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Dynamical system analysis of DBI scalar field cosmology in general symmetric teleparallel gravity.

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Topic Outlines:

Introduction:

In the year 1929, astronomer Edwin Hubble indicated the expanding behavior of the universe. He measured the distances and velocities of some galaxies and found a linear relation between them, which is known as Hubble's law.

In 1998, the observational studies on type Ia Supernovae done by the two scientific groups led by Riess and Perlmutter discovered that the universe is undergoing an accelerated expansion[a] [b].

[a] Riess, A.G. et al., "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant" *The Astronomical Journal*. 1998, 116:1009.

[b] Perlmutter, S. et al., "Measurement of Ω and Λ from 42 high red-shift supernovae" *The Astronomical Journal*. 1999, 517:565-585.

Scalar field:

Why scalar fields are important?

It is well known that the scalar fields are essential for cosmology as they can naturally explain inflation[a] in the very early universe, as well as late time acceleration via quintessence field [b].

Here we would try both focus on the scalar field which can be used both for early inflation and late time quintessence.

[a] Guth. A., "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems" *Physical Review D*. 1981, 23: 347–356.

[b] Ratra. B., Peebles. P. "Cosmological Consequences of a Rolling Homogeneous Scalar Field" *Physical Review D*. 1988, 37: 3406.

Scalar field:

Inflation:

Even with so much success the standard Big-Bang cosmology could not answer flatness problem, mono-pole problem, horizon problem. Inflation [a] successfully addresses the issue also gives falsifiable prediction like B-mode in polarization in CMB.

[a] Guth. A., "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems" *Physical Review D*. 1981, 23: 347–356.

Scalar field:

Quintessence:

As it turns out even though the cosmological constant solves the late-time acceleration problem the origin of Λ is not explained. As summing over zero modes gives an error of the order of 10^{-120}. This is known as a cosmological constant problem. Quintessence naturally addresses the problem by stating that like inflation a similar scalar field quintessence field [a],causes the decay of Λ explaining the tiny value of Λ.

[a] Ratra. B., Peebles. P. "Cosmological Consequences of a Rolling Homogeneous Scalar Field" *Physical Review D*. 1988, 37: 3406.

Scalar field:

Einstein-Hilbert action with scalar field:

It is well known that the Einstein-Hilbert action under scalar field potential in the presence of matter can be read as,

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\phi_\mu\phi^\mu - V(\phi)\right] + \mathcal{S}_m, \tag{1}$$

We will use the natural unit that is $\kappa^2 = 8\pi G = 1$. Also, ϕ denotes the scalar field.

Scalar field:

Effective ρ and P for scalar field:

One can find the pressure (P) and energy density (ρ) given by the following,

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \tag{2}$$

and

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi), \tag{3}$$

respectively.

Scalar field:

Friedman equation:

We can write the Friedman equation as follows,

$$3H^2 = \rho_m + \rho_\phi, \tag{4}$$

Deceleration equation:

The deceleration equation takes the form,

$$\dot{H} = -\frac{1}{2}(\rho_m + \dot{\phi}^2), \tag{5}$$

Conservation equation:

Also finally, the conservation equation becomes,

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{6}$$

Scalar field:

Background scalar field equation:

Given the scalar field Lagrangian, we can get the equation of motion(by using the Euler-Lagrange equation) given by the following,

$$\ddot{\phi} = -3H\dot{\phi} - \frac{dV}{d\phi}. \tag{7}$$

Power-Law potential

$$1 = \frac{\rho_m}{3H^2} + \frac{\dot{\phi}^2}{6H^2} + \frac{V}{3H^2}. \tag{8}$$

$$\lambda \equiv -\frac{V'(\phi)}{V(\phi)}, \tag{9}$$

$$f \equiv \frac{V''(\phi)}{V(\phi)} - \frac{V'(\phi)^2}{V(\phi)^2}, \tag{10}$$

$$V'(\phi) = -\lambda V(\phi), \tag{11}$$

$$V''(\phi) = \left(f + \lambda^2\right) V(\phi). \tag{12}$$

$$u = \frac{2\tan^{-1}(\lambda)}{\pi}, -1 < u < 1, \tag{13}$$

$$x^2 = \frac{\dot{\phi}^2}{6H^2}, \quad y^2 = \frac{V}{3H^2} \tag{14}$$

Equations on background

$$\frac{dx}{dN} = \begin{cases} \sqrt{\frac{3}{2}}y^2, & u = 1 \\ \sqrt{\frac{3}{2}}y^2 \tan\left(\frac{\pi u}{2}\right) + \frac{3}{2}x\left(x^2 - y^2 - 1\right), & -1 < u < 1 \\ -\sqrt{\frac{3}{2}}y^2, & u = -1 \end{cases}, \quad (15)$$

$$\frac{dy}{dN} = \begin{cases} -\sqrt{\frac{3}{2}}xy, & u = 1 \\ -\frac{1}{2}y\left(\sqrt{6}x\tan\left(\frac{\pi u}{2}\right) - 3x^2 + 3y^2 - 3\right), & -1 < u < 1 \\ \sqrt{\frac{3}{2}}xy, & u = -1 \end{cases}, \quad (16)$$

$$\frac{du}{dN} = -\frac{\sqrt{6}x(\cos(\pi u) - 1)}{\pi n}. \quad (17)$$

Plot

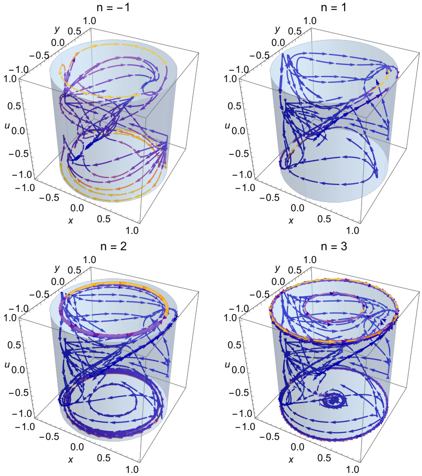

Figure 1: Compact 3D phase space of the system (15), (16), and (17), for $n = -1, 1, 2,$ and 3.

Literature Review cont.

Background metric:

We derived those formulas assuming that the FLRW metric gives the background. However, the background may get changed under scalar field perturbation. FLRW metric is the following;

$$ds^2 = -(cdt)^2 + a(t)^2[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)] \quad (18)$$

Metric perturbation decomposed in scalar,vector and 2 rank tensor:

We tackle such a change in geometry by writing down the perturbed metric by the following [Mukhanov, Physics Report,]

$$ds^2 = -(1+\alpha)dt^2 - 2a(t)(\beta,_i - S_i)dtdx^i$$
$$+ a^2(t)[(1+2\psi)\delta_{ij} + 2\partial_i\partial_j\gamma + 2\partial_{(i}F_{j)} + h_{ij}]dx^i dx^j, \quad (19)$$

Literature Review cont.

Meaning of ψ in form of Gauss–Codazzi equation:

where the inhomogeneous perturbation quantities α, β, ψ, γ, F_i, h_{ij} are functions of both t and $\bar{x}$(109). The quantity $\psi(t, \bar{x})$ is directly related to the 3-curvature of the spatial hypersurface,

$$^{(3)}R = -\frac{4}{a^2}\nabla^2\psi, \tag{20}$$

Literature Review cont.

Bardeen potential:

We note that to handle the metric perturbation, we somehow have to find a gauge invariant prescription to handle such a metric perturbation. Fortunately, Bardeen potential does that. Following [a] the Bardeen potentials are given by,

$$\Phi \equiv \alpha - \frac{d}{dt}[a(\beta + a\gamma)], \qquad \Psi \equiv -\psi + aH(\beta + a\dot{\gamma}), \tag{21}$$

We first note that for the single scalar field, both the potentials are the same.

[a] V. F. Mukhanov et al., *Physics Reports* **215**, 203-233 (1992).

Literature Review cont.

Equation for curvature perturbation:

We can find [a] Φ satisfies the following equation,

$$\Phi'' + 2\left(\mathcal{H} - \frac{\phi_0''}{\phi_0'}\right)\Phi' + 2\left(\mathcal{H}' - \mathcal{H}\frac{\phi_0''}{\phi_0'}\right)\phi - \mathcal{H}^{-2}\nabla^2\Phi = 0, \tag{22}$$

Where $\mathcal{H} = aH$, and $'$ denotes derivative with respect to the conformal time η given by

$$d\eta = \frac{dt}{a(t)}. \tag{23}$$

[a] V. F. Mukhanov et al., *Physics Reports* **215**, 203-233 (1992).

Curvature perturbation terms

Defining

$$Z = k^2(aH)^{-2}, \tag{24}$$

We also compactify the Z variable via,

$$\bar{Z} = \frac{Z}{1+Z} = \frac{k^2}{k^2+(aH)^2}, \quad Z = \frac{\bar{Z}}{1-\bar{Z}}. \tag{25}$$

When $\bar{Z} \to 1$, we have a singularity, so we change the "e folding time" from N to $\bar{N}$ through.
We use conformal time η instead of e-fold time $N = \ln a$. In making the transition from N to η, we use the relations

$$\frac{d}{dN} = \mathcal{H}^{-1}\frac{d}{d\eta}, \quad \frac{d^2}{dN^2} = \mathcal{H}^{-2}\frac{d^2}{d\eta^2} + q\mathcal{H}^{-1}\frac{d}{d\eta}. \tag{26}$$

$$\frac{d\bar{N}}{dN} = \frac{1}{1-\bar{Z}} = 1 + Z. \tag{27}$$

θ definition

In equation past we first note that Φ_k is generally complex (as it came from Fourier transformation) [1]. So, we write $\Phi_k = F_1 + iF_2$, where F_1 and F_2 are the real and imaginary parts of Φ_k, respectively. Moreover, the resulting equation has the structure

$$F'' + PF' + QF = 0, \tag{28}$$

that is the same for F_1 and F_2. Generically, we denote $F_i = r_i \cos\theta_i$ and $F_i' = r_i \sin\theta_i$ where $i = 1, 2$. So,

$$F' = F \tan\theta, \tag{29}$$

where $\frac{F'}{F} = \mathcal{Y}$ has a period of π.

$$\mathcal{Y}' = -\mathcal{Y}^2 - P\mathcal{Y} - Q, \tag{30}$$

or

$$\theta' = -\sin^2\theta - P\sin\theta\cos\theta - Q\cos^2\theta. \tag{31}$$

[1] Alho,Uggla, Wainwright,JCAP 09 (2019) 045, arXiv: 1904.02463

Short wavelength boundary (or sub-horizon boundary) ($k^2\mathcal{H}^{-2} \gg 1$)

$$\frac{dx}{d\bar{N}} = -\left(3x - \sqrt{\frac{3}{2}}\lambda\right)\left(1 - x^2\right)\left(1 - \bar{Z}\right), \tag{32a}$$

$$\frac{d\lambda}{d\bar{N}} = -\sqrt{6}xf\left(1 - \bar{Z}\right), \tag{32b}$$

$$\frac{d\bar{Z}}{d\bar{N}} = 2(3x^2 - 1)\bar{Z}\left(1 - \bar{Z}\right)^2. \tag{32c}$$

$$\frac{d\theta}{d\bar{N}} = -\left[\sin^2\theta + \left(7 - 3x^2 + \sqrt{6}\lambda\left(\frac{1 - x^2}{x}\right)\right)\sin\theta\cos\theta + \left(6\left(1 - x^2\right) + \sqrt{\frac{3}{2}}\lambda\left(\frac{1 - x^2}{x}\right)\right)\cos^2\theta\right]\left(1 - \bar{Z}\right) - \bar{Z} \tag{33}$$

5.12 and 5.19

Label	x	λ	$\bar{Z}$	θ	k_1	k_2	k_3	k_4	$a(t), H(t), \phi(t)$
$A_1(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$-\cos^{-1}(-\Delta_1)$	$\frac{1}{2}\left(\lambda^{*^2}-6\right)$	$\lambda^{*^2}-2$	$\Gamma_1+\left(8-\frac{3\lambda^{*^2}}{2}\right)\sin\left(2\sin^{-1}(\Delta_1)\right)$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$A_2(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$\cos^{-1}(\Delta_1)$	$\frac{1}{2}\left(\lambda^{*^2}-6\right)$	$\lambda^{*^2}-2$	$\Gamma_1+\left(8-\frac{3\lambda^{*^2}}{2}\right)\sin\left(2\cos^{-1}(\Delta_1)\right)$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$A_3(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$-\cos^{-1}(\Delta_1)$	$\frac{1}{2}\left(\lambda^{*^2}-6\right)$	$\lambda^{*^2}-2$	$\Gamma_1+\left(\frac{3\lambda^{*^2}}{2}-8\right)\sin\left(2\cos^{-1}(\Delta_1)\right)$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$A_4(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$\cos^{-1}(-\Delta_1)$	$\frac{1}{2}\left(\lambda^{*^2}-6\right)$	$\lambda^{*^2}-2$	$\Gamma_1+\left(\frac{3\lambda^{*^2}}{2}-8\right)\sin\left(2\sin^{-1}(\Delta_1)\right)$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$A_5(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$-\cos^{-1}(-\Delta_2)$	$\frac{1}{2}\left(\lambda^{*^2}-6\right)$	$\lambda^{*^2}-2$	$\Gamma_2+\left(8-\frac{3\lambda^{*^2}}{2}\right)\sin\left(2\sin^{-1}(\Delta_2)\right)$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$A_6(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$\cos^{-1}(\Delta_2)$	$\frac{1}{2}\left(\lambda^{*^2}-6\right)$	$\lambda^{*^2}-2$	$\Gamma_2+\left(8-\frac{3\lambda^{*^2}}{2}\right)\sin\left(2\cos^{-1}(\Delta_2)\right)$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$A_7(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$-\cos^{-1}(\Delta_2)$	$\frac{1}{2}\left(\lambda^{*^2}-6\right)$	$\lambda^{*^2}-2$	$\Gamma_2+\left(\frac{3\lambda^{*^2}}{2}-8\right)\sin\left(2\cos^{-1}(\Delta_2)\right)$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$A_8(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$\cos^{-1}(-\Delta_2)$	$\frac{1}{2}\left(\lambda^{*^2}-6\right)$	$\lambda^{*^2}-2$	$\Gamma_2+\left(\frac{3\lambda^{*^2}}{2}-8\right)\sin\left(2\sin^{-1}(\Delta_2)\right)$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$A_9(\lambda^*)$	-1	λ^*	0	0	-4	4	$\sqrt{6}\lambda^*+6$	$\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$A_{10}(\lambda^*)$	-1	λ^*	0	$-\pi$	-4	4	$\sqrt{6}\lambda^*+6$	$\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$A_{11}(\lambda^*)$	-1	λ^*	0	π	-4	4	$\sqrt{6}\lambda^*+6$	$\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$A_{12}(\lambda^*)$	-1	λ^*	0	$\sec^{-1}\left(-\sqrt{17}\right)$	4	4	$\sqrt{6}\lambda^*+6$	$\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$A_{13}(\lambda^*)$	-1	λ^*	0	$-\sec^{-1}\left(\sqrt{17}\right)$	4	4	$\sqrt{6}\lambda^*+6$	$\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$A_{14}(\lambda^*)$	1	λ^*	0	0	-4	4	$6-\sqrt{6}\lambda^*$	$-\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$A_{15}(\lambda^*)$	1	λ^*	0	$-\pi$	-4	4	$6-\sqrt{6}\lambda^*$	$-\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$A_{16}(\lambda^*)$	1	λ^*	0	π	-4	4	$6-\sqrt{6}\lambda^*$	$-\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$A_{17}(\lambda^*)$	1	λ^*	0	$\sec^{-1}\left(-\sqrt{17}\right)$	4	4	$6-\sqrt{6}\lambda^*$	$-\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$A_{18}(\lambda^*)$	1	λ^*	0	$-\sec^{-1}\left(\sqrt{17}\right)$	4	4	$6-\sqrt{6}\lambda^*$	$-\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
A_{19}	x_c	λ_c	1	$-\frac{\pi}{2}$	0	0	0	0	(3.18), (3.19), (3.20)
A_{20}	x_c	λ_c	1	$\frac{\pi}{2}$	0	0	0	0	(3.18), (3.19), (3.20)
$A_{21}(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	1	$-\frac{\pi}{2}$	0	0	0	0	(3.12), (3.13), (3.14)
$A_{22}(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	1	$\frac{\pi}{2}$	0	0	0	0	(3.12), (3.13), (3.14)
A_{23}	-1	λ_c	1	$-\frac{\pi}{2}$	0	0	0	0	(3.15), (3.16), (3.17)
A_{24}	1	λ_c	1	$-\frac{\pi}{2}$	0	0	0	0	(3.15), (3.16), (3.17)
A_{25}	-1	λ_c	1	$\frac{\pi}{2}$	0	0	0	0	(3.15), (3.16), (3.17)
A_{26}	1	λ_c	1	$\frac{\pi}{2}$	0	0	0	0	(3.15), (3.16), (3.17)
A_{27}	$-\frac{1}{\sqrt{3}}$	λ_c	1	$-\frac{\pi}{2}$	0	0	0	0	(3.21), (3.22), (3.23)
A_{28}	$\frac{1}{\sqrt{3}}$	λ_c	1	$-\frac{\pi}{2}$	0	0	0	0	(3.21), (3.22), (3.23)
A_{29}	$-\frac{1}{\sqrt{3}}$	λ_c	1	$\frac{\pi}{2}$	0	0	0	0	(3.21), (3.22), (3.23)
A_{30}	$\frac{1}{\sqrt{3}}$	λ_c	1	$\frac{\pi}{2}$	0	0	0	0	(3.21), (3.22), (3.23)

Table 9. Equilibrium points of the system (5.12) and (5.29).

Long wavelength boundary (or super-horizon boundary) ($k^2\mathcal{H}^{-2} \ll 1$)

$$\frac{dx}{d\bar{N}} = -\left(3x - \sqrt{\frac{3}{2}}\lambda\right)\left(1 - x^2\right)\left(1 - \bar{Z}\right), \tag{34a}$$

$$\frac{d\lambda}{d\bar{N}} = -\sqrt{6}xf\left(1 - \bar{Z}\right), \tag{34b}$$

$$\frac{d\bar{Z}}{d\bar{N}} = 2(3x^2 - 1)\bar{Z}\left(1 - \bar{Z}\right)^2. \tag{34c}$$

$$\frac{d\theta}{d\bar{N}} = -\Bigg[\sin^2\theta + 3\left(1 - x^2\right)\sin\theta\cos\theta + 18\left(1 - x^2\right)\left(\frac{f}{6} + \left(x - \frac{\lambda}{\sqrt{6}}\right)^2\right)\cos^2\theta\Bigg]\left(1 - \bar{Z}\right) - \bar{Z}\cos^2\theta, \tag{35}$$

5.12 and 5.67

Label	x	λ	$\bar{Z}$	θ	k_1	k_2	k_3	k_4	$a(t), H(t), \phi(t)$
$C_1(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$-\cos^{-1}\left(-\frac{2}{\sqrt{\lambda^{*4}-12\lambda^{*2}+40}}\right)$	$\frac{1}{2}\left(\lambda^{*2}-6\right)$	$\lambda^{*2}-2$	$3-\frac{\lambda^{*2}}{2}$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$C_2(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$\cos^{-1}\left(\frac{2}{\sqrt{\lambda^{*4}-12\lambda^{*2}+40}}\right)$	$\frac{1}{2}\left(\lambda^{*2}-6\right)$	$\lambda^{*2}-2$	$3-\frac{\lambda^{*2}}{2}$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$C_3(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$-\cos^{-1}\left(\frac{2}{\sqrt{\lambda^{*4}-12\lambda^{*2}+40}}\right)$	$\frac{1}{2}\left(\lambda^{*2}-6\right)$	$\lambda^{*2}-2$	$-\frac{1}{2}\lambda^{*2}+\frac{8\left(\lambda^{*2}-6\right)}{\lambda^{*4}-12\lambda^{*2}+40}+3$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$C_4(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$\cos^{-1}\left(-\frac{2}{\sqrt{\lambda^{*4}-12\lambda^{*2}+40}}\right)$	$\frac{1}{2}\left(\lambda^{*2}-6\right)$	$\lambda^{*2}-2$	$-\frac{1}{2}\lambda^{*2}+\frac{8\left(\lambda^{*2}-6\right)}{\lambda^{*4}-12\lambda^{*2}+40}+3$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$C_5(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	0	$\frac{1}{2}\left(\lambda^{*2}-6\right)$	$\frac{1}{2}\left(\lambda^{*2}-6\right)$	$\lambda^{*2}-2$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$C_6(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	$-\pi$	$\frac{1}{2}\left(\lambda^{*2}-6\right)$	$\frac{1}{2}\left(\lambda^{*2}-6\right)$	$\lambda^{*2}-2$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$C_7(\lambda^*)$	$\frac{\lambda^*}{\sqrt{6}}$	λ^*	0	π	$\frac{1}{2}\left(\lambda^{*2}-6\right)$	$\frac{1}{2}\left(\lambda^{*2}-6\right)$	$\lambda^{*2}-2$	$-\lambda^* f'(\lambda^*)$	(3.12), (3.13), (3.14)
$C_8(\lambda^*)$	-1	λ^*	0	0	4	0	$\sqrt{6}\lambda^*+6$	$\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$C_9(\lambda^*)$	-1	λ^*	0	$-\pi$	4	0	$\sqrt{6}\lambda^*+6$	$\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$C_{10}(\lambda^*)$	-1	λ^*	0	π	4	0	$\sqrt{6}\lambda^*+6$	$\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$C_{11}(\lambda^*)$	1	λ^*	0	0	4	0	$6-\sqrt{6}\lambda^*$	$-\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$C_{12}(\lambda^*)$	1	λ^*	0	$-\pi$	4	0	$6-\sqrt{6}\lambda^*$	$-\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
$C_{13}(\lambda^*)$	1	λ^*	0	π	4	0	$6-\sqrt{6}\lambda^*$	$-\sqrt{6}f'(\lambda^*)$	(3.15), (3.16), (3.17)
C_{14}	x_c	λ_c	1	$-\frac{\pi}{2}$	0	0	0	0	(3.18), (3.19), (3.20)
C_{15}	x_c	λ_c	1	$\frac{\pi}{2}$	0	0	0	0	(3.18), (3.19), (3.20)
C_{16}	0	0	1	$-\frac{\pi}{2}$	0	0	0	0	$e^{H_0(t-t_U)}, H_0, \phi_0$
C_{17}	0	0	1	$\frac{\pi}{2}$	0	0	0	0	$e^{H_0(t-t_U)}, H_0, \phi_0$
C_{18}	$-\frac{1}{\sqrt{3}}$	$-\sqrt{2}$	1	$-\frac{\pi}{2}$	0	0	0	0	(3.21), (3.22), (3.23)
C_{19}	$-\frac{1}{\sqrt{3}}$	$-\sqrt{2}$	1	$\frac{\pi}{2}$	0	0	0	0	(3.21), (3.22), (3.23)
C_{20}	$\frac{1}{\sqrt{3}}$	$\sqrt{2}$	1	$-\frac{\pi}{2}$	0	0	0	0	(3.21), (3.22), (3.23)
C_{21}	$\frac{1}{\sqrt{3}}$	$\sqrt{2}$	1	$\frac{\pi}{2}$	0	0	0	0	(3.21), (3.22), (3.23)

Table 11. Equilibrium points of system (5.12) and (5.67).

Figure 3: List of fixed points

Perturbation of the Stress-energy tensor

Scalar field under perturbation:

We also see that the perturbation changes the energy-momentum in the following way,

$$T^0_0 = -(\rho(t) + \delta\rho(t, \bar{x})) \quad (36)$$

$$T^0_i = -(\rho(t) + P(t))\partial_i v(t, \bar{x}) \quad (37)$$

$$T^i_j = (P(t) + \delta P(t, \bar{x}))\delta^i_j \quad (38)$$

where $v(t, \bar{x})$ being the velocity potential and $\delta\phi(t, \bar{x})$ is the perturbation of the scalar field.

Dynamical System

As Einstein's field equations are highly nonlinear, even after finding a differential equation about how **gauge-invariant** scalar potential (like **Bardeen potential**) changes, we can not hope to solve the exact differential equation to its full extent.

Why dynamical system?

For this case, we have to borrow ideas from dynamical system analysis,
We often take various auxiliary variables to study the time evolution of an autonomous dynamical system.

Dynamical system:

So that we can get an equation of the form $\mathbf{X}' = \mathbf{f(X)}$, where $\mathbf{X}$ is the column vector of the auxiliary variables, and $\mathbf{f(X)}$ denotes vector field for autonomous equations.

Dynamical System cont.

Fixed point:

Once we set up the differential equations, we can try to investigate the fixed points of such an autonomous system of equations. We note that by definition, fixed points are the points that satisfy $\mathbf{X_c} = \mathbf{0}$

Once we extract the fixed points $\mathbf{X_c}$ from the previous equation, we can see the effect of a linear perturbation around $\mathbf{X_c}$ keeping only the first order, and we can get $\mathbf{X_c}$ as $\mathbf{X} = \mathbf{X_c} + \mathbf{U}$, where $\mathbf{U}$ is the column vector of the perturbations due to auxiliary variables.

Dynamical System on background:

Stability of fixed points:

So in first-order one can write $\mathbf{U}' = \mathbf{Y} \cdot \mathbf{U}$. Finally, by finding the eigenvalues of matrix $\mathbf{Y}$ we can find the characteristics of the fixed points, like whether the point is stable (unstable) if the real parts of the eigenvalues are negative (positive); or saddle point if there is a change of sign.

Uniform curvature gauge:

One can also choose other gauges to write the differential equation.
For example, if one chooses the uniform curvature gauge [2], one can get an equation for the potential as follows,

$$\frac{d^2\Phi}{dN^2} + \frac{d\Phi}{dN}\left(\frac{V}{H^2}\right) + \Phi\left(\frac{V}{H^2}\right) + \Phi\left(\frac{V_{,\phi\phi} + 2\frac{\dot{\phi}}{H}V_{,\phi} + \left(\frac{\dot{\phi}}{H}\right)^2 V}{H^2}\right) - \mathcal{H}^{-2}\nabla^2\Phi = 0, \tag{39}$$

Where Φ is Sasaki-mukhanov variable.

[2] Alho. A., Uggla. C., Wainwright. J. "Dynamical systems in perturbative scalar field cosmology" *Classical and Quantum Gravity*, 2020, 37(22):225011

Comoving curvature perturbation $\mathcal{R}$

For single scalar field models, comoving curvature perturbation is defined as

$$\mathcal{R} \equiv \psi - \frac{H}{\dot{\phi}}\delta\phi \tag{40}$$

The name comes from the fact that this variable coincides with the 3-curvature perturbation of the spatial slice in the *comoving* gauge, which, for single scalar field models, is given by $\delta\phi = 0$. At the linear level, comoving curvature perturbation evolves according to the following equation [3]

$$\ddot{\mathcal{R}} + \frac{\left(a^3\frac{\dot{\phi}^2}{H^2}\right)^{\cdot}}{\left(a^3\frac{\dot{\phi}^2}{H^2}\right)}\dot{\mathcal{R}} - \frac{1}{a^2}\nabla^2\mathcal{R} = 0 \tag{41}$$

[3] Antonio De Felice, Shinji Tsujikawa, f(R) theories, Living Rev.Rel. 13 (2010) 3, arXiv : 1002.4928

Motivation for Q:

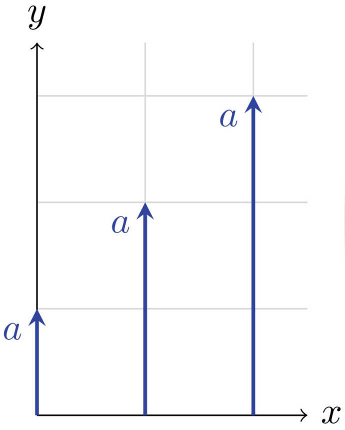

Figure 4: When the connection is not metric compatible we get $\frac{D}{d\lambda}(a.b) = Q_{\alpha\mu\nu}\frac{dx^{\alpha}}{d\lambda}a^{\mu}b^{\nu}$[pic credit:E.N. Saridakis "Metric-Affine Gravity"(CANTATA-2021)]

$f(Q)$ gravity (geometric trinity):

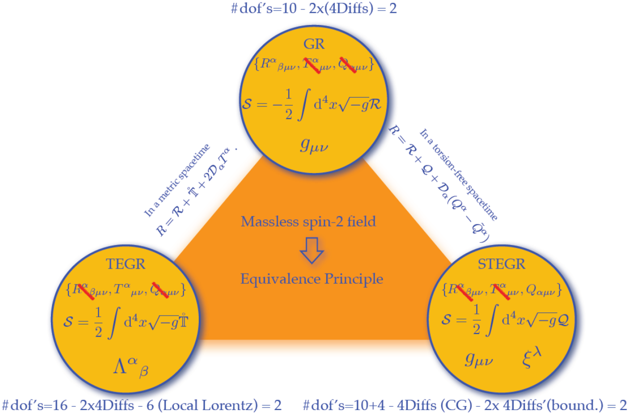

Figure 5: Geometric trinity: Relation between curvature, torsion and metric-affine gravity.[pic credit: T.S. Koivisto "Geometric Foundations of Gravity" (CANTATA-2021)]

The $f(Q)$ gravity:

The nonmetricity tensor is given by

$$Q_{\lambda\mu\nu} = \nabla_\lambda g_{\mu\nu} \tag{42}$$

One can also establish a superpotential related to the nonmetricity tensor as

$$P^\alpha{}_{\mu\nu} = \frac{1}{4}\left[-Q^\alpha{}_{\mu\nu} + 2Q_{(\mu}{}^\alpha{}_{\nu)} + Q^\alpha g_{\mu\nu} - \tilde{Q}^\alpha g_{\mu\nu} - \delta^\alpha_{(\mu} Q_{\nu)}\right], \tag{43}$$

where

$$Q_\alpha = Q_\alpha{}^\mu{}_\mu, \;\; \tilde{Q}_\alpha = Q^\mu{}_{\alpha\mu}. \tag{44}$$

are two independence traces.

The $f(Q)$ gravity:

Also, the nonmetricity scalar defined as:

$$Q = -Q_{\alpha\mu\nu}\, P^{\alpha\mu\nu}. \tag{45}$$

In the $f(Q)$ gravity, the total Einstein Hilbert action is given:

$$\mathcal{S} = \int \frac{1}{2}\, f(Q)\sqrt{-g}\, d^4x + \int \mathcal{L}_m \sqrt{-g}\, d^4x\,, \tag{46}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, and $\mathcal{L}_m$ is Lagrangian density of matter.
In order to find the field equations for this theory of gravity, we can set that the action (46) is constant in respect to variations over the metric tensor $g_{\mu\nu}$, resulting in

$$\frac{2}{\sqrt{-g}}\, \nabla_\gamma \left(\sqrt{-g}\, f_Q\, P^\gamma{}_{\mu\nu}\right) + \frac{1}{2} g_{\mu\nu} f + f_Q \left(P_{\mu\gamma i}\, Q_\nu{}^{\gamma i} - 2\, Q_{\gamma i \mu}\, P^{\gamma i}{}_\nu\right) = -T_{\mu\nu}, \tag{47}$$

where $f_Q = \frac{df}{dQ}$, and $T_{\mu\nu}$ is the standard energy-momentum tensor.

$f(Q)$ gravity:

In FLRW background, we get the following Friedmann equations governing the gravitational interactions under the $f(Q)$ gravity background in the presence of a scalar field,

$$3H^2 = \frac{1}{2f_Q}\left(-\rho_\phi + \frac{f}{2}\right) \tag{48}$$

$$\dot{H} + 3H^2 + \frac{\dot{f}_Q}{f_Q}H = \frac{1}{2f_Q}\left(p_\phi + \frac{f}{2}\right) \tag{49}$$

For the $f(Q)$ functional $f(Q) = -Q + \Psi(Q)$, we can rewrite the Friedmann equations (48)-(49) as (where we can recover ordinary GR by putting $\Psi = 0$)

$$3H^2 = \rho_\phi + \rho_{de} \tag{50}$$

$$\dot{H} = -\frac{1}{2}[\rho_\phi + p_\phi + \rho_{de} + p_{de}] \tag{51}$$

$f(Q)$ gravity:

Where ρ_{de} and p_{de} represent the energy density and pressure of the dark energy component evolving due to the geometry of spacetime,

$$\rho_{de} = -\frac{\Psi}{2} + Q\Psi_Q \tag{52}$$

$$p_{de} = -\rho_{de} - 2\dot{H}\left(\Psi_Q + 2Q\Psi_{QQ}\right) \tag{53}$$

$f(Q)$ gravity:

We define the following variables,

$$x^2 = \frac{\dot{\phi}^2}{6H^2}, \quad y^2 = \frac{V}{3H^2}, \quad \text{and} \quad s^2 = \Omega_{de} = \frac{\rho_{de}}{3H^2} \tag{54}$$

We have following constraint,

$$x^2 + y^2 + s^2 = 1 \tag{55}$$

Hence it follows that $\Omega_{de} = s^2 = 1 - x^2 - y^2 = 1 - \Omega_\phi$ and the constraint $0 \leq x^2 + y^2 \leq 1$.

$$\lambda = -\frac{V_{,\phi}}{V} \tag{56}$$

$f(Q)$ gravity:

We obtain the following dynamical system with respect to e-folding time $N = ln(a)$,

$$x' = -3x - x\frac{\dot{H}}{H^2} + \sqrt{\frac{3}{2}}\lambda y^2 \tag{57}$$

$$y' = \frac{1}{2y}\left[\frac{\dot{V}}{3H^2} - y^2\frac{2\dot{H}}{H^2}\right] \tag{58}$$

$$\lambda' = -\sqrt{6}\lambda^2 x(\Gamma - 1) \tag{59}$$

where $\Gamma = \frac{VV_{,\phi\phi}}{V_{,\phi}^2}$. Here using the equation (51), we have

$$\frac{\dot{H}}{H^2} = \frac{3x^2}{(\Psi_Q + 2Q\Psi_{QQ} - 1)} \tag{60}$$

Exponential potential:

We have taken the geometric part as $f(Q) = -Q + \alpha Q^n$
We assume the following form of exponential potential,

$$V(\phi) = V_0 e^{-\beta\phi} \tag{61}$$

The dynamical system of the equation becomes,

$$x' = -3x\left[1 + \frac{x^2}{n(1 - x^2 - y^2) - 1}\right] + \sqrt{\frac{3}{2}}\beta y^2 \tag{62}$$

$$y' = -xy\left[\sqrt{\frac{3}{2}}\beta + \frac{3x}{n(1 - x^2 - y^2) - 1}\right] \tag{63}$$

Fixed point analysis:

Table 1: Table shows the critical points and their behavior corresponding to the model $f(Q) = -Q + \alpha Q^n$ with exponential potential $V(\phi) = V_0 e^{-\beta\phi}$..

Critical Points (x_c, y_c)	Eigenvalues λ_1 and λ_2	Nature of critical point	q	ω
$O(0,0)$	$-3, 0$	Stable	-1	-1
$A(-1,0)$	$6-6n, 3+\sqrt{\frac{3}{2}}\beta$	Stable for $(n>1\ \&\ \beta<-\sqrt{6})$	2	1
$B(1,0)$	$6-6n, 3-\sqrt{\frac{3}{2}}\beta$	Stable for $(n>1\ \&\ \beta>\sqrt{6})$	2	1
$C\left(\frac{\beta}{\sqrt{6}}, \sqrt{1-\frac{\beta^2}{6}}\right)$	$\frac{1}{2}(\beta^2-6), (1-n)\beta^2$	Stable for $(-\sqrt{6}<\beta<0\ \&\ n>1)$ or $(0<\beta<\sqrt{6}\ \&\ n>1)$	$-1+\frac{\beta^2}{2}$	$-1+\frac{\beta^2}{3}$
$D\left(\frac{\beta}{\sqrt{6}}, -\sqrt{1-\frac{\beta^2}{6}}\right)$	$\frac{1}{2}(\beta^2-6), (1-n)\beta^2$	Stable for $(-\sqrt{6}<\beta<0\ \&\ n>1)$ or $(0<\beta<\sqrt{6}\ \&\ n>1)$	$-1+\frac{\beta^2}{2}$	$-1+\frac{\beta^2}{3}$

Phase portrait:

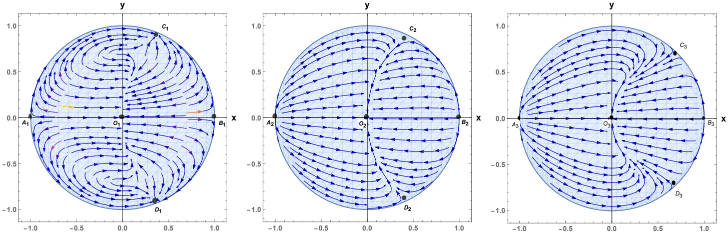

Figure 6: Phase-space plots for the case $n = 2$ (left panel above) and $n = -1$ (right panel above), with $\beta = 1$, and for the case $n = -1$ with $\beta = \sqrt{3}$ (below) corresponding to the exponential potential.

Ω vs N plot:

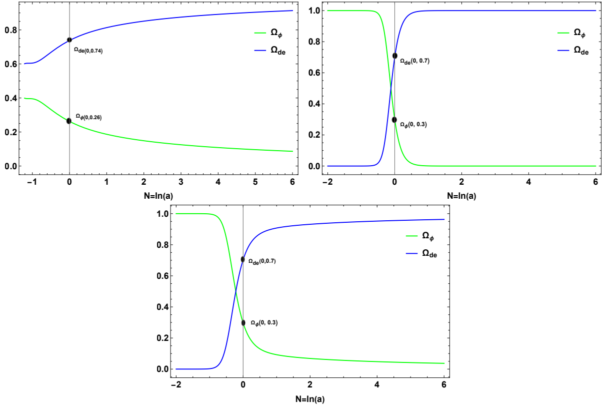

Figure 7: Evolutionary profile of the scalar field density and dark energy density for the case $n = 2$ (left panel) and $n = -1$ (middle panel), with $\beta = 1$, and for the case $n = -1$ with $\beta = \sqrt{3}$ (right panel) corresponding to the exponential potential.

q and ω plot:

PPC-2024

Sayantan Ghosh

Introduction:

Scalar fields:

Dynamical system in $f(Q)$ gravity:

DBI field

Dynamical system in DBI field.

Conclutions:

References:

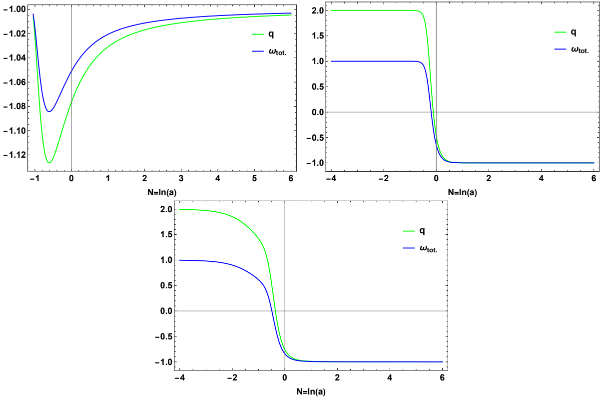

Figure 8: Evolutionary profile of the deceleration and the equation of state parameter for the case $n = 2$ (left panel) and $n = -1$ (middle panel), with $\beta = 1$, and for the case $n = -1$ with $\beta = \sqrt{3}$ (right panel) corresponding to the exponential potential.

Power law scaler field:

We assume the following form of power-law potential,

$$V(\phi) = V_0 \phi^{-k} \tag{64}$$

So the dynamical system reduces to the following autonomous system,

$$x' = -3x\left[1 + \frac{x^2}{n\left(1 - x^2 - y^2\right) - 1}\right] + \sqrt{\frac{3}{2}}\lambda y^2 \tag{65}$$

$$y' = \sqrt{\frac{3}{2}}\lambda xy\left[1 - \frac{\sqrt{6}x}{\lambda\{n\left(1 - x^2 - y^2\right) - 1\}}\right] \tag{66}$$

$$\lambda' = -\frac{\sqrt{6}}{k}\lambda^2 x \tag{67}$$

Fixed point analysis:

Table 2: Table shows the critical points and their behavior corresponding to the model $f(Q) = -Q + \alpha Q^n$ with potential $V(\phi) = V_0\phi^{-k}$.

Critical Points (x_c, y_c, z_c)	Eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$	Nature of critical point	q	ω
$O'(0,0,0)$	$(-3,0,0)$	Stable	-1	-1
$A'(0,y,0)$	$(-3,0,0)$	Stable	-1	-1
$B'(0,0,z)$	$(0,0,-3)$	Stable	-1	-1
$C'(1,0,0)$	$(3,0,6-6n)$	Unstable for $n \leq 1$ and N.H. for $n > 1$	2	1
$D'(-1,0,0)$	$(3,0,6-6n)$	Unstable for $n \leq 1$ and N.H. for $n > 1$	2	1

Compactified diagram:

We have also compactified the λ as a Poincare prescription that is $z = \frac{\lambda}{\lambda+1}$. (arXiv:2309.11198)

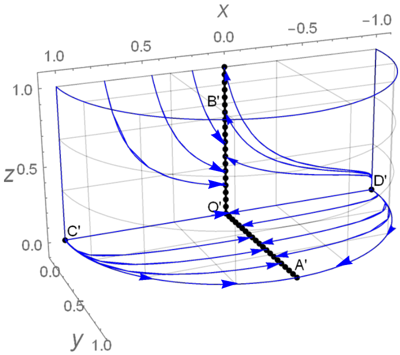

Figure 9: The 3-D phase-space trajectories plotted for a set of solutions to the autonomous system for the parameter value $n = -2$ and $k = 0.16$ corresponding to the power-law potential.

ω and q:

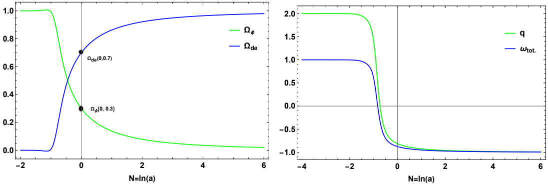

Figure 10: Evolutionary profile of scalar field density, dark energy density, deceleration, and the equation of state parameter for the parameter value $n = -2$ and $k = 0.16$ corresponding to the power-law potential.

DBI field

The Dirac Born Infeld theory was first proposed as a nonlinear extension of Maxwell's equations to "renormalize" the infinite self-energy of an electron.

It is well known that even in bosonic string theory, the quantization of the Polyakov action (conformal transformation of Nambu-Goto action) gives tachyon like a field which soon decays via spontaneous symmetry breaking.

It was first observed by Mazumdar et al. [a] that the decay of a non-BPS $D4$ branes to a stable $D3$ branes can give rise to tachyon field, which can act as an inflation field in the cosmological context

[a] A. Mazumdar, S. Panda, and A. Perez-Lorenzana, *Nuclear Physics B* **614.1-2** 101-116 (2001).

DBI field

In 2002, a series of three papers by Sen [a] [b] [c] showed how, in string theory, as well as in string field theory, tachyons occur naturally, and in (A. Sen, *JHEP* **07**, 065 (2002)) it has been shown that the effective field of such tachyons can be viewed as DBI scalar field theory.

[a] A. Sen, *Modern Physics Letters A* **17.27**, 1797-1804 (2002).
[b] A. Sen, *JHEP* **04**, 048 (2002).
[c] A. Sen, *JHEP* **07**, 065 (2002).

Padmanabhan [a] and Gibbons [b] showed how these DBI-type fields could be used to give inflation field-like behaviour. Alternative ways of getting the DBI field from other forms of string theory have been reviewed by Gibbons [c].

[a] T. Padmanabhan et al., *Phys.Rev. D* **66**, 021301 (2002).
[b] G. W. Gibbons, *Phys. Lett. B* **537**, 1-4 (2002)
[c] G. W. Gibbons, *Class.Quant.Grav.* **20**, S321-S346 (2003).

DBI field

The study of the DBI field in late time acceleration context has been done by Bhagla et al. [a]

[a] J. Bhagla et al., *Phys.Rev. D* **67**, 063504 (2003).

Gorini et al.[a] offered the alternative way of visualizing the DBI field as a modified Chaplygin gas.

[a] V. Gorini et al., *Phys.Rev. D* **69**, 123512 (2004).

We also note that DBI field has been proposed as an alternative to dark matter by Padmanabhan [a] this shows that the DBI field could indeed effect the late time cosmology.

[a] T. Padmanabhan et al., *Phys.Rev. D* **66**, 081301 (2002).

DBI field

It is also worth noting that as Silverstein and Tong [a] have shown, if one considers a D3-brane moving towards the horizon of AdS space, one can get a generalized DBI field in a strong coupling limit (as opposed to a weak coupling limit where the previous work has been done). In strong coupling limit, it can shown that DBI field gets extra contributions from the movement of the D3-brane and the lagrangian becomes $\mathcal{L}_{GDBI} = \frac{1}{f(\phi)}(\sqrt{1 + f(\phi)\partial\phi^2} - 1) - V(\phi)$.

[a] E. Silverstein, D. Tong , *Phys.Rev. D* **70**, 103505 (2004).

DBI field

Copeland [a] and Aguirregabiria [b] first studied the study of the DBI field in the dynamical system setting. In this paper also, we are closely following the treatment given in Copeland.

[a] E. J. Copeland et al., *Phys.Rev. D* **71**, 043003 (2005).
[b] Aguirregabiria et al.,*Phys.Rev. D* **69** , 123502 (2004).

Soon after that, Fang and Lu [a] considered a much more general type of potential beyond inverse square potential, the work later extended by Quiros et al. [b] to include much more general potentials. Guo [c] used exponential potential to form an autonomous dynamical system.

[a] W. Fang, H. Q. Lu, *Eur. Phys. J C* **68**, 567-572 (2010).
[b] Quiros et al., *Class.Quant.Grav.* **27**, 215021 (2010).
[c] Z. K. Guo et al., *Phys.Rev. D* **71**, 023501 (2005).

DBI field

Tachyon modes are typically given by Dirac-Born-Infeld (DBI) Lagrangian, which have the following form,

$$\mathcal{L}_{Tachyon} = V(\phi)\sqrt{1 + \partial\phi^2} \quad (68)$$

Where $\partial\phi^2 = \partial^\mu\phi\partial_\mu\phi$ and $V(\phi)$ is a potential function for the scalar field and $\partial^\mu\phi\partial_\mu\phi$ denotes the kinetic term for tachyon fields,

Cosmology in DBI field

From the Lagrangian, one can find the field equation for the Tachyon field from the Euler-Lagrangian equation as,

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V_{,\phi}}{V} = 0 \tag{69}$$

This is the modified Klein-Gordon equation for the DBI field. Cosmological (freedman equations) equations become,

$$3H^2 = \rho_\phi + \frac{V}{\sqrt{1-\dot{\phi}^2}} \tag{70}$$

Cosmology in DBI field

We note that for such cases the energy density (ρ) and pressure (p) are given by,

$$\rho_\phi = \frac{V}{\sqrt{1-\dot{\phi}^2}} \tag{71}$$

$$p_\phi = -V\sqrt{1-\dot{\phi}^2} \tag{72}$$

so the equation of state (ω_ϕ) is given by

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \dot{\phi}^2 - 1 \tag{73}$$

Dynamical system

We take the following form of the exponential potential, given by:

$$V(\phi) = V_0 e^{-\beta\phi} \tag{74}$$

as $\lambda = -\frac{V_{,\phi}}{V^{\frac{3}{2}}} = \frac{\beta}{\sqrt{V_0 e^{-\beta\phi}}}$, and $\Gamma = \frac{VV_{,\phi\phi}}{V_{,\phi}^2} = 1$

So the dynamical system of equations becomes,

$$x' = (x^2 - 1)(3x - \sqrt{3}\lambda y) \tag{75}$$

$$y' = -\frac{1}{2}y[\sqrt{3}\lambda xy + \frac{3x^2y^2}{[(n-1)\sqrt{1-x^2} - ny^2]}] \tag{76}$$

$$\lambda' = \frac{\sqrt{3}}{2}xy\lambda^2 \tag{77}$$

DBI field

We also note that q and ω expression as follows,

$$q = -1 - \frac{3x^2y^2}{(n-1)\sqrt{1-x^2} + y^2} \tag{78}$$

$$\omega = -1 - \frac{2x^2y^2}{(n-1)\sqrt{1-x^2} + y^2} \tag{79}$$

Table 3: Table shows the critical points and their behavior corresponding to the model $f(Q) = -Q + \alpha Q^n$ with potential $V(\phi) = V_0 e^{-\beta\phi}$.

Critical Points (x_c, y_c, z_c)	Eigenvalues $(\lambda_1,\ \lambda_2,\ \lambda_3)$	Nature of critical point	q	ω
$O(0,0,0)$	$(-3,0,0)$	Stable (NH)	-1	-1
$A(1,0,\lambda)$	$(6,0,0)$	Non-hyperbolic	-1	-1
$B(-1,0,\lambda)$	$(6,0,0)$	Non-hyperbolic	-1	-1
$C(0,y,0)$	$(-3,0,0)$	Stable (NH)	-1	-1
$D(0,0,\lambda)$	$(-3,0,0)$	Stable (NH)	-1	-1

Phase space analysis

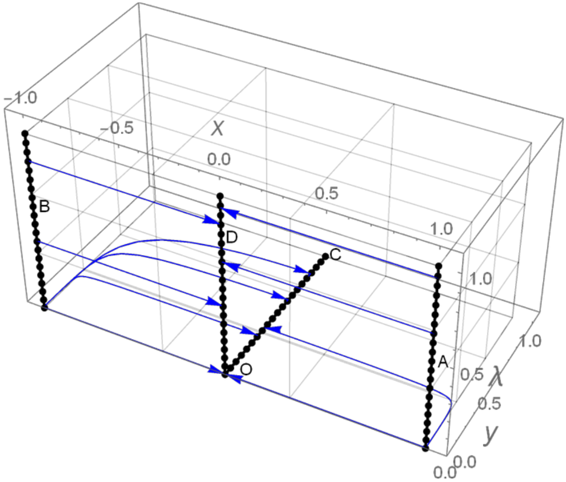

Figure 11: 3D phase portrait for exponential potential.

Phase space analysis

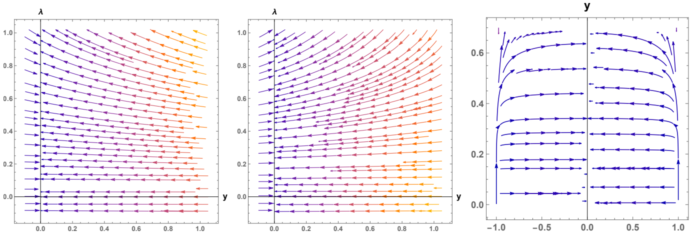

Figure 12: 2D phase portrait for the $x = 1, -1$ and xy planes and shows that even though the critical points are non-hyperbolic, one can still take some particular limits to see the matter dominated to de Sitter transition. The sufficient condition to show a de-Sitter transition is given by the equation $\frac{1-x}{y^4} \leq 2$

Power law potential

We assume the following form of power-law potential,

$$V(\phi) = V_0 \phi^{-k} \tag{80}$$

Then for this choice of potential, we obtain,
$\lambda = -\frac{V_{,\phi}}{V^{\frac{3}{2}}} = \frac{V_0 k \phi^{-k-1}}{V_0 \phi^{-k} (V_0 \phi^{-k})^{\frac{1}{2}}} = \frac{k}{\sqrt{V_0}} \phi^{-1+\frac{k}{2}}$
In particular, for $k = 2$ we get $\lambda = \frac{2}{\sqrt{V_0}}$.

Dynamical system for power law potential

Therefore the complete autonomous form of dynamical equationsfor the power-law potential becomes,

$$x' = (x^2 - 1)(3x - \sqrt{3}\lambda y) \tag{81}$$

$$y' = -\frac{1}{2}y^2[\sqrt{3}\lambda xy + \frac{3x^2 y}{[(n-1)\sqrt{1-x^2} - ny^2]}] \tag{82}$$

$$\lambda' = \frac{\sqrt{3}(k-2)}{2k}xy\lambda^2 \tag{83}$$

Fixed points

Table 4: Table shows the critical points and their behaviour corresponding to the model parameter $n = -1$ and potential $V(\phi) = V_0\phi^{-k}$ with $k \neq 2$.

Critical Points (x_c, y_c, z_c)	Eigenvalues $(\lambda_1,\ \lambda_2,\ \lambda_3)$	Nature of critical point	q	ω
$O'(0, 0, \lambda)$	$(-3, \sqrt{3}\lambda, 0)$	Stable for $\lambda < 0$ and Non-hyperbolic for $\lambda \geq 0$	-1	-1
$A'(x, y, 0)$	$(-3, 0, 0)$	Stable (NH)	-1	-1
$B'(0, y, 0)$	$(-3, 0, 0)$	Stable (NH)	-1	-1

For $k = 2$

In particular, for the case $k = 2$, the power-law case, it reduces to a following 2-dimensional dynamical system, as for the choice $k = 2$ we have $\lambda' = 0$.

$$x' = (x^2 - 1)(3x - 2\sqrt{3}y) \tag{84}$$

$$y' = -\frac{1}{2}y^2[2\sqrt{3}xy + \frac{3x^2y}{[(n-1)\sqrt{1-x^2} - ny^2]}] \tag{85}$$

Fixed points

Table 5: Table shows the critical points and their behaviour corresponding to the model parameter $n = -1$ with potential $V(\phi) = V_0\phi^{-k}$ having $k = 2$ and $V_0 = 1$.

Critical Points (x_c, y_c)	Nature of critical point	q	ω
$O''(0,0)$	Stable	-1	-1
$A''(0.806, 0.698)$	Saddle	0.362	-0.092

Phase space analysis

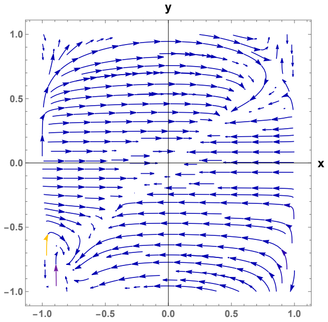

Figure 13: 2D phase portrait for polynomial potential for $k = 2$.

q and Ω

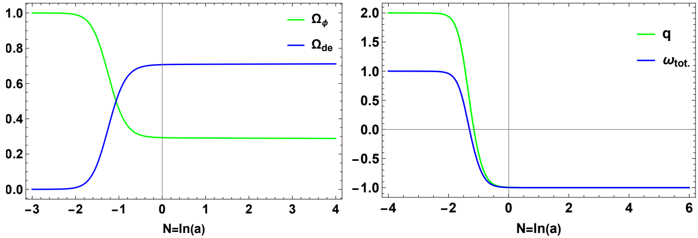

Figure 14: Evolutionary profile of scalar field density, dark energy density, deceleration, and the equation of state parameter for the exponential potential in DBI scalar field.

Conclusions:

The main features of this current study are highlighted below:

- We have discussed the importance of scalar field theories
- We have also discussed the origin of DBI fields in $f(Q)$ gravity.
- We have written the dynamical system equation in $f(Q)$ gravity.
- We have done the dynamical system analysis and plotted the q and ω for the fixed points.
- We have also drawn phase portraits for the analytical potential for ordinary scalar and DBI fields.
- In the future, we plan to extend the analysis for string-motivated K-essence type scalar fields and in the baryogenesis context.

References:

The work is based on mainly following three articles.

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Any questions, comments, suggestions? (pic credit: google image)