

Probing microscopic origins of axions by the chiral magnetic effect

Sang Hui Im (IBS-CTPU)

- K Choi, SHI, HJ Kim, H Seong, *JHEP 08 (2021) 058*, 2106.05816
- DK Hong, SHI, KS Jeong, D Yeom, 2207.06884

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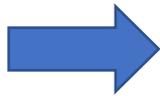
April 8, 2023

Outline

- Classification of possible UV physics for axions
- Distinctive patterns of low energy axion couplings to SM particles depending on the UV origins
- Measurement of the key discriminator (axion-electron coupling) by the chiral magnetic effect

Strong CP problem and QCD axion

$$y_u H Q_L u_R^c + y_d H^* Q_L d_R^c + \frac{g_s^2}{32\pi^2} \theta G \tilde{G}$$



$$\bar{\theta} = \theta + \arg \det (y_u y_d) < 10^{-10}$$

Non-observation
of neutron EDM
[Abel et al '20]

CPV in the QCD sector

while $\delta_{\text{CKM}} = \arg \det [y_u y_u^\dagger, y_d y_d^\dagger] \sim \mathcal{O}(1)$

The QCD vacuum energy is minimized at the CP-conserving point ($\bar{\theta} = 0$).

[Vafa, Witten '84]

$$V_{\text{QCD}} = -\Lambda_{\text{QCD}}^4 \cos \bar{\theta}$$

Promote $\bar{\theta}$ to a dynamical field (=QCD axion) : $\frac{g_s^2}{32\pi^2} \left(\theta + \frac{a}{f_a} \right) G \tilde{G}$

[Peccei, Quinn '77, Weinberg '78, Wilczek '78]

QCD axion lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{g_s^2}{32\pi^2} c_G \frac{a}{f_a} G^{\mu\nu} \tilde{G}_{\mu\nu} \\ + \frac{a}{f_a} \sum_{A=W,B,\dots} \frac{g_A^2}{32\pi^2} c_A F^{A\mu\nu} \tilde{F}_{\mu\nu} + \frac{\partial_\mu a}{f_a} \left(\sum_{\psi=q,\ell,\dots} c_\psi \psi^\dagger \bar{\sigma}^\mu \psi + \sum_{\phi=H,\dots} c_\phi \phi^\dagger i \overleftrightarrow{D}^\mu \phi \right)$$

$$U(1)_{PQ} : a(x) \rightarrow a(x) + \alpha$$

broken by $c_G \neq 0$ non-perturbatively

$$\Rightarrow m_a^2 \simeq c_G^2 \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

The axion couplings to the other SM particles

$c_W, c_B, c_q, c_\ell, c_H$ are UV model-dependent.

Axion-Like Particles (ALPs)

- Cousins of the QCD axion, while not being necessarily involved in solving the strong CP problem (so c_G can be 0)
- Ubiquitous in many BSM scenarios, in particular, string theory

[Arvanitaki, Dimopoulos, Dubovsky, Kaloper, Marsh-Russell, '09]

$$\frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 + \frac{a}{f_a} \sum_A \frac{g_A^2}{32\pi^2} c_A F^{A\mu\nu} \tilde{F}_{\mu\nu}^A + \frac{\partial_\mu a}{f_a} \left(\sum_\psi c_\psi \psi^\dagger \bar{\sigma}^\mu \psi + \sum_\phi c_\phi \phi^\dagger i \overleftrightarrow{D}^\mu \phi \right)$$

i) approximate shift symmetry $U(1)_{PQ}$ $a(x) \rightarrow a(x) + c$ ($c \in \mathbb{R}$)

: ALP can be naturally light.

ii) periodicity $\frac{a(x)}{f_a} \equiv \frac{a(x)}{f_a} + 2\pi$

: f_a characterizes typical size of ALP couplings

up to the dimensionless coefficients c_A, c_ψ, c_ϕ .

KSVZ-like models

Kim '79, Shifman, Vainshtein, Zakharov '80

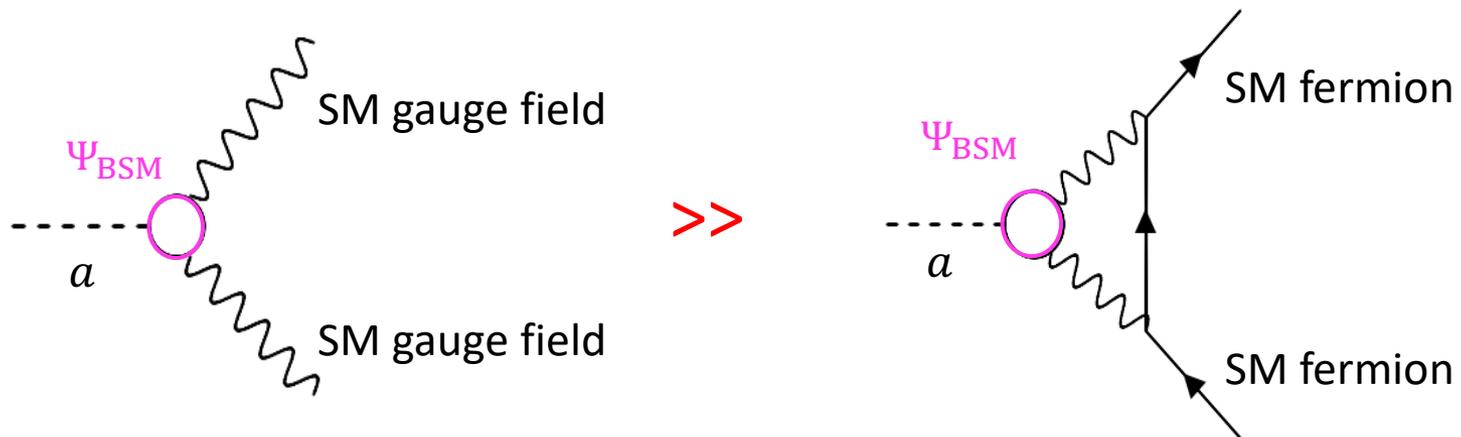
The axion couples to SM fields via a heavy BSM fermion charged under the SM gauge group.

$$y\Phi\Psi_{\text{BSM}}\Psi_{\text{BSM}}^c + \text{h.c.}$$

$$\Phi = \frac{1}{\sqrt{2}}(\rho + f_a)e^{ia/f_a}$$

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}f_a$$

$$m_\Psi = \frac{y}{\sqrt{2}}f_a$$



“KSVZ-like models”

: no tree-level couplings to the SM fermions

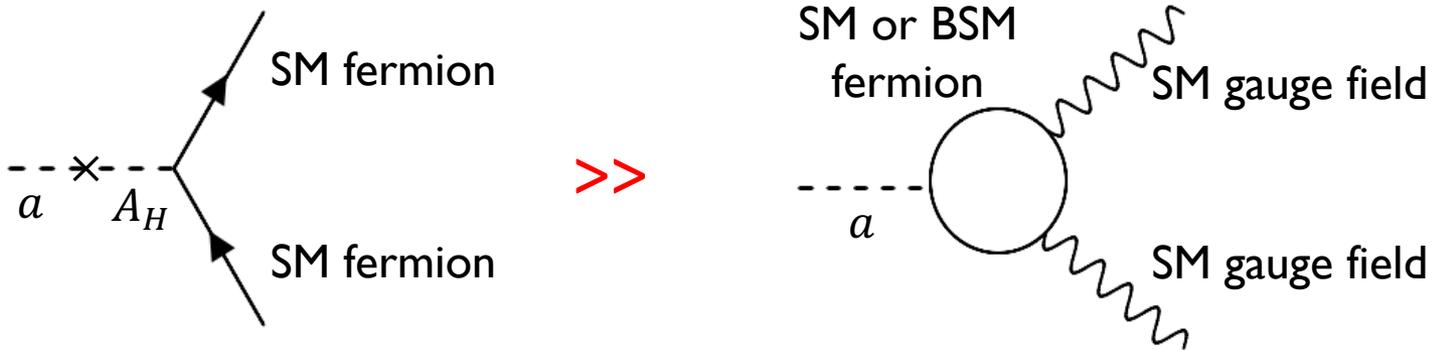
DFSZ-like models

Dine, Fischler, Srednicki '81, Zhitnitsky '80

The axion couples to the SM sector at tree-level (e.g. via Higgs portal).

$$y_u u_R^c Q_L H_u + y_d d_R^c Q_L H_d + y_e e_R^c L H_d + \lambda \Phi^2 H_u H_d + \text{h.c.}$$

$$\Phi = \frac{1}{\sqrt{2}}(\rho + f_a)e^{ia/f_a}$$



“DFSZ-like models”

: $O(1)$ tree-level couplings to the SM fermions

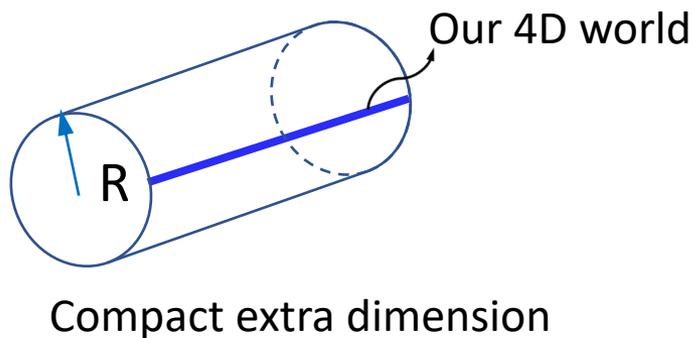
String-theoretic models

Witten '84

$$A_{[m_1 m_2 \dots m_p]}(x^\mu, y^m) = a(x^\mu) \Omega_{[m_1 m_2 \dots m_p]}(y^m) \quad \Omega : \text{harmonic } p\text{-form on the compact internal space}$$

4D axions identified as zero modes of higher-dimensional p -form gauge field

- Simplified 5D toy model



$$A_M(x^\mu, y) \quad M = 0, 1, 2, 3, 5$$

$$\Rightarrow A_5(x^\mu, y) = a(x^\mu) \Omega_5(y)$$

$$F_{MN} = \partial_M A_N - \partial_N A_M$$

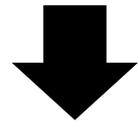
M_5 : 5D Planck mass (String scale)

$$S = \int d^5x \sqrt{-g} \left(-\frac{1}{4g_5^2} F^{MN} F_{MN} + \underbrace{c_1 \frac{\epsilon^{MNPQR}}{\sqrt{-g}} A_M G_{NP}^a G_{QR}^a}_{\text{5D Chern-Simon term}} + \underbrace{\frac{c_2}{M_5} F_{MN} \bar{\Psi} \gamma^M \gamma^N \Psi + \dots}_{\text{5D gauge-matter coupling}} \right)$$

5D Chern-Simon term
(axion-gauge field coupling)

5D gauge-matter coupling
(axion-matter coupling)

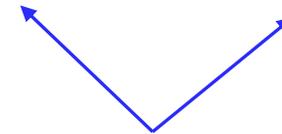
$$c_1 \sim c_2 \sim O(1)$$



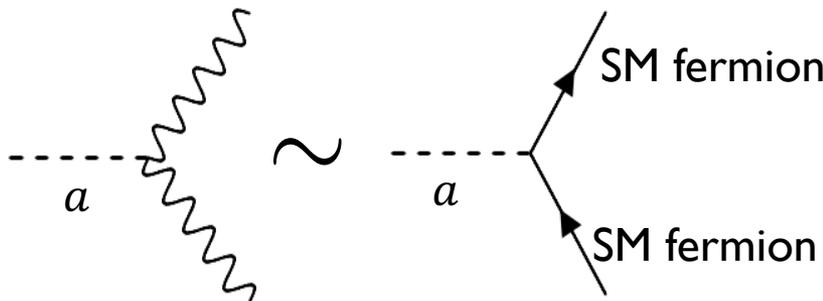
Integrating out the extra dimension

$$S_{\text{eff}} = \int d^4x \left(-\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \kappa M_5^2 \partial^\mu a \partial_\mu a + c_1 a G^{a\mu\nu} \tilde{G}_{\mu\nu}^a + c_2 \partial_\mu a \bar{\Psi} \gamma^\mu \gamma^5 \Psi + \dots \right)$$

$\kappa \sim O(1)$

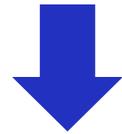


String-theoretic axion couplings to matter fields and gauge fields are comparable.



$$S_{\text{eff}} = \int d^4x \left(-\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \kappa M_5^2 \partial^\mu a \partial_\mu a + c_1 a G^{a\mu\nu} \tilde{G}_{\mu\nu}^a + c_2 \partial_\mu a \bar{\Psi} \gamma^\mu \gamma^5 \Psi + \dots \right)$$

$$\kappa \sim c_1 \sim c_2 \sim O(1)$$



Canonical
normalization

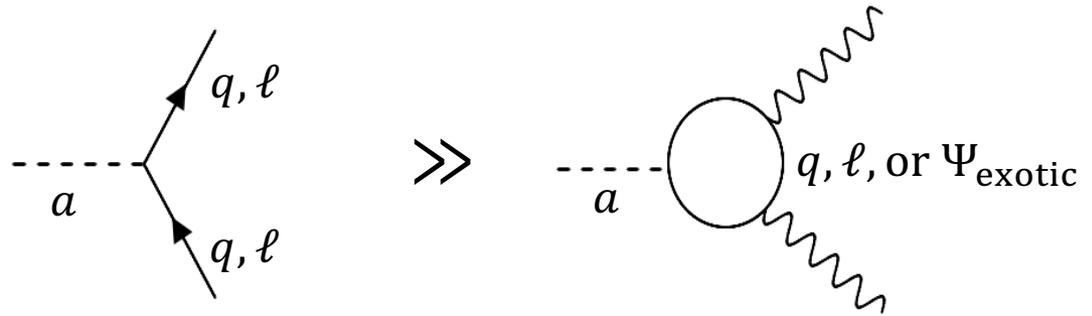
$$a \rightarrow \frac{a}{32\pi^2 f_a} \quad f_a = \frac{\sqrt{\kappa}}{32\pi^2} M_5$$

$$S_{\text{eff}} = \int d^4x \left(-\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \partial^\mu a \partial_\mu a + \frac{c_1}{32\pi^2} \frac{a}{f_a} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a + \underbrace{\frac{c_2}{32\pi^2} \frac{\partial_\mu a}{f_a} \bar{\Psi} \gamma^\mu \gamma^5 \Psi + \dots}_{\text{Axon-matter couplings}} \right)$$

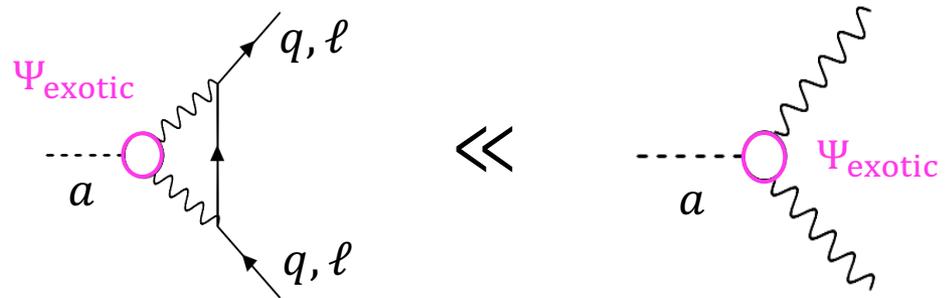
Axon-matter couplings are
suppressed by the 1-loop factor.

Summary: characteristic patterns of axion couplings to the SM depending on the microscopic origins

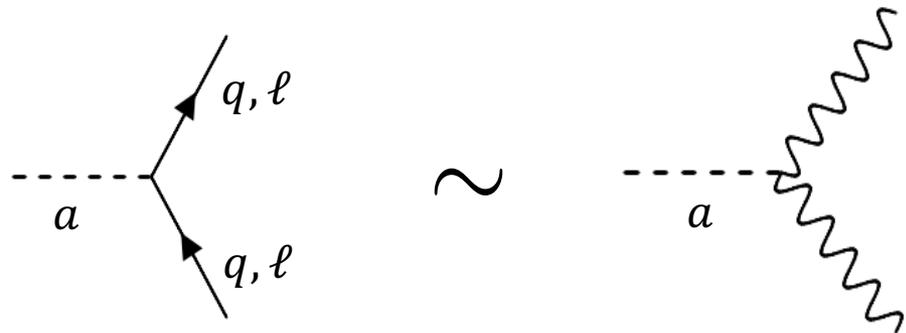
DFSZ-like models



KSVZ-like models



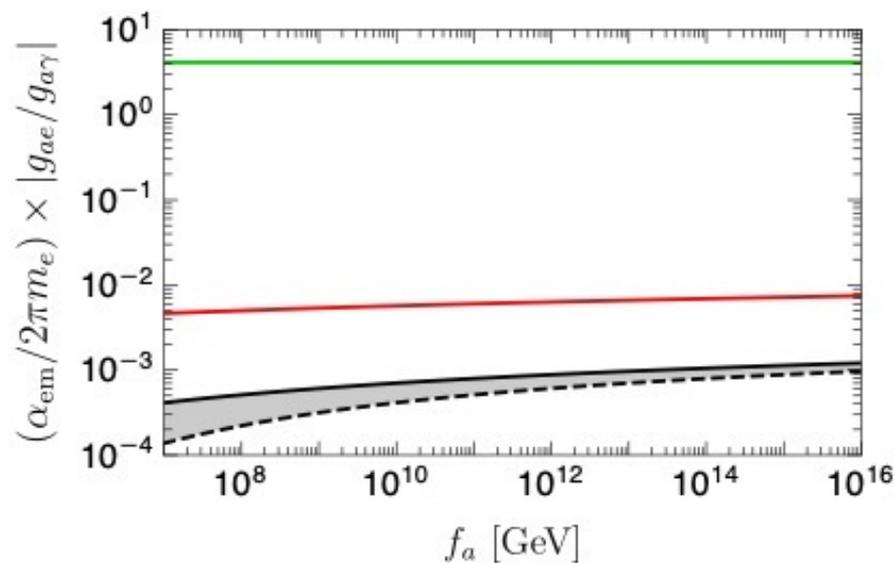
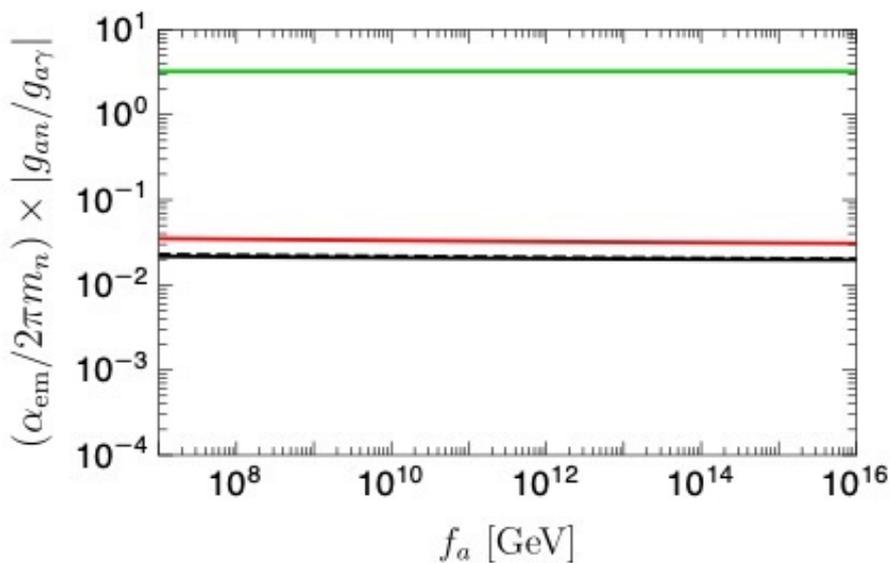
String-theoretic models



Distinguishing the models of an axion by coupling ratios

K Choi, SHI, HJ Kim, H Seong '21

For QCD axion ($c_G \neq 0$), $g_{ap} \sim \frac{m_p}{f_a}$ regardless of the classes of models

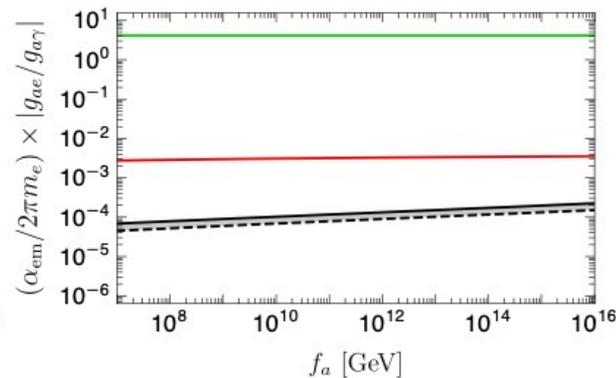
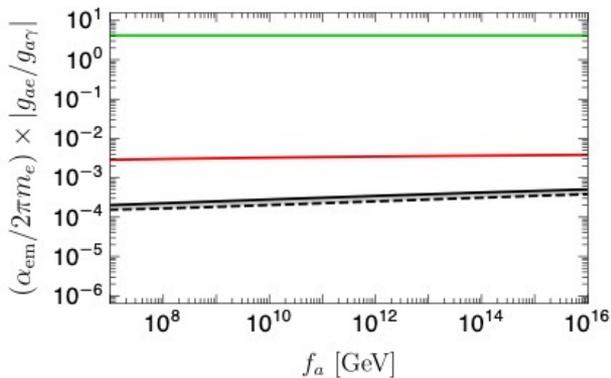
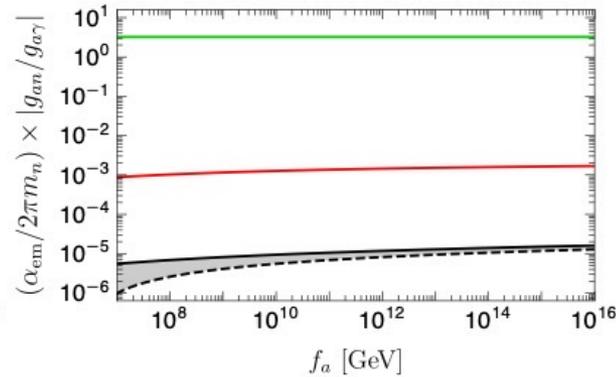
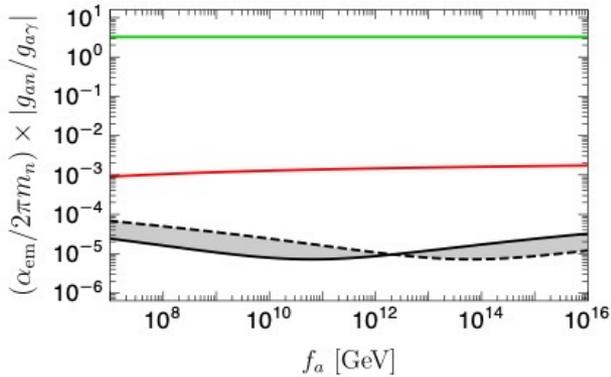
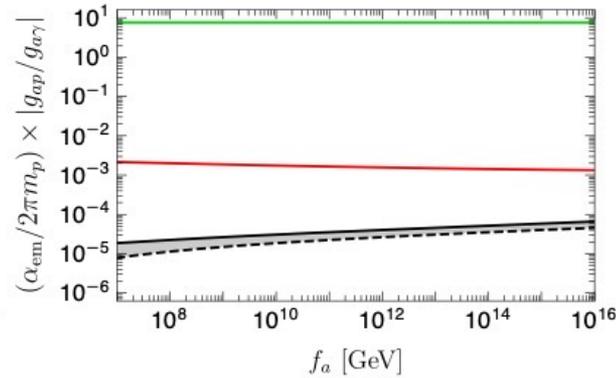
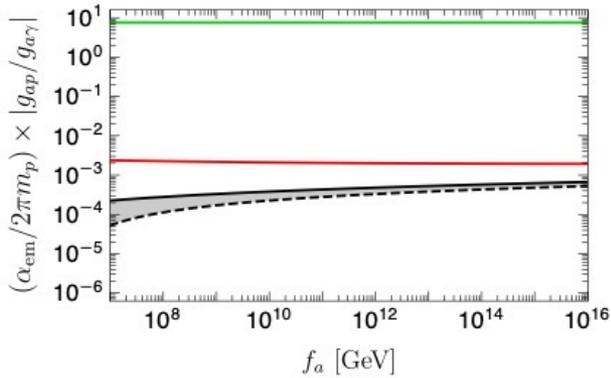


Green : DFSZ-like model

Red : String-theoretic model

Black : KSVZ-like model (dashed : $m_\Psi = 10^{-3} f_a$, solid : $m_\Psi = f_a$)

For ALPs with ($c_G = 0$),



Green : DFSZ-like model
 Red : String-theoretic model
 Black : KSVZ-like model
 (dashed : $m_\Psi = 10^{-3} f_a$,
 solid : $m_\Psi = f_a$)

$$c_W = 1 \quad (c_G = c_B = 0)$$

$$c_B = 1 \quad (c_G = c_W = 0)$$

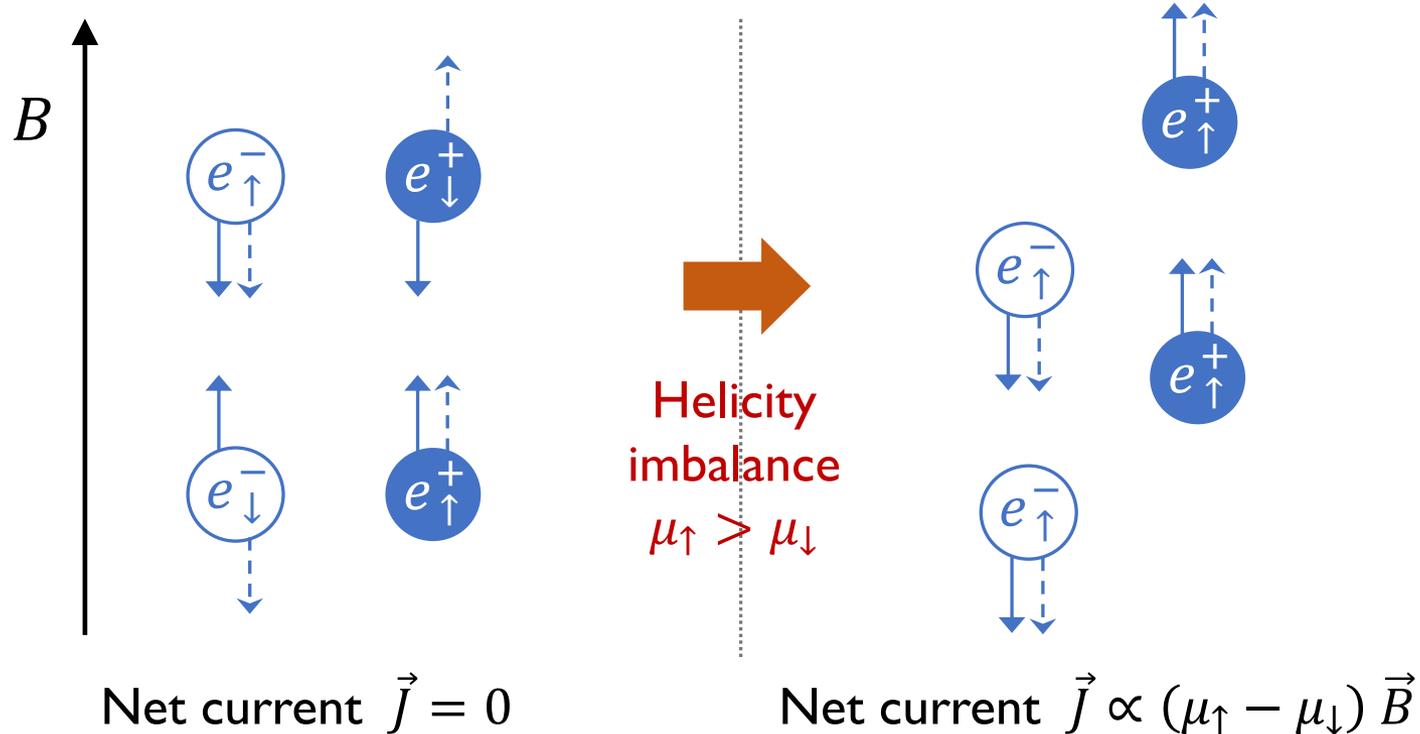
Take-home message I

- In principle, we have three possible classes of UV physics for an axion : KSVZ-like, DFSZ-like, and string-theoretic.
- Those three classes of UV physics may be experimentally distinguishable by measuring the ratio of an axion-fermion coupling to the axion-photon coupling.
- For the QCD axion, *the measurement of the axion-electron coupling is crucial for the distinction.*

Chiral Magnetic Effect (CME) in a nutshell

Kharzeev, McLerran, Warringa '08
Fukushima, Kharzeev, Warringa '08

— Momentum direction
- - - Spin direction



- The magnetic field aligns the spin directions depending on particles and antiparticles.
- The helicity imbalance causes a non-zero electric current along the B-field direction.

- Chiral chemical potential

$$\begin{aligned}\mathcal{L} \supset \mu_5(n_R - n_L) &= \mu_5 \left(\psi_R^\dagger \psi_R - \psi_L^\dagger \psi_L \right) \\ &= \mu_5 \bar{\Psi} \gamma^0 \gamma^5 \Psi\end{aligned}\quad \Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

It makes *helicity imbalance* for both massless and massive fermions.

- (Vector) chemical potential

$$\begin{aligned}\mathcal{L} \supset \mu(n_R + n_L) &= \mu \left(\psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L \right) \\ &= \mu \bar{\Psi} \gamma^0 \Psi\end{aligned}$$

It makes *charge imbalance* (i.e. particles vs antiparticles).

The *charge imbalance alone cannot induce a current*. However, μ might be still relevant for the magnitude of the current for a given helicity imbalance.

Time-varying axion field as a source for μ_5

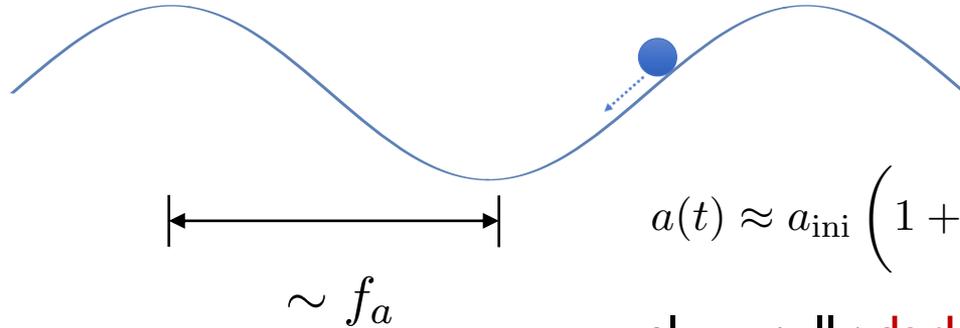
Suppose that we have an axion-fermion coupling:

$$c_\Psi \frac{\partial_\mu a}{f_a} \bar{\Psi} \gamma^\mu \gamma^5 \Psi \quad \longrightarrow \quad c_\Psi \frac{\dot{a}}{f_a} \bar{\Psi} \gamma^0 \gamma^5 \Psi$$

$$\mu_5(t) = c_\Psi \frac{\dot{a}}{f_a}$$

Cosmological evolution of an axion field

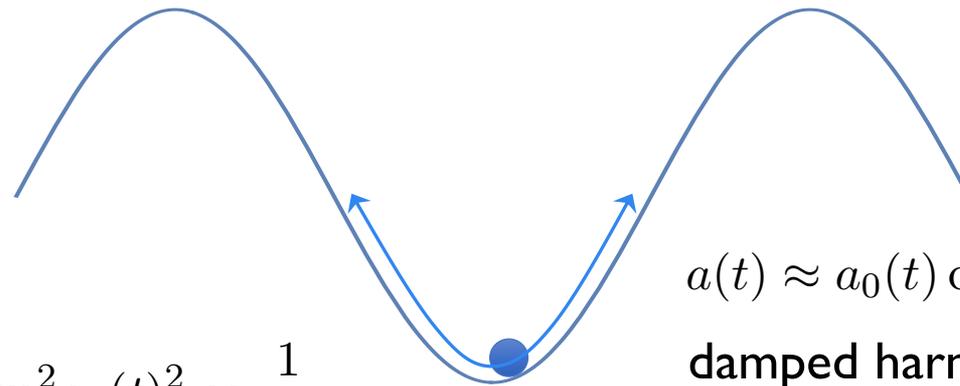
$$m_a \ll H(t)$$



$$a(t) \approx a_{\text{ini}} \left(1 + \frac{1}{20} \left[\frac{m_a^2}{H_{\text{ini}}^2} - \frac{m_a^2}{H(t)^2} \right] \right)$$

slow-roll : **dark energy**

$$m_a \gg H(t)$$

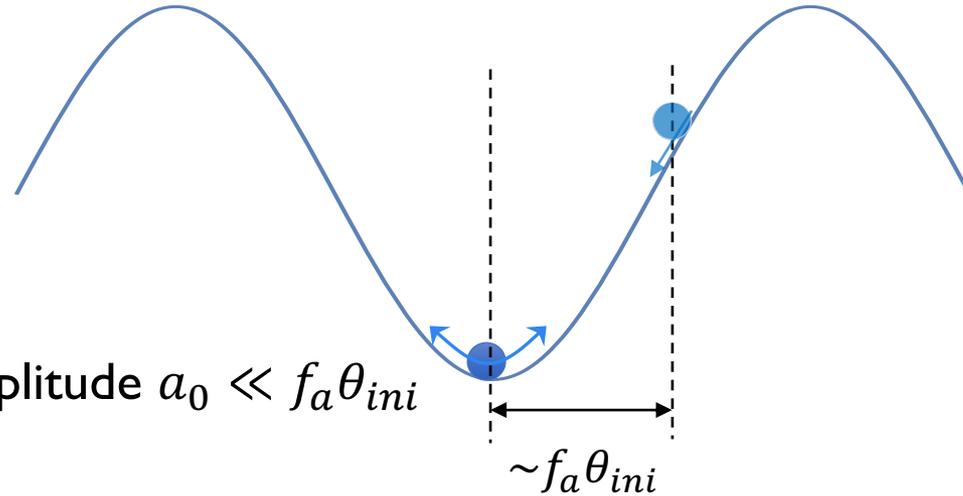


$$\rho_a \approx \frac{1}{2} m_a^2 a_0(t)^2 \propto \frac{1}{R^3}$$

$$a(t) \approx a_0(t) \cos(m_a t)$$

damped harmonic oscillator : **cold dark matter**

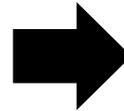
Misalignment production of axion dark matter and chiral chemical potential



Present oscillation amplitude $a_0 \ll f_a \theta_{ini}$

$$a(t, \vec{x}) \approx a_0 \cos m_a t$$

$$\rho_a = \frac{1}{2} m_a^2 a_0^2$$



$$\mu_5(t) \approx -c_\Psi \frac{\sqrt{2\rho_a}}{f_a} \sin m_a t$$

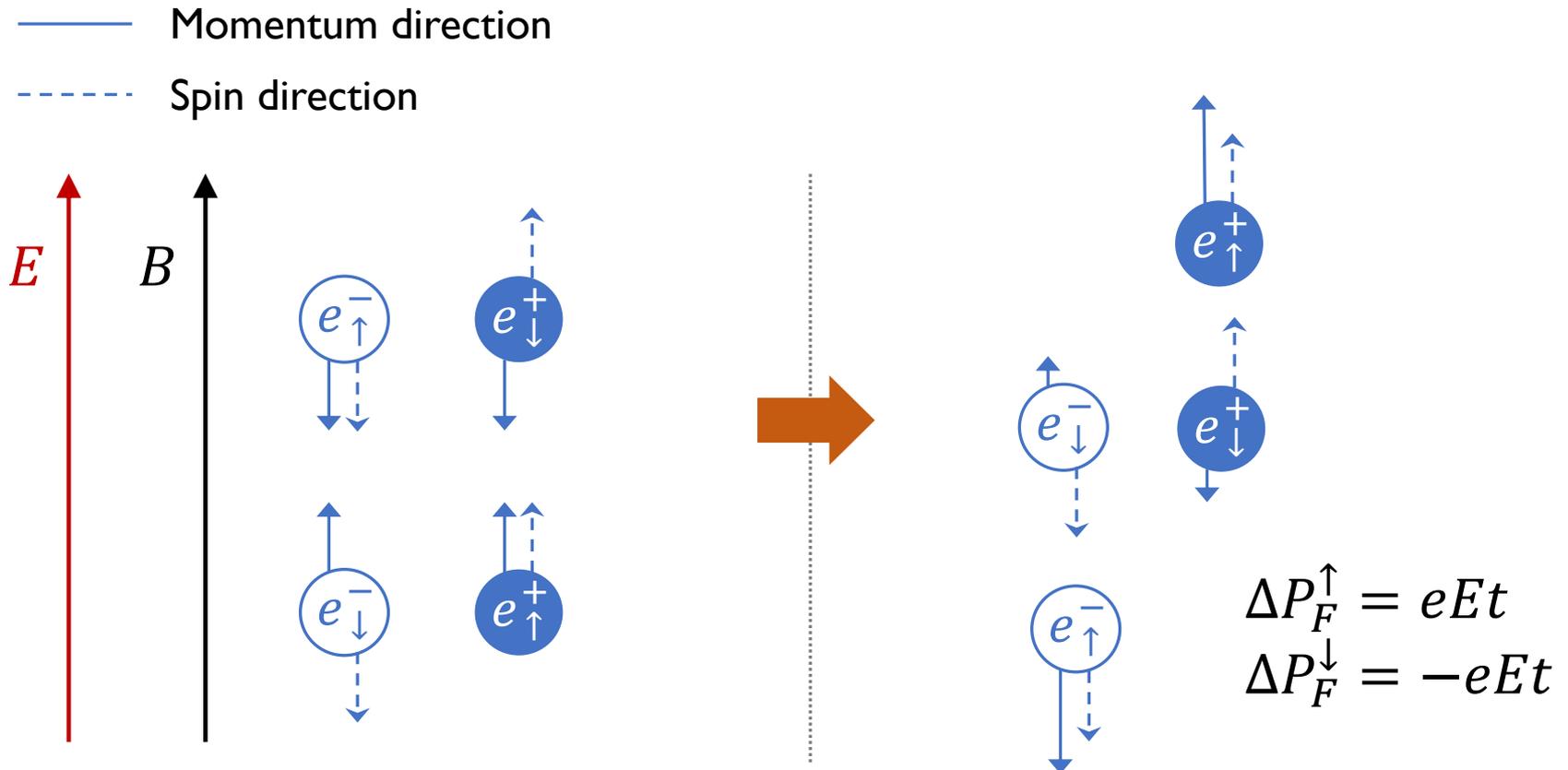
$$\mu_5(t) = c_\Psi \frac{\dot{a}}{f_a}$$

The axion dark matter background gives rise to an oscillating chiral chemical potential for fermions coupled to the axion.

CME-induced current

I. Energy balance argument

Nielsen and Ninomiya '83
Fukushima, Kharzeev, Warringa '08



Density change of
helicity-up fermion states

$$\underbrace{\frac{\Delta P_F^\uparrow}{2\pi}}_{\text{Longitudinal number density}} \cdot \underbrace{\frac{eB}{2\pi}}_{\text{Transverse number density in the lowest Landau level}} = \frac{e^2}{4\pi^2} \vec{E} \cdot \vec{B} t$$

Longitudinal number
density

Transverse number density
in the *lowest* Landau level

Aharonov and Casher '79

Density change of
helicity-down fermion states

$$\frac{\Delta P_F^\downarrow}{2\pi} \cdot \frac{eB}{2\pi} = -\frac{e^2}{4\pi^2} \vec{E} \cdot \vec{B} t$$

$$\blackrightarrow \frac{d}{dt} (n_\uparrow - n_\downarrow) = \frac{e^2}{2\pi^2} \vec{E} \cdot \vec{B}$$

which reproduces the chiral anomaly equation in massless limit

$$\partial_\mu (\bar{\Psi} \gamma^\mu \gamma^5 \Psi) = \frac{e^2}{2\pi^2} \vec{E} \cdot \vec{B} + 2m \bar{\Psi} i \gamma^5 \Psi$$

$$\frac{d}{dt}(n_{\uparrow} - n_{\downarrow}) = \frac{e^2}{2\pi^2} \vec{E} \cdot \vec{B}$$

$$\blackrightarrow \underbrace{\frac{\mathcal{E}}{2} \frac{d}{dt}(N_{\uparrow} - N_{\downarrow})}_{\text{Energy cost per unit time needed for making helicity imbalance}} = \frac{e^2}{4\pi^2} \mathcal{E} \int d^3x \vec{E} \cdot \vec{B}$$

Energy cost per unit time needed for making helicity imbalance

||

$$\int d^3x \vec{j} \cdot \vec{E}$$

This energy cost has to be supplied by the electric power.

$$\blackrightarrow \boxed{\vec{j} = \frac{e^2}{4\pi^2} \mathcal{E} \vec{B}}$$

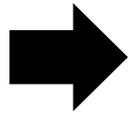
\mathcal{E} : Energy cost for converting a helicity-down state to a helicity-up state

Essentially

States lying in the lowest Landau level

$$\begin{aligned}
 j &= \int d\rho v \overbrace{dn^\uparrow} \overbrace{v} \\
 &= e \cdot \frac{eB}{2\pi} \cdot \left(\int_0^{p_F^\uparrow} \frac{dp_z}{2\pi} \frac{p_z}{\sqrt{p_z^2 + m^2}} - \int_0^{p_F^\downarrow} \frac{dp_z}{2\pi} \frac{p_z}{\sqrt{p_z^2 + m^2}} \right) \\
 &= \frac{e^2}{4\pi^2} \cdot \underbrace{\left(\sqrt{(p_F^\uparrow)^2 + m^2} - \sqrt{(p_F^\downarrow)^2 + m^2} \right)}_{\varepsilon} \cdot B
 \end{aligned}$$

Dirac eq. $\begin{pmatrix} -m & \mu - \vec{p} \cdot \vec{\sigma} + \mu_5 \\ \mu + \vec{p} \cdot \vec{\sigma} - \mu_5 & -m \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$ $h = \hat{p} \cdot \vec{\sigma}$ helicity
 $\vec{p} \cdot \vec{\sigma} = |p|h$

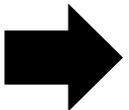


$$\mu = \sqrt{(|p|h - \mu_5)^2 + m^2}$$

$$p_F^\uparrow = p_F + \mu_5$$

$$p_F^\downarrow = p_F - \mu_5$$

where p_F is the Fermi momentum when $\mu_5 = 0$.



$$\mathcal{E} = \sqrt{(p_F^\uparrow)^2 + m^2} - \sqrt{(p_F^\downarrow)^2 + m^2} \simeq 2\mu_5 v_F$$

$$\vec{j} \simeq \frac{e^2}{2\pi^2} \mu_5 v_F \vec{B}$$

In the original formula by (FKW '08) the v_F dependence is missing (Hong, SHI, Jeong, Yeom '22).

From Deog Ki Hong's slide

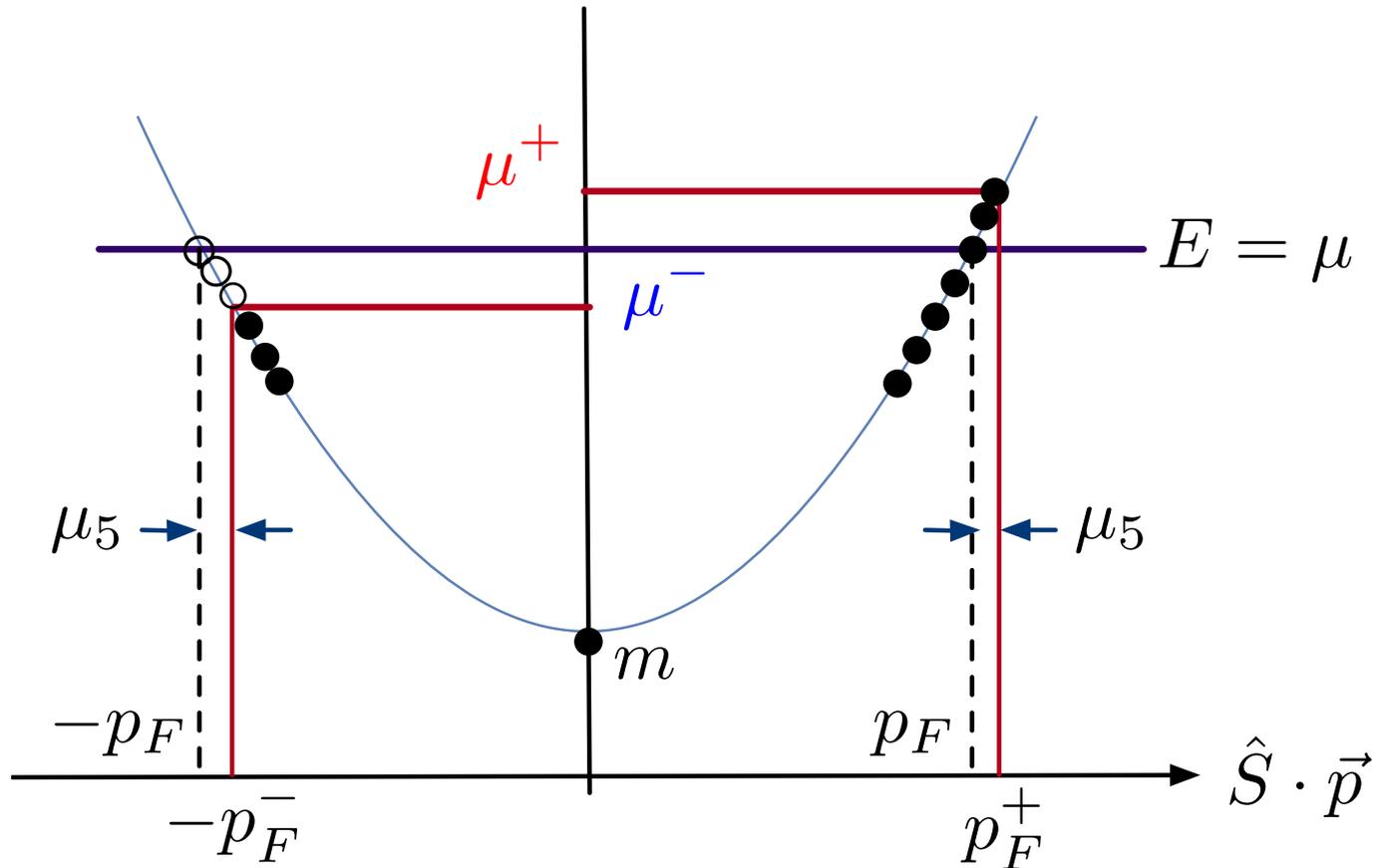


Figure: chiral medium

CME-induced current

II. Field-theoretic calculation

DK Hong, SHI, KS Jeong, D Yeom '22



$$\langle j^\mu \rangle = e \langle \bar{\Psi} \gamma^\mu \Psi \rangle = -e \int \frac{d^4 p}{(2\pi)^4} \text{Tr} [\gamma^\mu S_F^{n=0}(p, \mu, \mu_5)]$$

$$S_F^{n=0}(p, \mu, \mu_5) = \left[\frac{2i (\not{\tilde{p}}_{\parallel} + m) P_- H_+ e^{-p_{\perp}^2/|eB|}}{[(1+i\epsilon) p_0 + \mu_+]^2 - p_z^2 - m^2} + \frac{2i (\not{\tilde{p}}_{\parallel} + m) P_- H_- e^{-p_{\perp}^2/|eB|}}{[(1+i\epsilon) p_0 + \mu_-]^2 - p_z^2 - m^2} \right]$$

: the electron propagator in the lowest Landau level in a dense medium

P_- : spin projection operator

H_{\pm} : helicity projection operator

$$\mu_{\pm} = \sqrt{p_F^{\pm 2} + m^2}$$

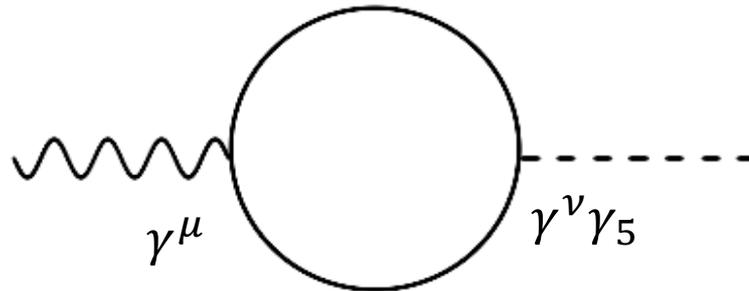
: chemical potential for
the helicity eigenstates

The field-theoretic calculation reproduces the CME formula obtained by the energy balance argument :

$$\begin{aligned}
 \langle j^3 \rangle &= \frac{e^2 B}{4\pi^2} \left[\int_0^{\mu_+} dp_0 \int_{p_z > 0} |p_z| \delta(p_0^2 - p_z^2 - m^2) - \int_0^{\mu_-} dp_0 \int_{p_z > 0} |p_z| \delta(p_0^2 - p_z^2 - m^2) \right] \\
 &= \frac{e^2 B}{4\pi^2} \left[\underbrace{\sqrt{(p_F + \mu_5)^2 + m^2} - \sqrt{(p_F - \mu_5)^2 + m^2}}_{\varepsilon} \right] = \frac{e^2 B}{2\pi^2} \mu_5 v_F [1 + \mathcal{O}(v_F^2, r^2)] ,
 \end{aligned}$$

CME and chiral anomaly in (1+1)D

Since only the electrons in the lowest Landau level (i.e. transverse zero modes) contribute to the CME current, the physics may be understood in terms of electrons moving in (1+1)D spacetime.



$$\Gamma^{\mu\nu}(q_1)\delta^{(2)}(q_1 + q_2) \equiv \int \prod_i d^2x_i e^{iq_i \cdot x_i} \langle 0 | T j^\mu(x_1) j_5^\nu(x_2) | 0 \rangle$$

In the vacuum,

$$\Gamma_{\text{vac}}^{\mu\nu}(q) = \frac{eB}{4\pi^2} (\epsilon^{\nu\alpha} q_\alpha q^\mu + \epsilon^{\mu\nu} q^2) H(q^2, m^2)$$

$$H(q^2, m^2) = \frac{1}{q^2} \left(1 - \frac{1}{\sqrt{\tau(1-\tau)}} \tan^{-1} \sqrt{\frac{\tau}{1-\tau}} \right) \simeq -\frac{1}{4m^2} (q \rightarrow 0)$$

$$\tau \equiv \frac{q^2}{4m^2} \quad : \text{No pole at } q^2 = 0 \text{ for massive electrons}$$

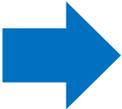
The chiral anomaly diagram vanishes as $q \rightarrow 0$ when there is no massless charged fermion. Coleman and Grossman '82

In a medium,

$$\Gamma^{\mu\nu}(q) = \frac{eB}{4\pi^2} (\epsilon^{\nu\alpha} q_\alpha q^\mu + \epsilon^{\mu\nu} q^2) F(q^2, m^2, \mu)$$

$$F(q^2, m^2, \mu) = \frac{1}{q^2} \left(\mathcal{O} \left(\frac{q^2}{m^2} \right) + \frac{\mathcal{P}F}{\mu} + \mathcal{O} \left(\frac{q}{\mu} \right) \right)$$

: a pole at $q^2 = 0$
even for massive electrons

 $\langle \partial_\nu j_5^\nu \rangle_A = ie \int \frac{d^2q}{4\pi^2} \lim_{q_0 \rightarrow 0} \lim_{q_3 \rightarrow 0} e^{iq \cdot x} q_\nu A_\mu(q) \Gamma^{\mu\nu}(q) = \frac{e^2 B}{4\pi^2} v_F \epsilon^{\mu\nu} F_{\mu\nu}$

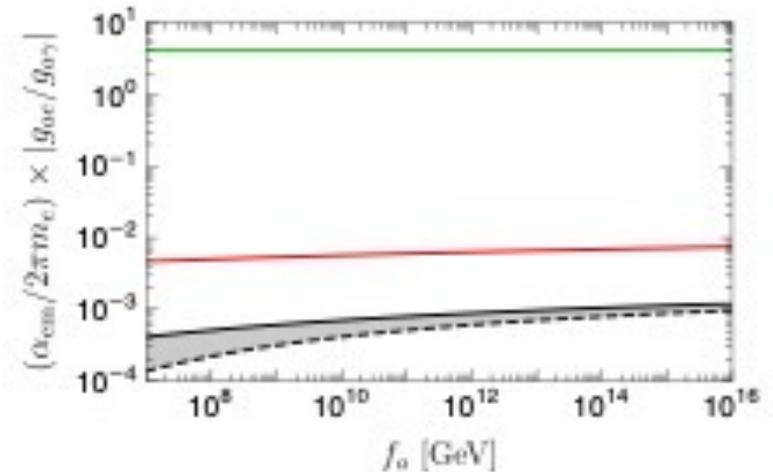
$$\langle j^3 \rangle = -e\mu_5 \lim_{q_0 \rightarrow 0} \lim_{q_3 \rightarrow 0} \Gamma^{30}(q) = \frac{e^2 B}{2\pi^2} v_F \mu_5$$

The CME can be understood as (1+1)D chiral anomaly in a medium induced by *gapless modes at the Fermi surface*.

The axion-electron coupling

$$\mathcal{L}_{\text{int}} = C_e \frac{\partial_\mu a}{f} \bar{\psi} \gamma^\mu \gamma_5 \psi$$

$$C_e \simeq \begin{cases} \mathcal{O}(1) & \text{DFSZ-like models} \\ \mathcal{O}(10^{-4} \sim 10^{-3}) & \text{KSVZ-like models} \\ \mathcal{O}(10^{-3} \sim 10^{-2}) & \text{string-theoretic axions} \end{cases}$$



The measurement of the axion-electron coupling will give us an important clue for underlying high energy physics.

K Choi, SHI, HJ Kim, H Seong '21

Detecting axion dark matter via the CME

DK Hong, SHI, KS Jeong, D Yeom '22

$$C_e \frac{\partial_\mu a}{f} \bar{\psi} \gamma^\mu \gamma_5 \psi \quad \longrightarrow \quad \mu_5 = C_e \frac{\dot{a}}{f}$$

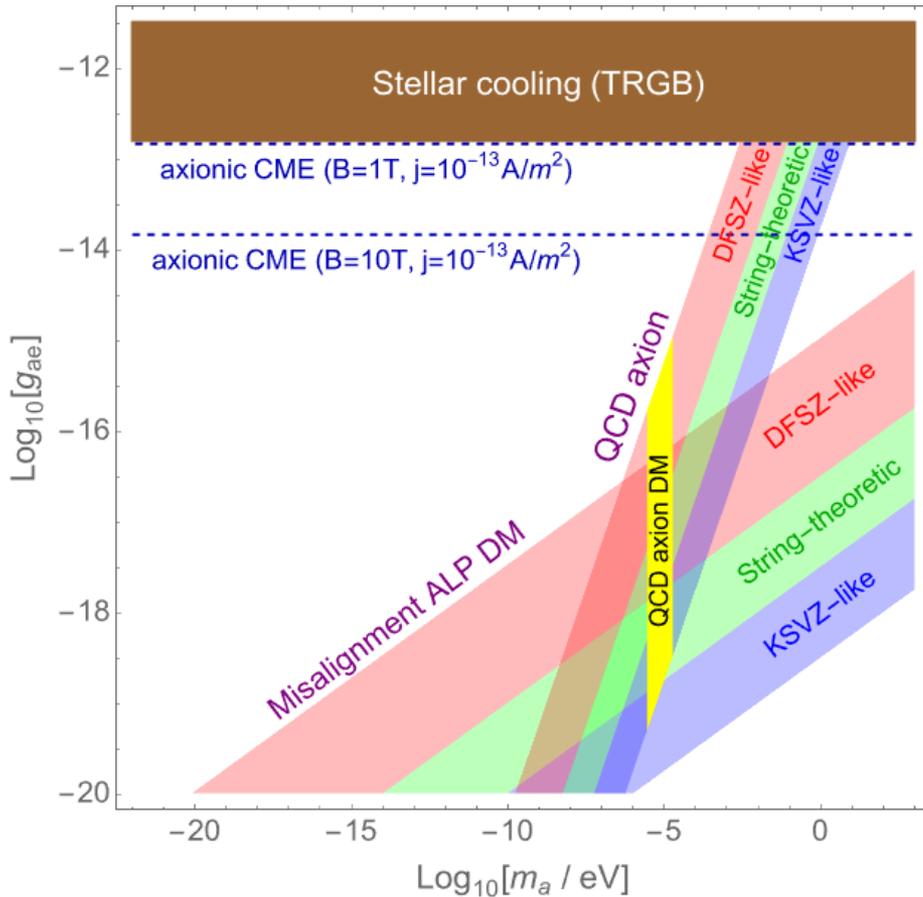
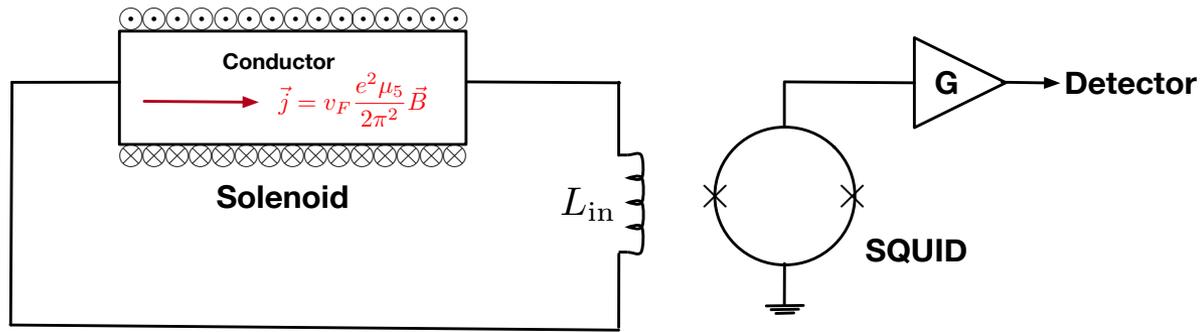
$$a(t) = \frac{\sqrt{2\rho_{\text{DM}}}}{m_a} \sin(m_a t)$$

The axion dark matter field induces an oscillating chiral chemical potential if the axion couples to the electrons.

$$\mu_5 = C_e \frac{\sqrt{\rho_{\text{DM}}}}{f} \cos(m_a t) \sim 0.25 \times 10^{-23} \text{ eV} \cdot \left(\frac{\rho_{\text{DM}}}{0.4 \text{ GeV cm}^{-3}} \right)^{1/2} \cdot \left(\frac{10^{12} \text{ GeV}}{f/C_e} \right)$$

$$\longrightarrow j^3 = \frac{e^2}{2\pi^2} \mu_5 v_F B = 6.8 \times 10^{-15} \text{ A m}^{-2} \cos(m_a t)$$

CME $\times \left(\frac{v_F}{10^{-2}} \right) \cdot \left(\frac{\rho_{\text{DM}}}{0.4 \text{ GeV cm}^{-3}} \right)^{1/2} \cdot \left(\frac{10^{12} \text{ GeV}}{f/C_e} \right) \cdot \left(\frac{B}{10 \text{ Tesla}} \right)$



Such a tiny CME-induced current may be measurable by exploiting a highly sensitive superconducting SQUID coil.

For this plot, we simply assume $j^{max} = 10^{-13} A$.

A dedicated study is needed for estimation of SNR in the measurement of such tiny oscillating electric current.

Conclusions

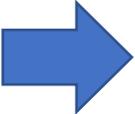
- Axions are theoretically well-motivated new particles which may be an important clue for underlying UV physics when they are discovered.
- The underlying UV physics may be distinguishable by precision measurements of low energy axion couplings.
- The measurement of the axion-electron coupling is particularly important for pinning down the microscopic origin of the QCD axion.
- The chiral magnetic effect (CME) offers an intriguing possibility for the measurement of the axion-electron coupling, when the axion comprises a major fraction of dark matter.
- We have newly computed the CME-induced current and claim that it is proportional to the Fermi velocity of the electrons.

Back-up slides

String-theoretic models

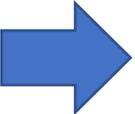
$$C_{[m_1 m_2 \dots m_p]}(x^\mu, y^m) = a(x^\mu) \Omega_{[m_1 m_2 \dots m_p]}(y^m) \quad \Omega : \text{harmonic } p\text{-form on the compact internal space}$$

4D axions identified as zero modes of higher-dimensional p -form gauge field

 SUSY-preserving compactification

$$\left\{ \begin{array}{l} T = \tau + ia \quad \text{Axion chiral superfield } (\tau : \text{volume modulus of } p\text{-cycle dual to } \Omega) \\ U(1)_{PQ} : a \rightarrow a + \text{const} \\ \quad \quad \quad : \text{remnant of a higher-dimensional gauge symmetry} \end{array} \right.$$

$$\delta C_{[m_1 m_2 \dots m_p]} = \partial_{[m_1} \Lambda_{m_2, \dots, m_p]}$$

 4D Low energy effective action

$$K = K_0(T + T^*) + Z_I(T + T^*)\Phi_I^*\Phi_I$$

$$\mathcal{F}_A = c_A T \quad Z_I \propto (T + T^*)^{\omega_I} \quad \omega_I \sim \mathcal{O}(1) \quad \text{scaling weight of } \Phi_I$$

$$c_A \sim \mathcal{O}(1)$$

$$K = K_0(T + T^*) + Z_I(T + T^*)\Phi_I^*\Phi_I$$

$$\mathcal{F}_A = c_A T \quad Z_I \propto (T + T^*)^{\omega_I} \quad \omega_I \sim \mathcal{O}(1) \quad c_A \sim \mathcal{O}(1)$$



$$T = \tau + ia$$

$$\mathcal{L}_{\text{eff}} = \frac{M_P^2}{4} \partial_\tau^2 K_0 (\partial_\mu a)^2 + \frac{\omega_I}{2\tau} \partial_\mu a \left(\psi_I^\dagger \bar{\sigma}^\mu \psi_I + \phi_I^\dagger i \overleftrightarrow{D}^\mu \phi_I \right) - \frac{1}{4} \frac{c_A \tau}{c_A g_A^2} F^{A\mu\nu} F_{\mu\nu}^A + \frac{c_A}{4} a F^{A\mu\nu} \tilde{F}_{\mu\nu}^A$$

$\sim \mathcal{O}(1)$ $\sim \mathcal{O}(1)$

$\tau = \frac{1}{c_A g_A^2} \sim \mathcal{O}(1)$

String-theoretic axion couplings to matter fields and gauge fields are comparable to each other.



Canonical normalization

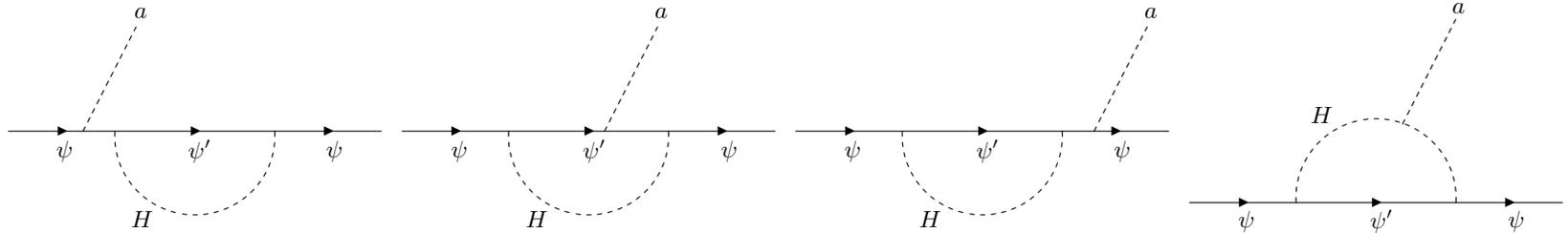
$$a \rightarrow \frac{a}{8\pi^2 f_a} \quad f_a = \frac{M_P}{8\pi^2} \sqrt{\frac{\partial_\tau^2 K_0}{2}}$$

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4g_A^2} F^{A\mu\nu} F_{\mu\nu}^A + \frac{1}{2} (\partial_\mu a)^2 + \omega_I \frac{c_A g_A^2}{16\pi^2} \frac{\partial_\mu a}{f_a} \left(\psi_I^\dagger \bar{\sigma}^\mu \psi_I + \phi_I^\dagger i \overleftrightarrow{D}^\mu \phi_I \right) + \frac{c_A}{32\pi^2} \frac{a}{f_a} F^{A\mu\nu} \tilde{F}_{\mu\nu}^A$$

$\sim \mathcal{O}(g^2/16\pi^2)$

Running of axion couplings by Yukawa interactions

K Choi, SHI, CB Park, S Yun '17, Camalich, Pospelov, Vuong, Ziegler, Zupan '20
 Heiles, König, Neubert '20, Chala, Guedes, Ramos, Santiago '20



$$\frac{\partial_\mu a}{f_a} \left(\sum_\psi c_\psi \psi^\dagger \bar{\sigma}^\mu \psi + \sum_{\alpha=1,2} c_{H_\alpha} H_\alpha^\dagger i \overleftrightarrow{D}^\mu H_\alpha \right) + \sum_A \frac{g_A^2}{32\pi^2} c_A \frac{a}{f_a} F^{A\mu\nu} \tilde{F}_{\mu\nu}^A$$

$$y_t u_3^c Q_{L3} H_u$$



$$\frac{dc_{Q_3}}{d \ln \mu} \approx \frac{\xi_y}{16\pi^2} y_t^2 n_t$$

$$\frac{dc_{u_3^c}}{d \ln \mu} \approx \frac{\xi_y}{8\pi^2} y_t^2 n_t$$

$$\frac{dc_{H_u}}{d \ln \mu} \approx \frac{3}{8\pi^2} y_t^2 n_t$$

$$n_t \equiv c_{Q_3} + c_{u_3^c} + c_{H_u}$$

$$\xi_y = \begin{cases} 1 & \text{for non-SUSY models} \\ 2 & \text{for SUSY models} \end{cases}$$

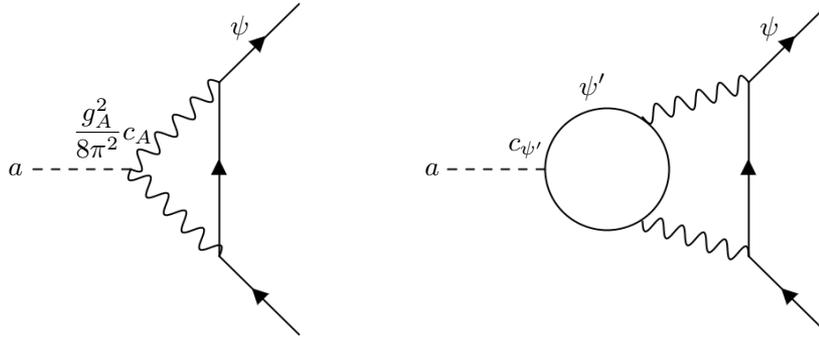
Running of axion couplings by gauge interactions

Srednicki '85, S Chang and K Choi '93

K Choi, SHI, CS Shin '20,

Chala, Guedes, Ramos, Santiago '20

Bauer, Neubert, Renner, Schnubel, Thamm '20



$$\frac{\partial_\mu a}{f_a} \left(\sum_\psi c_\psi \psi^\dagger \bar{\sigma}^\mu \psi + \sum_{\alpha=1,2} c_{H_\alpha} H_\alpha^\dagger i \overleftrightarrow{D}^\mu H_\alpha \right) + \sum_A \frac{g_A^2}{32\pi^2} c_A \frac{a}{f_a} F^{A\mu\nu} \tilde{F}_{\mu\nu}^A$$

$$\left. \frac{dc_\psi}{d \ln \mu} \right|_{\text{gauge}} = -\xi_g \sum_A \frac{3}{2} \left(\frac{g_A^2}{8\pi^2} \right)^2 \mathbb{C}_A(\psi) \tilde{c}_A \quad \tilde{c}_A \equiv c_A - \sum_{\psi'} c_{\psi'}$$

$$\left. \frac{dc_{H_\alpha}}{d \ln \mu} \right|_{\text{gauge}} = -\xi_H \sum_A \frac{3}{2} \left(\frac{g_A^2}{8\pi^2} \right)^2 \mathbb{C}_A(H_\alpha) \tilde{c}_A \quad \mathbb{C}_A(\Phi) : \text{quadratic Casimir}$$

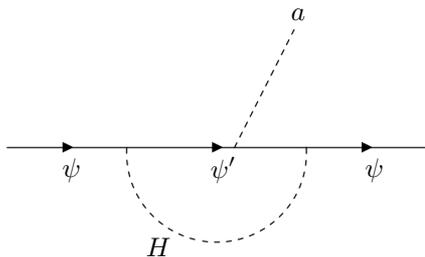
$$\xi_g = \begin{cases} 1 & \text{for non-SUSY models} \\ 2/3 & \text{for SUSY models} \end{cases}, \quad \xi_H = \begin{cases} 0 & \text{for non-SUSY models} \\ 2/3 & \text{for SUSY models} \end{cases}$$

Numerical results

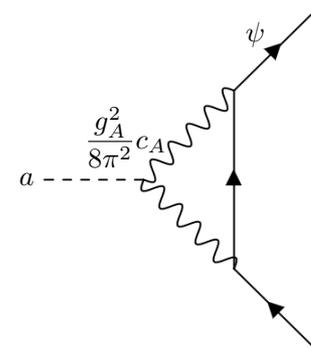
For $f_a = 10^{10}$ GeV, $t_\beta = 10$, and $m_{SUSY} = 10$ TeV,

$$\begin{aligned}
 C_u(2 \text{ GeV}) &\simeq C_u(f_a) - 0.28 n_t(f_a) + [17.7 \tilde{c}_G(f_a) + 0.52 \tilde{c}_W(f_a) + 0.036 \tilde{c}_B(f_a)] \times 10^{-3}, \\
 C_d(2 \text{ GeV}) &\simeq C_d(f_a) + 0.31 n_t(f_a) + [19.4 \tilde{c}_G(f_a) + 0.23 \tilde{c}_W(f_a) + 0.0047 \tilde{c}_B(f_a)] \times 10^{-3} \\
 C_e(m_e) &\simeq C_e(f_a) + 0.29 n_t(f_a) + [0.81 \tilde{c}_G(f_a) + 0.28 \tilde{c}_W(f_a) + 0.10 \tilde{c}_B(f_a)] \times 10^{-3}.
 \end{aligned}$$

$$\frac{y_t^2}{8\pi^2} n_t(f_a) \ln\left(\frac{f_a}{m_t}\right) \sim \text{a few} \times 0.1 n_t(f_a)$$



$$\left(\frac{g_A^2}{8\pi^2}\right)^2 \tilde{c}_A(f_a) \ln\left(\frac{f_a}{\mu}\right) \sim (10^{-4} - 10^{-2}) \tilde{c}_A(f_a)$$



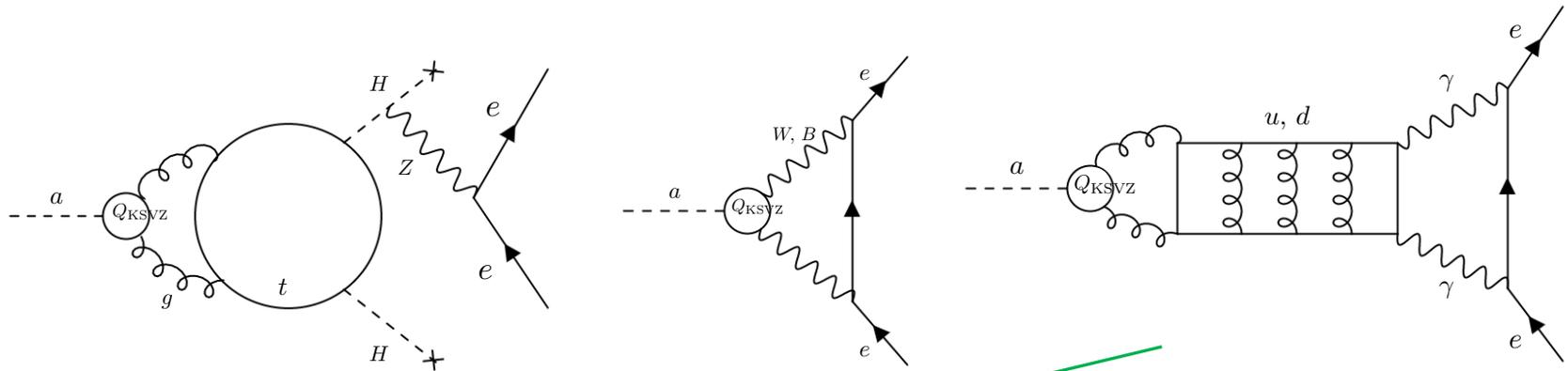
??

ΔC_e in KSVZ-like models

Srednicki '85

S Chang and K Choi '93

Bauer, Neubert, Renner, Schnubel, Thamm '20



$$C_e(m_e) \simeq \left[0.84 c_G - 0.03 c_G + 0.28 c_W + 0.10 c_B \right] \times 10^{-3} \quad (\text{KSVZ with MSSM})$$

$$C_e(m_e) \simeq \left[0.83 c_G - 0.03 c_G + 0.54 c_W + 0.13 c_B \right] \times 10^{-3} \quad (\text{KSVZ with SM}),$$

Previously ignored because it is at three-loop level.

$$\left(\frac{\alpha_s}{2\pi} \right)^3 y_t^2 c_G \ln \left(\frac{f_a}{m_t} \right) \sim 10^{-3} c_G$$

Consequences in low energy observables

Axion couplings to the photon, electron, neutron, and proton below GeV

$$\frac{1}{4}g_{a\gamma}a\vec{E}\cdot\vec{B} + \partial_\mu a \left[\frac{g_{ae}}{2m_e}\bar{e}\gamma^\mu\gamma_5e + \frac{g_{an}}{2m_n}\bar{n}\gamma^\mu\gamma_5n + \frac{g_{ap}}{2m_p}\bar{p}\gamma^\mu\gamma_5p \right]$$

$$g_{a\gamma} \simeq \frac{\alpha_{\text{em}}}{2\pi} \frac{1}{f_a} \left(c_W + c_B - \frac{2}{3} \frac{m_u + 4m_d}{m_u + m_d} c_G \right) \simeq \frac{\alpha_{\text{em}}}{2\pi} \frac{1}{f_a} \left(c_W + c_B - 1.92c_G \right),$$

$$g_{ap} \simeq \frac{m_p}{f_a} \left(C_u\Delta u + C_d\Delta d - \left(\frac{m_d}{m_u + m_d}\Delta u + \frac{m_u}{m_u + m_d}\Delta d \right) c_G \right),$$

$$\simeq \frac{m_p}{f_a} \left(0.90(3) C_u(2 \text{ GeV}) - 0.38(2) C_d(2 \text{ GeV}) - 0.48(3) c_G \right), \quad \underbrace{\langle p | \bar{u}\gamma^\mu\gamma_5u | p \rangle}_{s^\mu \Delta u}$$

$$g_{an} \simeq \frac{m_n}{f_a} \left(C_d\Delta u + C_u\Delta d - \left(\frac{m_u}{m_u + m_d}\Delta u + \frac{m_d}{m_u + m_d}\Delta d \right) c_G \right),$$

$$\simeq \frac{m_n}{f_a} \left(0.90(3) C_d(2 \text{ GeV}) - 0.38(2) C_u(2 \text{ GeV}) - 0.04(3) c_G \right), \quad \underbrace{\langle p | \bar{d}\gamma^\mu\gamma_5d | p \rangle}_{s^\mu \Delta d}$$

$$g_{ae} \simeq \frac{m_e}{f_a} C_e(m_e),$$

Cortona, Hardy, Vega, Villadoro '15

Taking into account the radiative corrections with the choice of parameters
 $f_a = 10^{10}$ GeV, $t_\beta = 10$, and $m_{SUSY} = 10$ TeV,

$$g_{ap} \simeq \frac{m_p}{f_a} \begin{cases} \mathcal{O}(1), & \text{DFSZ-like} \\ -0.48c_G + (0.5c_W + 0.05c_B) \times 10^{-3}, & \text{KSVZ-like} \\ -0.48c_G + 0.7\omega_I g_{GUT}^2 \times 10^{-2}, & \text{String} \end{cases}$$

$$g_{an} \simeq \frac{m_n}{f_a} \begin{cases} \mathcal{O}(1), & \text{DFSZ-like} \\ -0.03c_G + (0.5c_W - 0.15c_B) \times 10^{-4}, & \text{KSVZ-like} \\ -0.03c_G + 0.63\omega_I g_{GUT}^2 \times 10^{-2}, & \text{String} \end{cases}$$

$$g_{ae} \simeq \frac{m_e}{f_a} \begin{cases} \mathcal{O}(1), & \text{DFSZ-like} \\ (c_G + 0.4c_W + 0.15c_B) \times 10^{-3}, & \text{KSVZ-like} \\ (c_G + 0.4c_W + 0.15c_B) \times 10^{-3} + \omega_I g_{GUT}^2 \times 10^{-2}, & \text{String} \end{cases}$$

For the string-theoretic model, a universal scaling weight ω_I is assumed.

Ex) $\omega_I = \frac{1}{2}$, $\omega_I g_{GUT}^2 \sim 0.25$ in a type-IIB string Large Volume Scenario

Laboratory searches for axion DM -photonic probes

$$\frac{g_{a\gamma}}{4} a F \tilde{F} \quad \longrightarrow \quad \nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} \underbrace{-g_{a\gamma} \vec{B} \partial_t a}_{\vec{J}_{\text{eff}}} \quad \text{effective current}$$

Background axion DM field

$$a \approx a_0 \cos [m_a (t - \vec{v} \cdot \vec{x})]$$

$$\rho_a = \frac{1}{2} m_a^2 a_0^2 \quad |\vec{v}| \sim 10^{-3} c$$

$$\vec{J}_{\text{eff}} \approx g_{a\gamma} \sqrt{2\rho_a} \vec{B} \sin m_a t$$

The best experimental sensitivity on $g_{a\gamma}$ is obtained when $\rho_a = \rho_{DM}$.

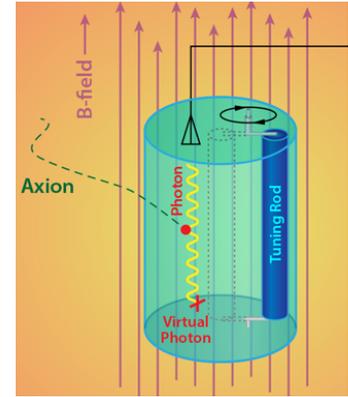
Misalignment axion DM

$$f_a \simeq 10^{17} \text{ GeV} \left(\frac{10^{-22} \text{ eV}}{m_a} \right)^{1/4} \sqrt{\frac{\rho_a}{\rho_{DM}}} \quad \longrightarrow \quad g_{a\gamma} = \frac{e^2}{8\pi^2} \frac{1}{f_a} c_{a\gamma}$$

Given axion DM mass,
 $g_{a\gamma}$ is determined for $c_{a\gamma} \sim O(1)$.

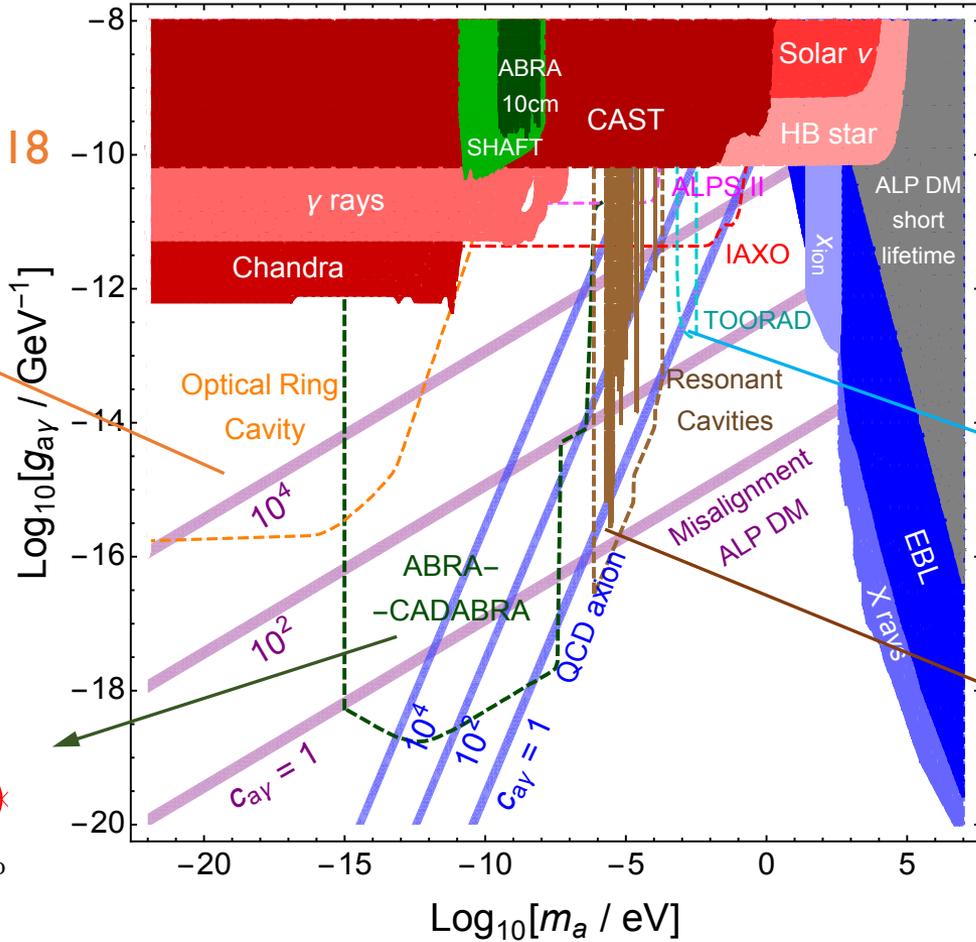
Current and future limits on $g_{a\gamma}$

Choi, SHI, Shin '20

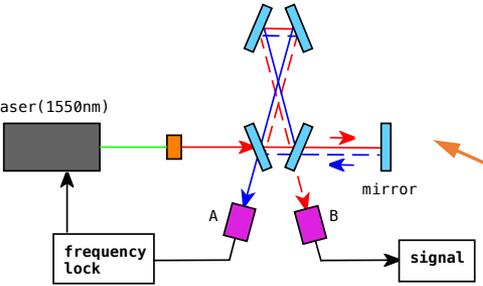


Marsh, Fong, Lenz, Smejkal, Ali '18

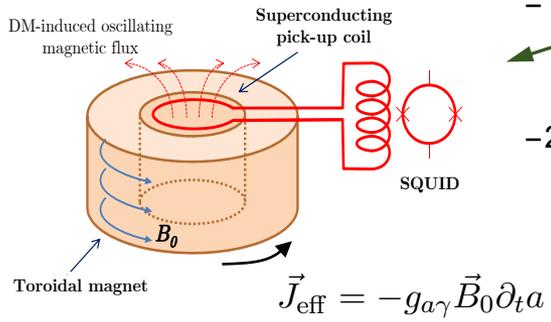
ADMX, IBS-CAPP, MADMAX...



Obata, Fujita, Michimura '18



Kahn, Safdi, Thaler '16



Laboratory searches for axion DM -nucleonic probes

$$g_{aN} \frac{\partial_\mu a}{2m_N} \bar{N} \gamma^\mu \gamma^5 N \quad \longrightarrow \quad \underbrace{g_{aN} \frac{\nabla a}{\gamma_N m_N}}_{\vec{B}_{\text{eff}}} \cdot \gamma_N \vec{S}_N \quad \gamma_N : \text{nucleon gyromagnetic ratio}$$

Background axion DM field

$$a \approx a_0 \cos [m_a (t - \vec{v} \cdot \vec{x})]$$

$$\rho_a = \frac{1}{2} m_a^2 a_0^2 \quad |\vec{v}| \sim 10^{-3} c$$

$$\vec{B}_{\text{eff}} \approx g_{aN} \frac{\sqrt{2\rho_a}}{\gamma_N m_N} \vec{v}_a \sin m_a t$$

The best experimental sensitivity on g_{aN} is obtained when $\rho_a = \rho_{DM}$.

Misalignment axion DM

$$f_a \simeq 10^{17} \text{ GeV} \left(\frac{10^{-22} \text{ eV}}{m_a} \right)^{1/4} \sqrt{\frac{\rho_a}{\rho_{DM}}} \quad \longrightarrow \quad g_{aN} = \frac{m_N}{f_a} c_{aq} \times \mathcal{O}(1)$$

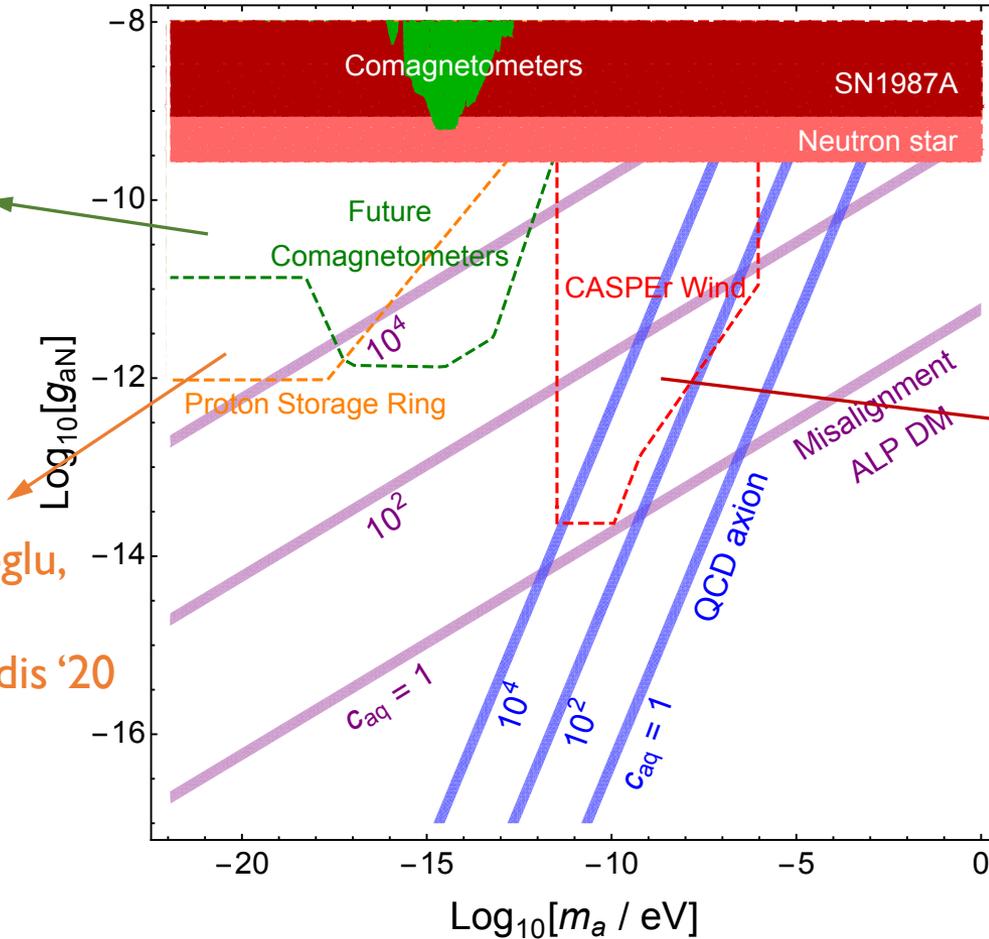
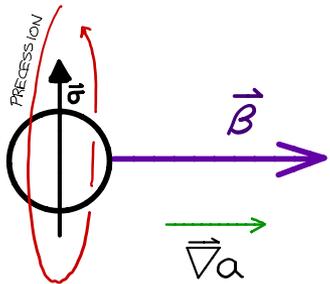
Given axion DM mass,
 g_{aN} is determined for $c_{aq} \sim \mathcal{O}(1)$.

Current and future limits on g_{aN}

Bloch, Hochberg,
Kuflik, Volansky '19

$$\frac{B_{\text{eff}}^e}{B_{\text{eff}}^N} \sim \frac{c_{ae} m_e}{c_{aN} m_N} \neq 1$$

Graham, Hacıomeroglu,
Kaplan, Omarov,
Rajendran, Semertzidis '20



Choi, Shi, Shin '20

Kimball et al '17

