CMB Constraints on Natural Inflation with Gauge Field Production

Khursid Alam

Supervisor: Prof. Koushik Dutta

Talk based on K. Alam, K. Dutta, an Nur jaman, arXiv: 2405.10155

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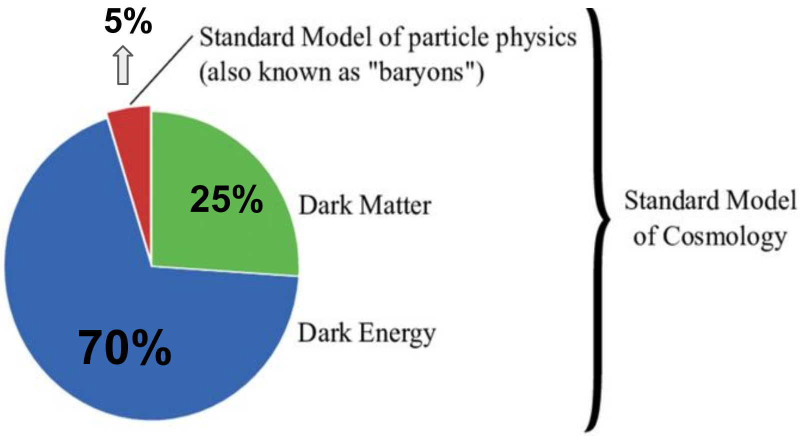
IISER KOLKATA

Outline

- ☆ Motivation of Inflation
- ☆ Background dynamics of the inflaton field in the presence of gauge fields
- ☆ Scalar and tensor power spectrum in the presence of the gauge fields
- ☆ Calculation of spectral index and tensor to scalar ratio and compare with observation
- ☆ Conclusion

Problem in Standard cosmology



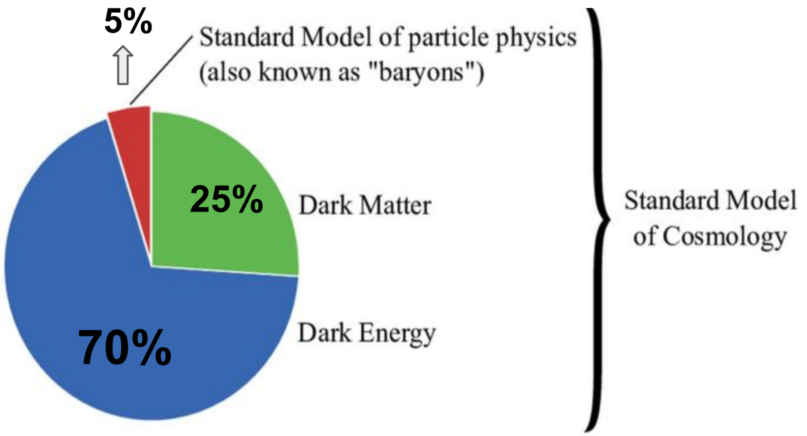
- ☆ Homogeneity problem
- ☆ Flatness problem
- ☆ Root of the CMB fluctuation

$$\frac{\Delta T}{T} = 10^{-5}$$

T is background temperature
T= 2.7 K

- ☆ Origin of the primordial black holes formation. We do not explain it in my talk.
- ☆ etc…

Problem in Standard cosmology



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Background CMB temperature

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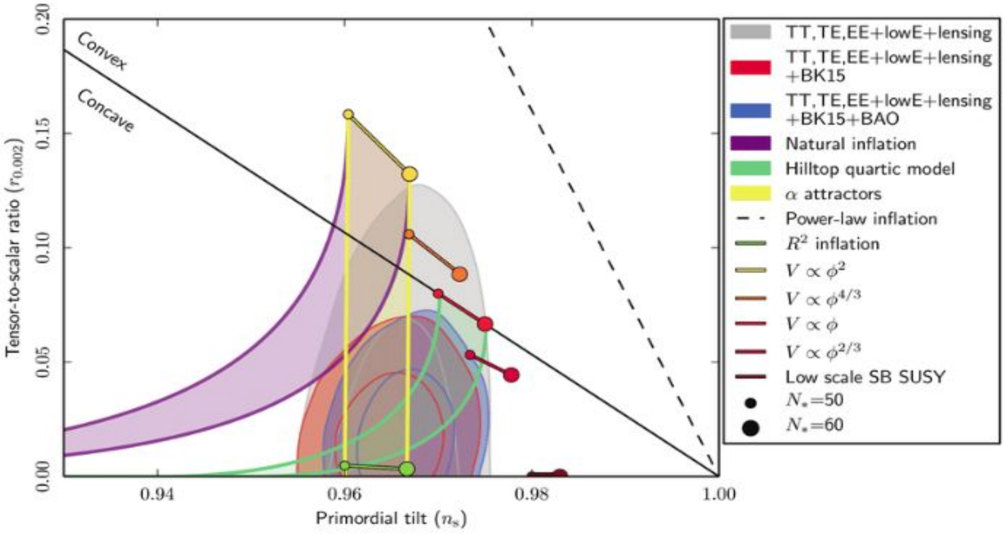
Introducing inflation can resolve all the issues previously noted.

Predictions of n_s and r based on various observations and theoretical models



Planck Collaboration, arXiv:1807.06211

Figure: Plot represents the predictions of n_s and r based on various observation and theoretical models

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right]$$

⇒ Natural inflation model

Note: Natural inflation model is under tension for $f \lesssim 1 m_{\rm pl}$ where $m_{\rm pl}$ is the planck mass

Action of the inflaton, the gauge field, and their interaction

The action for a pseudo-scalar inflaton ϕ, coupled to a massless Abelian gauge field A_μ,

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{R}{2} - \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{4f} \phi F^{\mu\nu} \tilde{F}_{\mu\nu} \right]$$

Einstein-Hilbert action

Kinetic and potential term of the inflation field

Kinetic term of gauge field

Interaction term between gauge and inflation field

Inflation potential

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f} \right) \right]$$

Note: Natural inflation model is under tension but after introducing the gauge field, would it be survive the natural inflation model.

M. M. Anber and L. Sorbo, arXiv:astro-ph/0606534, and arXiv:0908.4089 , W. D. Garretson, G. B. Field, and S. M. Carroll, arXiv:hep-ph/9209238

Dynamical equations for the fields

The action for a pseudo-scalar inflaton ϕ, coupled to a massless Abelian gauge field A_μ,

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{R}{2} - \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{4f} \phi F^{\mu\nu} \tilde{F}_{\mu\nu} \right]$$

After varying this action

$$\phi'' + 2\mathcal{H}\phi' - \nabla^2\phi + a^2 \frac{\partial V}{\partial \phi} = a^2 \frac{\alpha}{f} \mathbf{E} \cdot \mathbf{B} \ ,$$

$$3\,\mathcal{H}^2 = \left[\frac{1}{2}\phi'^2 + \frac{1}{2}(\nabla\phi)^2 + a^2\, V(\phi) \ + \ \frac{a^2}{2}\ (\mathbf{E}^2 + \mathbf{B}^2) \right] ,$$

$$\mathbf{A}'' - \nabla^2 \mathbf{A} + \nabla(\nabla . \mathbf{A}) = \frac{\alpha}{f} \phi' \, (\nabla \times \mathbf{A}) - \frac{\alpha}{f} \, (\nabla \phi) \times \mathbf{A}',$$

$$(\nabla . \mathbf{A})' = \frac{\alpha}{f} \, (\nabla \phi) \, . \, (\nabla \times \mathbf{A}) \ ,$$

Set of equations for the fields

$$\mathbf{E} = -\frac{1}{a^2} \mathbf{A}', \qquad \mathbf{B} = \frac{1}{a^2} \boldsymbol{\nabla} \times \mathbf{A}$$

Electric and magnetic field in term of gauge field

M. M. Anber and L. Sorbo, arXiv:astro-ph/0606534, and arXiv:0908.4089 , W. D. Garretson, G. B. Field, and S. M. Carroll, arXiv:hep-ph/9209238

Gauge field equation in k-space

Assumption: $\nabla\phi = 0$ and $\nabla . A = 0$

$$\mathbf{A}'' - \nabla^2 \mathbf{A} - \frac{\alpha}{f}\phi_0' \left(\nabla \times \mathbf{A}\right) = 0$$

Dynamical equation of the gauge field in k-space is,

$$\left[\frac{\partial^2}{\partial\tau^2} + (k^2 \mp 2\, a\, H\, \xi\, k)\right] A_k^{\pm}(\tau) = 0,$$

where $\pm$ two helicity of gauge field

$$\Downarrow$$

$$\xi = \frac{\alpha\, \dot{\phi}_0}{2\, f\, H}.$$

Mode which satisfies the condition $k/a\,H < 2|\xi|$ has an exponential solution and the analytical solution of the above equation is,

$$A_k^{+} \simeq \frac{1}{\sqrt{2\,k}} \left(\frac{k}{2\,\xi\, a\, H}\right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}}$$

M. M. Anber and L. Sorbo, arXiv:0908.4089; N. Barnaby and M. Peloso, arXiv:1011.1500; N. Barnaby, R. Namba, and M. Peloso, arXiv:1102.4333

Background dynamics of the inflation field in the presence of the gauge field

Background dynamic of inflaton field

Background dynamics of the inflaton field is

Source term which depend on the gauge field production

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + \frac{\partial V}{\partial \phi_0} = \frac{\alpha}{f}\langle \boldsymbol{E}.\boldsymbol{B}\rangle = -\frac{\alpha}{4\, f\, \pi^2\, a^4}\int dk\, k^3\, \frac{d}{d\tau}\left\{|A_k^+|^2 - |A_k^-|\right\}$$

$$3H^2 = \frac{1}{2}\dot{\phi}_0^2 + V(\phi_0) + \frac{1}{2}\langle \boldsymbol{E}^2 + \boldsymbol{B}^2\rangle = \frac{1}{2}\dot{\phi}_0^2 + V(\phi_0) + \frac{1}{8\,\pi^2\, a^4}\int dk\, k^2 \sum_{\lambda=\pm} |A_k'^{\lambda}|^2 + k^2|A_k^{\lambda}|^2$$

Volume average of the gauge field energy density

$$\left[\frac{\partial^2}{\partial \tau^2} + (k^2 \mp 2\, a\, H\, \xi\, k)\right] A_k^{\pm}(\tau) = 0,$$

Time evolution of ξ and the source term

$$\xi = \frac{\alpha\,\dot{\phi}_0}{2\,f\,H}.$$ → Definition of ξ

Mode which has an exponential growth which has to satisfy the condition $k/a\,H < 2|\xi|$

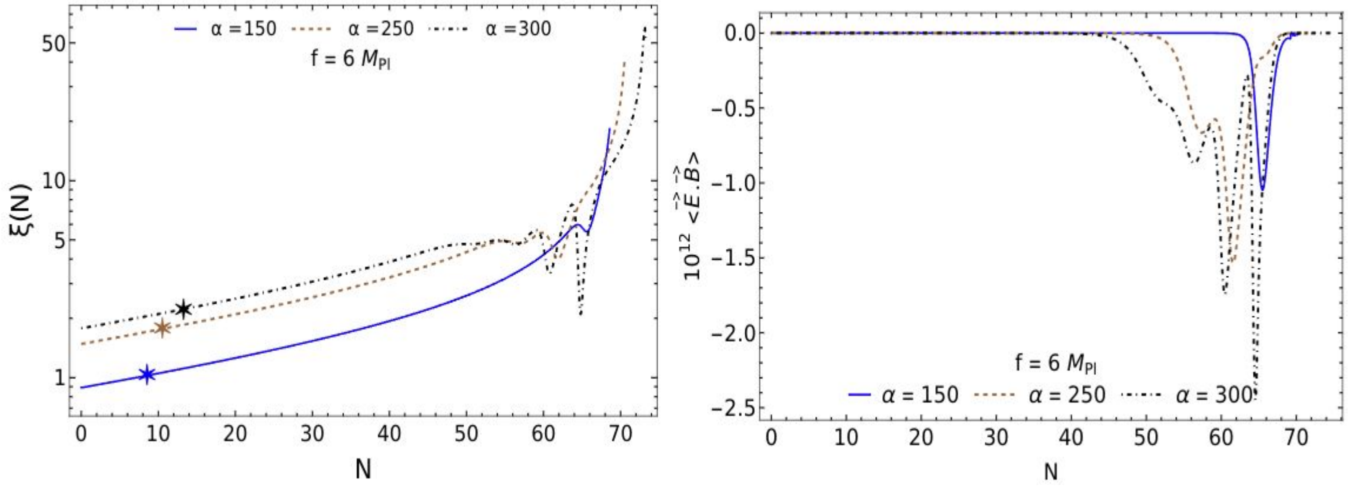


Figure: The left panel plot is time evolution of ξ and the right panel figure is time evolution of source term

Note: As time goes not more number of gauge fields will be excited

Time evolution of the slow-roll parameter and Inflaton dynamics

$$\epsilon(N) = -\frac{\dot{H}}{H^2}$$ → Definition of the slow-roll parameter

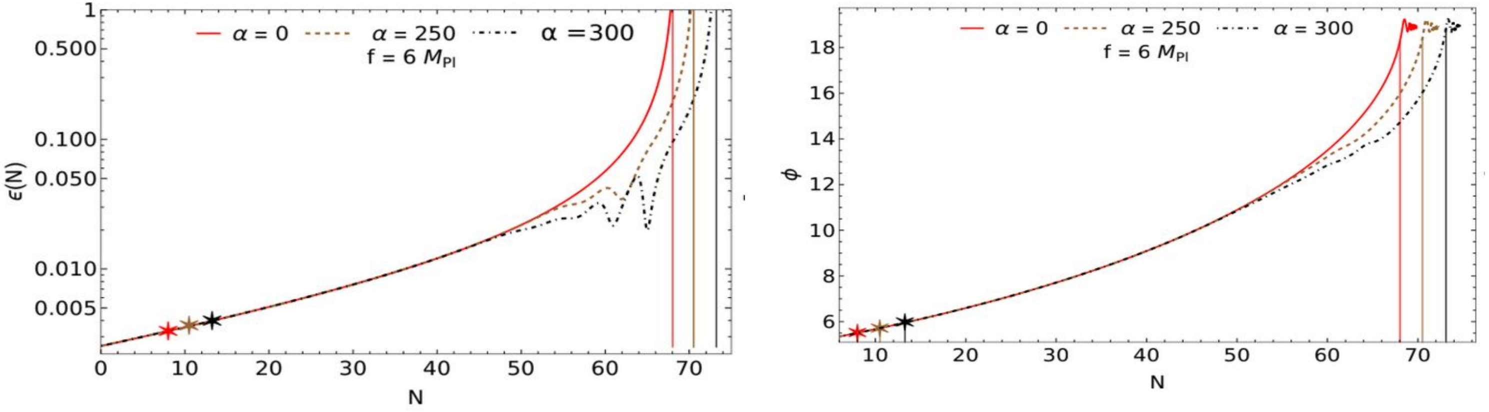


Figure: The left panel illustrates the time evolution of the first slow-roll parameter, while the right panel depicts the evolution of the Hubble parameter.

In the plot, the stars ('⋆') of different colours correspond to 60 e-foldings before the end of inflation for different choices of α

Note: The duration of inflation is extended in the presence of a gauge field, and for different choices of parameter α (for a given value of f), the CMB scales ($k = 0.05 Mpc^{-1}$) probe different parts of the axion potential.

Study the perturbation dynamics of the inflation field in the presence of the gauge field

Plot of the scalar power spectrum in the presence of the gauge field

$$P_\zeta(k) = P_\zeta(k)_{\text{vac}} + P_\zeta(k)_{\text{source}} = \underbrace{\left(\frac{H^2}{2\pi\dot{\phi}_0}\right)^2}_{\text{vaccum}} + \underbrace{\left(\frac{\alpha}{f}\frac{\langle \mathbf{E}.\mathbf{B}\rangle}{3\beta H\dot{\phi}_0}\right)^2}_{\text{sourced}}$$

where $\beta \equiv 1 - 2\pi\xi\frac{\alpha}{f}\frac{\langle \mathbf{E}.\mathbf{B}\rangle}{3H\dot{\phi}_0}$

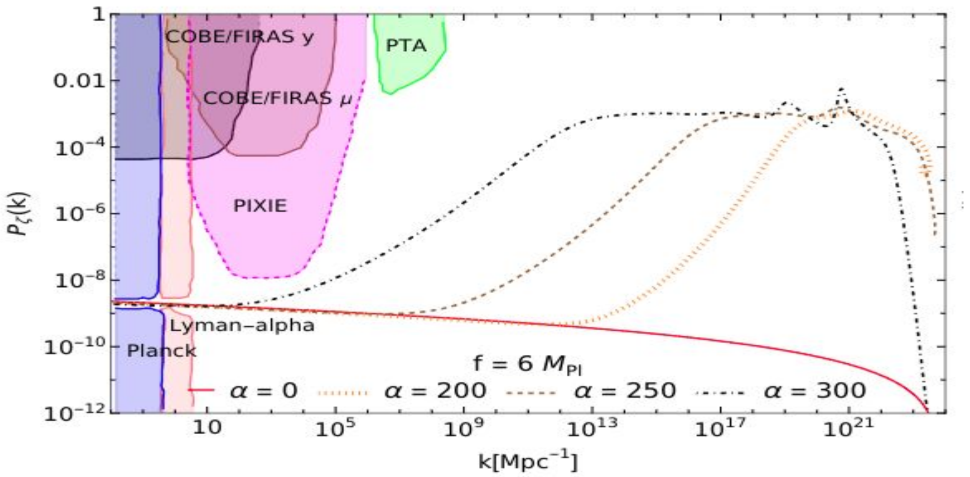


Figure: The amplitude of the scalar power spectrum is plotted against comoving wavenumbers for different values of the coupling constant α and f = 6. Several colored contours show existing (continuous) and projected future (dotted) constraints.

M. M. Anber and L. Sorbo, arXiv:0908.4089; N. Barnaby, E. Pajer, and M. Peloso, arXiv:1110.3327 ; A. Linde, S. Mooij, and E. Pajer, arXiv:1212.1693

Plot of the tensor power spectrum in the presence of the gauge field

$$P_h^{\pm} = \frac{H^2}{\pi^2}\left(\frac{k}{k_0}\right)^{n_T}\left[1 + 2\,H^2 f_h^{\pm}(\xi) e^{4\pi\xi}\right]$$

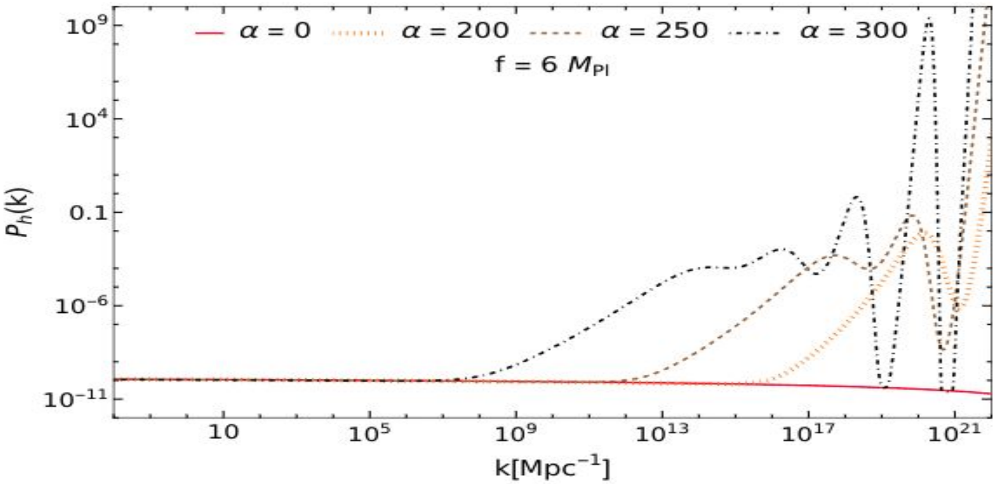


Figure: The amplitude of the scalar power spectrum is plotted against comoving wavenumbers for different values of the coupling constant α and f= 6.

N. Barnaby, R. Namba, and M. Peloso, arXiv:1102.4333; L. Sorbo, arXiv:1101.1525; N. Barnaby, E. Pajer, and M. Peloso, arXiv:1110.3327 .

Calculation of spectral index and tensor to scalar ratio

As we know the scalar and tensor power spectrum, we can calculate the inflationary observable, namely scalar spectral index n_s and tensor to scalar ratio r.

Definition of scalar spectral index n_s and tensor to scalar ratio,

$$n_s - 1 \equiv \left.\frac{d\ln P_\zeta(k)}{d\ln k}\right|_{k_*} \quad \text{and} \quad r \equiv \left.\frac{P_h}{P_\zeta}\right|_{k_*} = \left.\frac{P_h^+ + P_h^-}{P_\zeta}\right|_{k_*}$$

Plot of the spectral index and tensor to scalar ratio

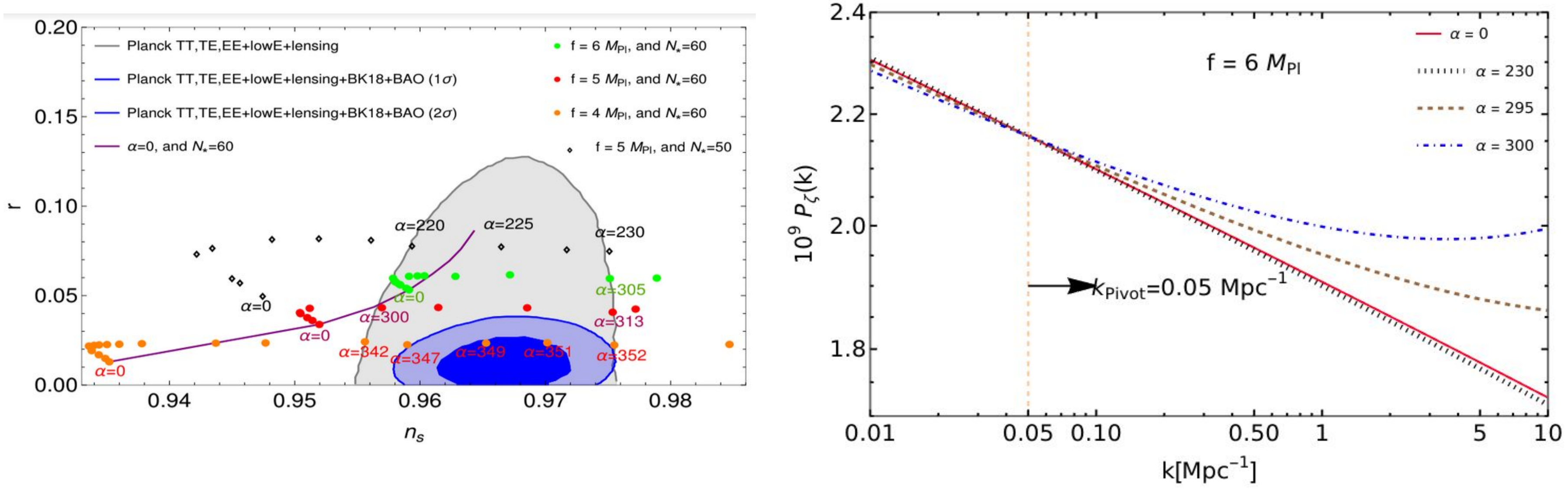


Figure: The left panel figure represent the scalar spectral index vs tensor to scalar ratio for various value of the coupling constant α , while the right panel plot illustrates the scalar power spectrum around the CMB scale

Note: Point which was outside the contour without gauge fields that point is again inside the contour for certain range of α

Running of the spectral index and calculation of non-gaussianity

In this model, the non-gaussianity is of the equilateral type, and it can be calculated by using the formula

$$f_{NL}^{equil} = \left. \frac{f_3(\xi)\,(P_\zeta(k)_{vac})^3\; e^{6\pi\xi}}{(P_\zeta(k))^2} \right|_{k=k_*}$$

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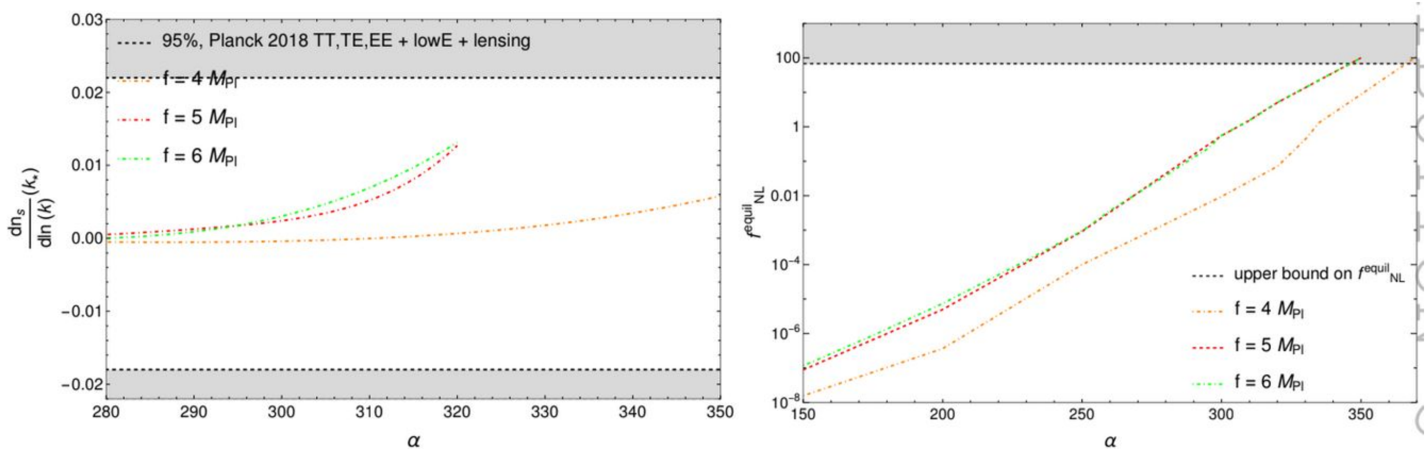

Figure: the left panel figure represents the running of the spectral index and the right panel figure represents the non-gaussianity of the system.

Note: the range of coupling constant which is allowed in ns-r plane is also allowed for running of spectral index and non-gaussianity.

Conclusion

- Presence of the gauge field start to influence the inflaton field dynamics later stage of inflation.

- Duration of the inflation is going to be prolonged due to the presence of the gauge field.

- The extended duration of inflation helps alleviate the tension in the natural inflation model for a specific range of coupling constant.

Future Plan

- In this work, we have considered massless abelian gauge fields. We can do similar thing by considering the non abelian gauge field.
- We observe the growth of the scalar power spectrum during the later stages of inflation which can produce the primordial black holes.
- To get the scalar and tensor power spectrum, we have used the semi analytical expression. We have not done the full numeric. To do the full numeric, we have to learnt lattice. I am now working on it.

THANK YOU ....