

Renormalization-group improved Higgs to two gluons decay rate

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The partial decay width of the Higgs-boson to gluons $\Gamma_{H \rightarrow gg}$ decay is given as,

$$\Gamma_{H \rightarrow gg} = \frac{\sqrt{2} G_F}{M_H} |C_1|^2 \text{Im} \Pi^{GG}(-M_H^2 - i\delta), \quad (1)$$

The coefficient C_1 is known up to N⁴LO and its perturbative expansion is given by,

$$C_1 = -\frac{1}{3} a_s \left(1 + \sum_{n=1} c_n a_s^n(\mu^2) \right), \quad (2)$$

where $a_s \equiv \frac{\alpha_s^{n_f}}{4\pi}$ where n_f are number of light flavours. The absorptive part of the vacuum polarization is computed at N⁴LO and written in the following form,

$$\frac{4\pi}{N_A q^4} \text{Im} \Pi^{GG}(q^2) \equiv G(q^2) = 1 + \sum_{n=1} g_n a_s^n, \quad (3)$$

The decay width of $H \rightarrow gg$ can be written as,

$$\Gamma = \left[\sqrt{2} G_F M_H^3 / 72\pi \right] x^2(\mu) S[x(\mu), L(\mu)], \quad (4)$$

where the perturbative expansion $S[x(\mu), L(\mu)]$ in the so called “fixed-order perturbation theory” (FOPT) is written as,

$$S_{\text{FOPT}}[x(\mu), L(\mu)] = \sum_{n=0}^{\infty} \sum_{k=0}^n T_{n,k} x^n L^k. \quad (5)$$

where $x(\mu) = \frac{\alpha_s^{n_f}(\mu)}{\pi}$ and $L(\mu) = \ln(\mu^2/m_t^2(\mu))$.

Suppose for the decay process the perturbation series $S[x, L]$

$$S^{NLO} = T_{0,0} + (T_{1,0} + T_{1,1}L) x$$

$$S^{N^2LO} = S^{NLO} + (T_{2,0} + T_{2,1}L + T_{2,2}L^2) x^2$$

$$S^{N^3LO} = S^{N^2LO} + (T_{3,0} + T_{3,1}L + T_{3,2}L^2 + T_{3,3}L^3) x^3$$

$$S^{N^4LO} = S^{N^3LO} + (T_{4,0} + T_{4,1}L + T_{4,2}L^2 + T_{4,3}L^3 + T_{4,4}L^4) x^4.$$

These NLO and higher-order expressions exhibit scale dependence as the magnitude of L increases.

In the RGSPT, the FOPT expansion of the function $S[x(\mu), L(\mu)]$ is equivalent to writing the following new expansion,

$$S(x, L) = \sum_{n=0}^{\infty} x^n S_n(xL), \quad (6)$$

function $S_n(xL)$ is defined by,

$$S_n(xL) \equiv \sum_{k=n}^{\infty} T_{k,k-n}(xL)^{k-n}. \quad (7)$$

For $n = 0 - 4$,

$$S_0(xL) \equiv T_{0,0} + T_{1,1}xL + T_{2,2}x^2L^2 + T_{3,3}x^3L^3 + \dots = \sum_{n=0}^{\infty} T_{n,n}x^n L^n$$

$$S_1(xL) \equiv T_{1,0} + T_{2,1}xL + T_{3,2}x^2L^2 + \dots = \sum_{n=1}^{\infty} T_{n,n-1}(xL)^{n-1}$$

$$S_2(xL) \equiv T_{2,0} + T_{3,1}xL + T_{4,2}x^2L^2 + \dots = \sum_{n=2}^{\infty} T_{n,n-2}(xL)^{n-2}$$

$$S_3(xL) \equiv T_{3,0} + T_{4,1}xL + T_{5,2}x^2L^2 + \dots = \sum_{n=3}^{\infty} T_{n,n-3}(xL)^{n-3}$$

$$S_4(xL) \equiv T_{4,0} + T_{5,1}xL + T_{6,2}x^2L^2 + \dots = \sum_{n=4}^{\infty} T_{n,n-4}(xL)^{n-4}$$

The main feature of the RGSPT is the explicit all-orders summations of all RG-accessible logarithms in the function $S_n(xL)$.

The functions $S_n(u)$, where $u = xL$ can be derived in a closed analytical form using the RG invariance of $\Gamma(H \rightarrow gg)$ decay width:

$$\mu^2 \frac{d}{d\mu^2} \{\Gamma_{H \rightarrow gg}\} = 0. \quad (8)$$

$$(1 - \beta_0 u) \frac{dS_n}{du} - u \sum_{\ell=0}^{n-1} \beta_{\ell+1} \frac{dS_{n-\ell-1}}{du} + 2 \sum_{\ell=0}^{n-1} \gamma_{\ell} \frac{dS_{n-\ell-1}}{du} - \sum_{\ell=0}^n (n - \ell + 2) \beta_{\ell} S_{n-\ell} = 0. \quad (9)$$

The new RGS expansions now can be written as,

$$S_{RGSPT}^{N^4LO} = S_0(xL) + xS_1(xL) + x^2S_2(xL) + x^3S_3(xL) + x^4S_4(xL) \quad (10)$$

The above RGSPT expansions exhibit good stability and reduced sensitivity to RG scale μ .

Scale and scheme dependence in the FOPT and the RGSPT₉

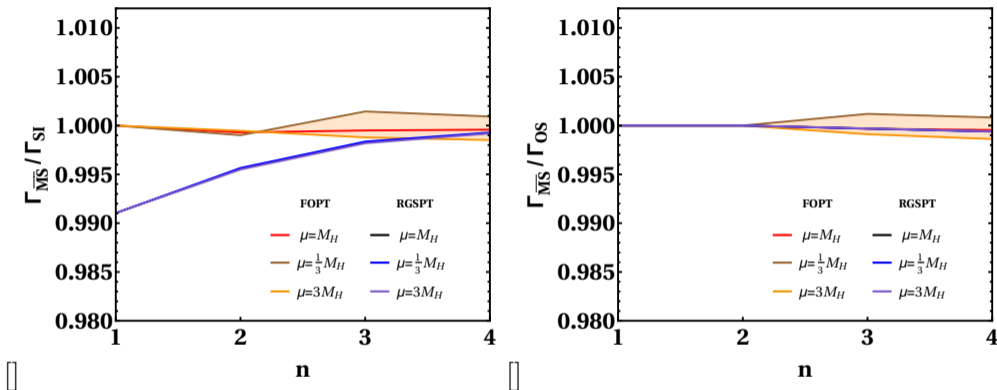


Figure 1: The variation of $\Gamma_{\overline{MS}}/\Gamma_{SI}$ and $\Gamma_{\overline{MS}}/\Gamma_{OS}$ at RG scales $\mu = \frac{1}{3}M_H, M_H,$ and $3M_H$ in the FOPT and RGSPT up to order $n = 4$.

Asymptotic Padé approximant improved $H \rightarrow gg$ decay rate¹⁰

Considering a generic perturbative expansion of the form,

$$S \equiv 1 + R_1x + R_2x^2 + R_3x^3 + R_4x^4 + \dots, \quad (11)$$

where the coefficients $\{R_1, R_2, R_3, R_4\}$ are known and the coefficients $\{R_5, \dots\}$ are unknown.

The Padé approximant to a generic perturbative expansion is denoted by,

$$\begin{aligned} S_{[N|M]} &\equiv \frac{1 + a_1x + a_2x^2 + \dots + a_Nx^N}{1 + b_1x + b_2x^2 + \dots + b_Mx^M} \\ &= 1 + R_1x + R_2x^2 + R_3x^3 + R_4x^4 + \dots + R_{N+M+1}x^{N+M+1} + \dots \end{aligned} \quad (12)$$

The asymptotic error in the Padé approximant prediction is given by,

$$\frac{R_{N+M+1}^{Padé} - R_{N+M+1}}{R_{N+M+1}} = -\frac{M!A^M}{[N + M + aM + b]^M} \quad (13)$$

In this work, we choose $a = b = 0$ which provide the best predictions. Our APAP estimate of the true value R_5 is,

$$\begin{aligned} R_5 &= \frac{(-R_3^3 + 2R_2R_3R_4 - R_1R_4^2)}{(1 + \delta)(R_2^2 - R_1R_3)} \\ &= \frac{8R_2^2(R_3^3 - 2R_2R_3R_4 + R_1R_4^2)}{(R_1^4 - 2R_1^2R_2 - 7R_2^2)(R_2^2 - R_1R_3)}. \end{aligned} \quad (14)$$

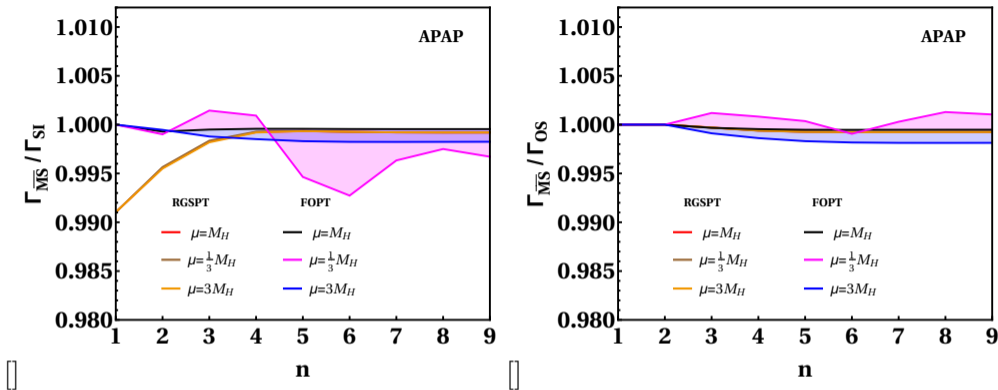


Figure 2: The variation of $\Gamma_{\overline{MS}}/\Gamma_{SI}$ and $\Gamma_{\overline{MS}}/\Gamma_{OS}$ at RG scales $\mu = \frac{1}{3}M_H, M_H,$ and $3M_H$ in the FOPT and RGSPT up to order $n = 9$.

We apply the APAP formalism to the generalized Borel transform (eq.15) of the FOPT expansion of the Higgs to gluons decay width. This method is referred to as Padé-Borel approximant (PBA).

$$B[S](u) = \sum_n^{\infty} \left(\frac{d_1}{n!} + \frac{d_2}{n!^2} + \frac{d_3}{n!^3} + \frac{d_5}{n!^5} \right) R_n u^n, \quad (15)$$

where $d_{1,2,3,5}$ are the scheme-dependent real constants given in table 1.

Schemes	d_1	d_2	d_3	d_5
$\overline{\text{MS}}$	0.5	1.5	0	1.2
OS	1	0	1.623	0
SI	0.87	0	1.6	0

The numerical values of the constants $d_{1,2,3,5}$.

Our predictions for the $\Gamma(H \rightarrow gg)$ decay width at the order N⁵LO in the APAP formalism in the FOPT are,

$$\begin{aligned}\Gamma_{\text{N}^5\text{LO}}^{\overline{\text{MS}}} &= \Gamma_0 \left(1.837 \pm 0.047_{\alpha_s(M_Z), 1\%} \pm 0.0004_{M_t} \pm 0.0066_{M_H} \pm 0.0009_{\text{P}} \pm 0.007_{\text{s}} \right), \\ \Gamma_{\text{N}^5\text{LO}}^{\text{SI}} &= \Gamma_0 \left(1.837 \pm 0.046_{\alpha_s(M_Z), 1\%} \pm 0.0004_{M_t} \pm 0.0066_{M_H} \pm 0.0026_{\text{P}} \pm 0.007_{\text{s}} \right), \\ \Gamma_{\text{N}^5\text{LO}}^{\text{OS}} &= \Gamma_0 \left(1.838 \pm 0.047_{\alpha_s(M_Z), 1\%} \pm 0.0004_{M_t} \pm 0.0066_{M_H} \pm 0.0023_{\text{P}} \pm 0.007_{\text{s}} \right).\end{aligned}$$

Our predictions for the $\Gamma(H \rightarrow gg)$ decay width at the order N⁵LO in the APAP formalism in the RGSPT are,

$$\Gamma_{\text{RGSN}^5\text{LO}}^{\overline{\text{MS}}} = \Gamma_0 \left(1.840 \pm 0.047_{\alpha_s(M_Z), 1\%} \pm 0.0005_{M_t} \pm 0.0066_{M_H} \pm 0.0002_{\mu} \pm 0.0007_{\text{P}} \right),$$

$$\Gamma_{\text{RGSN}^5\text{LO}}^{\text{SI}} = \Gamma_0 \left(1.841 \pm 0.047_{\alpha_s(M_Z), 1\%} \pm 0.0005_{M_t} \pm 0.0066_{M_H} \pm 0.0002_{\mu} \pm 0.0018_{\text{P}} \right),$$

$$\Gamma_{\text{RGSN}^5\text{LO}}^{\text{OS}} = \Gamma_0 \left(1.842 \pm 0.047_{\alpha_s(M_Z), 1\%} \pm 0.0005_{M_t} \pm 0.0066_{M_H} \pm 0.0002_{\mu} \pm 0.0019_{\text{P}} \right).$$

- ▶ The dependence of the perturbative expansion on the RG scale μ is considerably reduced in the RGSPT.
- ▶ We have calculated the higher-order corrections to the $\Gamma(H \rightarrow gg)$ decay width in the APAP and PBA formalisms.
- ▶ The predictions of PBA method are found to be in agreement with that of APAP method.
- ▶ The RGSPT expansion continue to show greater stability against the RG scale at higher orders in the APAP as well as the PBA frameworks.
- ▶ The uncertainty due to truncation of the series is 0.6% at N⁴LO, and reduces to 0.4% at N⁵LO in the FOPT.

Thank You