Impact of Finite Temperatures and Ultra strong Magnetic Fields on Anisotropic Magnetized White Dwarfs in γ-metric formalism

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భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
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Outline of the talk

- Magnetized White Dwarfs
- Density Dependent Anisotropic Magnetic Field
- Anisotropic EoS for magnetized electron gas at finite temperature
- Equation of State of Anisotropic Magnetized Hot White Dwarfs
- Stellar Structure Equations of Anisotropic Magnetized Hot White Dwarfs
- Numerical Results
- Summary

Magnetized White Dwarfs

- Magnetized white dwarfs are formed as the remnants of low to intermediate mass stars upto ~ 10 $M_\odot$ and supported by relativistic electron degeneracy pressure against gravitational collapse.
- Observations have revealed that 10% of all identified white dwarfs have high magnetic fields ~ 1 MG. Some of them have surface magnetic fields as high as 10^9 Gauss (PG 1031+234), comparable to the magnetic fields of neutron stars.
- Moreover, observations have shown that ultra magnetized white dwarfs are ultra massive and expected to have a magnetic field strength ~ 10^{12}-10^{15} Gauss at their centres.
- The facts that highly massive white dwarfs are strongly magnetized and the existence of super-Chandrasekhar white dwarfs reveal some connection between the maximum mass and the magnetic field strength.
- The theoretical research is continuing on the equation of state (EoS) and mass-radius relations of these super-Chandrasekhar white dwarfs.

Density Dependent Anisotropic Magnetic Field

- In our work, we consider the magnetic field is density dependent which is given as,

$$B_D = B_s + B_0 \left(1 - \exp\left\{ -\alpha \left(\frac{n_e}{n_0} \right)^{\beta} \right\} \right)$$

$B_D = B/B_c$ = Dimensionless magnetic field at electronic number density n_e

B_s = Surface magnetic field strength

n_0 = Central electron number density

α, β and B_0 are constant

Parameter	Value
B_s	10^9 Gauss
α	0.8
β	0.9
n_0	$n_e\,(\mathrm{r} = 0)/10$

Anisotropic EoS for magnetized electron gas at finite temperature

- In the presence of magnetic field, the total energy of degenerate electrons is quantized into Landau levels is given as,

$$E_{\nu,p_z} = \nu\hbar\omega_c + \frac{p_z^2}{2m_e}$$

$$\nu = n + \frac{1}{2} + s_z$$

- For extremely strong magnetic field i.e. $\hbar\omega_c \geq m_e c^2$, the degenerate electrons become highly relativistic. Then the total energy of relativistic degenerate electrons is derived from Dirac equation is given as,

$$E_{\nu,p_z} = [p_z^2 c^2 + m_e^2 c^4 (1 + 2\nu B_D)]^{1/2}$$

$B_D = B/B_c =$ Dimensionless magnetic field.

$B_c =$ Critical magnetic field $= \hbar\omega_c = \hbar\frac{|e|B_c}{m_e c} = m_e c^2$

$\Rightarrow B_c = \frac{m_e^2 c^3}{|e|\hbar} = 4.414 \times 10^{13}\, gauss.$

Anisotropic EoS for magnetized electron gas at finite temperature

- In the presence of strong magnetic field, the thermodynamics quantities of degenerate electron gas system at finite temperature are given as,

- The number density of electrons at a given temperature and magnetic field is given as, (Ref: Strickland et al. 2012)

$$n_e = \sum_{\nu=0} \frac{2\pi}{h^2} m_e^2 c^2 B_D g_\nu \int f(E) \frac{dp_z}{h} \quad ,$$

$$= \frac{2\pi}{h^2} m_e^2 c^2 B_D \sum_{\nu=0}^{\infty} g_\nu \int_{-\infty}^{+\infty} \frac{1}{e^{\beta(E_{\nu,p_z} - \mu)} + 1} \frac{dp_z}{h},$$

$$= \frac{(m_e c^2)^3 B_D}{2\pi^2 (\hbar c)^3} \sum_{\nu=0}^{\nu_{max}} g_\nu \int_{t(\nu,p_z=0)}^{\infty} \frac{\tilde{\beta}(1+\tilde{\beta}t)\, dt}{(e^{t-\eta}+1)(\sqrt{(1+\tilde{\beta}t)^2 - (1+2\nu B_D)})}$$

IMP. FORMULAE:

At T > 0 K,

The Fermi distribution function is given by,

$$f(E) = \frac{1}{exp\, \beta(E-\mu)+1}$$

Where, $\beta = \frac{1}{k_B T}$.

- For numerical computation, we taken the maximum energy level when

$$(E_F - \mu) = 30\, k_B \mathrm{T} \ ,$$

This gives the maximum Landau level to be ,

$$\nu_{\max} = \frac{1}{2B_D}\left(\frac{(30 k_B T + \mu)^2}{m_e^2 c^4} - 1\right)$$

For p_z=0, we have

$$t_{(\nu,p_z=0)} = \frac{1}{\tilde{\beta}}\left(\sqrt{1 + 2\nu B_D} - 1\right)$$

Thus,

$$\nu_{\max} = \frac{1}{2B_D}\left(\left(\tilde{\beta}(30+\eta)+1\right)^2 - 1\right).$$

Anisotropic EoS for magnetized electron gas at finite temperature

- The energy density of electrons in magnetic field at a given temperature is given by,

$$\varepsilon_e = \sum_{\nu=0} \frac{2\pi}{h^2} m_e^2 c^2 B_D g_\nu \int E_{\nu,p_z} f(E) \frac{dp_z}{h}$$

$$= \frac{2\pi}{h^2} m_e^2 c^2 B_D \sum_{\nu=0}^{\infty} g_\nu \int_{-\infty}^{+\infty} \frac{E_{\nu,p_z}}{e^{\beta(E_{\nu,p_z}-\mu)}+1} \frac{dp_z}{h},$$

$$= \frac{(m_e c^2)^4 B_D}{2\pi^2 (\hbar c)^3} \sum_{\nu=0}^{\nu_{max}} g_\nu \int_{t(\nu,p_z=0)}^{\infty} \frac{\tilde{\beta}(1+\tilde{\beta}t)^2 dt}{(e^{t-\eta}+1)\sqrt{(1+\tilde{\beta}t)^2-(1+2\nu B_D)}}$$

IMP. FORMULAE:

Some dimensionless quantities are defined as,

$$\eta = \frac{\mu - m_e c^2}{k_B T} = \frac{\tilde{\mu}}{k_B T},$$

$$\tilde{E} = E_{\nu,pz} - m_e c^2,$$

$$t = \frac{\tilde{E}}{k_B T},$$

$$\tilde{\beta} = \frac{k_B T}{m_e c^2}.$$

Anisotropic EoS for magnetized electron gas at finite temperature

- The parallel pressure (pressure along the direction of the magnetic field) of the electron gas is given as,

$$P_{\parallel e} = \sum_{\nu=0} \frac{2\pi}{h^3} m_e^2 c^2 B_D g_\nu \int \frac{c^2 p_z{}^2}{E_{\nu,p_z}} f(E)\, dp_z\,,$$

$$= \frac{(m_e c^2)^4 B_D}{2\pi^2 (\hbar c)^3} \sum_{\nu=0}^{\nu_{max}} g_\nu \int_{t(\nu,p_z=0)}^{\infty} \frac{\sqrt{(1+\tilde{\beta} t)^2 - (1+2\nu B_D)}\, dt}{(e^{t-\eta}+1)(\sqrt{(1+\tilde{\beta} t)^2 - (1+2\nu B_D)}}$$

- The perpendicular pressure (pressure perpendicular to the direction of the magnetic field) of the electron gas is also given as,

$$P_{\perp e} = \sum_{\nu=0} \frac{2\pi}{h^3} m_e^2 c^2 (B_D)^2 g_\nu \left(\frac{m_e^2 c^4}{2}\right) \int \frac{2\nu}{E_{\nu,p_z}} f(E)\, dp_z\,,$$

$$= \frac{(m_e c^2)^4 B_D}{2\pi^2 (\hbar c)^3} \sum_{\nu=0}^{\nu_{max}} g_\nu \int_{t(\nu,p_z=0)}^{\infty} \frac{2\nu\tilde{\beta}\, dt}{(e^{t-\eta}+1)\sqrt{(1+\tilde{\beta} t)^2 - (1+2\nu B_D)}}$$

Equation of State of Anisotropic Magnetized Hot White Dwarfs

- We can get full EoS of an anisotropic magnetized white dwarf by considering both matter and field contribution in account.
- The total energy density, total parallel pressure and total perpendicular pressure of an anisotropic magnetized white dwarf are given as,

A. $$\varepsilon_T = \varepsilon_e + n_e(m_p + m_n)c^2 + \frac{B^2}{8\pi}$$

B. $$P_{\parallel T} = P_{\parallel e} - \frac{B^2}{8\pi}$$

C. $$P_{\perp T} = P_{\perp e} + \frac{B^2}{8\pi}$$

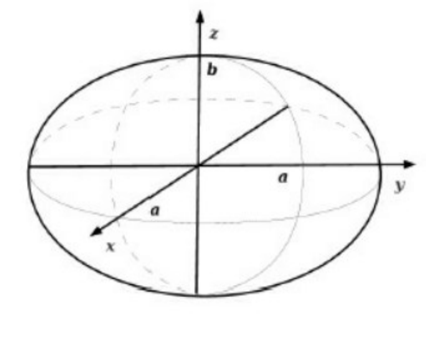
D. $$P_{avg} = (P_{\parallel T} + 2P_{\perp T})/3$$

Stellar Structure Equations of Anisotropic Magnetized Hot White Dwarfs

- Due to anisotropy in pressures along both parallel and perpendicular directions of the magnetic field, the magnetized white dwarf undergoes axisymmetric deformation.
- We study the stellar structure equations of deformed magnetized white dwarf by using **γ** -metric formalism.

Ref: Zubairi, et. al., "Stellar structure models of deformed neutron stars." [Int. J. Mod. Phys. Conf. Ser., 45, (2017)]

- The **γ**-metric formalism is the small axisymmetric deviation from spherical Schwarzschild metric.
- The **γ** -metric which describe the deformed compact object with axisymmetric is given as,



$$ds^2 = -\left[1 - \frac{2M(r)}{r}\right]^{\gamma} dt^2 + \left[1 - \frac{2M(r)}{r}\right]^{-\gamma} dr^2 + r^2 \sin^2\theta \, d\phi^2 + r^2 \, d\theta^2$$

- To show how the magnetized EoS at finite temperature relates to the stellar structure equations and the related anisotropy, we have defined **γ** -metric as the ratio between the total central parallel pressure and total central perpendicular pressure respectively.

$$\gamma = \frac{P_{\parallel T0}}{P_{\perp T0}}$$

$\gamma = \frac{z}{r}$ = gamma metric

z = polar radius

r = equatorial radius

Stellar Structure Equations of Anisotropic Magnetized Hot White Dwarfs

- In General Relativity , Einstein's field equations are given as,

$$G_{\mu v} = R_{\mu v} - \frac{1}{2} R g_{\mu v} = \frac{8\pi G}{c^4} T_{\mu v}$$

$G_{\mu v}$ = Einstein tensor, $R_{\mu v}$ = Ricci tensor

R = Ricci curvature scalar, $g_{\mu v}$ = metric tensor

$T_{\mu v}$ = Energy- momentum tensor of the anisotropic deformed magnetized WD

$$T_{\mu v} = \begin{pmatrix} \varepsilon_T & 0 & 0 & 0 \\ 0 & P_{\perp T} & 0 & 0 \\ 0 & 0 & P_{\perp T} & 0 \\ 0 & 0 & 0 & P_{\parallel T} \end{pmatrix}$$

- The stellar structure equations resulting from Einstein's field equations using the metric described above and the energy-momentum tensor derived from the anisotropic magnetized EoS, are presented as follows,

$$\frac{dm}{dr} = 4\pi r^2 \gamma \left(\frac{\epsilon_{\parallel T} + \epsilon_{\perp T}}{2} \right),$$

$$\frac{dP_{\parallel T}}{dz} = -\frac{(\epsilon_{\parallel T} + P_{\parallel T}) \left[\frac{r}{2} + 4\pi r^3 P_{\parallel T} - \frac{r}{2} \left(1 - \frac{2m}{r}\right)^{\gamma} \right]}{\gamma r^2 \left(1 - \frac{2m}{r}\right)^{\gamma}},$$

$$\frac{dP_{\perp T}}{dr} = -\frac{(\epsilon_{\perp T} + P_{\perp T}) \left[\frac{r}{2} + 4\pi r^3 P_{\perp T} - \frac{r}{2} \left(1 - \frac{2m}{r}\right)^{\gamma} \right]}{r^2 \left(1 - \frac{2m}{r}\right)^{\gamma}}. \quad (17)$$

Ref: Terrero, et. al., Physical Review D, 99, (2019)

NUMERICAL COMPUTATION:

- We have solved the equation of state (EoS) at finite, nonzero temperatures, specifically 0, 10^6, 10^7, and 10^8 K, for various values of dimensionless central magnetic field strengths, i.e., B_{DC} = 1, 2, 3, 4, 5, 6, 7, and 8 (in units of B_c), which are considered density-dependent inside the white dwarf.
- Then we have solved the stellar structure equations in the presence of gamma-metric formalism by using below mentioned boundary conditions.

➢ Boundary conditions for computation:

- In the numerical approach, we start from a point at the centre with $\varepsilon_{T_0} = \varepsilon_T(\mathrm{r}=0)$, $P_{\perp \mathrm{T}_0} = P_{\perp \mathrm{T}}(r=0)$, $P_{\parallel T_0} = P_{\parallel}(r=0)$ from the EoS.
- The equatorial and polar radii of the star R and Z= γR are defined by $P_{\parallel T}(Z) = 0$ and the mass of the star is M=M(R).

Numerical Results:

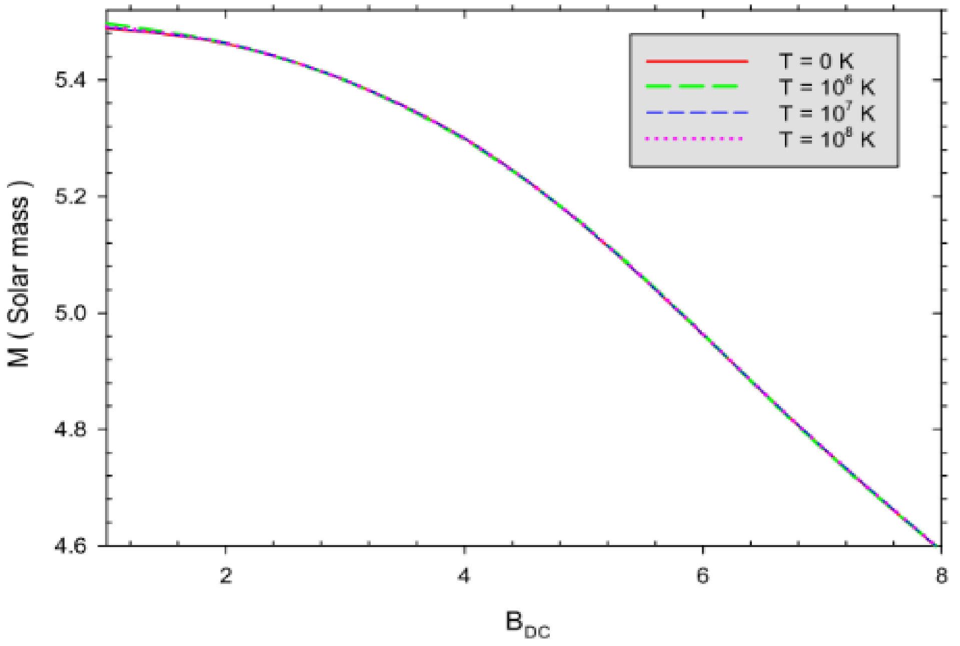


Figure 1. Plot of Mass as a function of the central magnetic field strength of anisotropic magnetized white dwarfs for a fixed central electron density $\sim 3.0 \times 10^{-6}\ fm^{-3}$ at temperatures $T = 0$, 10^6, 10^7 and 10^8 K, respectively.

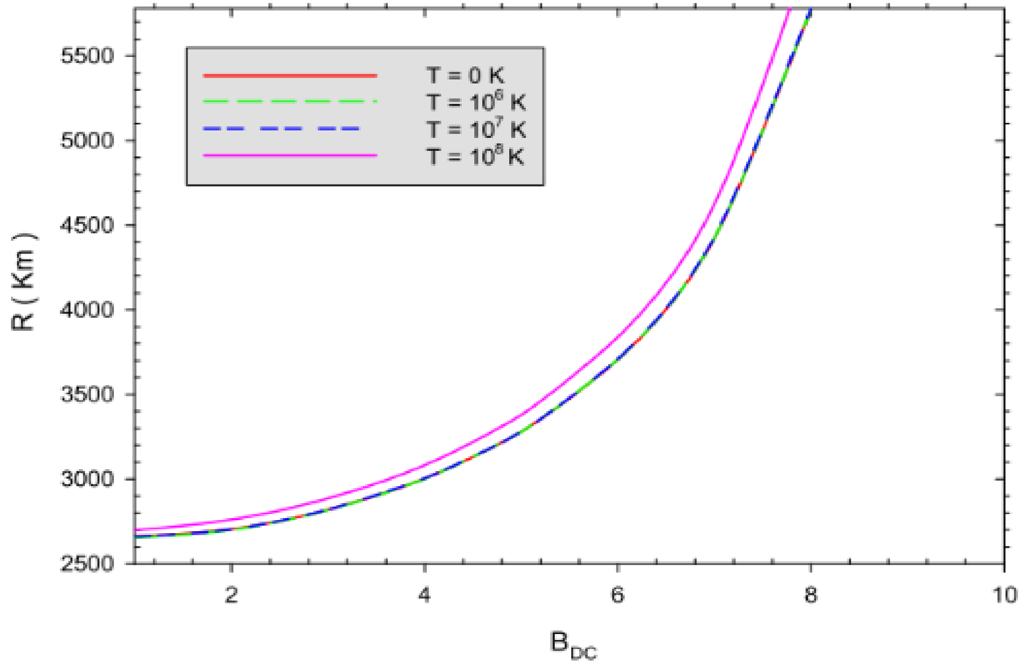


Figure 2. Plot of equatorial radius as a function of the central magnetic field strength of anisotropic magnetized white dwarfs for a fixed central electron density $\sim 3.0 \times 10^{-6}\ fm^{-3}$ at temperatures $T = 0$, 10^6, 10^7 and 10^8 K, respectively.

Ref: R Sahoo, et. al., J. Astrophys. Astr., 45, 28(2024)

Numerical Results:

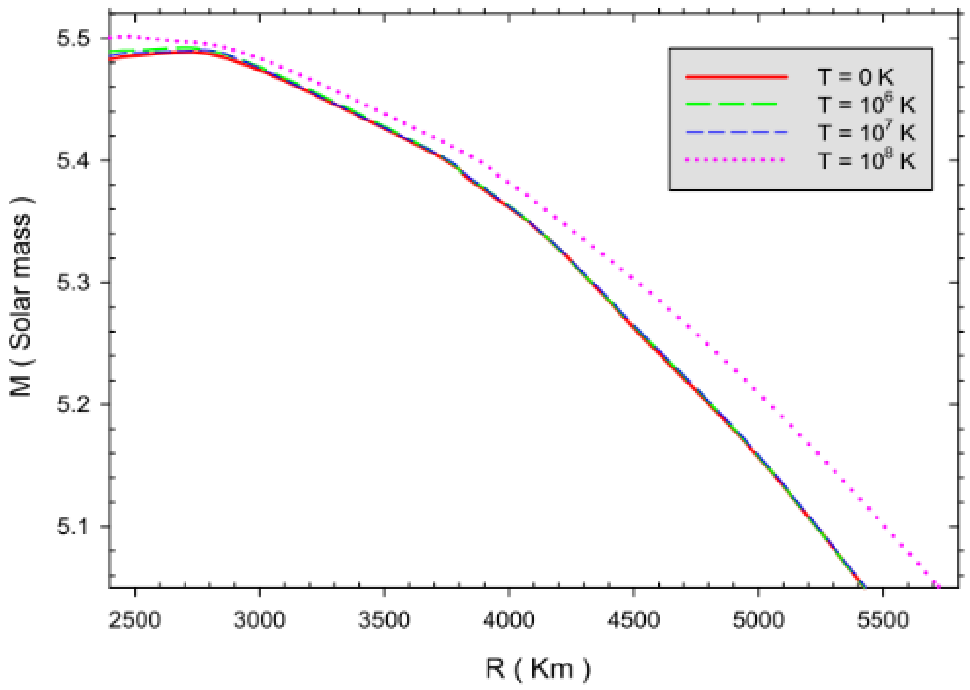


Figure 3. Mass vs. equatorial radius at temperatures $T = 0, 10^6, 10^7$ and 10^8 K for $B_{DC} = 1$.

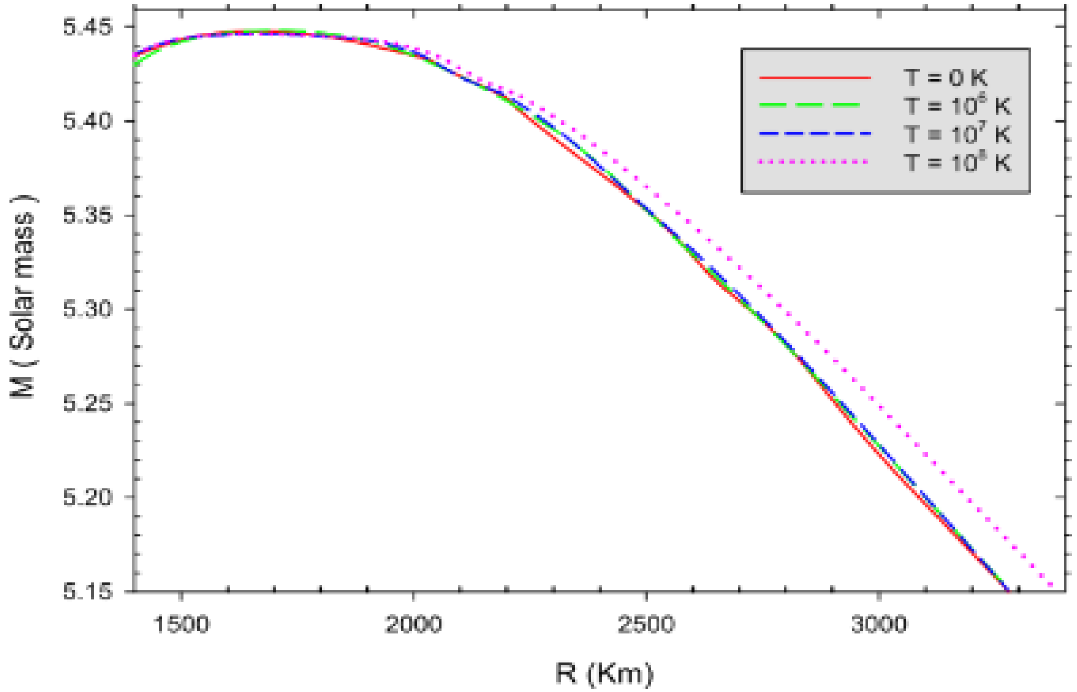


Figure 4. Mass vs. equatorial radius at temperatures $T =$ 0, Q $10^6, 10^7$ and 10^8 K for $B_{DC} = 5$.

Ref: R Sahoo, et. al., J. Astrophys. Astr., 45, 28(2024)

Numerical Results:

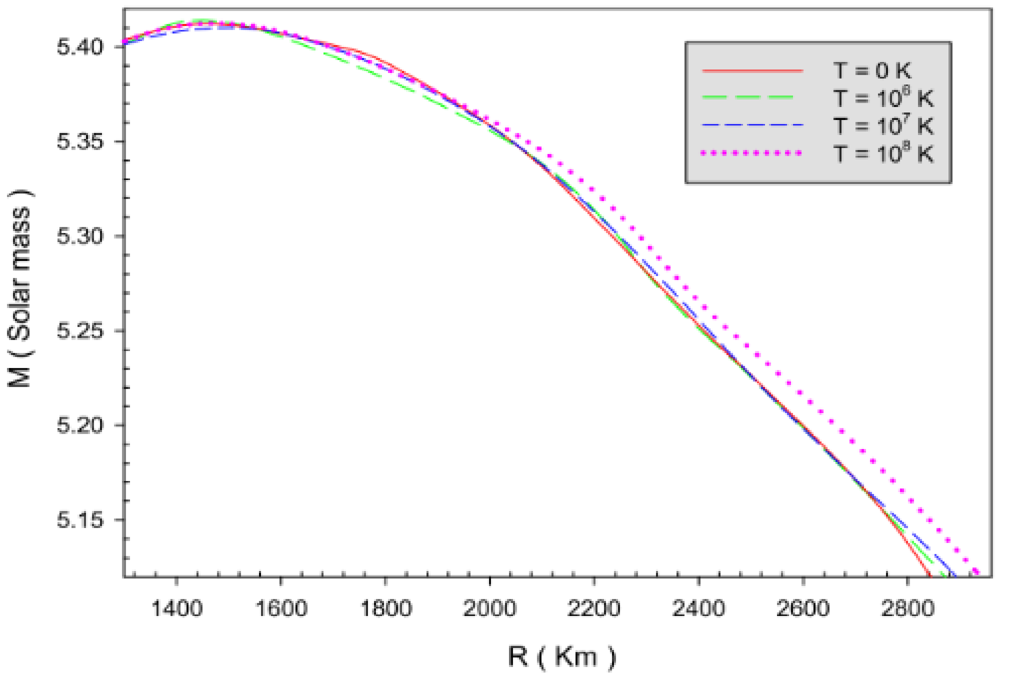


Figure 5. Mass vs. equatorial radius at temperatures $T = 0, 10^6, 10^7$ and 10^8 K for $B_{DC} = 8$.

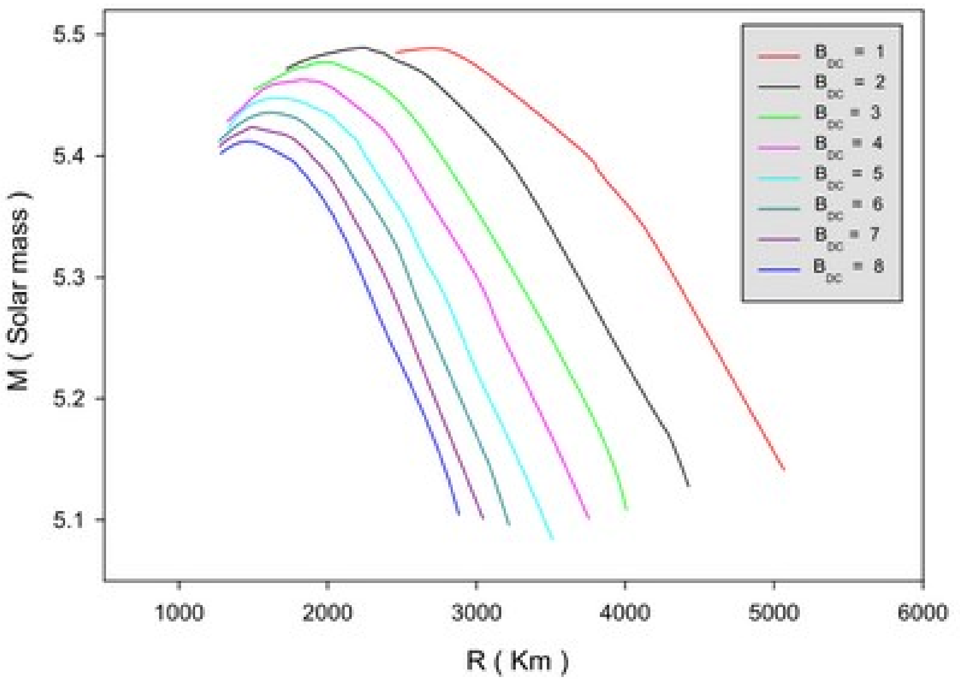


Figure 6. Mass vs. equatorial radius for $B_{DC} = 1, 2, 3, 4, 5, 6, 7$ and 8 at $T = 0$ K.

Ref: R Sahoo, et. al., J. Astrophys. Astr., 45, 28(2024)

Summary:

- The effect of finite non-zero temperature and ultra strong magnetic field on the masses and radii of anisotropic deformed magnetized white dwarfs in the γ –metric formalism is investigated.
- We found stable super-Chandrasekhar masses of white dwarfs (above ~5 $M_\odot$).
- At a fixed central electron density and temperature, the masses decrease monotonically as the central magnetic field increases, and equatorial radii increase monotonically.
- We also observed that the maximum mass and its corresponding equatorial radius decrease with the increase of the central magnetic field for all temperature. Moreover, the maximum mass occurs at a higher central density as the magnetic field increases. This shows that increasing the magnetic field (hence increasing anisotropy) softens the EoS and makes the star more compact.
- The finite temperature has an opposing effect to that of the magnetic field by decreasing the anisotropy of the system, thereby making EoS stiffer and star less compact.


THANK YOU