Study Of Angular Observables In B_c Decays

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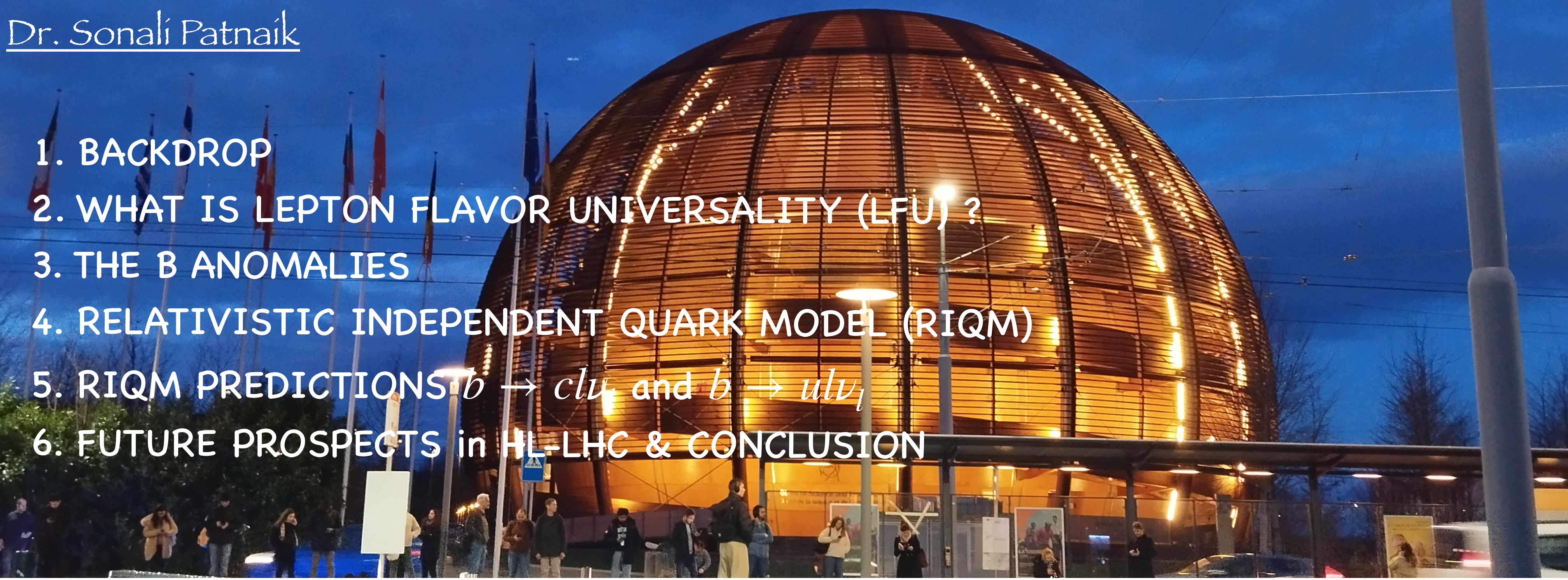


1. BACKDROP
2. WHAT IS LEPTON FLAVOR UNIVERSALITY (LFU) ?
3. THE B ANOMALIES
4. RELATIVISTIC INDEPENDENT QUARK MODEL (RIQM)
5. RIQM PREDICTIONS $b \to c l \nu_l$ and $b \to u l \nu_l$
6. FUTURE PROSPECTS in HL-LHC & CONCLUSION

Flavor Physics

Why are we sitting here?????

A story full of successes

1950's Discovery of parity violation

1960's CP violation in K decays

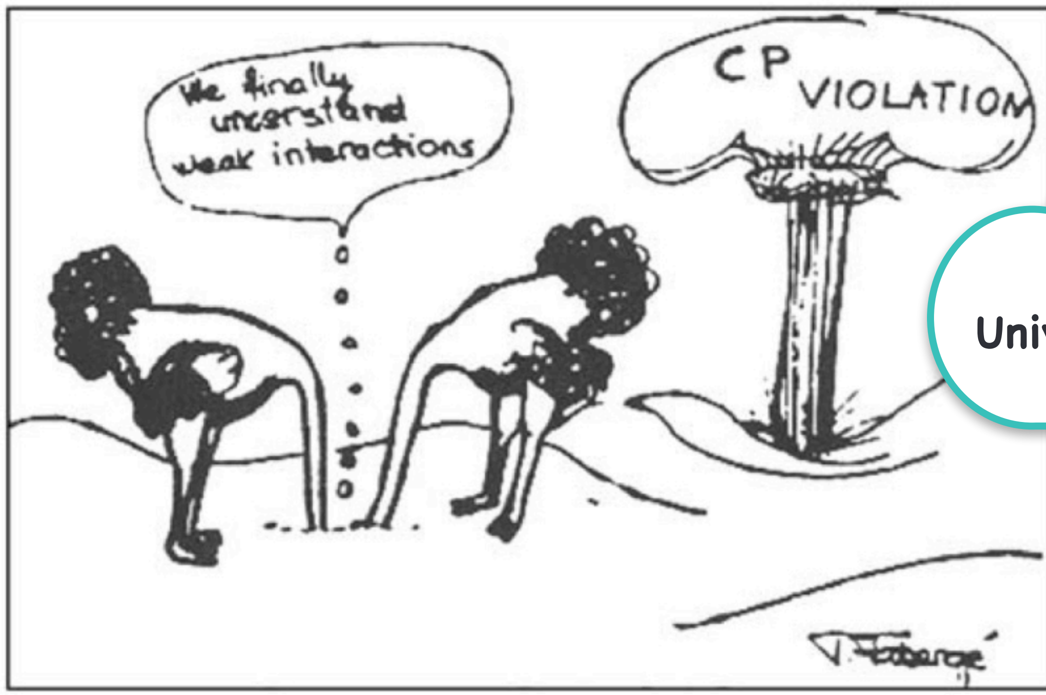
1970's Discovery of J/ψ and charm quark

1980's Inference on top quark mass from B mixing

2000's CP violation in B decays

2010's Penta- and tetra-quarks

2020's CP violation in D decays



Cartoon presented by N. Cabibbo at the Berkeley conference in 1966

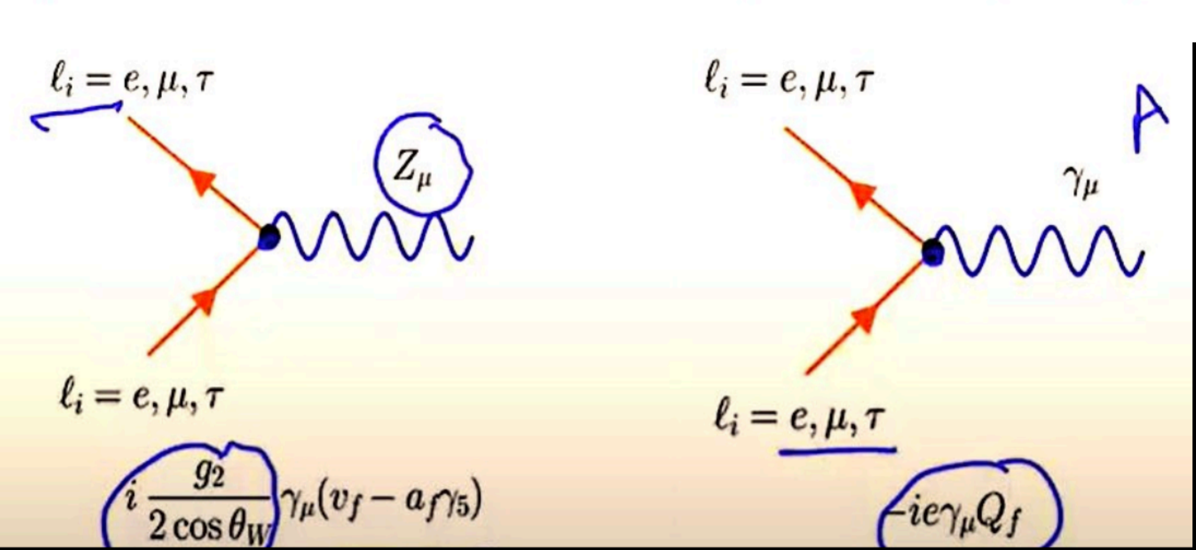
Lepton Flavor Universality Violation

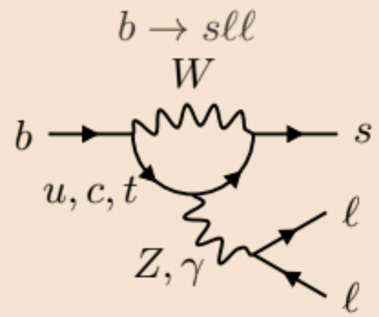
What is Lepton Flavor Universality ?

In the SM:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs+Yukawa}}$$

- LFU: e, μ, τ are all the same $(\gamma, W, Z) \rightarrow$ expect $\Gamma_e = \Gamma_\mu = \Gamma_\tau$
- LFUV: $m_e \neq m_\mu \neq m_\tau$



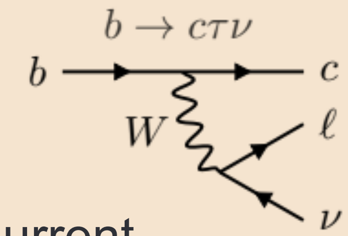


Neutral Current

LFU ratios:

$$R_{X_s} = \frac{\mathcal{B}(B \to X_s \mu\mu)}{\mathcal{B}(B \to X_s ee)}$$

$$X_s = K, K^*, K_S, \phi$$



Charged Current

LFU ratios:

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$

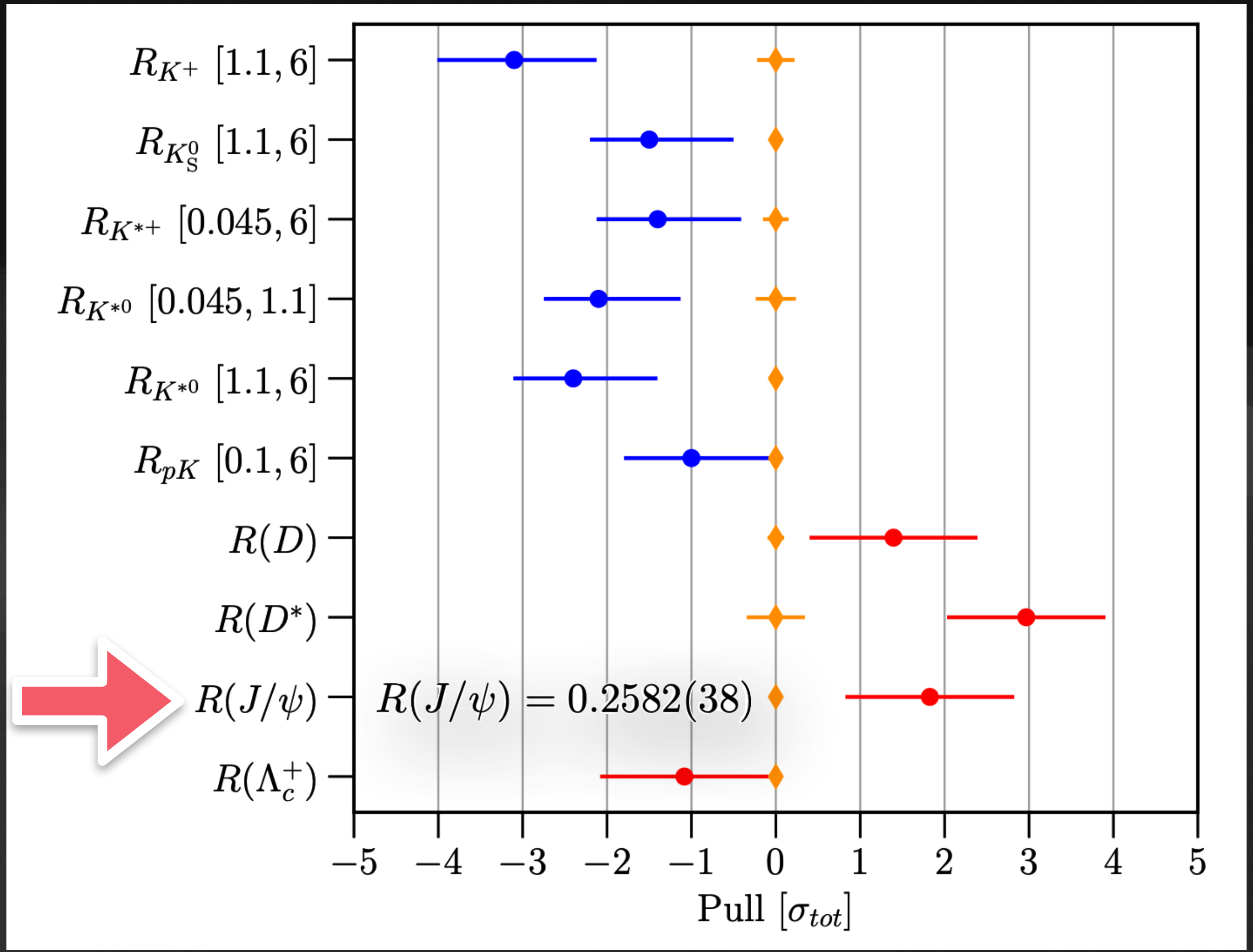
$$\ell = \mu, e$$

- Excess of τ leptons

B Anomalies

LFU Observables

- ★ Markers in blue (red) corresponds to decay $b \to s$ and $b \to c$ transitions
- ★ Markers in orange indicate theoretical predictions with Zero pull to themselves
- ★ Experimental value is then offset its deviation from SM in σ



$$R_X = \frac{\mathcal{B}(B_c^- \to X\tau^-\bar{\nu}_\tau)}{\mathcal{B}(B_c^- \to X\mu^-\bar{\nu}_\mu)}, \quad for \quad X = (\eta_c, J/\psi).$$

$$R^{\text{Exp}}_{J/\psi} = 0.71 \pm 0.17(stst.) \pm 0.18(syst.)$$

$$R^{\text{CMS}}_{J/\psi} = 0.17^{+0.18}_{-0.17}\text{stat}{}^{+0.21}_{-0.22}\text{syst}{}^{+0.19}_{-0.18}\text{theory}$$

$$R_{J/\psi} = 0.52 \pm 0.20$$

Why B_c decays ?

1. It is the lowest bound state having two heavy quarks (b and c)

2. The B_c meson lie intermediate in mass an size between charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) family where the heavy quark interactions are understood well.

3. As it has open flavours, B_c decays weakly but not via strong and radiative modes, and therefore it is long lived .

4. The data available in this sector is scant. The masses of Bc excited states and even the ground state of Bc* have not yet been determined.

5. The fundamental mechanism for creating the $\bar{b}c$ system is considerably more complex, At least proportional to the fourth power of the strong coupling constant, α_S^4: $q\bar{q}gg \rightarrow (\bar{b}c)b\bar{c}$

6. It provides a fertile ground as well as challenging for both theoretical and experimental studies as it has neutrinos in their final states

Angular Observables

Observation of τ - polarization $B \to D^* \tau \nu_\tau$

Belle Collaboration measurement of $P_\tau(D^*)$

$$P_\tau(D^*) = -0.38 \pm 0.51\,(\text{stat.})^{0.21}_{-0.16}\,(\text{syst.})\,,$$
$$F_L(D^*) = 0.60 \pm 0.08\,(\text{stat.}) \pm (0.04)\,(\text{syst.})$$

SM PREDICTIONS

$$P_\tau(D^*) = -0.497 \pm 0.013\,,$$
$$F_L(D^*) = 0.441 \pm 0.006 \quad \text{or} \quad 0.457 \pm 0.010\,.$$

$$P_\tau = \frac{\Gamma(\lambda_\tau = 1/2) - \Gamma(\lambda_\tau = -1/2)}{\Gamma(\lambda_\tau = 1/2) + \Gamma(\lambda_\tau = -1/2)}\,,$$

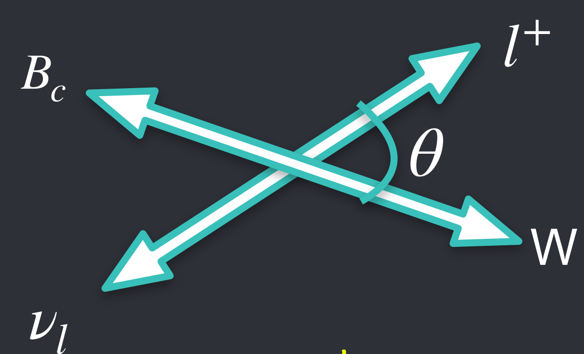
$$P_L = \frac{U + L - \tilde{U} - \tilde{L} - \tilde{S}}{U + L + \tilde{U} + \tilde{L} + \tilde{S}}$$

Predictions of Forward-Backward Asymmetry

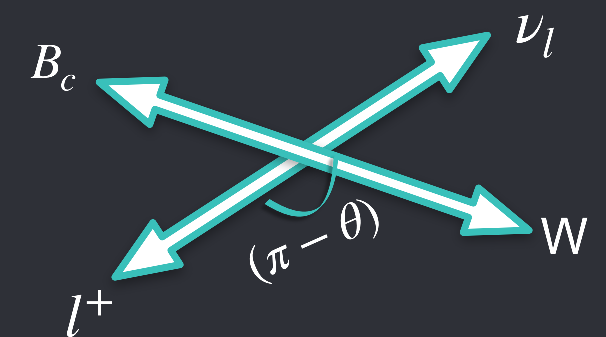
$$A^{FB}(q^2) = \frac{\left(\int_0^1 - \int_{-1}^0\right) d\cos\theta_\ell \frac{d^2\Gamma}{dq^2\, d\cos\theta_\ell}}{\left(\int_0^1 + \int_1^0\right) d\cos\theta_\ell \frac{d^2\Gamma}{dq^2\, d\cos\theta_\ell}}$$

$$A_{FB} = \frac{d\Gamma(\theta) - d\Gamma(\pi - \theta)}{d\Gamma(\theta) + d\Gamma(\pi - \theta)}$$

$$A_{FB} = \frac{3}{4}\left[\frac{\pm P + 4\tilde{S}L}{U + \tilde{U} + L + \tilde{L} + \tilde{S}}\right]$$



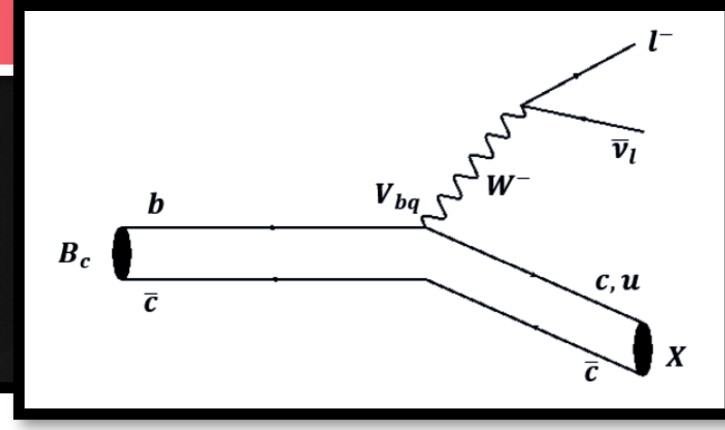
Forward Event



Backward Event

B_C IS THE MOST CRUCIAL PROBE IN THE UPCOMING FUTURE HL-COLLIDERS ERA

THE PRECISE MEASUREMENTS OF B_C DECAYS CAN PLAY AN IMPORTANT ROLE IN TESTING SM AND SEARCHING FOR THE EVIDENCE OF NP.



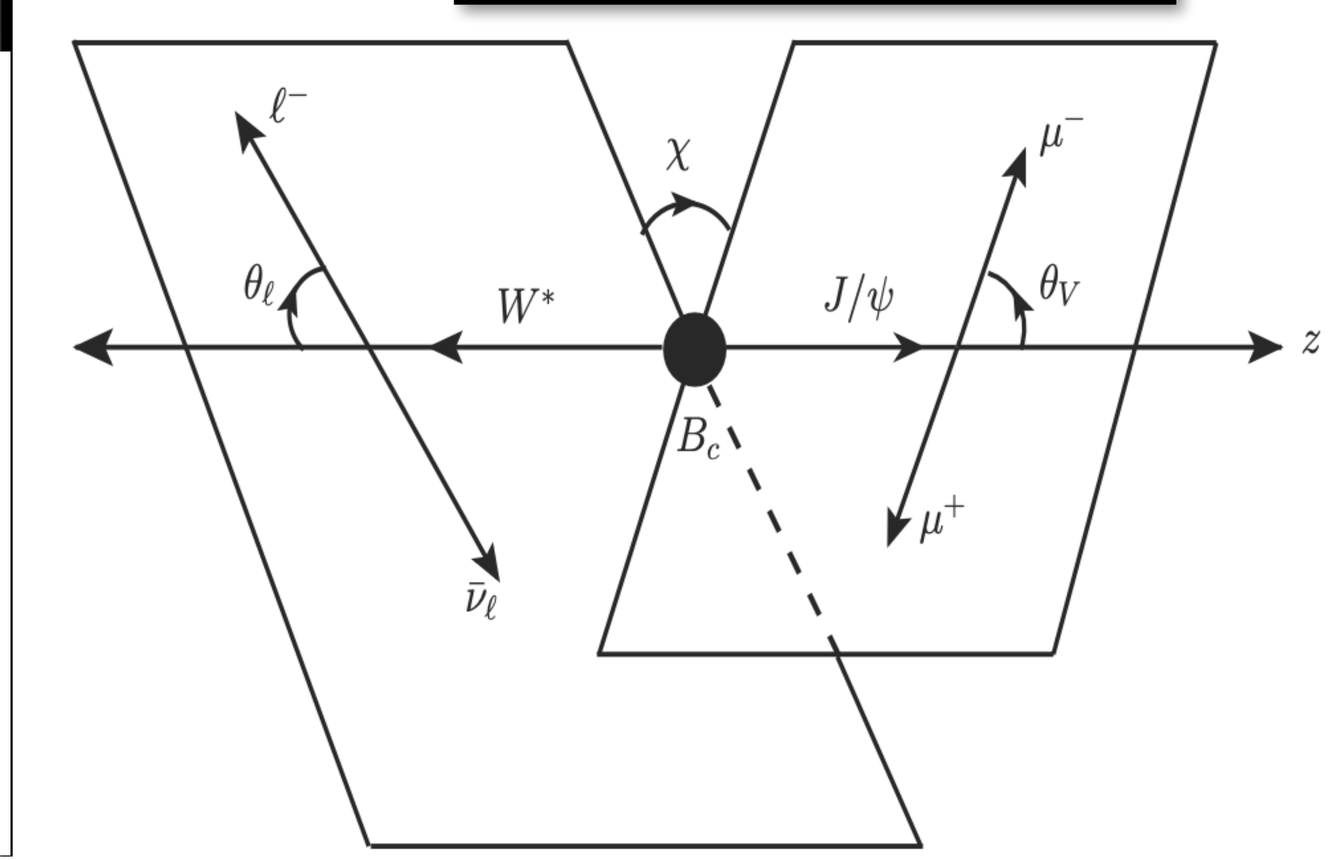
ANGULAR ASYMMETRIES

$$B_c \to \eta_c(J/\psi)l\nu_l \quad B_c \to D(D^*)l\nu_l$$

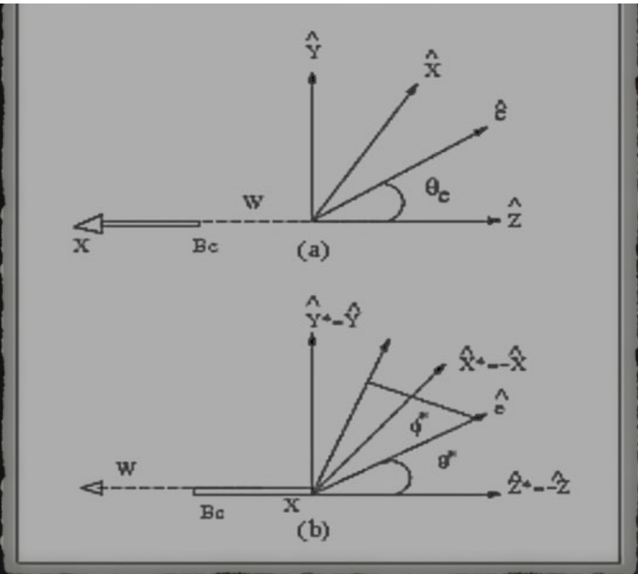
$\theta_l =$ The angle between the direction of the charged lepton in the virtual W frame and the W in the B frame

$\theta_V =$ The angle between the D in the D^* frame and the D^* in the B frame

$\chi =$ The angle between the decay planes formed by the virtual W and the D^* in the B frame



$b \to c l \nu_l \quad b \to u l \nu_l$



$$\vec{H} = H_+ \hat{e}_+ + H_- \hat{e}_- + H_0 \hat{e}_0$$

$$\hat{e}_\pm = \frac{1}{\sqrt{2}}(\mp \hat{x} - i\hat{y}); \quad \hat{e}_0 = \hat{z}$$

$$\frac{d\Gamma_i}{dq^2} = \frac{\mathcal{G}_f^2}{(2\pi)^3} |V_{bq'}|^2 \frac{(q^2 - m_l^2)^2}{12 M^2 q^2} |\vec{k}| H_i$$

$$\begin{aligned}
H_U &= Re(H_+ H_+^\dagger) + Re(H_- H_-^\dagger) && : Unpolarized-transversed\,, \\
H_L &= Re(H_0 H_0^\dagger) && : Longitudinal\,, \\
H_P &= Re(H_+ H_+^\dagger) - Re(H_- H_-^\dagger) && : Parity-odd\,, \\
H_S &= 3Re(H_t H_t^\dagger) && : Scalar\,, \\
H_{SL} &= Re(H_t H_0^\dagger) && : Scalar-Longitudinal\ Interference\,.
\end{aligned}$$

For Pseudoscalar Meson In Final state

$$\Gamma_P = \Gamma_L + \tilde{\Gamma}_L + \tilde{\Gamma}_S$$

$0^- \to 0^-$ transition state

$$\frac{d\tilde{\Gamma}_i}{dq^2} = \frac{m_l^2}{2q^2} \frac{d\Gamma_i}{dq^2}$$

For Vector meson In Final State

$$\Gamma_V = \Gamma_U + \Gamma_L + \tilde{\Gamma}_U + \tilde{\Gamma}_L + \tilde{\Gamma}_S + \Gamma_P$$

$0^- \to 1^-$ transition state

RELATIVISTC INDEPENDENT QUARK MODEL

A quark potential model succeeds when it reasonably reproduces available observed data in different hadron sectors. Regardless of the Lorentz structure of the interacting potential used, a phenomenological model is considered reliable if it describes confinement & constituent-level dynamics within the hadron core and predicts various hadronic properties, including decays. Therefore, it is crucial to extend the applicability of a quark model to a broader range of observed data.

Interaction Harmonic Potential: $U(r) = \frac{1}{2}(1+\Upsilon_0)(ar^2 + V_0)$, $\pounds = \bar{\psi}_q[\frac{1}{2} i\Upsilon^{\mu}\partial_{\mu} - U(r) - m_q]\,\psi_q(r)$

$(a, V_0) \equiv (0.017166\text{GeV}^3, -0.1375\text{GeV})$

$(m_b, m_c, m_u) \equiv (4.77659,\ 1.49276,\ 0.07875)\text{GeV}$

$(E_b, E_c) \equiv (4.76633,\ 1.57951)\ \text{GeV}$

$$\psi_q^+(\vec{r}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} \frac{i g_q(r)}{r} \\ (\vec{\sigma}.\hat{r})\frac{f_q(r)}{r} \end{pmatrix} \chi_\lambda \qquad \psi_q^-(\vec{r}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} i(\vec{\sigma}.\hat{r})\frac{f_q(r)}{r} \\ g_q(r)/r \end{pmatrix} \tilde{\chi}_\lambda$$

Meson State: $|\, B_c(\vec{P}\,,\ S_{B_c}\rangle = \hat{\Lambda}_{Bc}\,(\vec{P}, S_B)\ |\,(\vec{pb}, \lambda_b);(\vec{pc},\lambda_c)\,\rangle$

$$\hat{\Lambda}_{Bc}\,(\vec{P}, S_B) = \frac{\sqrt{3}}{\sqrt{N_{B_c}(\vec{P})}} \sum_{\delta_b,\delta_{\bar{c}}} \varsigma_{b,\bar{c}}^{B_c}(\lambda_b, \lambda_{\bar{c}}) \int d^3\vec{p}_b\, d^3\vec{p}_{\bar{c}}\ \delta^{(3)}\left(\vec{p}_b + \vec{p}_{\bar{c}} - \vec{P}\right)\mathcal{G}_{B_c}\left(\vec{p}_b, \vec{p}_{\bar{c}}\right)$$

$\mathcal{G}_{B_c}(\vec{p}_b, \vec{p}_{\bar{c}}) = \sqrt{G_b(\vec{p}_b)\tilde{G}_{\bar{c}}(\vec{p}_{\bar{c}})}$

Meson Normalization: $N(\vec{P}) = \int d\vec{pb}\,|\ G(\vec{p}\,b\,, \vec{P} - \vec{p}\,b\,)|^2$

For $0^- \rightarrow 0^-$ transition

$$\mathcal{H}_\mu(B_c \rightarrow (\bar{c}c/\bar{u}c)_{S=0}) = (p+k)_\mu F_+(q^2) + q_\mu F_-(q^2)$$

$$\langle S_X(\vec{k})|V_0|S_{B_c}(0)\rangle = \frac{(E_{p_b}+m_b)(E_{p_{c/u}}+m_{c/u}) + |\vec{p}_b|^2}{\sqrt{(E_{p_b}+m_b)(E_{p_{c/u}}+m_{c/u})}}$$

For $0^- \rightarrow 1^-$ transition

$$\begin{aligned}\mathcal{H}_\mu(B_c \rightarrow (\bar{c}c/\bar{u}c)_{S=1}) = \frac{1}{(M+m)}\epsilon^{\sigma^\dagger}\{&g_{\mu\sigma}(p+k)qA_0(q^2)\\ &+ (p+k)_\mu(p+k)_\sigma A_+(q^2)\\ &+ q_\mu(p+k)_\sigma A_-(q^2)\\ &+ i\epsilon_{\mu\sigma\alpha\beta}(p+k)^\alpha q^\beta V(q^2)\} \quad (4)\end{aligned}$$

$$\langle S_X(\vec{k},\hat{\epsilon}^*)|V_0|S_{B_c}(0)\rangle = 0$$

$$\langle S_X(\vec{k},\hat{\epsilon}^*)|V_i|S_{B_c}(0)\rangle = \frac{i(E_{p_b}+m_b)(\hat{\epsilon}^*\times\vec{k})_i}{\sqrt{(E_{p_b}+m_b)(E_{p_b+k}+m_{c/u})}}$$

$$\langle S_X(\vec{k},\hat{\epsilon}^*)|A_i|S_{B_c}(0)\rangle = \frac{(E_{p_b}+m_b)(E_{p_b+k}+m_{c/u}) - \frac{|\vec{p}_b|^2}{3}}{\sqrt{(E_{p_b}+m_b)(E_{p_b+k}+m_{c/u})}}$$

$$\langle S_X(\vec{k},\hat{\epsilon}^*)|A_0|S_{B_c}(0)\rangle = \frac{-(E_{p_b}+m_b)(\hat{\epsilon}^*.\vec{k})}{\sqrt{(E_{p_b}+m_b)(E_{p_b+k}+m_{c/u})}}$$

LORENTZ INVARIANT FORM FACTORS

$$f_{\pm}(q^2) = \frac{1}{2M}\sqrt{\frac{ME_k}{N_{B_c}(0)N_X(\vec{k})}}\int d\vec{p}_b \mathcal{G}_{B_c}(\vec{p}_b, -\vec{p}_b)\mathcal{G}_X(\vec{k}+\vec{p}_b, -\vec{p}_b)$$
$$\times \frac{(E_{p_b}+m_b)(E_{p_{c/u}}+m_{c/u}) + |\vec{p}_b|^2 \pm (E_{p_b}+m_b)(M \mp E_k)}{E_{p_b}E_{p_{c/u}}(E_{p_b}+m_b)(E_{p_{c/u}}+m_{c/u})}$$

$B_c \to \eta_c(D)$

$$V(q^2) = \frac{M+m}{2M}\sqrt{\frac{ME_k}{N_{B_c}(0)N_X(\vec{k})}}\int d\vec{p}_b \mathcal{G}_{B_c}(\vec{p}_b, -\vec{p}_b)\mathcal{G}_X(\vec{k}+\vec{p}_b, -\vec{p}_b)$$
$$\times \sqrt{\frac{(E_{p_b}+m_b)}{E_{p_b}E_{p_{c/u}}(E_{p_{c/u}}+m_{c/u})}}$$

$$A_0(q^2) = \frac{1}{(M-m)}\sqrt{\frac{Mm}{N_{B_c}(0)N_X(\vec{k})}}\int d\vec{p}_b \mathcal{G}_{B_c}(\vec{p}_b, -\vec{p}_b)\mathcal{G}_X(\vec{k}+\vec{p}_b, -\vec{p}_b)$$
$$\times \frac{(E_{p_b}+m_b)(E^0_{p_{c/u}}+m_{c/u}) - \frac{|\vec{p}_b|^2}{3}}{\sqrt{E_{p_b}E_{p_{c/u}}(E_{p_b}+m_b)(E_{p_{c/u}}+m_{c/u})}}$$

$$A_{\pm}(q^2) = \frac{-E_k(M+m)}{2M(M+2E_k)}\left[T \mp \frac{3(M \mp E_k)}{(E_k^2 - m^2)}\{I - A_0(M-m)\}\right]$$

$B_c \to J/\psi(D^*)$

Predictions of LFUV, τ-polarization & Forward-Backward Asymmetry

Ratio of branching fractions $(\mathcal{R})(l = e, \mu)$	RIQM	CQM [46]	PQCD [43]	LQCD [22]	LCSR [82]
$\mathcal{R}_{\eta_c} = \frac{\mathcal{B}(B_c \to \eta_c \tau \nu_\tau)}{\mathcal{B}(B_c \to \eta_c l \nu_l)}$	0.43	0.26	0.34	···	0.32 ± 0.02
$\mathcal{R}_{J/\psi} = \frac{\mathcal{B}(B_c \to J/\psi \tau \nu_\tau)}{\mathcal{B}(B_c \to J/\psi l \nu_l)}$	0.21	0.24	0.28	0.2582(38)	0.23 ± 0.01

Ratio of branching fractions $(\mathcal{R})$	RIQM	CQM [46]	LQCD [33]
$\mathcal{R}_D = \frac{\mathcal{B}(B_c \to D \tau \nu_\tau)}{\mathcal{B}(B_c \to D \mu \nu_\mu)}$	0.81	0.63	0.682(37)
$\mathcal{R}_{D^*} = \frac{\mathcal{B}(B_c \to D^* \tau \nu_\tau)}{\mathcal{B}(B_c \to D^* \mu \nu_\mu)}$	0.91	0.56	···

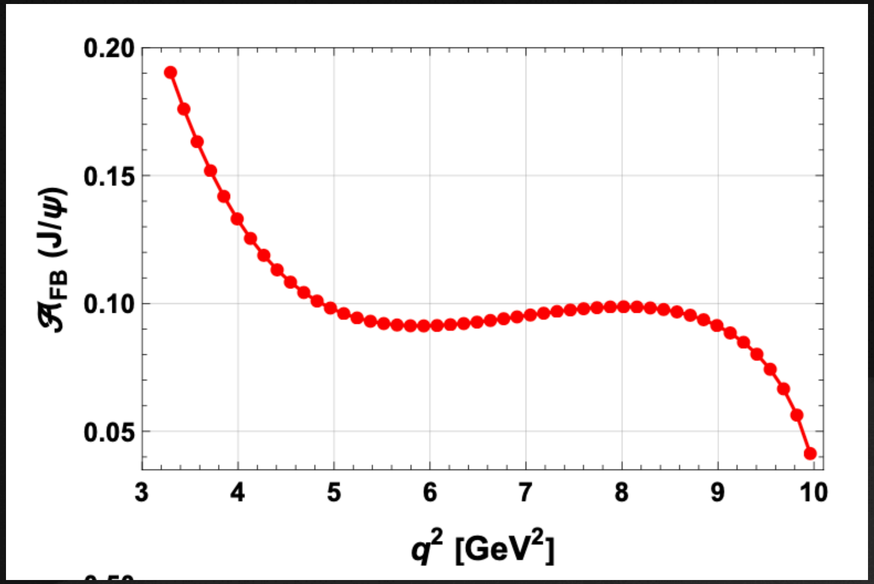
P_τ	RIQM	PQCD [32]	Lattice + PQCD [3
$P_\tau(\eta_c)$	-0.28 ± 0.0001	0.37 ± 0.01	-0.36 ± 0.01
$P_\tau(J/\psi)$	-0.56 ± 0.0003	-0.55 ± 0.01	-0.53 ± 0.01
$P_\tau(D)$	-0.47 ± 0.0001	···	···
$P_\tau(D^*)$	0.14 ± 0.0004	···	···

Decay process	$A_{FB}(l^-)$	$A_{FB}(l^+)$
$B_c \to \eta_c e \nu$	2.049×10^{-7}	2.049×10^{-7}
$B_c \to \eta_c \tau \nu$	0.357	0.357
$B_c \to J/\psi e \nu$	0.180	0.180
$B_c \to J/\psi \tau \nu$	0.093	−0.255
$B_c \to D e \nu$	2.55×10^{-8}	2.55×10^{-8}
$B_c \to D \tau \nu$	0.210	0.210
$B_c \to D^* e \nu$	0.394	−0.394
$B_c \to D^* \tau \nu$	0.137	−0.328

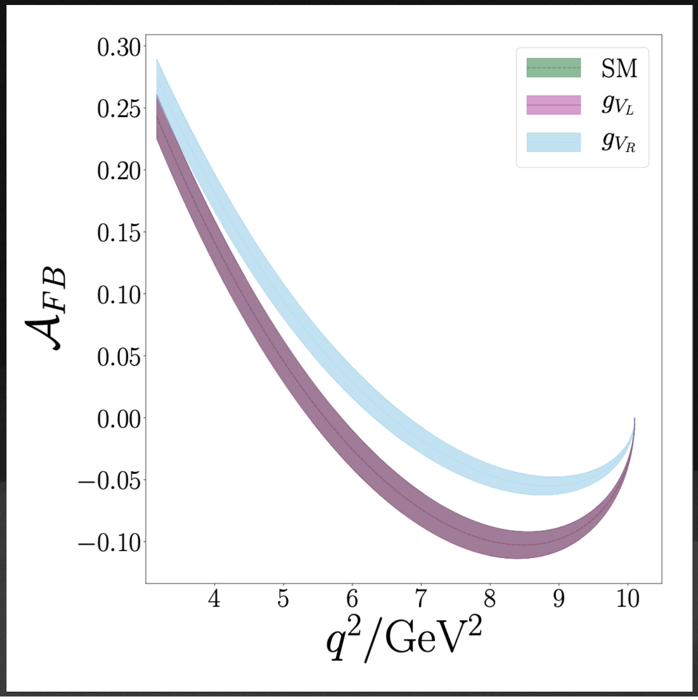
LQCD = 0.058 (12)

RIQM PREDICTIONS

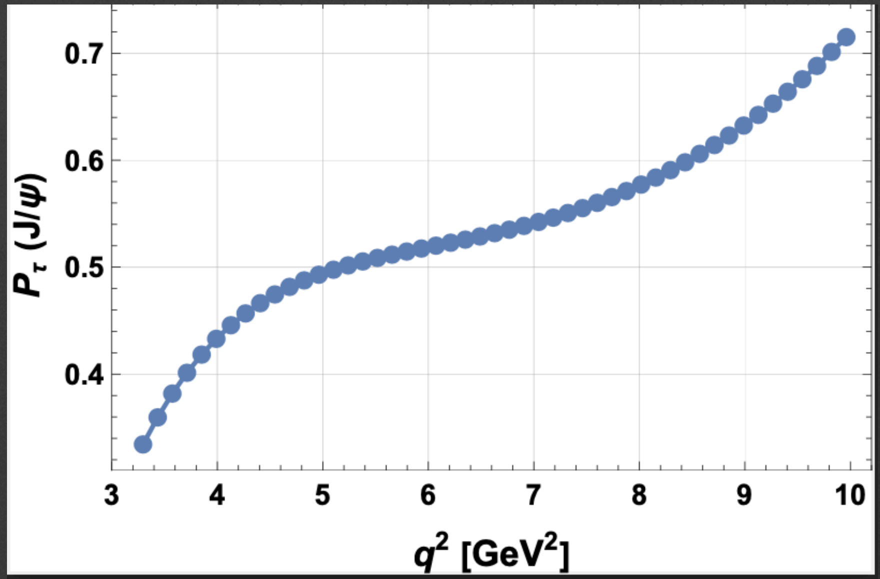
LATTICE PREDICTIONS (HPQCD Collaboration)

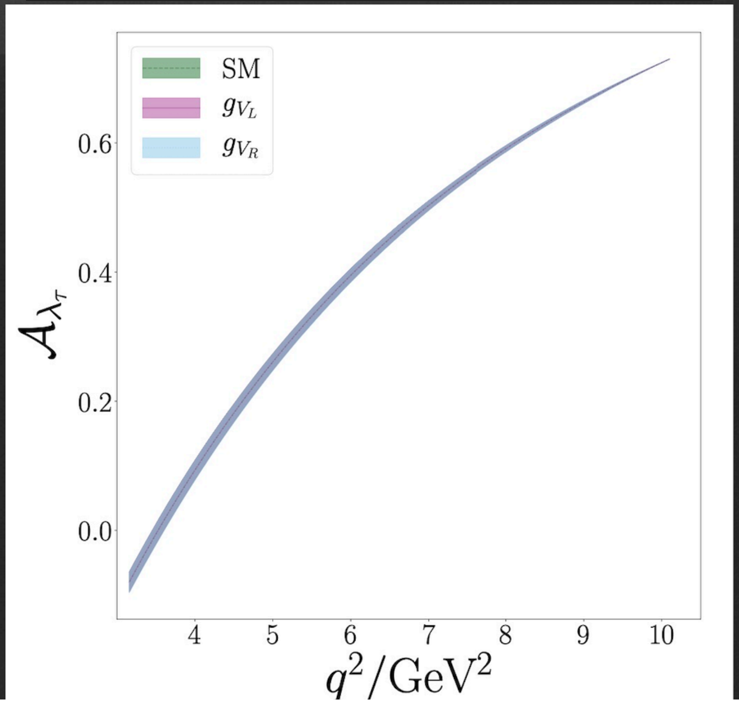


Comparison of q^2 distribution spectra Of AFB and τ polarization



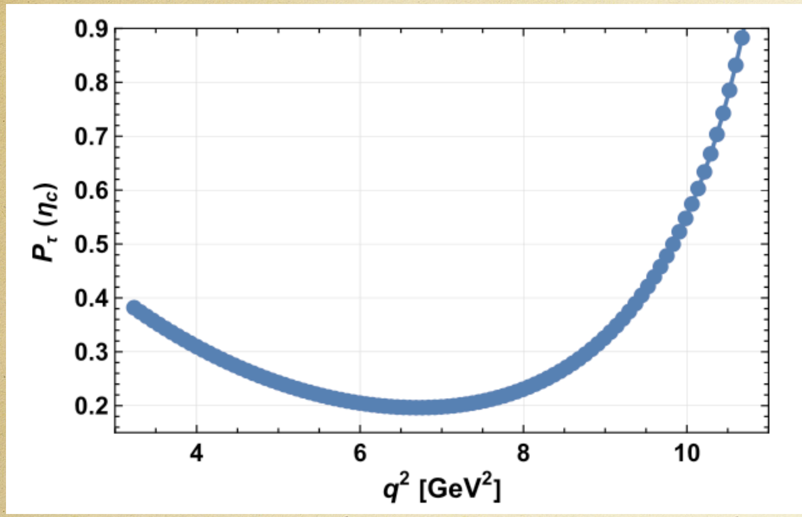
Our results "Qualitatively" agree with the "Lattice Predictions"

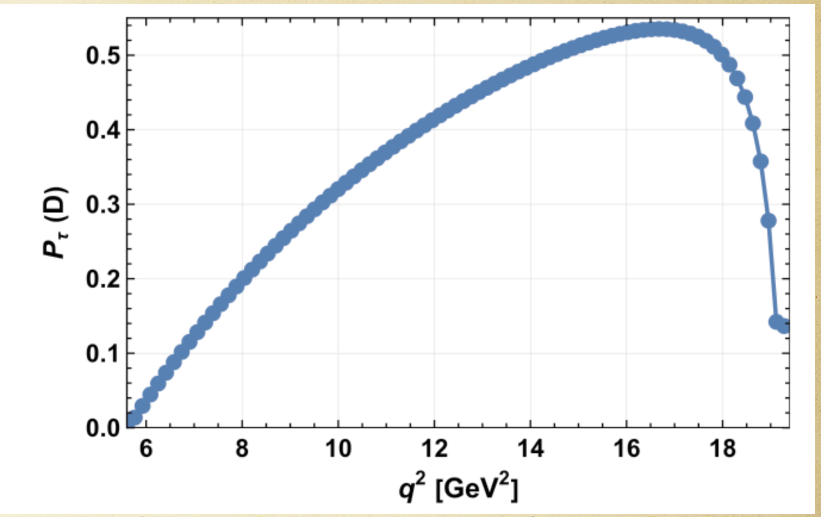




q^2- distribution spectra OF P_τ polarization

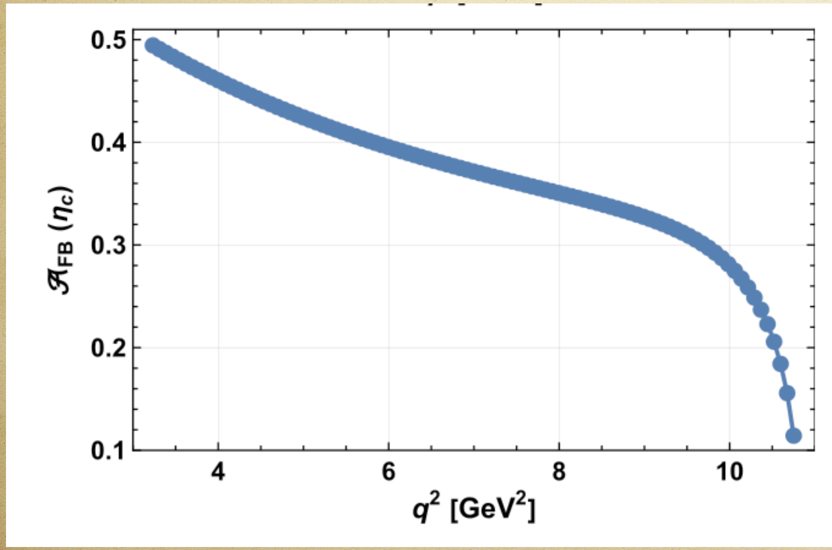
(For η_c and D)

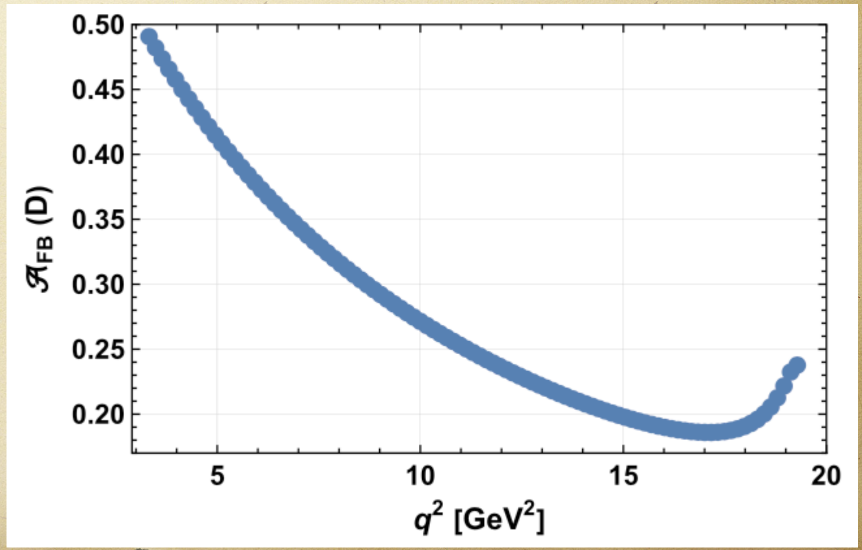




q^2- distribution spectra OF AFB

(For η_c and D)





Opportunities in flavor physics observables in HL- LHC era

Observable	Legacy ($9\,\mathrm{fb}^{-1}$)		2026 ($23\,\mathrm{fb}^{-1}$)	U2 ($300\,\mathrm{fb}^{-1}$)
$\sin 2\phi_1$, with $B^0 \to J/\psi K^0_S$	0.015	[29]	0.011	0.003
ϕ_s, with $B^0_s \to J/\psi K^+K^-$ [mrad]	23	[188]	14	4
$\phi_s^{s\bar{s}s}$, with $B^0_s \to \phi\phi$ [mrad]	80	[65]	39	11
ϕ_3	4°	[32]	1.5°	0.35°
$\|V_{ub}\|/\|V_{cb}\|$	6%	[189]	3%	1%
$\mathcal{R}_{\mu^+\mu^-}$	90%	[76]	34%	10%
R_K $(1 < q^2 < 6\,\mathrm{GeV}^2/c^4)$	0.1	[95]	0.025	0.007
R_{K^*} $(1 < q^2 < 6\,\mathrm{GeV}^2/c^4)$	0.1	[95]	0.031	0.008
$R(D^*)$	0.022	[138]	0.0072	0.002
$R(J/\psi)$	0.24	[144]	0.071	0.02
$\Delta A_{CP}(KK - \pi\pi)$ $[10^{-5}]$	85	[190]	17	3.0

Up to *Nature* whether our " Wish for *Discovery* " is Granted ... or Not ...

Thank You !

BACKUP

NEW COLLIDERS

Lepton Colliders

$e^- \ e^+ \ p \ \bar{p}$

$\mu^- \ \mu^+$

HADRON Colliders

ilc

$\gamma\gamma$

HE-LHC

CEPC

Muon Accelerator Program

SppC

CLIC

Laser Wakefield Accelerators

FCC hh ee he

MESON	MESON MASS (GEV)	
	PREDICTED	EXPERIMENTAL
$D^{\pm *}$	2.0149	2.0101
$D^{\pm}$	1.8538	1.8694
$D_s^{\pm *}$	2.0731	2.1103
$D_s^{\pm}$	1.9149	1.9690
$B^{\pm *}$	5.3292	5.3246
$B^{\pm}$	5.2643	5.2786
B_s^{0*}	5.3720	5.4256
B_s^{0}	5.3055	5.3786
$B_c^{\pm *}$	6.3142	-
$B_c^{\pm}$	6.2707	6.2749

$(a, V_0) \equiv (0.017166\ GeV^3, -0.1375\ GeV)$

Quark q	m_q (GeV)
u	0.07875
d	0.07875
s	0.31575
c	1.49276
b	4.77659

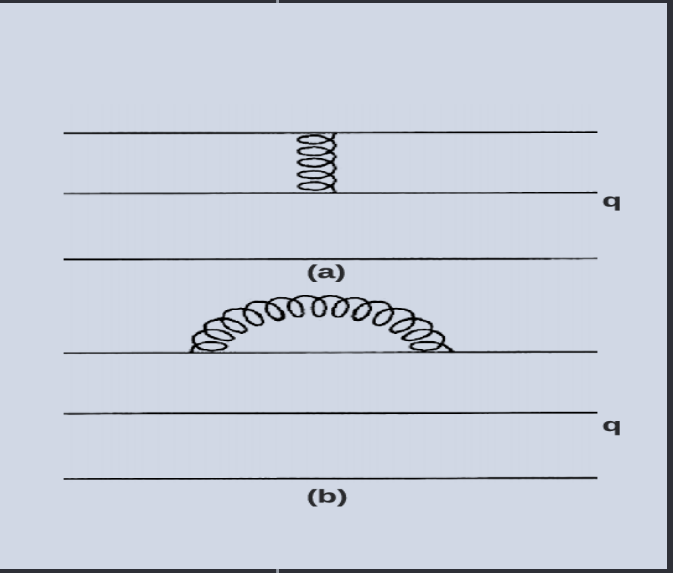
- *Barik and Dash: Phys. Rev.D. 33, 1925 (1986)*
  - *Pramana-J. Phys: 29(6), 543-557 (1987)*

GLUONIC CORRECTION

- *Inside the mesons quark and antiquark are bound. The binding is due to exchange of gluons.*

Taking one gluon exchange the interaction lagrangian density is in the form of

$$\pounds_I^g = \sum_{\infty} J_i^{\mu\,a} A_\mu^a(x)$$



(One gluon exchange contribution to the energy conservation)

Where $A_\mu^a(x)$ is the vector-gluon fields and $J_i^{\mu\,a}$ is the colour current

Since at small distance the quarks should be almost free, it is reasonable to calculate the shift in the energy of meson core using first oder perturbation theory.

Energy shift is in the form:

$$(\Delta E)_g = (\Delta E)_g^{\epsilon} + (\Delta E)_g^{M}$$

$$(\Delta E_M)_g^{\epsilon} = \alpha_s \sum_{i,j} \left\langle \sum_a \lambda_i^a \lambda_j^a \right\rangle \frac{1}{\sqrt{\pi} R_{ij}} \left(1 - \frac{\alpha_i + \alpha_j}{R_{ij}^2} + \frac{3\alpha_i \alpha_j}{R_{ij}^4}\right)$$

$$(\Delta E_M)_g^{M} = \alpha_s \sum_{i<j} \left\langle \sum_a \lambda_i^a \lambda_j^a \sigma_i \sigma_j \right\rangle \frac{256}{9\sqrt{\pi}} \frac{1}{(3E_i' + m_i')(3E_j' + m_j')} \frac{1}{R_{ij}^3}.$$

So total energy of meson in its ground state is

$$E_M = E_M^0 + (\Delta E)_g^{\epsilon} + (\Delta E)_g^{M}$$

CENTER OF MASS CORRECTION

- *The independent motion of quarks inside the hadron core does not lead to a state of definite total momentum*
- *The energy associated with the spurious centre of mass motion must provide a further correction to the hadron energy obtained*
- *This prescription was given by:*

1. Wong C W Phys.Rev. D 24, 1416 (1981)
2. Duck I Phys.Lett. 77, 223, 1978
3. Bertelski J et al. Phys.Rev.D. 29, 1035 (1984)

The static meson core state is decomposed into plane-wave momentum eigen state:

$$|M(x)> = \int \frac{d^3P}{W_M(P)} \exp(iP.X)\varphi_M(P)\, |M(P)>$$

The inverse relation is:

$$|M(P)> = \frac{1}{(2\pi)^3} \frac{W_M(P)}{\varphi_M(P)} \int d^3X exp(-iP.X) |M(X)>$$

- *The normalisation is as follows:*

$$\langle M(P)|M(P')\rangle = (2\pi)^3 2E_p \delta(P - P')$$

$$\varphi_M^2 = \frac{W_n(P)}{(2\pi)^3} \tilde{I}_M(p)$$

- *This permits ready estimates of the centre of mass-momentum P*

$$\langle P^2\rangle = \int d^3p \check{I}_m(P) P^2$$

$$= \sum_q \langle p^2 \rangle_q \quad \text{Where} \quad \langle p^2 \rangle = \frac{\left(11E'_q + m'_Q\right)(E'^2_q - m'^2_q)}{6(3E'_q + m'_q)}$$

- *Here* $\langle p^2\rangle_q$ *is the average value of the square of the individual quark-momentum*

- *Mass of Meson* : $m_M = [\{E_M^0 + (\Delta E)_g^{\in} + (\Delta E)_g^{*M}\}^2 - \langle P^2 \rangle]^{1/2}$