Faint light of old neutron stars and detectability at the James Webb Space Telescope and exploring some analytics of multiscatter case

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Motivation

Flash slide: Dark matter and its probes

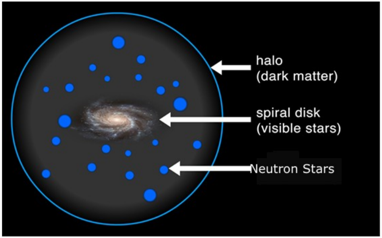
- Evidence for the existence of **dark matter** (DM) for the past 90 years and its $\sim$ **27 percent** participation in the universe content has increased the necessity to know more about its properties, nature and formation.
- The hunt for **possible candidates** that could constitute DM has been ongoing for a long time.
- The **Neutron Stars** (NSs) act as a laboratory for the DM particles as they scatter with constituents inside NSs and loses their kinetic energy.
- We explore the **capture of DM inside old Neutron Stars (NSs)**, which causes a rise in its temperature, and will have a detectability prospect at **James Webb Space Telescope (JWST)**.
- These **NSs act as a probe for DM** as we found the **maximum uncertainty in temperature** that could arise by varying possible NSs and DM parameters.

Capture of Dark Matter in old Neutron Star and its heating

Neutron Star as probes for Dark Matter

Neutron Stars (NS)

- NSs are compact objects with high energy density. Old NSs with (age $\geq 10^9$ yrs) are expected to have cooled down to surface temperatures of $\mathcal{O}$ (100 K).
- NS mass is of the order of **few solar masses** and radius about **10 km**.



- These old NSs could be present **inside the dark matter halo**, which causes an interaction between the constituent of NSs and dark matter (DM) via elastic scattering.
- It **results in the capture of DM** as it loses enough energy not to be able to leave, resulting in the **heating of NS**.
- If DM is thermalized, it can heat the **NS due to annhilation**.

Setup of our work

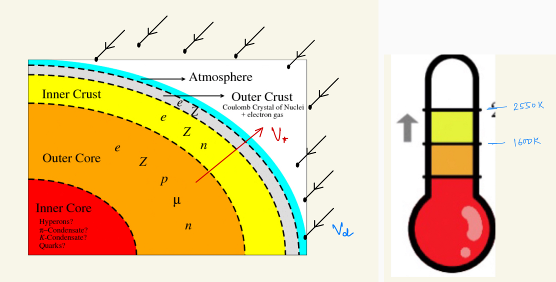


Figure 1: Capture of DM in NS causing the rise in its temperature
Roca-Maza et. al, (2011)

Astrophysical uncertainties in the properties of DM and NS

- Equation of state (EoS): R, M, v_{esc}
- DM density (ρ_χ)
- DM dispersion velocity (v_d)
- NS velocity (v_*)
- Probe in JWST.

Parametric Dependence

- $C^g_\star$ is the **Geometric Capture Rate**. It is the maximal rate of capture occurring when all DM particles incident upon a NS are captured.
- Mathematically, this rate in terms of DM energy density (ρ_χ), DM mass (m_χ) and NS mass and radius (M, R) related through (v_{esc}) is given by [1]:

$$C^g_\star = \pi R^2 \frac{\rho_\chi}{m_\chi} \frac{\langle v \rangle_0}{1 - v^2_{esc}} \sqrt{\frac{3\pi}{8}} \frac{v^2_{esc}}{v_\star v_d} \mathrm{Erf}\left(\sqrt{\frac{3}{2}} \frac{v_\star}{v_d}\right) \tag{1}$$

- The parametric dependence of observed **NS temperature due to DM capture** is:

$$T^\infty_{\mathrm{kin}} \approx 1787\,\mathrm{K}\left[\frac{\alpha_{\mathrm{kin}}}{0.08}\left(\frac{\rho_\chi}{0.42\,\mathrm{GeV/cm^3}}\right)\left(\frac{220\,\mathrm{km/s}}{\mathrm{v}_\star}\right)\mathrm{Erf}\left(\frac{270\,\mathrm{km/s}}{\mathrm{v_d}}\frac{\mathrm{v}_\star}{220\,\mathrm{km/s}}\right)\right]^{1/4}$$

$$T^\infty_{\mathrm{KA}} \approx 2518\,\mathrm{K}\left[\frac{\alpha_{\mathrm{KA}}}{0.33}\left(\frac{\rho_\chi}{0.42\,\mathrm{GeV/cm^3}}\right)\left(\frac{220\,\mathrm{km/s}}{\mathrm{v}_\star}\right)\mathrm{Erf}\left(\frac{270\,\mathrm{km/s}}{\mathrm{v_d}}\frac{\mathrm{v}_\star}{220\,\mathrm{km/s}}\right)\right]^{1/4}$$

with,

$$\alpha_{\mathrm{kin}} = (\gamma-1)(\gamma^2-1)/\gamma^4 \quad \text{and} \quad \alpha_{\mathrm{KA}} = \gamma(\gamma^2-1)/\gamma^4 \quad \text{and} \quad \gamma = (1-2\mathrm{GM/R})^{-1/2} \tag{4}$$

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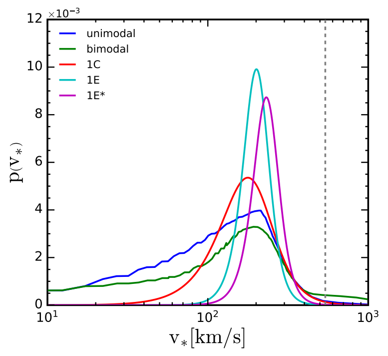
[1] N. F. Bell, G. Busoni, and S. Robles, JCAP 06, 054(2019)

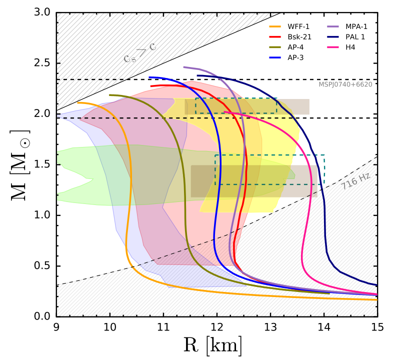
Computational Inputs

Dark Matter Parameters ρ_χ and v_d

- We consider values for **dark matter energy density** at solar radius to be $\rho_\chi = 0.39 - 0.52\,\mathrm{GeV/cm^3}$.
- **The dispersion velocity** is obtained through the relation $v_d = \sqrt{3/2}\,v_0$, where the local circular velocities $v_0 = 214 - 258\,\mathrm{km/s}$.

Neutron Star Parameters





- Left plot shows the **probability distribution of old NSs** in solar vicinity. [2] The dashed vertical line marks the Galactic escape velocity, taken here to be 540 km/s.
- Right plot shows the NSs equations-of-state considered. [3]

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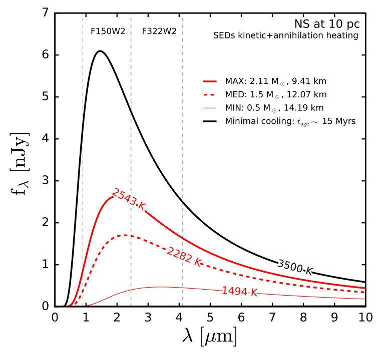
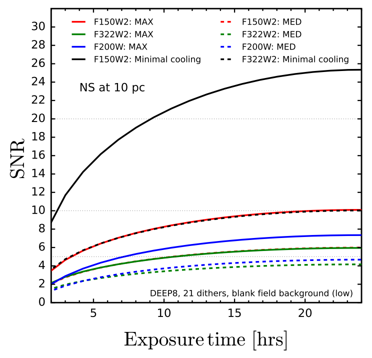
[2] Eran O. Ofek 2009 PASP 121 814, N. Sartore A&A 510, A23 (2010)

[3] Ozel, https://doi.org/10.1146/annurev-astro-081915-023322

Results

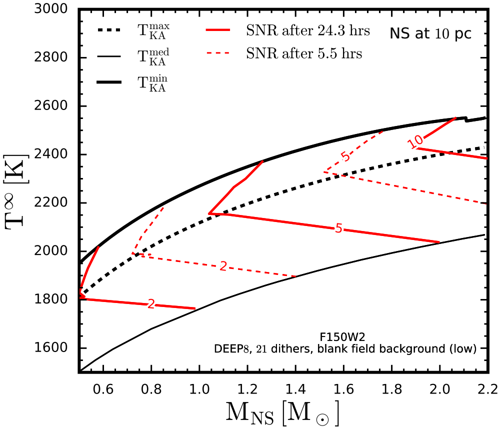
NS Spectral Energy Distribution (SED) and SNR senstivity at JWST





- The SEDs plot contains the JWST **F150W2 and F322W2 filter** information, which shows that these **filters are best suited** for observational prospects of NSs considered.
- The SNR plot shows that filter F150W2 will attain an **SNR of 10 within 24hrs** of exposure time.
- The thick black line is representative of **minimal cooling model for NS** of age $\sim$ 15 Myrs with black-body temperature 3500 K, with mass (radius) 1.5 $M_\odot$ (12 km)

Maximum Heating of NS in Kinetic + Annhilation (KA) Case



- This shows that the temperature of **NSs situated at 10 pc varies from ∼1600-2550 K** by considering all the NSs and DM parameters.

Analytics of Multiscatter Case

Multiscatter Capture Rate

- The **higher DM mass particle ($\geq 10^6$ GeV)** will have more momentum, thus the multi-scatter[4] effect become important. In this situation, the capture rate C_N will further depend upon the **number of scatters** N suppressed by $p_N(\tau)$, the probability that a DM particle will collide exactly N times as it crosses a star.

$$p_N(\tau) = 2\frac{(N+1)}{\tau^2}\left(1 - \frac{\Gamma(N+2,\tau)}{\Gamma(N+2)}\right) \tag{5}$$

- Here, **optical depth** $\tau \sim \sigma/\sigma_{sat}$ where, $\sigma_{sat} = \pi R^2/N_n$, N_n is number of scattering sites and $\Gamma(a,b)$ is incomplete gamma function.

- Here, $\boldsymbol{f(u)}$ is the velocity distribution for DM, $\boldsymbol{w^2 = u^2 + v_{esc}^2}$, with $\boldsymbol{u}$ being the DM velocity and $\boldsymbol{v_{esc}}$ depends upon NS parameters and $\boldsymbol{g_N(w)}$ is the capture probability.

$$C_N = \pi R^2 p_N(\tau)\int_0^\infty dw \frac{f(u)}{u^2} w^3\, g_N(w), \tag{6}$$

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[4] Bramante et al Phys. Rev. D 96, 063002

Optical depth and extinction factor

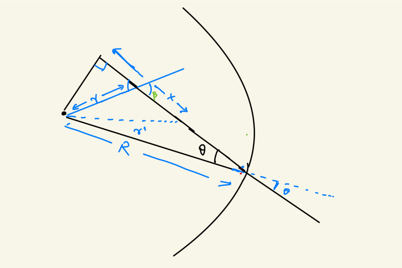
- The **optical depth** can be extended to account for **scattering inside the celestial body** [5]. For a fixed cross-section σ and some distribution of particles $n(r')$, it is defined as

$$\tau(r,z) = \int_{rz}^{\sqrt{R^2 - r^2(1-z^2)}} dx \, \sigma n(r')$$

where $r'^2 = x^2 + r^2(1-z^2)$ and $z = \cos\beta$

- The **flux of DM particles traversing the celestial body is significantly reduced over its path length**. To account for this effect, we add an **extinction factor** on top of optical depth defined as $\eta(r)$

$$\eta(r) = \int_{-1}^{1} dz e^{-\tau(r,z)}$$



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[5] Giorgio Busoni et al JCAP10(2017)037

General Capture Rate

- In general, **DM particle could also scatter inside the compact object** for which one has to take care of the forward scattering and **allowed incident angle for which the scattering will happen**.
- For the present, we consider a constant cross-section and the **profile to be gaussian $n(r) = n_0 e^{-r^2/R^2}$**, where n_0 is the number density for full geometric cross-section.
- The **escape velocity will no longer be a constant** for a particular NS, it will become a function of r. For Gaussian profile, the $\boldsymbol{v_{esc}}$ becomes

$$v_{esc}^2(r) = \frac{2GM}{R\left(\sqrt{\pi}\mathrm{Erf}(1) - 2/e\right)} \left(\frac{\sqrt{\pi}\mathrm{Erf}(\mathrm{r}/\mathrm{R})}{r/R} - \frac{2}{e} \right)$$

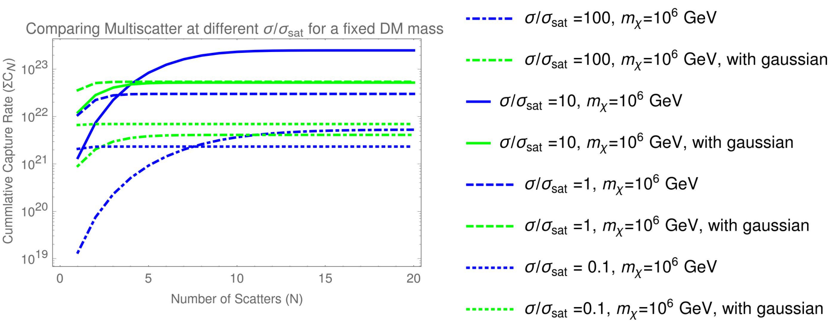
- Now, **using extinction factor along with the modified Poisson probability and the velocity limits on DM** we calculated **the generalized capture rate as**

$$\begin{aligned} C \quad &= \quad \frac{\pi R^2}{N} n_\chi \int d^3r \; n(r) P(N, r/R, \sigma/\sigma_{sat}) \eta(r) \\ \Bigg(\quad (\alpha - \tilde{\eta}) \quad & \int_{u_{min}}^{u_{max}} du \; v_{esc}^2(r) f_{v_*}(u)/u - \tilde{\eta} \int_{u_{min}}^{u_{max}} du \; f_{v_*}(u) u \Bigg) \end{aligned} \tag{7}$$

Some preliminary Results

Comparing different multiscatter scenarios

- For DM mass 10^6 GeV, NS parameters $M = M_\odot, R = 10$ km , $v_* = 200$ Km/s and DM parameters $v_d = \sqrt{3/2}\,214$ km/s , $n_\chi = 0.39 GeV/cm^3$



Conclusion

Summary and Further Work In Progress

- The radiation emitted from the NS due to KA heating can heatup the NSs **upto 2550K**.
- The NSs at 10pc with these surface temperature will be detectable by **F150W2 filter** of JWST at **SNR 10 within 24 hrs of exposure time**.
- This minimal effect of astrophysical uncertainties on the final kinetic temperature shows that we can use **NSs to constrain the properties of DM**.
- We found the robustness of using **old NSs as probes of DM** and found that the maximal temperature varies **at most by** $\sim 40\%$.
- We extended the analysis of **multiscatter DM capture to include the effect of scattering inside the compact object** and found the **general equation for multiscatter capture rate**.
- We are further implementing **simulations for the whole trajectory of DM particles inside NS** for a better understanding.

THANK YOU

Backup Slides

1. Velocity Distributions

- We include the probability distribution acting on NS velocity in the calculation of capture rate, which basically modifies the $f_{v\star}(u_\chi)$ to $\int dv_\star f_{NS}(v_*) f_{v_\star}(u_\chi)$.

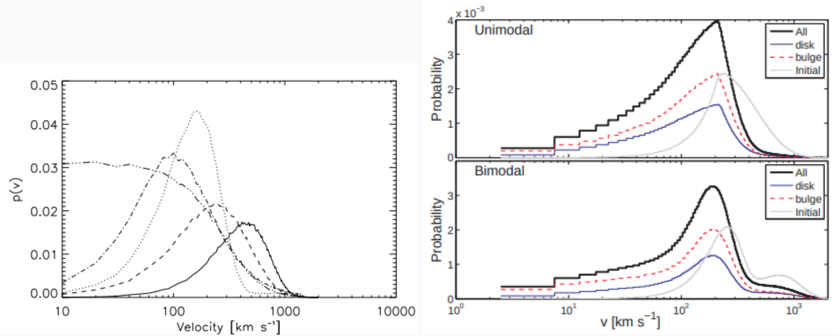


Figure 2: The velocity distribution obtained from simulations.
Image credits: Sartore et.al, 2009 (left) and Ofek,2009 (right)

2. DM halo distribution

- We take the local DM velocity modulus distribution in the Galactic rest frame for two haloes in the eagle HR and apostle IR simulations. This basically varies the ρ_χ and DM velocity (v_d) which effectively vary the velocity distribution function of NS $f_{v\star}(u_\chi)$. [6]

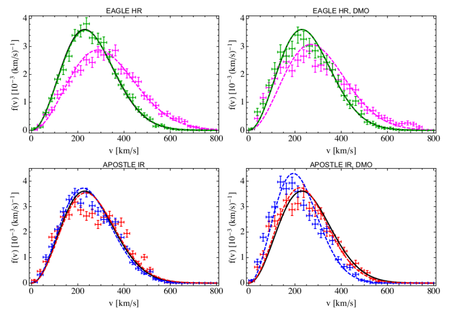


Figure 3: eagle HR simulation (top panel), apostle IR simulation (bottom panel) and right panel shows same haloes with DMO simulation, Image Credits: Bozorgnia et al , 2016

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[6] Bozorgnia et. al, JCAP05(2016)024

3. Geometric Capture Rate Averaging of Terms

For each set of (M, R) given by EoS *i* we compute the DM geometric capture rate averaging over DM phase space parameters through

$$C^{\mathrm{g}}_{i,j}(M,R) = \kappa \sum_{k,l} \int dv_\star p_j(v_\star) C^{\mathrm{g}}_\star(i, v_\star, v_d^k, \rho_\chi^l) \; , \qquad (9)$$

where, $j = 1, 5$ corresponds to the different velocity probability distributions (p_j) of NS discussed above, and the averaging coefficient $\kappa = (k_{\max}\, l_{\max})^{-1}$. Integers $k_{\max}$(=2) and $l_{\max}$(=2) denote the number of values we sample for parameters v_d and ρ_χ, respectively

4. Background Modelling

- The background model ‘high’ corresponds to considering only 90th percentile of the above distribution. Similarly, the models ‘medium’ and ‘low’ correspond to 50th and 10th percentile, respectively.
- We show SNRs for the MAX scenario corresponding to low (solid), medium (dashed), and high (dotted) background models. As mentioned in the main text the maximum attainable SNR within a day of observation varies by at most 2 units.

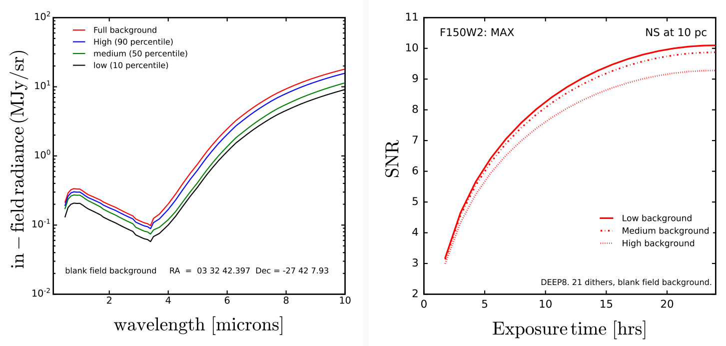
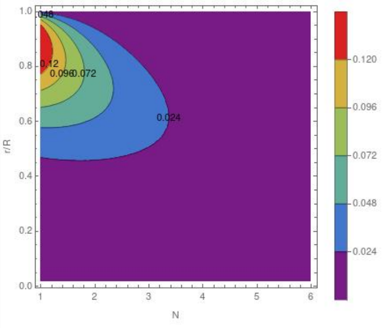
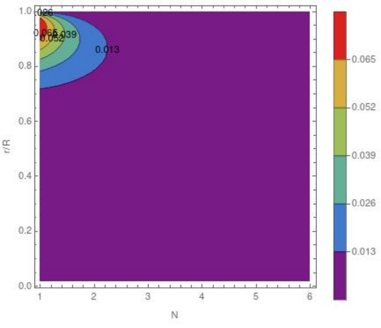


Figure 4: Left Panel: Background as function of wavelength. **Right Panel:** Dependence of SNR on background percentile.

Effect of including optical depth and extinction factor

- The left plot shows the **probability of multiscatter $p_N(\tau)$ when the optical depth has functional dependence**. In contrast, the right plot shows the **probability multiplied with an extinction factor** as a function of distance from centre r/R and number of scattering N.
- These results are for a fixed geometric cross-section, $\sigma/\sigma_{sat} = 10$, and **gaussian profile** of particles inside NS.
- The plot also shows that including **extinction factor will require fewer scatters** and **scattering is shifted more towards the surface** as we include the extinction factor.





Effect of increasing cross section

- These plots show the **probability of multiscatter multiplied with extinction factor as a function of r/R and N**. The left plot is for the case when $\sigma/\sigma_{sat} = 1$, and the right plot corresponds to $\sigma/\sigma_{sat} = 10$.