β-decay of neutron-rich thermally fissile Th and U isotopes

Tabassum Naz and Shakeb Ahmad

Aligarh Muslim University, India

Introduction

The investigation of the structure of thorium (Th) and uranium (U) nuclei have been at the focus of attention in nuclear physics due to the thermally fissile nature of some of their isotopes. The series of U and Th isotopes with $N = 154 - 172$ are identified as thermally fissile on the basis of the fission barrier height and neutron separation energy. All these isotopes are stable against α-decay and among these, the one with low mass have large β half-life time. However, the isotopes with $N = 126 - 140$ are β-stable but decay through α mode. So, in order to study the fission, the potential energy surface (PES) as a function of deformation plays a crucial role. The PES of Th and U, characterised by a two-humped barrier structure, has been extensively been studied both theoretically and experimentally. So, we tried to study the decay properties of Th and U isotopes. We have used the covariant density functional theory (CDFT) with density dependentinteractions(DD-ME1,DD-ME2,DD-PC1) in order to calculated the different properties.

Thoretical Formalism

Covariant density functional theory (CDFT) is based on density dependent vertices and one additional parameter characterising the range of the forces which are density dependent meson-exchange (DD-ME), and a point-coupling (DD-PC)[1] effective interactions. The basic difference of these two models lies on the treatment of the interaction range, the mesons, and the density dependence. Both DD-ME and DD-PC are density-dependent models but DD-ME has a finite interaction range, while DD-PC uses a zero-range interaction with one additional gradient term in the scalar-isoscalar channel. Each of these models is represented here by their corresponding parameter sets as DD-ME1, DD-ME2 and DD-PC1. The meson-exchange phenomenology describes nucleus as a system of Dirac nucleons, interacting via the exchange of mesons with finite masses, leading to finite-rang interaction. The standard Lagrangian density of meson-exchange (finite-range) model with medium dependence vertices is written as:

{

$$\begin{aligned}\mathcal{L} &= \overline{\psi}\left[\gamma(i\partial - \mathbf{g}_\omega\omega - \mathbf{g}_\rho\tilde{\rho}\tilde{\tau} - e\mathbf{A}) - m - \mathbf{g}_\sigma\sigma\right]\psi \\ &+ \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma{}^2\sigma^2 - \frac{1}{4}\mathbf{\Omega}_{\mu\nu}\mathbf{\Omega}^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^2 \\ &- \frac{1}{4}\vec{\mathbf{R}}_{\mu\nu}\vec{\mathbf{R}}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^2 - \frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}.\end{aligned}$$

The isoscalar-scalar σ meson, the isoscalar-vector ω meson, and the isovector-vector ρ meson build the minimal set of meson fields for a quantitative description of nuclei. The density-dependent of the meson-nucleon couplings is parameterized. The couplings of the σ and ω mesons to the nucleon field are defined as

$$\mathbf{g}_i(\rho) = \mathbf{g}_i(\rho_{\mathbf{sat}})f_i(x) \quad \textbf{for} \quad i = \sigma, \omega$$

where

$$f_i(x) = a_i\frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}$$

is a function of $x = \rho/\rho_{\rm sat}$, and $\rho_{\rm sat}$ denotes the baryon density at saturation in symmetric nuclear matter. The eight real parameters in but constrained as follows:

$$f_i(1) = 1, \quad f_\sigma''(1) = f_\omega''(1), \quad f_i''(0) = 0.$$

These five constraints reduce the number of independent parameters to three. Three additional parameters in the isoscalar channel are $g_\sigma(\rho_{\rm sat})$, $g_\omega(\rho_{\rm sat})$, and m_σ. The functional form of the density-dependence for the ρ-meson coupling is suggested by a Dirac-Brueckner calculations of asymmetric nuclear matter

$$\mathbf{g}_\rho(\rho) = \mathbf{g}_\rho(\rho_{\mathbf{sat}})exp\left[-a_\rho(x-1)\right]$$

The effective Lagrangian for the density-dependent point-coupling model that includes the isoscalar-scalar, isoscalar-vector and isovector-vector four-fermion interactions is given by

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma.\partial - m)\psi \\ &- \frac{1}{2}\alpha_s(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\hat{\rho})(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \\ &- \frac{1}{2}\alpha_{TV}(\hat{\rho})(\bar{\psi}\vec{\tau}\gamma^\mu\psi)(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\ &- \frac{1}{2}\delta_S(\partial_v\bar{\psi}\psi)(\partial^v\bar{\psi}\psi) - e\bar{\psi}\gamma\cdot\mathbf{A}\frac{1-\tau_3}{2}\psi\end{aligned}$$

The functional form of the point-couplings chosen is

$$\alpha_i(\rho) = a_i + (b_i + c_i x)\, e^{-d_i x}, \qquad (i = S, V, TV)$$

where $x = \rho/\rho_{\rm sat}$, and $\rho_{\rm sat}$ denotes the nucleon density at saturation in symmetric nuclear matter.

Pairing correlation plays an important role in studying the nuclear structure of open-shell nuclei. The model provides a unified description of particlehole (ph) and particleparticle (pp) correlations on a mean-field level by using two average potentials: the self-consistent mean field that encloses all the long range ph correlations, and a pairing field Δ which sums up the pp-correlations. The pairing force in coordinate representation

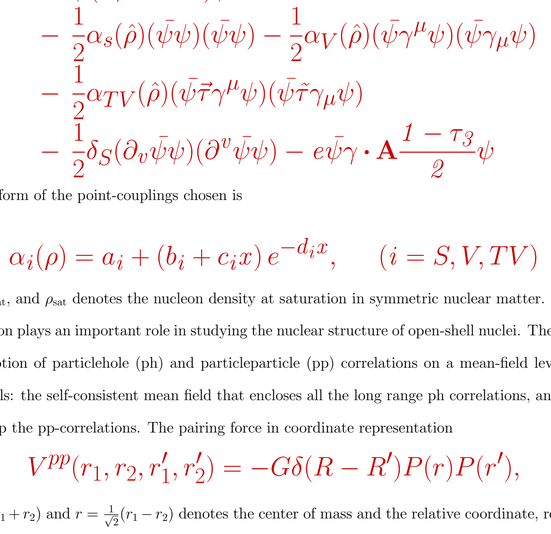
$$V^{pp}(r_1, r_2, r_1', r_2') = -G\delta(R - R')P(r)P(r'),$$

where $R = \frac{1}{\sqrt{2}}(r_1 + r_2)$ and $r = \frac{1}{\sqrt{2}}(r_1 - r_2)$ denotes the center of mass and the relative coordinate, respectively, P(r) is the Fourier transform of gaussian function ($P(k) = e^{-a^2k^2}$)

$$P(r) = \frac{1}{(4\pi a^2)^3/2}e^{-r^2/2a^2}$$

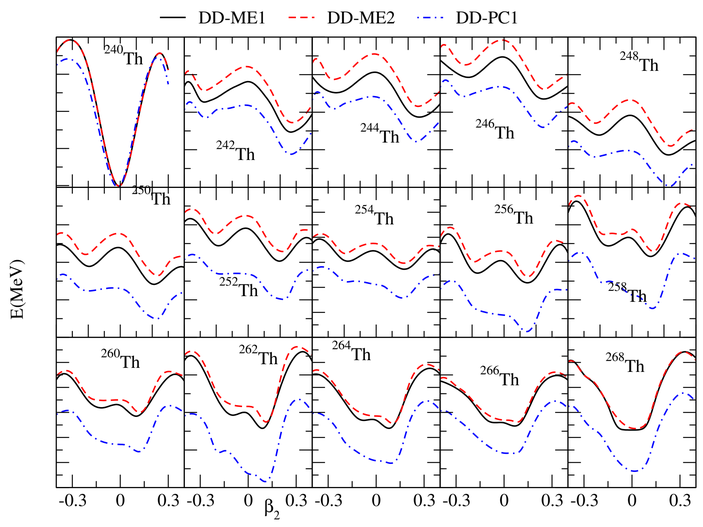
Results and Discussions

Potential Energy Surface

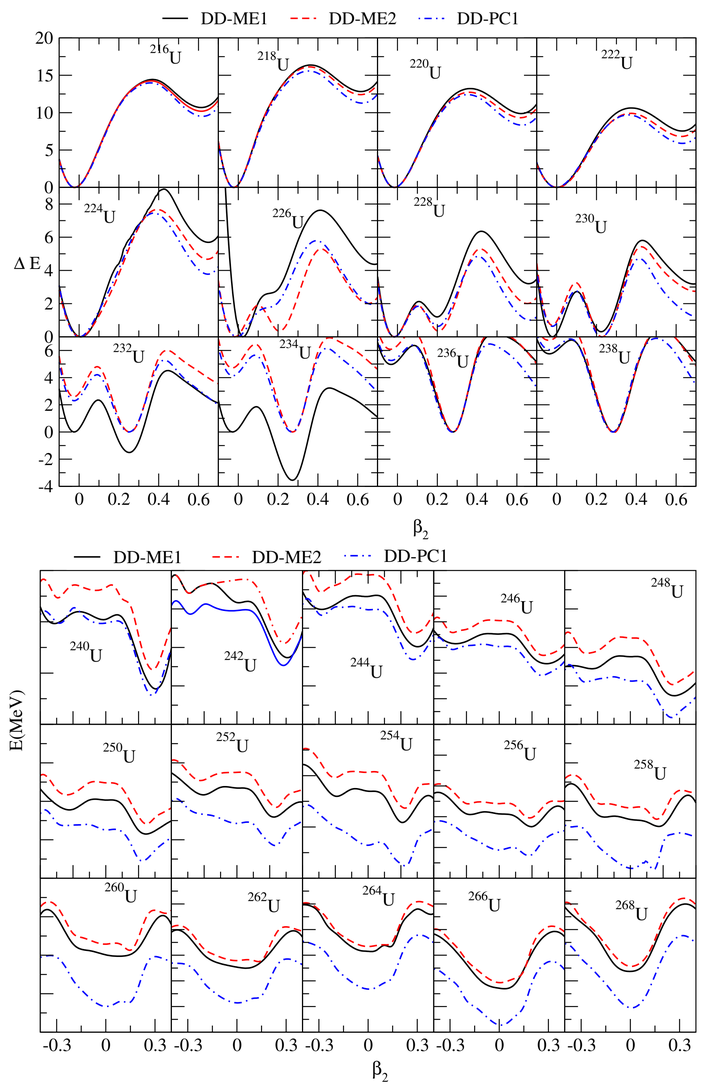


The potential energy surface map as a function of quadrupole deformation parameter can be performed by the constraint calculation.

$$\langle H'\rangle = \langle H\rangle + \frac{1}{2}C_\mu\left(\langle Q_2\rangle - \mu_2\right)^2$$



There is a shallow third hump in $^{230-232}Th$ isotopes which is supported by high resolution cross-section measurements. The third minima on the PES is mainly caused due to the large proton gap at the fermi surface at Z=90.

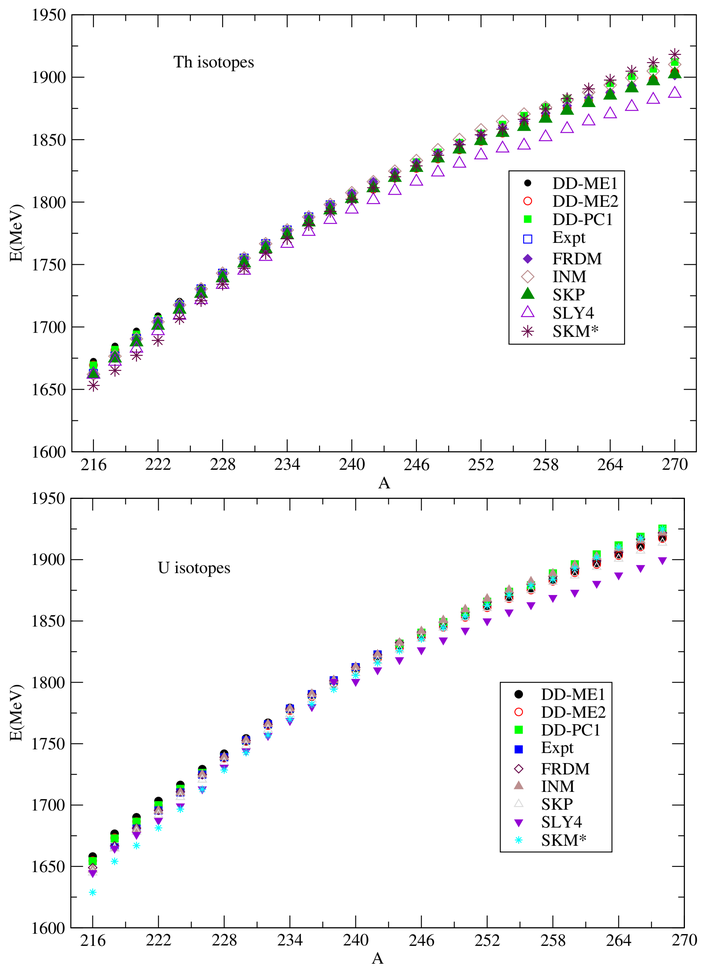


Evolution of single-nucleon shell structure with deformation is the origin of different shapes and shape transitions. $^{216-230}U$ are found to be spherical and $^{232-268}U$ are deformed neutron-rich isotopes.

Fission Barrier Height

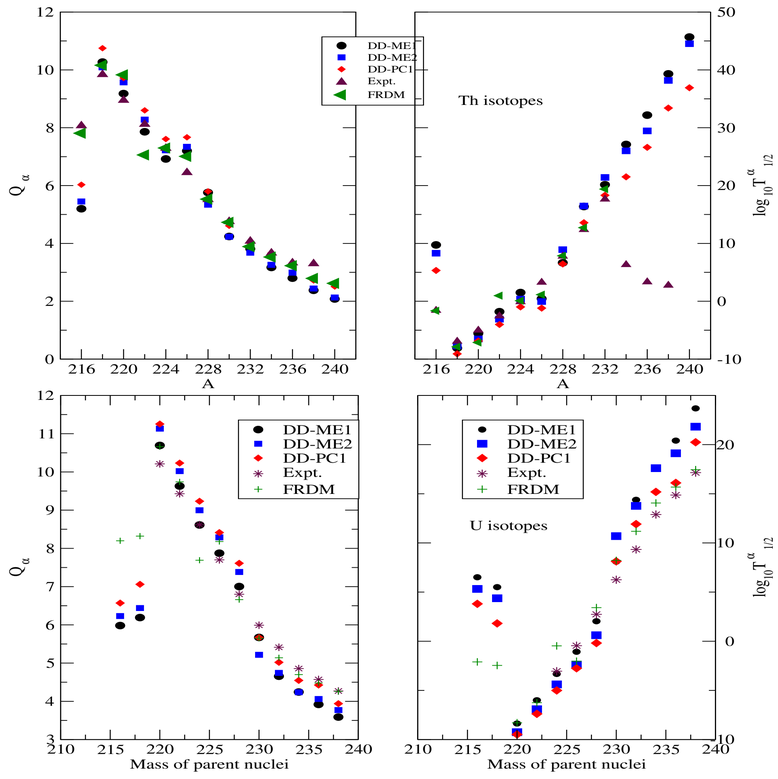
Nuclei	Expt. [64]	DD-ME1	DD-ME2	DD-PC1	FRDM [64]
^{228}Th	6.5	5.4	5.42	4.97	7.43
^{230}Th	7.0	5.47	5.38	4.98	7.57
^{232}Th	6.3	5.75	5.81	5.41	7.63
^{234}Th	6.65	6.03	6.13	5.5	7.44
^{232}U	5.4	6.04	5.99	5.27	6.61
^{234}U	5.8	6.77	6.91	6.09	6.79
^{236}U	5.75	7.1	7.36	6.47	6.65
^{238}U	5.9	7.47	7.65	6.9	4.89

Binding Energy



Decay Modes

Q_α and $T^\alpha_{1/2}$



The Q_α energy can be evaluated by using the relation

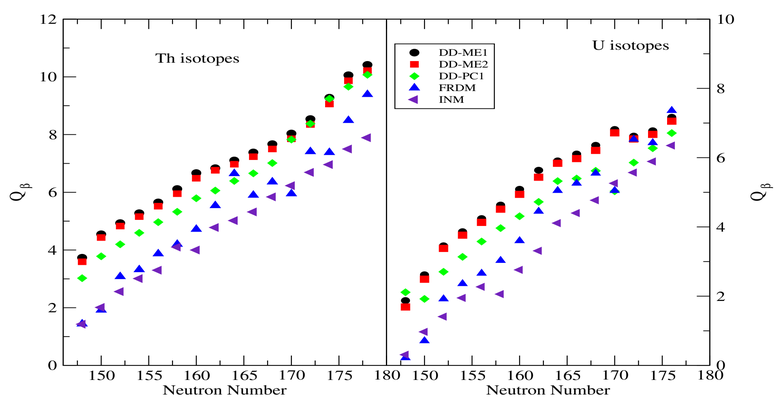
$$Q_\alpha(N, Z) = BE(N, Z) - BE(N-2, Z-2) - BE(^4He) \qquad (1)$$

where BE(N,Z), BE(N-2,Z-2) and BE(^4He) are the BEs of parent, daughter and alpha particle (= 28.296 MeV) with neutron number N and proton number Z. After calculating the Q_α values of the nuclei, we can estimate the $T^\alpha_{1/2}$(s) using the phenomenological formula of Viola and Seaborg [2]

$$log_{10}T^\alpha_{1/2}(s) = \frac{(aZ - b)}{\sqrt{Q_\alpha}} - (cZ + d) + h_{log} \qquad (2)$$

The Q_α of even-even isotopes of thorium decreases with the increase in the number of neutrons which means nuclei become stable against the α-decay. Accordingly, the $T^\alpha_{1/2}$(s) of these isotopes increases linearly with increase in mass number.

Q_β



In order to calculate the $T_{1/2}(\beta)$, we have calculated the Q_β by using the relation

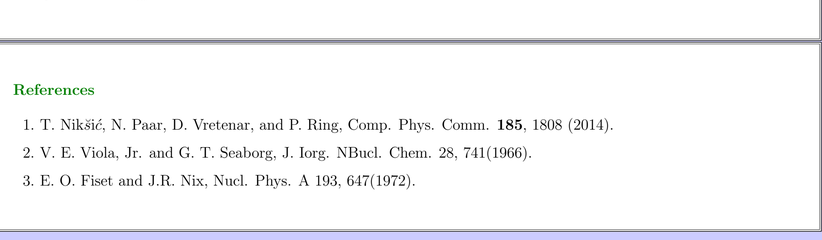
$$Q_\beta(N, Z) = BE(N, Z+1) - BE(N, Z) \qquad (3)$$

and by using the empirical formula of Fiset and Nix [3]

$$T_\beta = (540 \times 10^5)\frac{me^5}{\rho_{DOS}(W_\beta^6 - me^6)}s$$

, we have calculated the half-lives of neutron rich Th and U isotopes.

The half-life time $T^\beta_{1/2}$



From the results, it is clear that neutron-rich thorium and uranium isotopes undergo β-decay, which mean once a thermally fissile nuclei is formed (artificially in lab. or naturally by supernovae explosion), it immediately decay. All the interactions show almost same behaviour.So, for practical utility purposes, isotopes with greater half-life time are choosen.

Conclusion

In this work, we have carried out :

Systematic investigation to study the bulk properties and the microscopic structure of neutron rich even-even $^{216-268}Th$ and $^{218-268}U$ nuclei.

The investigations have been done within covariant density functional theory, and the explicit density-dependent effective interactions as DD-ME1, DD-ME2, and DD-PC1 were used in the calculations.

Proper treatment to the pairing correlations is taken care in the formalisms.

We have calculated the binding energy, rms radii, quadrupole deformation parameter to understand the ground state properties of these nuclei.

The potential energy surface have been studied to understand the structure and the behaviour of these nuclei.

References

1. T. Nikšić, N. Paar, D. Vretenar, and P. Ring, Comp. Phys. Comm. **185**, 1808 (2014).
2. V. E. Viola, Jr. and G. T. Seaborg, J. Iorg. NBucl. Chem. 28, 741(1966).
3. E. O. Fiset and J.R. Nix, Nucl. Phys. A 193, 647(1972).