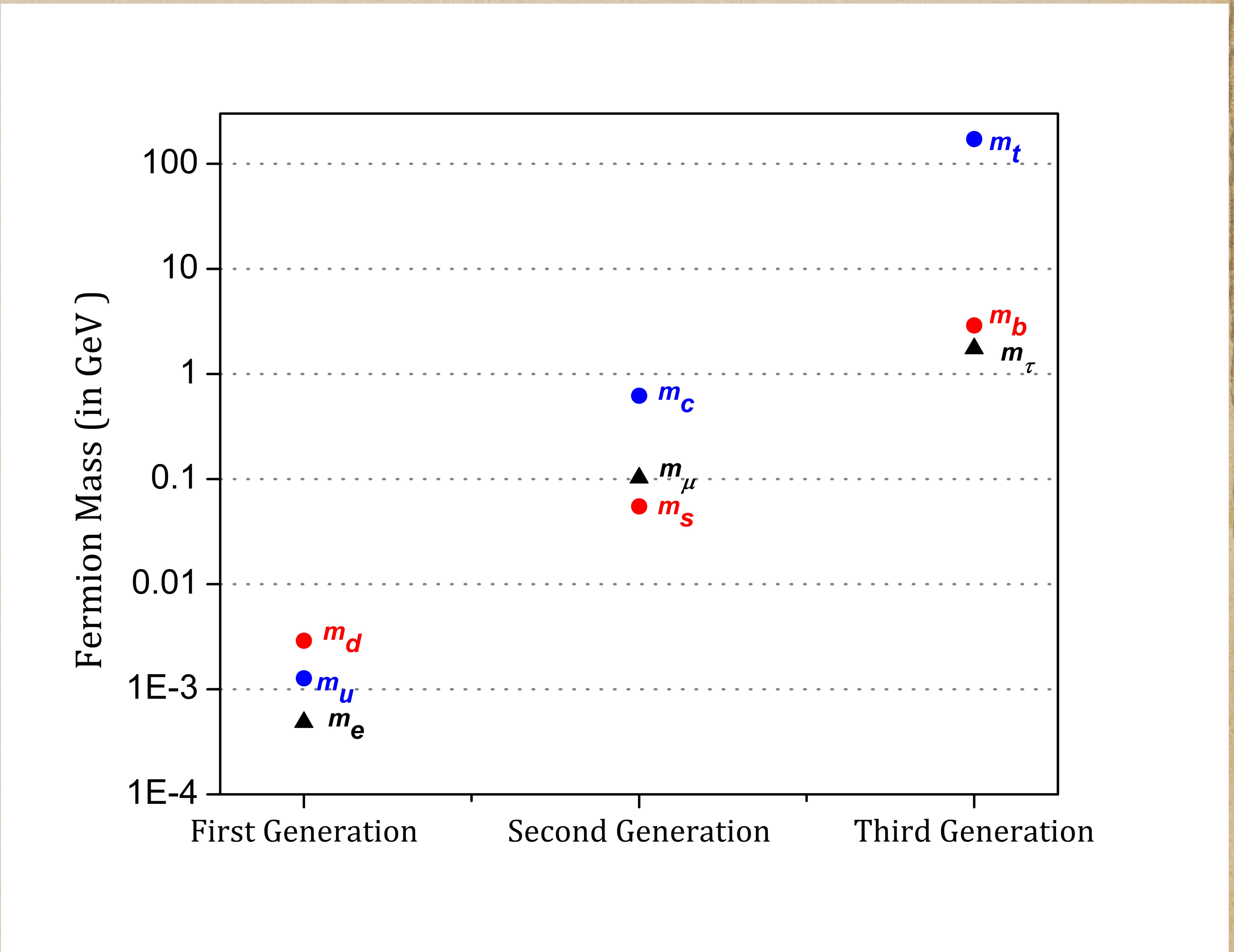
Outline

- Introduction
- Radiative mass generation
- Symmetry deconstruction
- Phenomenological analysis
- Summary

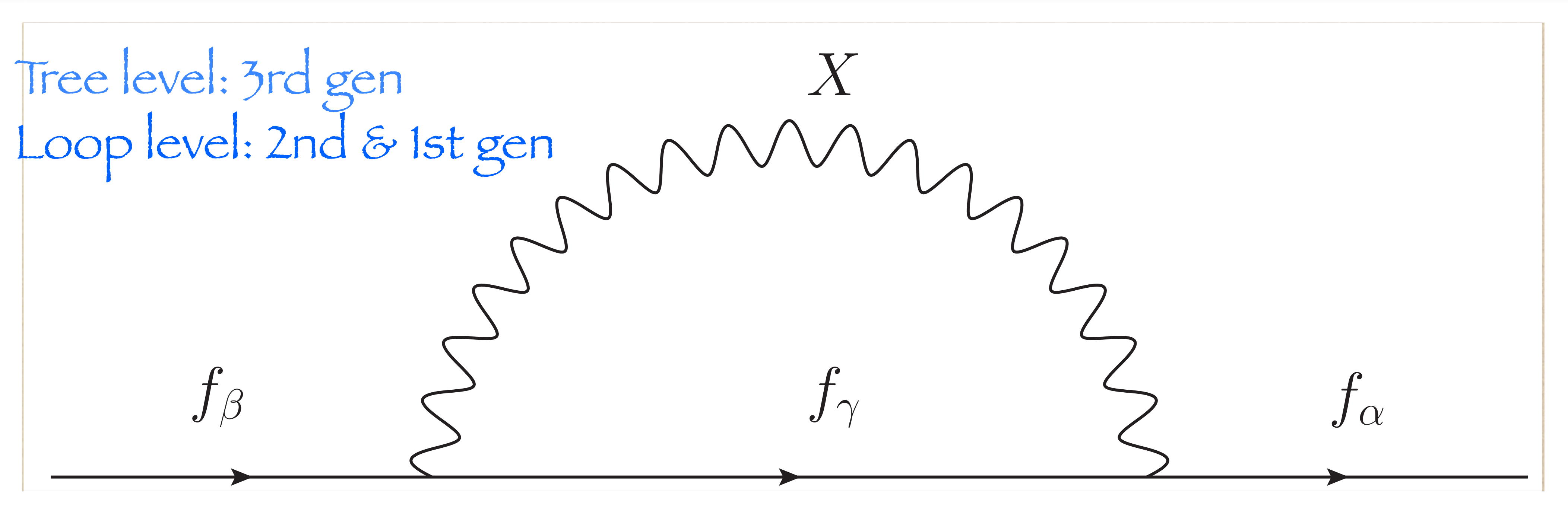
Introduction

- Elementary fermions: "Quarks and Leptons" each comes in three generations (families/flavour)
- The masses of the charged fermions are highly **hierarchical**. Also, quark mixing elements.
- The masses and mixings are the **incalculable** parameters of the theory.
- Masses of different generations have certain **correlations**.



Radiative mass generation

- Mass generation through quantum corrections (self-energy corrections). At leading order only third gen fermions are taken massive.



- Here Yukawa couplings can be chosen $\mathcal{O}(1)$
- loop supression $\propto \dfrac{1}{16\pi^2}$: Intergeneration Mass Hierarchy
- Masses become computable parameters.

Balakrishna P.R.L(*1988*) and few more, Dobrescu etal JHEP (2008) , Weinberg P.R.D (2020), …

Steps:

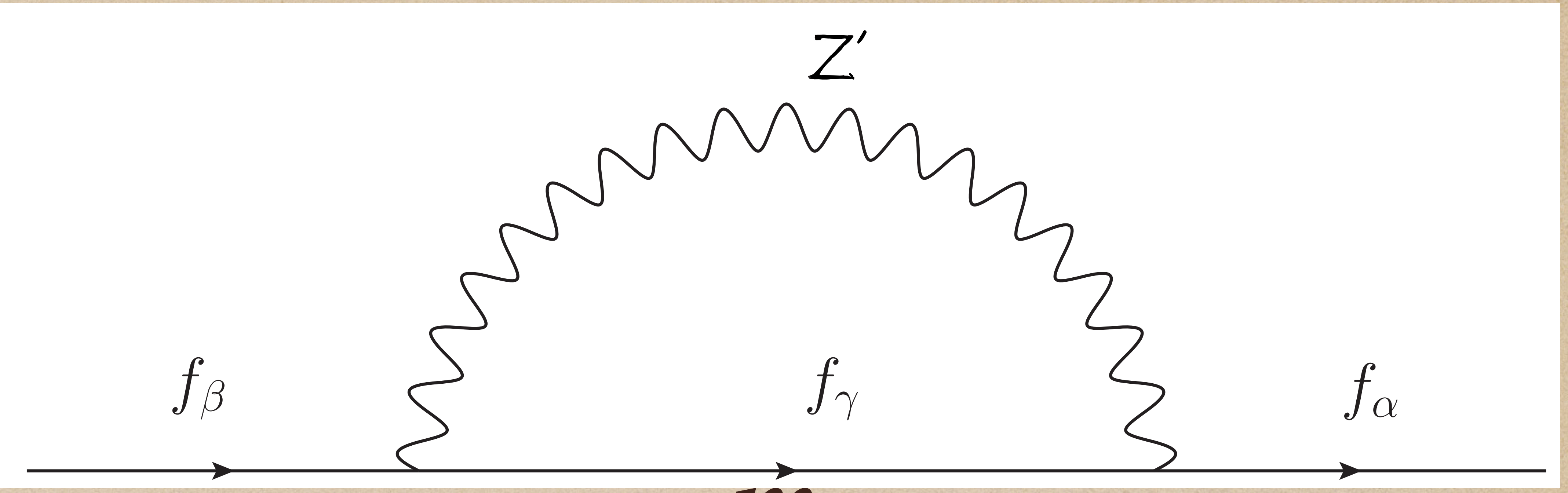
1. Forbid tree level masses for lighter fermions by imposing new symmetries and new fields.

2. Postulate Flavour changing couplings to induce loop masses.

3. Check cancellations of divergences for loop masses.

   (getting infinities doesn't mean non-renormalizability)

Radiative Models with Z' : toy model



$m_{3,4}$

- Tree level Rank 1 mass matrices

$$\mathscr{L}_m \supset \mu_{Li}\,\bar{f}_{Li}F_R + \mu_{Ri}\,\bar{F}_L f_{Ri} + m_F\,\bar{F}_L F_R + h.c$$

$$\mathscr{M} = \begin{pmatrix} 0_{3\times3} & \mu_L \\ \mu_R & M_F \end{pmatrix} \quad \Longrightarrow \quad M^{(0)}_{ij} = -\frac{\mu_{Li}\mu_{Rj}}{M_F};$$

- FCNCs are induced through

$$\mathscr{L}_g \supset g' Z'_\mu\,(q_{Li}\,\bar{f}_{Li}\gamma^\mu f_{Li} + q_{Ri}\,\bar{f}_{Ri}\gamma^\mu f_{Ri}) \quad \longrightarrow \quad q_{L,R} \longrightarrow Q^{(0)}_{L,R} = U^{(0)\dagger}_{L,R}\,q_{L,R}\,U^{(0)}_{L,R}$$

- Finite loop masses

$$(\delta M)_{ij} = \frac{g'^2}{4\pi^2}\,q_{Li}\,M^{(0)}_{ij}q_{Rj}\left(b_0[M^2_{Z'}, m_3^2] - b_0[M^2_{Z'}, m_F^2]\right)$$

$$M^{(1)}_{ij} = M^{(0)}_{ij} + \delta M_{ij} = M^{(0)}_{ij}(1 + C\,q_{Li}q_{Ri}),$$

Doesn't induce first generation masses

- All SM fermion masses are induced by

$$M^{(1)}_{ij} = M^{(0)}_{ij} + \delta M_{1ij} + \delta M_{2ij}$$

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FCNC constraints

$$j^\mu_{Z'} = g' \sum_{f=u,d,e} \left(\left(X^f_L\right)_{ij} \bar{f}_{Li}\gamma^\mu f_{Lj} + \left(X^f_R\right)_{ij} \bar{f}_{Ri}\gamma^\mu f_{Rj} \right),$$

With $X^f_{L,R} = U^{f\dagger}_{L,R}\, q_{L,R}\, U^f_{L,R}$,

- **Quark flavour violations**

$$\mathcal{H}^{\rm eff}_M = \sum_{i=1}^{5} C^M_i Q^M_i + \sum_{i=1}^{3} \tilde{C}^M_i \tilde{Q}^M_i$$

$$Q^{q_iq_j}_1 = \bar{q}^\alpha_{jL}\gamma_\mu q^\alpha_{iL}\bar{q}^\beta_{jL}\gamma^\mu q^\beta_{iL}\,,$$
$$Q^{q_iq_j}_2 = \bar{q}^\alpha_{jR}q^\alpha_{iL}\bar{q}^\beta_{jR}q^\beta_{iL}\,,$$
$$Q^{q_iq_j}_3 = \bar{q}^\alpha_{jR}q^\beta_{iL}\bar{q}^\beta_{jR}q^\alpha_{iL}\,,$$
$$Q^{q_iq_j}_4 = \bar{q}^\alpha_{jR}q^\alpha_{iL}\bar{q}^\beta_{jL}q^\beta_{iR}\,,$$
$$Q^{q_iq_j}_5 = \bar{q}^\alpha_{jR}q^\beta_{iL}\bar{q}^\beta_{jL}q^\alpha_{iR}\,.$$

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$K^0 - \bar{K}^0$ mixing:

$$C^1_K = \frac{g'^2}{M^2_{Z'}}\left[\left(X^d_L\right)_{12}\right]^2, \qquad \tilde{C}^1_K = \frac{g'^2}{M^2_{Z'}}\left[\left(X^d_R\right)_{12}\right]^2,$$

$$C^5_K = -4\frac{g'^2}{M^2_{Z'}}\left(X^d_L\right)_{12}\left(X^d_R\right)_{12}$$

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For $\mathcal{O}(1)$ **couplings:** $M_{Z'} \geq 10^7 - 10^8 GeV$

Observables	Experimental limit	Observables	Experimental limit
$\mathrm{Re}C^1_K$	$[-9.6, 9.6] \times 10^{-13}$	$\lvert C^1_D\rvert$	$< 7.2 \times 10^{-13}$
$\mathrm{Re}\tilde{C}^1_K$	$[-9.6, 9.6] \times 10^{-13}$	$\lvert \tilde{C}^1_D\rvert$	$< 7.2 \times 10^{-13}$
$\mathrm{Re}C^4_K$	$[-3.6, 3.6] \times 10^{-15}$	$\lvert C^4_D\rvert$	$< 4.8 \times 10^{-14}$
$\mathrm{Re}C^5_K$	$[-1.0, 1.0] \times 10^{-14}$	$\lvert C^5_D\rvert$	$< 4.8 \times 10^{-13}$
$\lvert C^1_{B_d}\rvert$	$< 2.3 \times 10^{-11}$	$\lvert C^1_{B_s}\rvert$	$< 1.1 \times 10^{-9}$
$\lvert \tilde{C}^1_{B_d}\rvert$	$< 2.3 \times 10^{-11}$	$\lvert \tilde{C}^1_{B_s}\rvert$	$< 1.1 \times 10^{-9}$
$\lvert C^4_{B_d}\rvert$	$< 2.1 \times 10^{-13}$	$\lvert C^4_{B_s}\rvert$	$< 1.6 \times 10^{-11}$
$\lvert C^5_{B_d}\rvert$	$< 6.0 \times 10^{-13}$	$\lvert C^5_{B_s}\rvert$	$< 4.5 \times 10^{-11}$

Flavour observables	Experimental limit
$\mathrm{BR}[\mu \to e]$	$< 7.0 \times 10^{-13}$
$\mathrm{BR}[\mu \to 3e]$	$< 1.0 \times 10^{-12}$
$\mathrm{BR}[\tau \to 3\mu]$	$< 2.1 \times 10^{-8}$
$\mathrm{BR}[\tau \to 3e]$	$< 2.7 \times 10^{-8}$
$\mathrm{BR}[\mu \to e\gamma]$	$< 4.2 \times 10^{-13}$
$\mathrm{BR}[\tau \to \mu\gamma]$	$< 4.4 \times 10^{-8}$
$\mathrm{BR}[\tau \to e\gamma]$	$< 3.3 \times 10^{-8}$

Symmetry deconstruction: Optimising flavour violations

- In the massless limit, Lagrangian will have a global $U(3)_L \times U(3)_R$ symmetry

- At tree level:

Mass Lagrangian

$$\mathscr{L}_m \supset m_3\, \bar{f}_{L3} f_{R3} + m_4\, \bar{f}_{L4} f_{R4} + h.c,$$

$$\mathscr{L}_m^{(0)} : U(2)_L \times U(2)_R$$

Gauge Lagrangian

$$Q_{L,R}^{(0)} = U_{L,R}^{(0)\dagger}\, q_{L,R}\, U_{L,R}^{(0)}$$

If $Q_{L,R}^{(0)} \neq \mathrm{Diag}(q, q, q')$

$\mathscr{L}_{Z'}^{(0)}$ doesn't respect $U(2)_L \times U(2)_R$

- At 1-loop level:

$$M_{ij}^{(1)} = M_{ij}^{(0)} + \delta M_{ij} = M_{ij}^{(0)}(1 + C\, q_{Li} q_{Ri}),$$

$$\mathscr{L}_m^{(1)} : U(1)_L \times U(1)_R$$

For $(Q_{L,R}^{(1)})_{12}, (Q_{L,R}^{(1)})_{13} \neq 0$ Then

$\mathscr{L}_{Z'}^{(1)}$ breaks $U(1)_L \times U(1)_R$

- At 2-loop level:

$$U(1)_L \times U(1)_R \xrightarrow{\text{at 2-Loop}} U(1)_{FN}$$

Choice of gauge charges

$$\left(Q_L^{(1)}\right)_{12} = \frac{(q_{L2}-q_{L1})(q_{L3}-q_{L1})}{\sqrt{N}\,(q_{L3}-q_{L2})}\left(\frac{\mu_{L1}}{\mu_{L3}}\left(U_L^{(1)}\right)_{32} - \frac{\mu_{L1}}{\mu_{L2}}\left(U_L^{(1)}\right)_{22}\right) \qquad \left(Q_L^{(1)}\right)_{13} = \frac{(q_{L2}-q_{L1})(q_{L3}-q_{L1})}{\sqrt{N}\,(q_{L3}-q_{L2})}\left(\frac{\mu_{L1}}{\mu_{L3}}\left(U_L^{(1)}\right)_{33} - \frac{\mu_{L1}}{\mu_{L2}}\left(U_L^{(1)}\right)_{23}\right)$$

$$\left(Q_L^{(1)}\right)_{23} = (q_{L3}-q_{L2})\left(U_L^{(1)}\right)^*_{32}\left(U_L^{(1)}\right)_{33} - (q_{L2}-q_{L1})\left(U_L^{(1)}\right)^*_{12}\left(U_L^{(1)}\right)_{13}\,.$$

- Suppressed Q_{12} and first gen. masses can be obtained if we choose

$$q_L = q_R = \begin{pmatrix} 1-\epsilon & 0 & 0 \\ 0 & 1+\epsilon & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$\mathscr{L}_{Z'}^{(0)}:\ U(1)_L \times U(1)_R$

$\mathscr{L}_{Z'}^{(1)}$ doesn't respect $U(1)_L \times U(1)_R$

- $\epsilon = 0$ doesn't generate first generation masses and doesn't contribute to flavour violations involving first two family fermions.

A $U(1)_F$ Model

- Two loop corrected mass matrix

$$\left(M_f^{(2)}\right)_{ij} = \left(M_f^{(1)}\right)_{ij} + \left(\delta\mathscr{M}^{(1)}\right)_{ij}$$

$$\left(M_f^{(2)}\right)_{ij} = \left(M_f^{(0)}\right)_{ij}\left(1 + \frac{g'^2}{4\pi^2} q_{Li}\, q_{Rj}\, (b_0[M_{Z'}, m_{f3}^{(1)}] - b_0[M_{Z'}, m_F])\right)$$
$$+ \left(\delta M_f^{(0)}\right)_{ij}\left(1 + \frac{g_X^2}{4\pi^2} q_{Li}\, q_{Rj}\, b_0[M_{Z'}, m_{f3}^{(1)}]\right)$$
$$+ \frac{g'^2}{4\pi^2} q_{Li}\, q_{Rj}\, (U_{fL}^{(1)})_{i2} (U_{fR}^{(1)})^*_{j2}\, m_{f2}^{(1)}\, (b_0[M_X, m_{f2}^{(1)}] - b_0[M_X, m_{f3}^{(1)}]),$$

With

$$\left(M_f^{(0)}\right)_{ij} = -\frac{\mu_{fi}\mu'_{fj}}{M_F},$$

$$\mu_{fi} = y_{fi}\langle H_{fi}\rangle, \qquad \mu'_{fi} = y'_{fi}\langle \eta_i^*\rangle,$$

- 25 real parameters

Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	$U(1)_F$
Q_{Li}	$(3, 2, \frac{1}{6})$	$(1-\epsilon, 1+\epsilon, -2)$
u_{Ri}	$(3, 1, \frac{2}{3})$	$(1-\epsilon, 1+\epsilon, -2)$
d_{Ri}	$(3, 1, -\frac{1}{3})$	$(1-\epsilon, 1+\epsilon, -2)$
L_{Li}	$(1, 2, -\frac{1}{2})$	$(1-\epsilon, 1+\epsilon, -2)$
e_{Ri}	$(1, 1, -1)$	$(1-\epsilon, 1+\epsilon, -2)$
ν_{Ri}	$(1, 1, 0)$	$(1-\epsilon, 1+\epsilon, -2)$
$U_{L,R}$	$(3, 1, \frac{2}{3})$	0
$D_{L,R}$	$(3, 1, -\frac{1}{3})$	0
$E_{L,R}$	$(1, 1, -1)$	0
H_{ui}	$(1, 2, -\frac{1}{2})$	$(1-\epsilon, 1+\epsilon, -2)$
H_{di}	$(1, 2, \frac{1}{2})$	$(1-\epsilon, 1+\epsilon, -2)$
η_i	$(1, 1, 0)$	$(1-\epsilon, 1+\epsilon, -2)$

Numerical fitting

$$\chi^2 = \sum_i \left(\frac{O^i_{\rm th} - O^i_{\rm exp}}{\sigma_i} \right)^2$$

$i = 1,\ldots,13$

(13 observables)

O^i_{th}: Theoretical values

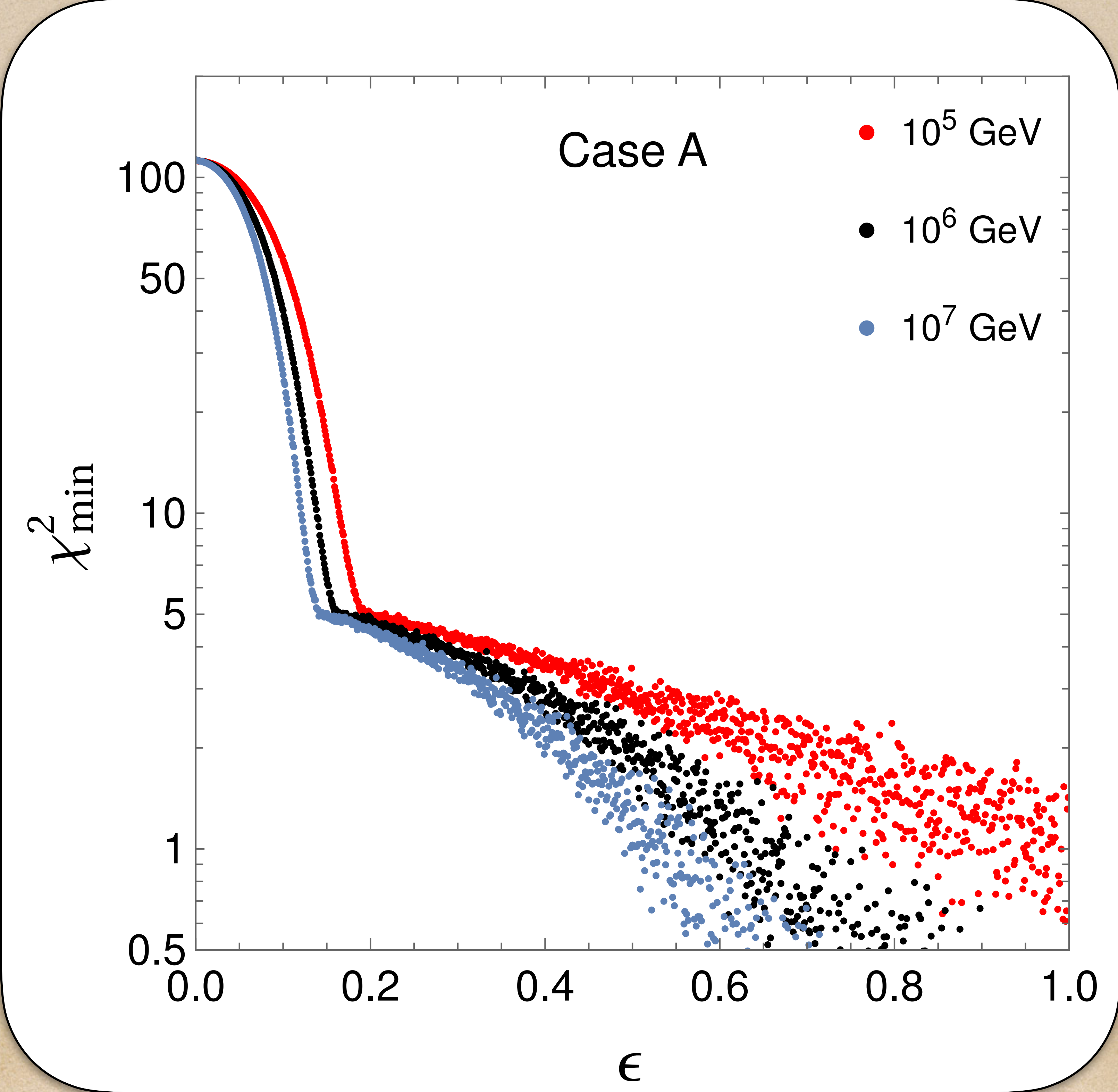
O^i_{th}: Experimental values

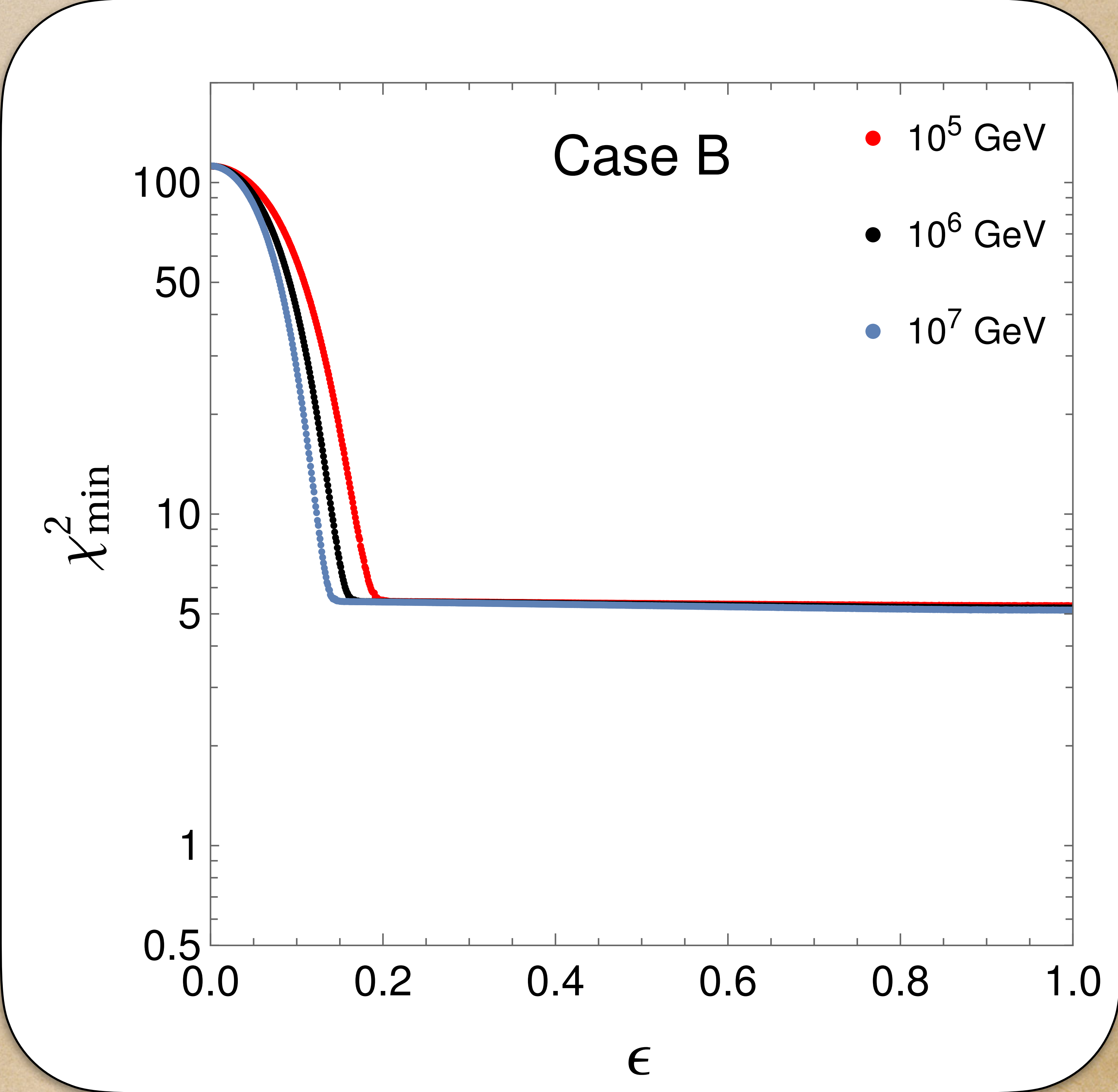
σ^i: Errors of i'th obs.

We obtain solutions for two cases:

1. Case A : Ordered μs
$|\mu_{f1}| < |\mu_{f2}| < |\mu_{f3}|,$
$|\mu'_{f1}| < |\mu'_{f2}| < |\mu'_{f3}|.$

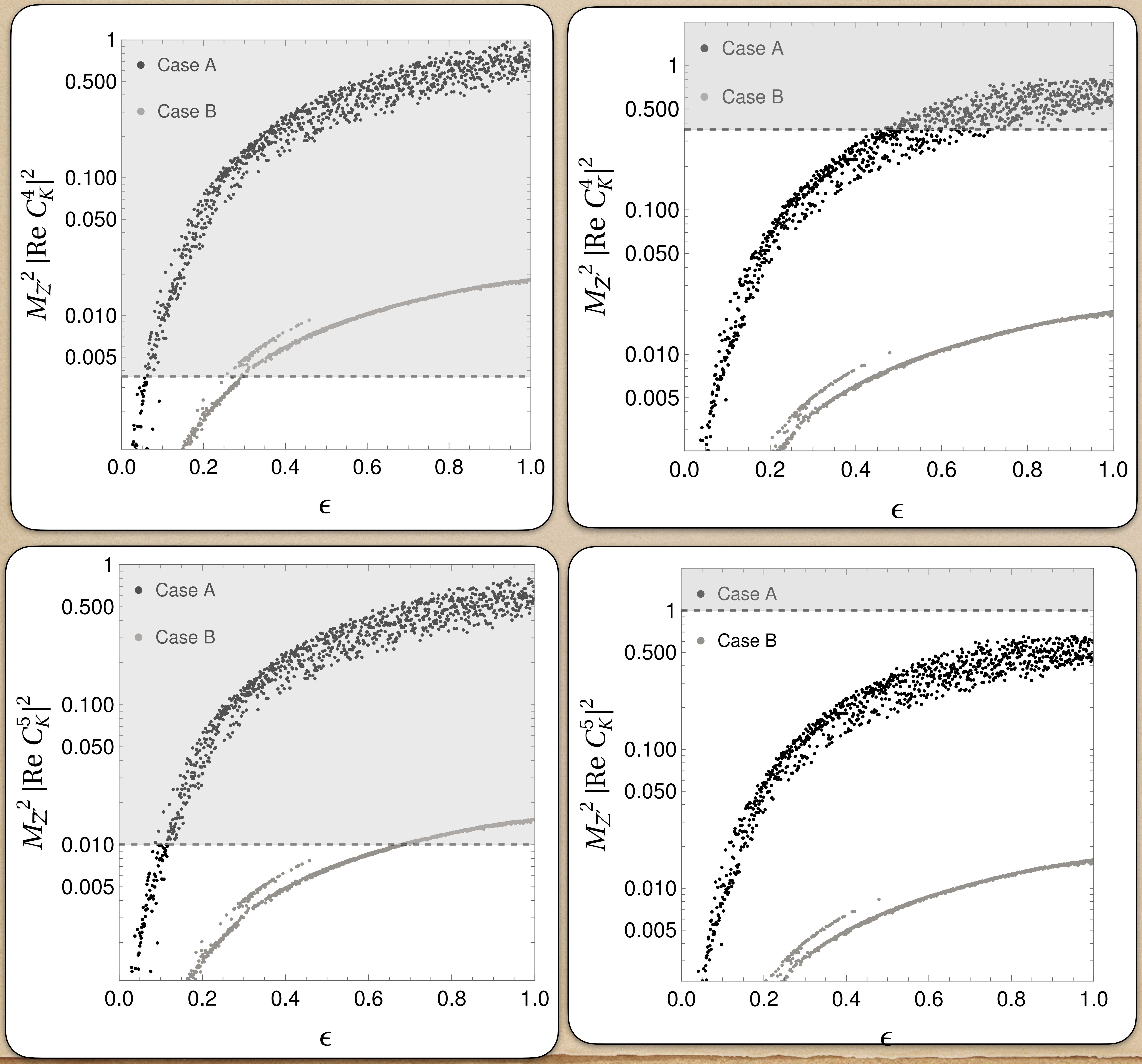
2. Case B : Strongly Ordered μs
$\frac{\mu_{d1}}{\mu_{d2}}, \frac{\mu'_{d1}}{\mu'_{d2}} < 0.1$ and Case A



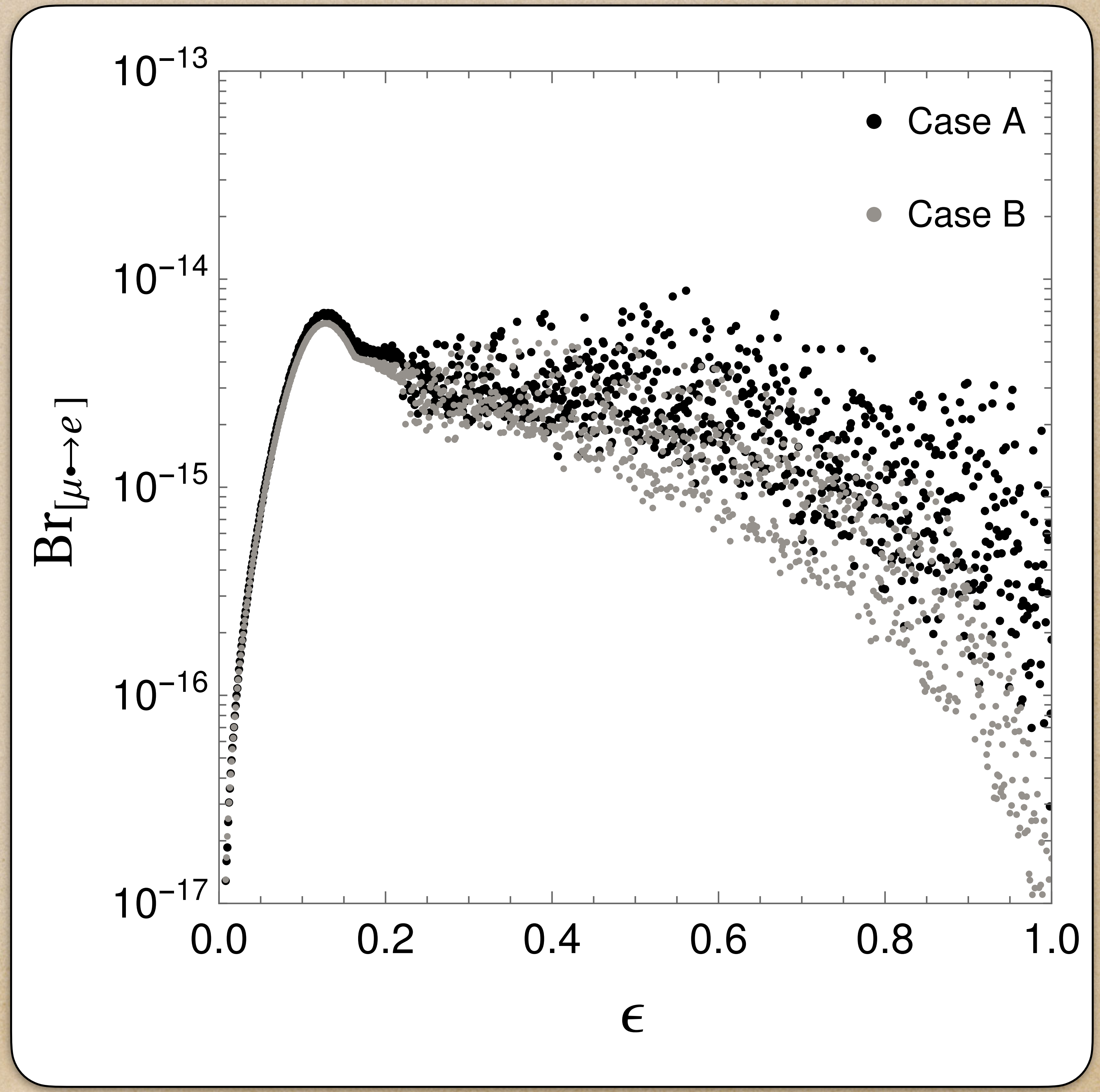


Phenomenological analysis

$M_{Z'} = 10^6 GeV$ $M_{Z'} = 10^7 GeV$



- For LFV process, stringent constraints are given by $\mu \to e$ conversion in nuclei (SINDRUM II) and $\mu \to e\gamma$.



Summary

- Radiative mechanism explains the origin of hierarchy as well as makes masses computable parameters (partially).

- Two important improvements compared to our previous work.

    1. First gen masses are generated at two loop level.

    2. A single U(1) can incorporate the mechanism.

- Flavour deconstruction analysis predicts optimum flavour violations. Our model predicts $M_{Z'} = 10^6$ GeV or higher.

Thank You